Initial conditions for evolution of double parton distributions

Emilia Lewandowska

Institute of Nuclear Physics Polish Academy of Science

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K. Golec-Biernat, E. Lewandowska, arXiv:1311.7392 [hep-ph]

Introduction

Double parton distribution functions (DPDF)

- are used in the description of double parton scattering (DPS)
- evolve with hard scales through QCD evolution equations known in the leading logarithmic approximation (LLA)
- obey nontrivial momentum and valence quark number sum rules which are conserved by the evolution.

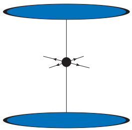
Problem of this talk: how to specify initial conditions for the evolution equations which obey these sum rules?

Outline

- Parton distribution functions
- Evolution equations
- Sum rules
- Initial conditions
- Summary

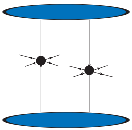
Parton distribution functions

- Single parton scattering



PDF: $D_f(x, Q)$

- Double parton scattering



DPDF: $D_{f_1 f_2}(x_1, x_2, Q_1, Q_2)$

- Two hard scales: Q_1, Q_2 and two flavours: f_1, f_2 (including gluon).
- Sum of partons' momenta cannot exceed total nucleon momentum

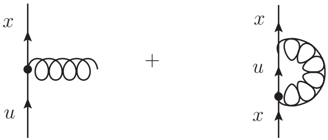
$$x_1 + x_2 \leq 1$$

QCD evolution equations for single PDF

- General form of evolution equations for single PDF ($t = \ln Q^2$)

$$\partial_t D_f(x,t) = \sum_{f'} \int_0^1 du\, \mathcal{K}_{ff'}(x,u,t)\, D_{f'}(u,t)$$

- The integral kernels describe real and virtual parton emission



$$\mathcal{K}_{ff'}(x,u,t) = \mathcal{K}^R_{ff'}(x,u,t) - \delta(u-x)\,\delta_{ff'}\,\mathcal{K}^V_f(x,t)$$

Evolution equations (cont.)

- The real emission kernels

$$\mathcal{K}^{R}_{ff'}(x,u,t) = \frac{1}{u} P_{ff'}\left(\frac{x}{u}, t\right) \theta(u-x)$$

- Splitting functions

$$P_{ff'}(z,t) = \frac{\alpha_s(t)}{2\pi} P^{(0)}_{ff'}(z) + \frac{\alpha_s^2(t)}{(2\pi)^2} P^{(1)}_{ff'}(z) + \ldots$$

- Well known DGLAP evolution equations for single PDF

$$\partial_t D_f(x,t) = \sum_{f'} \int_x^1 \frac{dz}{z} P_{ff'}(z,t)\, D_{f'}\left(\frac{x}{z}, t\right) - D_f(x,t) \sum_{f'} \int_0^1 dz\, z\, P_{f'f}(z,t)$$

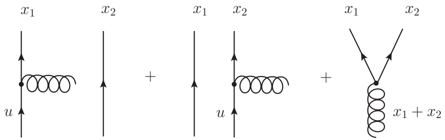
Evolution equations for DPDF

- Evolution of DPDF with equal hard scales $Q_1 = Q_2 \equiv Q$:

$$D_{f_1 f_2}(x_1, x_2, Q_0, Q_0) \;\rightarrow\; D_{f_1 f_2}(x_1, x_2, Q, Q) \;\equiv\; D_{f_1 f_2}(x_1, x_2, t = \ln Q^2)$$

- Evolution equations with three terms:

$$\begin{aligned}
\partial_t\, D_{f_1 f_2}(x_1, x_2, t) \;&=\; \sum_{f'} \int_0^{1-x_2} du\, \mathcal{K}_{f_1 f'}(x_1, u, t)\, D_{f' f_2}(u, x_2, t) \\
&+\; \sum_{f'} \int_0^{1-x_1} du\, \mathcal{K}_{f_2 f'}(x_2, u, t)\, D_{f_1 f'}(x_1, u, t) \\
&+\; \sum_{f'} \mathcal{K}^R_{f' \to f_1 f_2}(x_1, x_1 + x_2, t)\, D_{f'}(x_1 + x_2, t)
\end{aligned}$$



Additional term

- The third splitting term contains single PDF

$$\frac{\alpha_s(Q)}{2\pi}\sum_{f'}\frac{1}{x_1+x_2}P_{f'\to f_1 f_2}\left(\frac{x_1}{x_1+x_2}\right)D_{f'}(x_1+x_2,Q)$$

- The DPDF evolution equations need to be solved together with the DGLAP equations for single PDF.

- Initial conditions for both DPDF and PDF have to be specified:

$$D_{f_1 f_2}(x_1,x_2,Q_0) \qquad\qquad D_f(x,Q_0)$$

Momentum sum rule

- Momentum sum rule for PDF (conserved by DGLAP eqs.)

$$\sum_f \int_0^1 dx\, x\, D_f(x, Q) = 1$$

- By analogy: momentum sum rule for DPDF

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, \frac{D_{f_1 f_2}(x_1, x_2, Q)}{D_{f_2}(x_2, Q)} = (1 - x_2)$$

- The ratio in red is a conditional probablility to find a parton with the momentum fraction x_1 while the second parton fraction x_2 is fixed.

- Momentum sum rule is a relation between DPDF and PDF

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, D_{f_1 f_2}(x_1, x_2, Q) = (1 - x_2)\, D_{f_2}(x_2, Q) \qquad (1)$$

Valence number sum rule

- Valence number sum rule for PDF (N_i= no. of valence quarks)

$$\int_0^1 dx \left\{ D_{q_i}(x, Q) - D_{\bar{q}_i}(x, Q) \right\} = N_i$$

- Valence number sum rule for DPDF

$$\begin{aligned} I_{q_i f_2} &= \int_0^{1-x_2} dx_1 \left\{ D_{q_i f_2}(x_1, x_2, Q) - D_{\bar{q}_i f_2}(x_1, x_2, Q) \right\} \\ &= \begin{cases} N_i \, D_{f_2}(x_2, Q) & \text{for } f_2 \neq q_i, \bar{q}_i \\ (N_i - 1) \, D_{f_2}(x_2, Q) & \text{for } f_2 = q_i \\ (N_i + 1) \, D_{f_2}(x_2, Q) & \text{for } f_2 = \bar{q}_i \end{cases} \qquad (2) \end{aligned}$$

- Relations (1) and (2) are conserved by the evolution equations once imposed at some initial scale Q_0.

Initial conditions

- In practice: initial DPDF are built out of the existing single PDFs, e.g. (Gaunt, Stirling, Korotkikh, Snigirev)

$$D_{f_1 f_2}(x_1, x_2) = D_{f_1}(x_1)\, D_{f_2}(x_2)\, \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+n_1}(1 - x_2)^{2+n_2}}$$

- Symmetric input with respect to the parton interchange

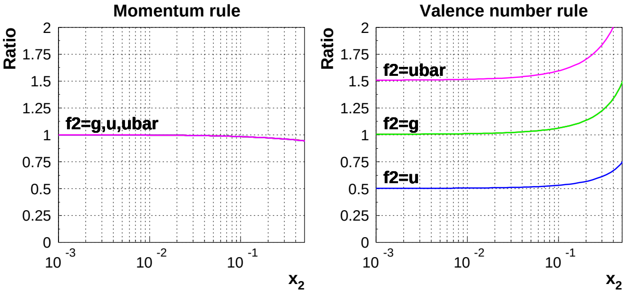
$$D_{f_1 f_2}(x_1, x_2) = D_{f_2 f_1}(x_2, x_1)$$

and positive definite.

Sum rules with the symmetric input

$$\mathrm{Ratio}_{\mathrm{Mom}}(x_2, f_2) = \frac{(1-x_2)\, D_{f_2}(x_2)}{\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, D_{f_1 f_2}(x_1, x_2)} = 1$$

$$\mathrm{Ratio}_{\mathrm{Val}}(x_2, f_2) = \frac{N_{uf_2}\, D_{f_2}(x_2)}{\int_0^{1-x_2} dx_1 \left\{ D_{uf_2}(x_1, x_2) - D_{\bar{u} f_2}(x_1, x_2) \right\}} = 1$$



Valence quark number sum rule is violated.

How to *exactly* fulfill the sum rules?

- Asymmetric ansatz to fulfill the momentum sum rule:

$$D_{f_1 f_2}(x_1, x_2) \;=\; \frac{1}{1-x_2} D_{f_1}(\frac{x_1}{1-x_2}) \cdot D_{f_2}(x_2)$$

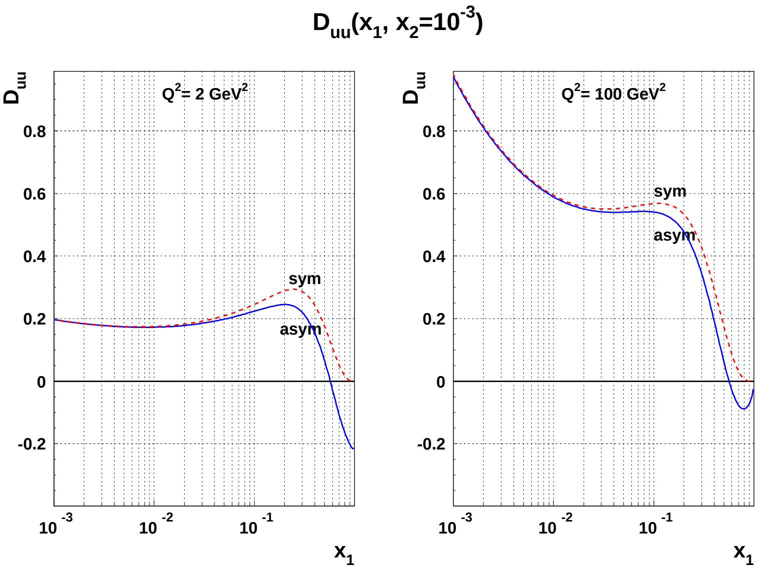
- Corrections for identical quark flavours/antiflavours to obey the valence number sum rule:

$$D_{f_i f_i}(x_1, x_2) \;=\; \frac{1}{1-x_2} \left\{ D_{f_1}(\frac{x_1}{1-x_2}) - \frac{1}{2} \right\} D_{f_1}(x_2)$$

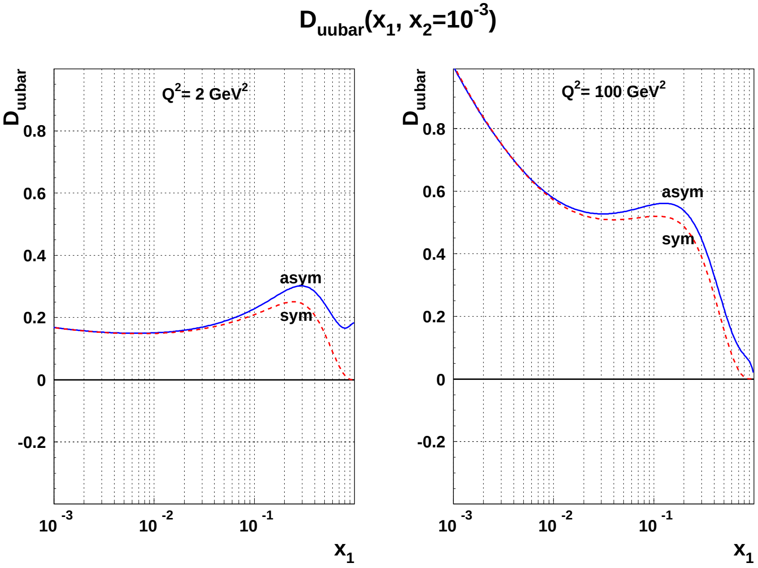
$$D_{f_i \bar{f}_i}(x_1, x_2) \;=\; \frac{1}{1-x_2} \left\{ D_{f_1}(\frac{x_1}{1-x_2}) + \frac{1}{2} \right\} D_{\bar{f}_1}(x_2)$$

- DPDF for identical flavours, $D_{f_i f_i}(x_1, x_2)$, are not positive definite. This is the price to pay for the construction with single PDFs !

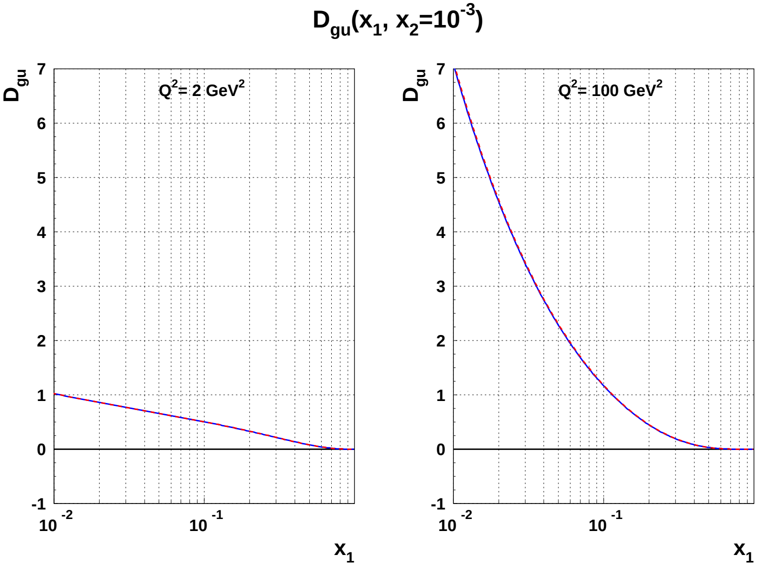
Symmetric vs. asymmetric input for D_{uu}



Symmetric vs. asymmetric input for $D_{u\bar{u}}$



Symmetric vs. asymmetric input for D_{gu}



Summary

- If input DPDF are constructed from known single PDF:

	symmetric input	asymmetric input
Parton symmetry	yes	no
Positivity	yes	no
Sum rules	no	yes

- Alternative: specify positive initial DPDF and generate initial PDF using sum rules. Unfortunately, no experimental knowledge on DPDF.

- However, for $x_1, x_2 \to 0$ factorized form is a good approximation:

$$D_{f_1 f_2}(x_1, x_2, Q) \approx D_{f_1}(x_1, Q)\, D_{f_2}(x_2, Q)$$

BACKUP

Evolution equations for DPDF

- For unequal hard scales, $Q_1 < Q_2$, two step evolution:

$$D_{f_1 f_2}(x_1, x_2, Q_0, Q_0) \;\rightarrow\; D_{f_1 f_2}(x_1, x_2, Q_1, Q_1) \;\rightarrow\; D_{f_1 f_2}(x_1{}^{fixed}, x_2, Q_1, Q_2)$$

- Single PDF evolution, $Q_1 \rightarrow Q_2$, with respect to the second variable x_2 in the second step:

$$\partial_{t_2} D_{f_1 f_2}(x_1, x_2, t_1, t_2) = \sum_{f'} \int_0^{1-x_1} du\, \mathcal{K}_{f_2 f'}(x_2, u, t)\, D_{f_1 f'}(x_1, u, t, t_2)$$