Evolution kernels for parton shower Monte Carlo

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Outline:

- Motivations: `KrkMC` project – construction of NLO parton shower
- New regularization scheme and kernel calculation
- Results
- Summary

Cracow Epiphany Conference, 8-10 January 2015

Motivations: KrkMC project construction of NLO shower

Past: only LO parton showers (early Pythia, Herwig)

- LO DGLAP evolution
- LO hard process

Currently available: only NLO-improved (N+LO) parton showers (MC@NLO, POWHEG)

- LO DGLAP evolution
- NLO corrections in hard process

Ongoing: KrkMC project (Jadach, et al.) = construct fully NLO parton shower

- NLO DGLAP evolution
- NLO corrections in hard process

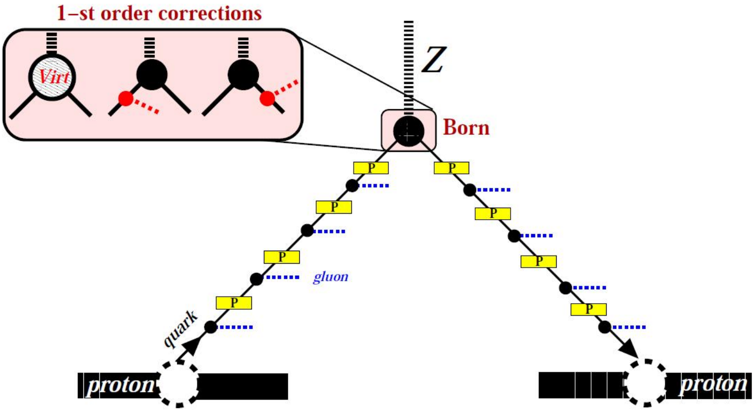
- future: maybe possibile to include N^2LO hard process corrections (N+NLO)

KrkMC project

arXiv:1002.0010, arXiv:1111.5368
arXiv:1209.4291, arXiv:1102.5083
arXiv:1401.5087

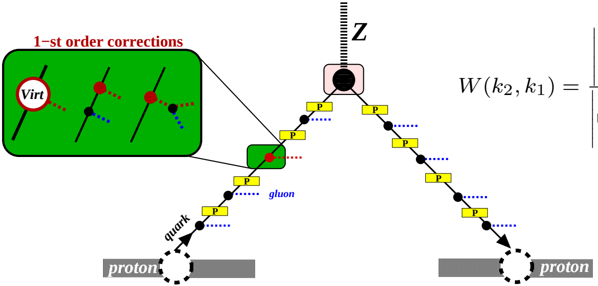
- Reformulate factorization (modify $\overline{\text{MS}}$ scheme for Monte Carlo use – *exclusive level*)
- Recalculate *evolution kernels* (splitting functions) in the modified scheme
  - inclusive and exclusive distributions
  - keep track of relation to $\overline{\text{MS}}$
- Method for implementing NLO corrections on top of LO shower
  - NLO corrections in the *hard process* (see A. Siodmok talk)
  - NLO corrections in the *shower*

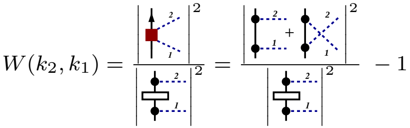
NLO corrections to hard process

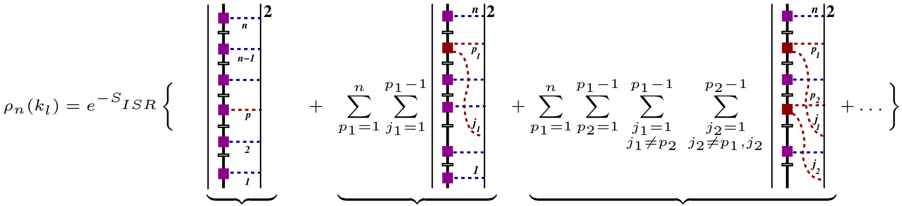


$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j \in F} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j \in B} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega}$$

NLO corrections in the MC ladder (gluons out of quarks)







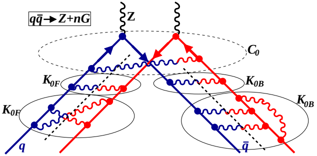
virtual + soft corrections

1 NLO real correction (insertion)

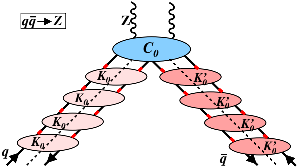
2 NLO real corrections (insertions)

Collinear factorization and evolution kernels

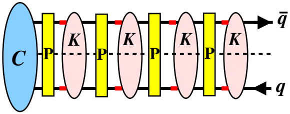
In *axial gauge* perturbative expansion of squared matrix element can be reorganized in form of *generalized ladder expansion* in terms of 2PI kernels C_0 and K_0.



Raw factorization:
C_0 kernel is finite, K_0 contains all mass singularities.



Full factorization:
obtained by introducing projection operator $\mathbb{P}$ decoupling C_0 and K_0 kernels



Evolution kernels extracted as $\frac{1}{\epsilon}$ coefficients:

$$\Gamma = \frac{1}{1-K} = \frac{1}{1-\mathbb{P}\left(K_0\cdot\frac{1}{1-(1-\mathbb{P})\cdot K_0}\right)} = x\,\mathrm{PP}\left\{\int \frac{d^nk}{(2\pi)^n}\delta\left(x-\frac{kn}{pn}\right)\left[\frac{\not{n}}{4kn}K_0\not{p}\right]\right\}$$

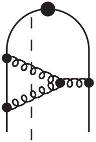
Kernel calculation – CFP approach

- Start from CFP scheme [Nucl.Phys. B175 (1980) 27] :
  - *Axial gauge* (physical interpretation, spurious singularities)
  - *PV regularization* applied to gluon propagator:

$$\frac{1}{l^2}\left(g^{\mu\nu} - \frac{l^\mu n^\nu + l^\nu n^\mu}{ln}\right) \to \frac{1}{l^2}\left(g^{\mu\nu} - \frac{l^\mu n^\nu + l^\nu n^\mu}{[ln]_{PV}}\right)$$

$$\frac{1}{[ln]_{PV}} = \frac{ln}{(ln)^2 + \delta^2 (lp)^2}$$

  - leads to $\frac{1}{\epsilon^3}$ poles (canceling between real and virtual graphs) – very problematic for 4-dimensional MC implementation, e.g. (sing. terms)



$$\sim \left[\frac{P_{qq}(x)}{\epsilon^3} - 2I_0 \frac{P_{qq}(x)}{\epsilon^2} + \frac{p_{qq}(x)}{\epsilon}\left(-2I_1 + 4I_0 + 2I_0 \ln x - 2I_0 \ln(1-x) + \text{finite}\right)\right]$$

$$I_0 = \int_0^1 \frac{dx}{[x]_{PV}} \sim -\ln\delta, \qquad I_1 = \int_0^1 dx \frac{\ln x}{[x]_{PV}} \sim -\frac{1}{2}\ln^2\delta$$

  - unintegrated distributions not available

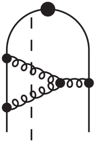
Kernel re-calculation – modified approach

- Monte Carlo (MC) scheme [Phys.Lett. B732 (2014) 218, JHEP 1108 (2011) 012] :
  - *Axial gauge*
  - *new/modified* use of PV regularization (NPV):

> Apply PV regularization to **all** singularities in the plus variable $l_+ = \frac{ln}{pn}$, not only to the axial denominators of the gluon propagators.

$$d^m l \, l_+^{-1+\epsilon} \to d^m l \left[\frac{1}{l_+}\right]_{PV} \left(1 + \epsilon \ln l_+ + \epsilon^2 \frac{1}{2} \ln^2 l_+ + \ldots\right)$$

  - No $\frac{1}{\epsilon^3}$ terms in real and virtual graphs (replaced by $\frac{1}{\epsilon} I_1$ and $\frac{1}{\epsilon^2} I_0$)



$$\sim \left[\frac{p_{qq}(x)}{\epsilon}\Big(2I_1 + 4I_0 + 2I_0 \ln x - 2I_0 \ln(1-x) + \text{finite terms}\Big)\right]$$

  - $\frac{1}{\epsilon^2}$ terms present *only in virtual graphs* (up to certain subtleties) – no need for real-virtual cancellations
  - Real and virtual diagrams do not depend on the upper phase space limit (connected to evolution variable)

Example: virtual integral in NPV regularization

$$\int \frac{d^m l}{(2\pi)^m} \frac{f(l_+)}{l^2(l-q)^2(l-p)^2} =$$
$$= \frac{-i}{16\pi^2 q^2}\left(\frac{4\pi}{-q^2}\right)^{-\epsilon} \frac{\Gamma(1-\epsilon)}{-\epsilon}\left[\int_0^x dy f(l_+) z^\epsilon (1-z)^\epsilon \left(1+2\epsilon \ln\frac{1-y}{1-z}\right)\frac{1}{1-y}\right.$$
$$\left. + 2\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}(1-x)^{-\epsilon}\int_x^1 dy f(l_+)(1-y)^{-1+2\epsilon}\right]$$

kinematics: $p^2 = (p-q)^2 = 0,\ q^2 < 0,\ x = \frac{q_+}{p_+},\ y = \frac{l_+}{p_+},\ z = \frac{y}{x},\ m = 4 + 2\epsilon$

- Singularities in $f(l_+)$ (at $y = 0$ and $y = x$) originate from gluon propagator and are regularized by PV.
- Singularity at $y = 1$ (NOT from gluon propagator) in NPV is also regularized by PV.

$$(1-y)^{-1+2\epsilon} \to (1-y)^{2\epsilon}\frac{1}{[1-y]_{PV}}$$

⚠ In NPV scheme l_+ integration must be performed as the last one.

Example: virtual non-axial integral

The non-axial integrals are also affected by the change of scheme.

$$J_3^{\mathrm{F}} = \int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2(q-l)^2(p-l)^2}$$

The PV regularization:

$$J_3^{\mathrm{F}} \sim \left[-\frac{1}{\epsilon^2} + \frac{\pi^2}{6} \right]$$

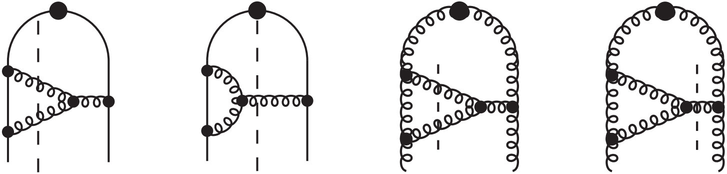
The NPV prescription:

$$J_3^{\mathrm{F}} \sim \left[-\frac{2I_0 + \ln(1-x)}{\epsilon} - 4I_1 + 2I_0 \ln(1-x) + \frac{\ln^2(1-x)}{2} \right]$$

$$I_0 = \int_0^1 \frac{dx}{[x]_{PV}} \sim -\ln\delta, \qquad I_1 = \int_0^1 dx \frac{\ln x}{[x]_{PV}} \sim -\frac{1}{2}\ln^2\delta$$

Results

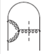
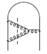
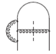
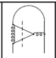
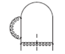
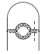
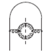
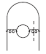
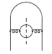
- The non-singlet P_{qq} and the most singular singlet P_{gg} graphs have been calculated earlier as a proof of the new methodology. [Phys.Lett. B732 (2014) 218, JHEP 1108 (2011) 012]



- `Axiloop` Mathematica package for calculating NLO kernels has been developed [Gituliar: http://www.gituliar.org/axiloop/]

Results: P_{qq}

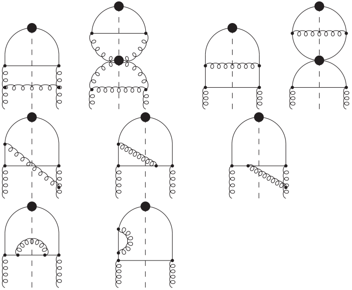
Phys.Lett. B732 (2014) 218,
JHEP 1108 (2011) 012

			SUM			SUM				SUM			SUM
	$(d):\ ^1\!/_2\, C_F C_A$			$(c):\ C_F^2 - {}^1\!/_2\, C_F C_A$			$(e):\ C_F^2$	$(f):\ ^1\!/_2\, C_F C_A$			$(g):\ C_F T_F$		
Double poles													
p_{qq}	-6	0	-6	-6	0	-6	6	$44/3$	$-22/3$	$22/3$	$-8/3$	$4/3$	$-4/3$
$p_{qq}\ \ln x$	4	0	4	4	0	4	-8	0	0	0	0	0	0
$p_{qq}\ \ln(1-x)$	8	0	8	0	0	0	0	-16	8	-8	0	0	0
$p_{qq}\ I_0$	16	0	16	8	0	8	-8	-16	8	-8	0	0	0
Single poles													
p_{qq}	-7	-4	-11	-7	0	-7	7	0	$103/9$	$103/9$	0	$-10/9$	$-10/9$
$p_{qq}\ \ln x$	0	$-3/2$	$-3/2$	0	$-3/2$	$-3/2$	0	0	$11/3$	$11/3$	0	$-2/3$	$-2/3$
$p_{qq}\ \ln(1-x)$	-3	8	5	-3	0	-3	3	$22/3$	$-34/3$	-4	$-4/3$	$4/3$	0
$p_{qq}\ \ln^2 x$	2	-1	1	2	-1	1	-2	0	0	0	0	0	0
$p_{qq}\ \ln x \ln(1-x)$	2	4	6	2	0	2	-4	0	-4	-4	0	0	0
$p_{qq}\ \ln^2(1-x)$	4	-2	2	0	0	0	0	-8	6	-2	0	0	0
$p_{qq}\ \mathrm{Li}_2(1)$	8	-2	6	4	0	4	-4	0	-4	-4	0	0	0
$p_{qq}\ \mathrm{Li}_2(1-x)$	-2	2	0	2	-2	0	0	0	0	0	0	0	0
$1-x$	$-5/2$	$3/2$	-1	$-7/2$	$-15/2$	-11	3	$22/3$	-4	$10/3$	$-4/3$	0	$-4/3$
$(1-x)\ \ln x$	2	0	2	2	0	2	-4	0	0	0	0	0	0
$(1-x)\ \ln(1-x)$	4	0	4	0	0	0	0	-8	4	-4	0	0	0
$1+x$	$-1/2$	$1/2$	0	$1/2$	$-1/2$	0	0	0	0	0	0	0	0
$(1+x)\ \ln x$	0	$1/2$	$1/2$	0	$-7/2$	$-7/2$	0	0	0	0	0	0	0
Spurious poles													
$p_{qq}\ I_0$	0	8	8	0	0	0	0	0	-4	-4	0	0	0
$p_{qq}\ I_0 \ln x$	4	4	8	4	0	4	-4	0	-4	-4	0	0	0
$p_{qq}\ I_0 \ln(1-x)$	12	-4	8	4	0	4	-4	-8	4	-4	0	0	0
$p_{qq}\ I_1$	-12	4	-8	-4	0	-4	4	0	4	4	0	0	0
$(1-x)\ I_0$	8	0	8	4	0	4	-4	-8	4	-4	0	0	0

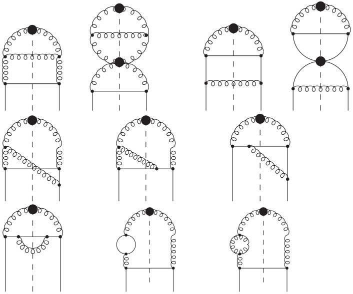
Results cross-checked with: Curci et al. 1980, Heinrich 1998, Vogelsang 1996

Results: calculation of P_{gg}, P_{qg} and P_{gq} NLO kernels completed

P_{qg} diagrams



P_{gq} diagrams



$$
\begin{aligned}
P_{qg}^{(1)} &= C_F T_f \Big\{ 4 - 9x - (1-4x)\ln x - (1-2x)\ln^2 x + 4\ln(1-x) \\
&+ \big[2\ln^2\big(\frac{1-x}{x}\big) - 4\ln\big(\frac{1-x}{x}\big) - \frac{2}{3}\pi^2 + 10 \big] p_{qg}(x) \Big\} \\
&+ N_C T_f \Big\{ \frac{182}{9} + \frac{14}{9}x + \frac{40}{9x} + \big(\frac{136}{3}x - \frac{38}{3}\big)\ln x - 4\ln(1-x) - (2+8x)\ln^2 x \\
&+ \big[-\ln^2 x + \frac{44}{3}\ln x - 2\ln^2(1-x) + 4\ln(1-x) + \frac{\pi^2}{3} - \frac{218}{9} \big] p_{qg}(x) \\
&+ 2 p_{qg}(-x) S_2(x) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
P_{gq}^{(1)} &= C_F^2 \Big\{ -\frac{5}{2} - \frac{7}{2}x + \big(2 + \frac{7}{2}x\big)\ln x - \big(1 - \frac{1}{2}x\big)\ln^2 x - 2x\ln(1-x) \\
&- \big[3\ln(1-x) + \ln^2(1-x) \big] p_{gq}(x) \Big\} \\
&+ C_F N_C \Big\{ \frac{28}{9} + \frac{65}{18}x + \frac{44}{9}x^2 - \big(12 + 5x + \frac{8}{3}x^2\big)\ln x + (4+x)\ln^2 x + 2x\ln(1-x) \\
&+ \big[-2\ln x \ln(1-x) + \frac{1}{2}\ln^2 x + \frac{11}{3}\ln(1-x) + \ln^2(1-x) - \frac{\pi^2}{6} + \frac{1}{2} \big] p_{gq}(x) \\
&+ S_2(x) p_{gq}(-x) \Big\} \\
&+ C_F T_f \Big\{ -\frac{4}{3}x - \big[\frac{20}{9} + \frac{4}{3}\ln(1-x) \big] p_{gq}(x) \Big\}
\end{aligned}
$$

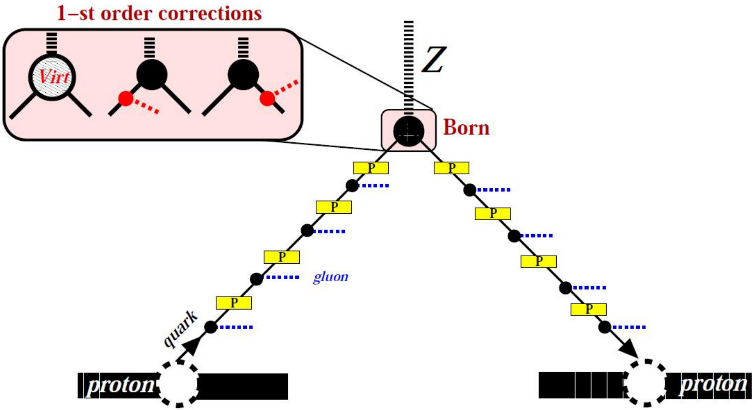
- Inclusive results agree with the literature.
- Some more work on the unintegrated distributions required.

Summary

- We introduced New PV (NPV) prescription by applying PV regulator to *all* singularities in the momentum plus component.
- We have calculated NLO DGLAP kernels in the MC-friendly scheme in *inclusive* and *exclusive* form.
- We have showed explicitly that NPV prescription works in practice by reproducing the inclusive $\overline{\text{MS}}$ splitting functions.
- In the new scheme there are no $\frac{1}{\epsilon^3}$ poles, most of the singularities cancel separately between real and virtual graphs.
- Most of the real diagrams can be calculated in 4-dimensions and are usable for the MC simulations.
- Some more calculations need: unintegrated distributions in variables matching the ordering variable of the LO shower.

Thank you

NLO corrections to hard process



$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega}$$

NLO correction to hard process

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j \in F} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j \in B} \frac{\tilde{\beta}_1(q_1, q_2, \bar{k}_j)}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s}, \hat{\theta})/d\Omega}$$

- soft+virtual NLO correction (kinematics independent!)

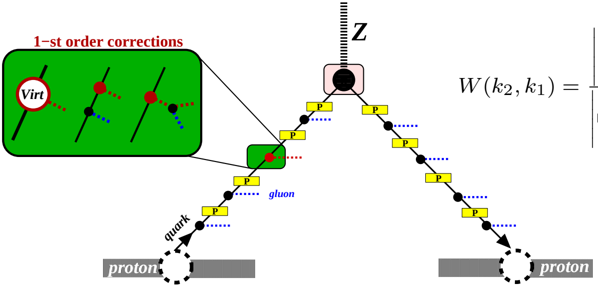
$$\Delta_{S+V} = \frac{C_F \alpha_s}{\pi} \left(\frac{2}{3}\pi^2 - \frac{5}{4} \right)$$

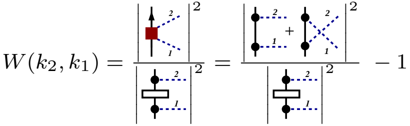
- real correction (with subtraction)

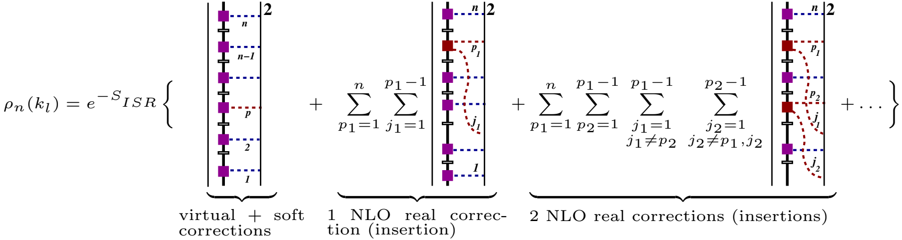
$$\tilde{\beta}_1(q_1, q_2, k) = \Big[\frac{(1-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \theta_F) + \frac{(1-\alpha)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \theta_B) \Big]$$
$$- \theta_{\alpha > \beta} \frac{1 + (1-\alpha-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}) - \theta_{\alpha < \beta} \frac{1 + (1-\alpha-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}).$$

- summation over all partons!

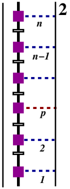
NLO corrections in the MC ladder (gluons out of quarks)

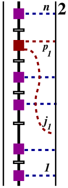


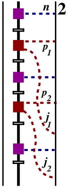




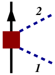
NLO corrections in the MC ladder (gluons out of quarks)

$$\rho_n(k_l) = e^{-S_{ISR}} \Bigg\{$$


$$+ \sum_{p_1=1}^{n} \sum_{j_1=1}^{p_1-1}$$


$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2\neq p_1, j_2}}^{p_2-1}$$


$$+ \dots \Bigg\}$$

- From the numerical point of view *one* or *two* **NLO insertions** 

are sufficient (restrict number of p_i sums).
- Sums over j_k can be also restricted (like in the case of hard process) using variable

$$u_{pj} = \eta_p - \eta_j + \lambda \ln(1 - z_j)$$

and choosing only the hardest (or two hardest) in u_{pj} emissions.