# Precise determination of $\pi\pi$ scattering amplitudes for $D$ and $F$ waves

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# Outline

# Definition of the problem

Lack of correct partial wave amplitudes for $D$ and $F$ waves in the processes $\pi\pi \to \pi\pi$, $K\bar{K}$ and $\eta\eta$ in the $I^G J^{PC} = 0^+2^{++}$ and the $1^+3^{--}$ sectors to study the $f_2$ and $\rho_3$ mesons respectively.

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- Further analysis presented a very precise determination for position of the $\sigma$ pole in:
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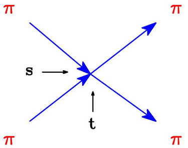
# Dispersion relations with imposed crossing symmetry

Crossing Symmetry:




$\longrightarrow \quad \overrightarrow{T_s}(s,t) = \hat{C}_{st}\,\overrightarrow{T_t}(t,s)$

Once-subtracted Dispersion Relations:

$$\mathrm{Re} f_\ell^I(s)^{out} = \sum_{I'=0}^{2} C_{st}^{II'} a_0^{I'} + \sum_{I'=0}^{2}\sum_{\ell'=0}^{3}\int_{4m_\pi^2}^{S_{max}} ds' K_{\ell\ell'}^{II'}(s,s')\mathrm{Im} f_{\ell'}^{I'}(s')^{in} + d_\ell^I(s),$$

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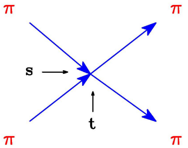


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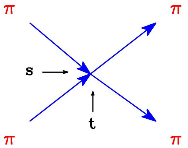


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# Improving the paremeters for $0^+2^{++}$ and $1^+3^{--}$ sectors

The total $\chi^2$ was composed of six parts:

$$\chi^2 = \sum_{j=1}^{2} \chi^2_{Data}(j) + \sum_{j'=0}^{3} \chi^2_{DR}(j') \quad (1)$$

where $j = 1, 2$ itemizes the $D$ and $F$ partial waves, respectively and $j' = 0, ..., 3$ itemizes all partial waves. Corresponding $\chi^2_{Data}(j)$ and $\chi^2_{DR}(j')$ are expressed by

$$\chi^2_{Data}(j) = \sum_{i=1}^{N^j_\delta} \frac{(\delta_i^{exp} - \delta_i^{th})^2}{(\Delta\delta_i^{exp})^2} + \sum_{i=1}^{N^j_\eta} \frac{(\eta_i^{exp} - \eta_i^{th})^2}{(\Delta\eta_i^{exp})^2} \quad (2)$$

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# Conclusions

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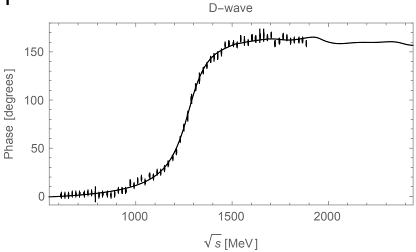


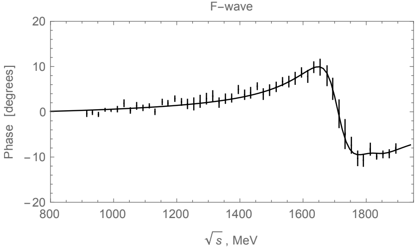

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- Achieved to new the amplitudes for $D$ and $F$ waves which very well describe the experimental data.
- New amplitudes fulfilled crossing symmetry very well.
- The dominant and the ineffective states of $0^+2^{++}$ sector:

| States: | $f_2(1270)$ | $f_2(1430)$ | $f_2(1525)$ | $f_2(1600)$ | $f_2(1730)$ |
|---|---|---|---|---|---|
| $f_2(1810)$ | $f_2(1960)$ | $f_2(2000)$ | $f_2(2020)$ | $f_2(2240)$ | $f_2(2410)$ |

- The dominant and the ineffective states of $1^+3^{--}$ sector:

| States: | $\rho_3(1690)$ | $\rho_3(1950)$ |
|---|---|---|

- Values of the $\chi^2$ for re-fitted (after fitting) amplitudes: n.d.f= 430

| | $\chi^2$ | $\chi^2_{Data}(D)$ | $\chi^2_{Data}(F)$ | $\chi^2_{DR}$ | $\chi^2/n.d.f$ |
|---|---|---|---|---|---|
| re-fitted | 882.5 | 247.6 | 483.7 | 151.1 | 2.05 |

Thank You