Monte Carlo study of NLO correction to QCD evolution kernel due to the change of the factorization scale

M. Fabiańska[(a)] **and S. Jadach**[(b)]

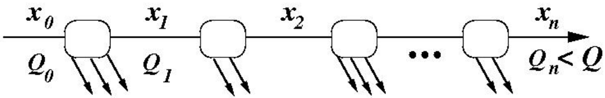
(a) Institute of Physics, Jagellonian University, Kraków
(b) Theory Division, Institute of Nuclear Physics PAN, Kraków

Partly supported by the grants of *Narodowe Centrum Nauki* DEC-2011/03/B/ST2/02632 and UMO-2012/04/M/ST2/00240 and FP7 grant PITN-GA-2010-264564 (LHCPhenoNet)

Cracow Epiphany Conference, January 9-11th, 2014

Introduction

- In the collinear factorization factorization scale Q limits transverse momenta of the emitted particles.
- Possible choices: virtuality, max. transverse momentum, max. rapidity, μ of the dimensional regularizations, total energy in the hard process $\sqrt{\hat{s}}$, etc.
- Redefinition of the factorization scale may involve factor $z \equiv$ relative loss of the energy of the emitter: $\widehat{Q} = z^{\sigma} Q$, $z = x_n/x_0$, $\sigma = \pm 1, \pm 2$, for various Q's.
- Many examples:
  (i) change from μ to virtuality (Altarelli, Ellis, Marinelli, 1980)
  (ii) space/time-like ladder (Curci, Furmanski, Petronzio, 1979)
  (iii) change from angular- to kT-ordering (Catani, Ciafaloni, Fiorani, Marchesini, 1988)



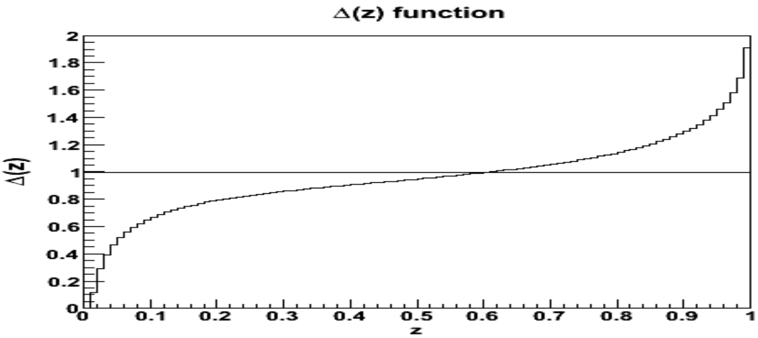
The aim of the study

- The aim of the present study is to show that: the redefinition of the factorization scale $Q \to zQ$ in the ladder can be traded exactly for the NLO correction to the LO evolution kernel, $P(z) \to P(z) + (2C_F\alpha_S/\pi)\Delta(z)$
- The above observation was done/exploited in classic CFP, CCFM and other papers.
- Here, the above mechanism will be demonstrated *numerically* in the context of the Monte Carlo parton shower.

Numerical Markovian Monte Carlo framework

- x_0 is generated according to $D(x_0) = 3(1 - x_0)^2$, $(\int D = 1)$.
- $t_i = \ln(Q_i)$ and $z_i = \frac{x_i}{x_{i-1}}$ are generated in a loop according to the following distributions:
  - $P(t_i) = L\lambda e^{-L\lambda t_i} \theta(t_i - t_{i-1}), \quad \lambda = \frac{2C_F \alpha_S}{\pi}$,
  - $p(z_i) = L^{-1} \frac{1+z_i^2}{2(1-z_i)} \theta(1 - z_i - \delta), \quad L = \left(\ln \frac{1}{\delta} - \frac{3}{4}\right)$.
- Markovian process (loop) is terminated at $i = N$, when $t_{N+1} > T$.
- The redefinition of the factorization scale $Q \rightarrow \hat{Q} = zQ$ corresponds to a redefinition of $T = \ln(Q)$ and is realized step by step in the loop by shifting down the exit point: $T \rightarrow T + \ln(z_{i-1})$ at the i-th step, $i = 2, 3, ...N$.
- Alternatively the same is also realized by adding the NLO correction directly to the evolution kernel.
- The corresponding MC weight is: $WT = \prod_{i=1}^{N} \frac{P(z_i)+\lambda\Delta(z_i)}{P(z_i)}$.

$\Delta(z)$ function of CFP

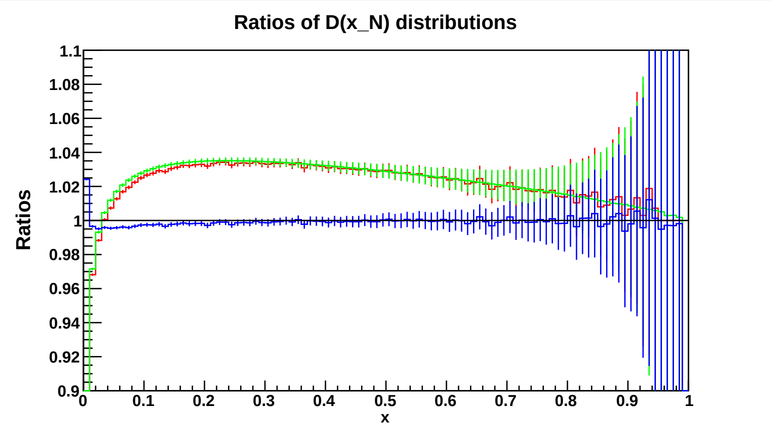


$$\Delta(z) = \{P_+ \otimes (\ln \times P)\}(z) = \int_0^1 dz_1 dz_2 \, (P(z_1))_+ \, \ln(z_2) P(z_2) \delta(z - z_1 z_2)$$

$$= \frac{1+z^2}{2(1-z)} \ln z \left[\ln \frac{(1-z)^2}{z} + \frac{3}{2} \right] + \frac{1+z}{8} \ln^2 z - \frac{1-z}{4} \ln z,$$

Note that $\int_0^1 \Delta(z) = 0$.

Results



Ploted ratios are: red: $\frac{(b)}{(a)}$, green: $\frac{(c)}{(a)}$, blue: $\frac{(b)}{(c)}$

(a) LO approximation,

(b) decreasing factorization scale $T \to T + \ln(z_i)$,

(c) NLO correction in the kernel $P(z) \to P(z) + \lambda\Delta(z)$.

Summary

- **It was shown numerically, in the framework of the simple Markovian Monte Carlo parton shower, that the redefinition of the factorization scale $Q \to zQ$ can be replaced or compensated by means of introduction of the well defined NLO correction in the evolution kernel** $P(z) \to P(z) + (2C_F\alpha_S/\pi)\,\Delta(z)$
- **Function $\Delta(z)$ was identified for the 1-st time in the Curci-Furmanski-Petronzio paper (1980).**
- **The above result can be exploited in the construction of the parton shower MC with complete NLO corrections (KRKMC project).**