Calculation of QCD NLO Splitting Functions in the light-cone gauge: a new regularization prescription

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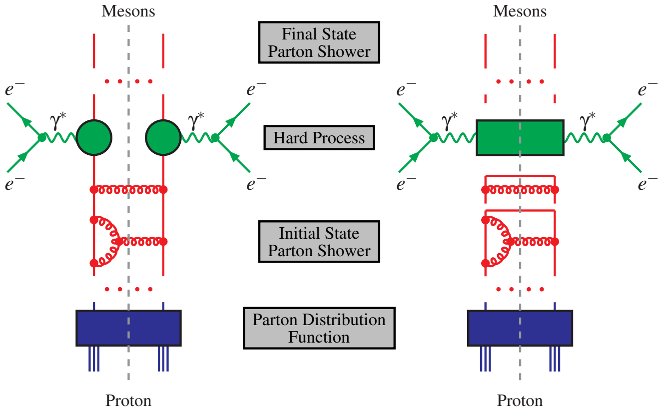
Collinear Factorization Theorem in light-cone gauge

R.Ellis, H.Georgi, M.Machacek, H.Politzer, and G.Ross, *Nucl. Phys. B152, 285 (1979)*



According to collinear factorization theorem **Parton Shower** is **a product of Splitting Functions**.

NLO Parton Shower for LHC

NLO Parton Shower MC for QCD does not exist!

In this talk I discuss calculation of **virtual** contributions to the **exclusive** NLO Splitting Functions suitable for NLO Parton Shower MC in QCD [KrKMC].

Brief history of Monte-Carlo for QCD:

- LO Hard Process + LO Parton Shower
  Pythia, Herwig (1980s)
- NLO Hard Process + LO Parton Shower
  MC@NLO, PowHEG (2000s)
- NLO Hard Process + NLO Parton Shower
  KrKMC [S.Jadach et al. *JHEP 08 012 (2011)*, *PhysRev D87 034029 (2013)*]

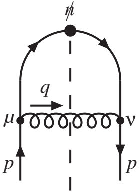
Definition

G.Curci, W.Furmanski, and R.Petronzio, *Nucl. Phys. B175, 27 (1980)*

Splitting Functions are defined in $m = 4 + 2\epsilon$ dimensions in terms of Feynman rules (light-cone gauge) and projection operators:

$$W^{(\mathrm{NLO})}(x, \epsilon) = \alpha_s^2 \, \mathrm{Tr}\left[\frac{\not{n}}{4\, p \cdot n} K \not{p} \right]$$

For example, LO Splitting Function reads:



$$= \frac{\alpha_s}{\mu_f^{2\epsilon}} \mathrm{Tr}\left(\frac{\not{n}}{4\, p \cdot n} \frac{\not{p} - \not{q}}{(p-q)^2} \gamma^\mu \not{p} \gamma^\nu \frac{\not{p} - \not{q}}{(p-q)^2} \right) d_{\mu\nu}(q) \delta^+(q^2)$$

$$d_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu n_\nu + n_\mu q_\nu}{q \cdot n}$$

Loop-Momentum Integration

Calculation of the virtual corrections to NLO SFs requires evaluation of the following loop integrals:

$$\int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{\{1, l^\mu, l^\mu l^\nu\}}{l^2(l+p_1)^2(l+p_2)^2} \frac{1}{l\cdot n}, \qquad l_+ \equiv l\cdot n$$

where

- $m = 4 + 2\epsilon$
- n is a gauge-fixing constant 4-vector (in our case $n^2 = 0$)
- p_1, p_2 are external momenta 4-vectors

Feynman denominators lead to IR and UV singularities which appear in the final result as $1/\epsilon$ terms.
Axial denominator is a source of spurious (unphysical) singularities and thus needs a special regularization prescription. A common choice is Principal Value prescription.

Principal Value vs Dimensional Regularization

Dimensional Regularization

(Transition to *m*-dimensional space-time is required.)

$$\int_0^1 \mathrm{d}l_+ \frac{1}{l_+} \to \int_0^1 \mathrm{d}^m l_+ \frac{1}{l_+} \overset{m\to 1+\epsilon}{\equiv} \int_0^1 \mathrm{d}l_+ \frac{1}{l_+^{1-\epsilon}} = \frac{1}{\epsilon}$$

Principal Value Prescription

$$\int_0^1 \mathrm{d}l_+ \frac{1}{l_+} \to \int_0^1 \mathrm{d}l_+ \left(\frac{1}{l_+}\right)_{\mathrm{PV}} \equiv \int_0^1 \mathrm{d}l_+ \frac{l_+}{l_+^2 + \delta^2} = -\ln\delta$$

In contrast to Dim Reg, Pr Val works in 4 dimensions.

That makes it possible to use expressions calculated in PV for Monte-Carlo simulations.

Standard Principal Value Approach

Use Principal Value prescription for axial denominators ONLY...

$$d^{\mu\nu} = g^{\mu\nu} - (l^{\mu} n^{\nu} + n^{\mu} l^{\nu}) \frac{1}{l_+} \quad \rightarrow \quad d^{\mu\nu}_{\mathrm{PV}} = g^{\mu\nu} - (l^{\mu} n^{\nu} + n^{\mu} l^{\nu}) \frac{l_+}{l_+^2 + \delta^2}$$

... and Dimensional Regularization for all others!

$$\frac{1}{l_+} \rightarrow \frac{1}{l_+^{1-\epsilon}}$$

$$\Downarrow$$

$$\int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{1}{l^2 (l+p)^2 (l+k)^2} = \int_0^1 \mathrm{d}z_1 \mathrm{d}z_2 \int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{1}{(l^2 + l \cdot A + B^2)^3} \simeq \frac{1}{\epsilon^2}$$

$$k^2 \neq 0 \qquad p^2 = (p-k)^2 = 0$$

New Principal Value Approach

Solution of the $1/\epsilon^2$ problem!

Use Principal Value prescription for all singularities in l_+!

(not only for axial denominators)

$$\frac{1}{l_+} \to \frac{l_+}{l_+^2 + \delta^2} \quad \text{and} \quad \frac{1}{l_+^{1-\epsilon}} \to \frac{l_+}{l_+^2 + \delta^2}\left(1 + \epsilon \ln l_+ + O(\epsilon^2)\right)$$

$$\Downarrow$$

$$\int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{1}{l^2 (l+p)^2 (l+k)^2} = \int_0^1 \mathrm{d}z_1 \mathrm{d}z_2 \int \frac{\mathrm{d}l_+^{PV} \mathrm{d}l_- \mathrm{d}^{m-2} l_\perp}{(2\pi)^m} \frac{1}{(l^2 + l \cdot A + B^2)^3} \simeq \frac{\ln \delta}{\epsilon}$$

$$k^2 \neq 0 \qquad p^2 = (p-k)^2 = 0$$

Example: three-point scalar Feynman integral

$$\mathcal{I}_3 = \int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{1}{l^2(l+k)^2(l+p)^2}$$

Principal Value approach

$$\mathcal{I}_3^{\mathrm{PV}} = \frac{i}{(4\pi)^2|k^2|}\left(\frac{4\pi}{|k^2|}\right)^{\epsilon}\Gamma(1+\epsilon)\left(-\frac{1}{\epsilon^2}+\frac{\pi^2}{6}\right)$$

New Principal Value approach

$$\mathcal{I}_3^{\mathrm{NPV}} = \frac{i}{(4\pi)^2|k^2|}\left(\frac{4\pi}{|k^2|}\right)^{\epsilon}\Gamma(1+\epsilon)\left(\frac{-2\ln\delta+\ln(1-x)}{\epsilon}\right.$$
$$\left.+2\ln^2\delta-2\ln\delta\ln(1-x)+\frac{\ln^2(1-x)}{2}+\frac{\pi^2}{6}\right), \qquad x \equiv k\cdot n$$

Properties of the Solution

- NLO SFs suitable for Parton Shower MC
- No higher-order ϵ poles in real and virt diagrams
- No dependence on the hard scale Q
- Full agreement with inclusive calculations

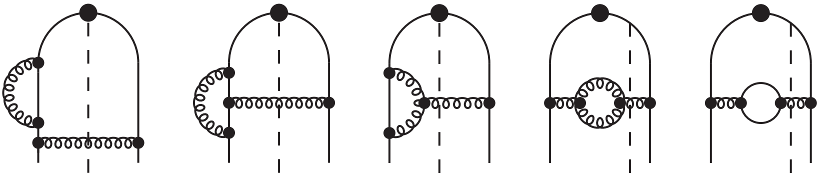
  [G.Curci et al. 1980; G.Heinrich 1998]

Axiloop: Features

Fully automated tool for symbolic calculation of Splitting Functions in light-cone gauge written in Mathematica.

- Index contraction and trace calculation
- One-loop integration with various prescriptions in light-cone and covariant gauges
- Passarino-Veltman reduction of tensor integrals with separated IR and UV poles
- One-particle final state integration

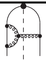
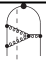
Non-Singlet Splitting Functions



For the all depicted non-singlet graphs we calculated:

- **UV counter-term**
- **Inclusive SF**
  (crosscheck with [G.Curci et al. 1980, G.Heinrich 1998])
- **Exclusive SF** – for Monte Carlo!

Cross-check with CFP: Inclusive SF

			SUM
Single poles			
p_{qq}	-7	-4	-11
$1-x$	$-5/2$	$3/2$	-1
$1+x$	$-1/2$	$1/2$	0
$p_{qq}\ \ln x$	0	$-3/2$	$-3/2$
$(1-x)\ \ln x$	2	0	2
$(1+x)\ \ln x$	0	$1/2$	$1/2$
$p_{qq}\ \ln(1-x)$	-3	8	5
$(1-x)\ \ln(1-x)$	4	0	4
$p_{qq}\ \ln^2 x$	2	-1	1
$p_{qq}\ \ln x \ln(1-x)$	2	4	6
$p_{qq}\ \ln^2(1-x)$	4	-2	2
$p_{qq}\ \mathrm{Li}_2(1-x)$	-2	2	0
$p_{qq}\ \mathrm{Li}_2(1)$	8	-2	6
$p_{qq}\ I_1$	-12	4	-8
$p_{qq}\ I_0$	0	8	8
$(1-x)\ I_0$	8	0	8
$p_{qq}\ I_0 \ln x$	4	4	8
$p_{qq}\ I_0 \ln(1-x)$	12	-4	8

Sum of real and virt corrections to NLO SF calculated in NPV agrees with inclusive results from [G.Curci et al. 1980].

Summary

Done:

- **Complete calculation of non-singlet case (virtual + real)**
- **New IR regularization scheme** defined in 4 dimensions
- Resulting **exclusive** SFs are better suitable for MC!
- **Axiloop** package written in Mathematica for fully automatic analytical calculations

In progress:

- Singlet splitting functions

Future:

- Coefficient functions (hard process)
- Two-loop virtual corrections