Solution of a resummed BK evolution equation

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The Balitzky-Kovchegov evolution equation

Integral form in momentum space:

$$\phi(x,k) = \phi_0(x,k)$$

$$+\bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \int_0^\infty \frac{\mathrm{d}l^2}{l^2} \left(\frac{l^2 \phi(\frac{x}{z}, l^2) - k^2 \phi(\frac{x}{z}, k^2)}{|k^2 - l^2|} + \frac{k^2 \phi(\frac{x}{z}, k)}{\sqrt{4l^4 + k^4}} \right)$$

$$-\bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \phi^2(\frac{x}{z}, k)$$

The Balitzky-Kovchegov evolution equation

A new form given in [Kutak, Golec-Biernat, Jadach, Skrzypek, **JHEP 1202** (2012) **117**] introduces a new scale μ to sum low q emissions:

$$\phi(x,k) = \phi_0(x,k)$$

$$+\bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2 q}{\pi q^2} \phi(\frac{x}{z}, |k+q|^2) \theta(q^2 - \mu^2)$$

$$+\bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2 q}{\pi q^2} \Big(\phi(\frac{x}{z}, |k+q|^2) \theta(\mu^2 - q^2) - \theta(k^2 - q^2) \phi(\frac{x}{z}, k) \Big)$$

The Balitzky-Kovchegov evolution equation

The final resummed form:

$$\phi(x,k) = \tilde{\phi}_0(x,k) + \bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2 q}{\pi q^2} \theta(q^2 - \mu^2) \Delta_R(z,k,\mu) \cdot$$

$$\cdot \left(\phi(\frac{x}{z}, |k+q|^2) - q^2 \delta(q^2 - k^2) \phi^2(\frac{x}{z}, q^2) \right)$$

with

$$\Delta_R(z,k,\mu) = \exp\left(-\bar{\alpha}_s \log\frac{1}{z} \log\frac{k^2}{\mu^2} \right)$$

The comparison

We want to compare solutions of the two forms of the BK equation:

$$\phi(x, k_2) = \phi_0(x, k_2)$$
$$+\bar{\alpha}_s \int_{\frac{x}{x_0}}^{1} \frac{\mathrm{d}z}{z} \int_{\mu^2}^{q_f^2} \frac{\mathrm{d}l_2}{l_2} \left(\frac{l_2 \phi(\frac{x}{z}, l_2) - k_2 \phi(\frac{x}{z}, k_2)}{|k_2 - l_2|} + \frac{k_2 \phi(\frac{x}{z}, k_2)}{\sqrt{4 l_2^2 + k_2^2}} \right)$$
$$-\frac{\bar{\alpha}_s}{\pi R^2} \int_{\frac{x}{x_0}}^{1} \frac{\mathrm{d}z}{z} \phi^2 \left(\frac{x}{z}, k_2 \right)$$

$$\Phi(x, k_2) = \tilde{\Phi}_0(x, k_2)$$
$$+\bar{\alpha}_s \int_{\frac{x}{x_0}}^{1} \frac{\mathrm{d}z}{z} \Delta_R(z, k_2) \int_{\mu^2}^{q_f^2} \frac{\mathrm{d}q_2}{\pi q_2} \int_0^{\pi} \mathrm{d}\varphi \Phi \left(\frac{x}{z}, m(S(q_2, k_2, \varphi), \mu^2, q_f^2) \right)$$
$$-\frac{\bar{\alpha}_s}{\pi R^2} \int_{\frac{x}{x_0}}^{1} \frac{\mathrm{d}z}{z} \Delta_R(z, k_2) \Phi^2 (\frac{x}{z}, k_2)$$

The method

- introduce a finite upper limit, q_f
- work with logarithms: $\tilde{z} = \log z$, $\widetilde{q^2} = \log q^2$, etc., so $\int \frac{\mathrm{d}z}{z} \rightarrow \int \mathrm{d}\tilde{z}$
- consider the difference between sides of the equation as a function of ϕ:

$$D_\phi(x,k) = \phi_0(x,k) - \phi(x,k) + \bar{\alpha}_s \int_{\tilde{x}-\widetilde{x_0}}^{0} \mathrm{d}\tilde{z} \int_{\widetilde{\mu^2}}^{\widetilde{q_f^2}} \mathrm{d}\widetilde{l^2}\,(\ldots)$$

The method

- expand the solution into products of Chebyshev polynomials:

$$\phi(\tilde{x}, \tilde{k}) = \sum_i c_i b_i(\tilde{x}, \tilde{k})$$

- define similar expanion of the squared function:

$$\phi^2(\tilde{x}, \tilde{k}) = \sum_i d_i b_i(\tilde{x}, \tilde{k})$$

- rearrange D_ϕ so that the integrals can be precalculated

$$L_i(x, k) = \int_{\tilde{x}-\widetilde{x_0}}^{0} \mathrm{d}\tilde{z} \int_{\widetilde{\mu^2}}^{\widetilde{q_f^2}} \mathrm{d}\widetilde{l^2}\,(\ldots)$$

$$Q_i(x, k) = \int_{\tilde{x}-\widetilde{x_0}}^{0} \mathrm{d}\tilde{z}\phi^2(\frac{x}{z}, k^2)$$

$$D_\phi(x, k) = \phi_0(x, k) - \phi(x, k) + \bar{\alpha}_s \sum_i c_i L_i(x, k) - \frac{\bar{\alpha}_s}{\pi R^2} \sum_i d_i Q_i(x, k)$$

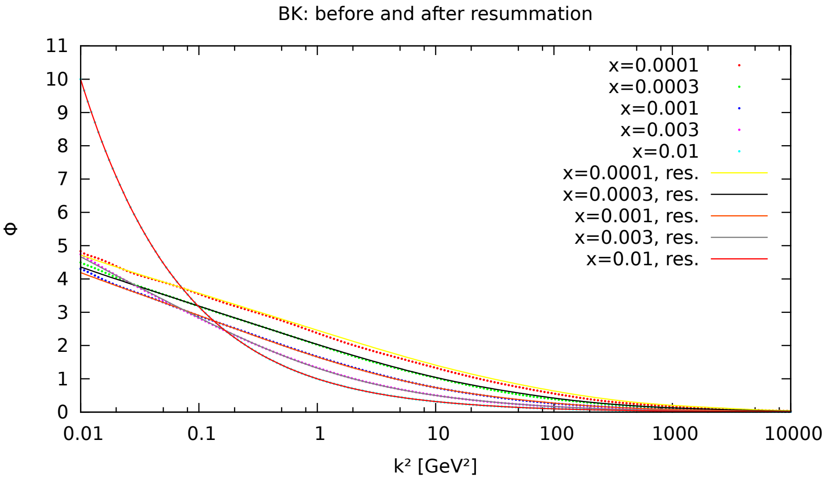
The method

- approximate the norm $\|D_\phi\|$ using a finite set of points (x_j, k_j)
- implement $\|D_\phi\|$ as a function of c within linear algebra, except that the nonlinear term ϕ^2 is expanded to b_i by a pair of DCT transforms

$$d \leftarrow \mathrm{DCT} \leftarrow \phi^2(x,k) \leftarrow \phi(x,k) \leftarrow \mathrm{DCT} \leftarrow c$$

- find the optimal coefficients c to build a solution ϕ

The result


BK: before and after resummation
Φ
k² [GeV²]
x=0.0001
x=0.0003
x=0.001
x=0.003
x=0.01
x=0.0001, res.
x=0.0003, res.
x=0.001, res.
x=0.003, res.
x=0.01, res.
0.01
0.1
1
10
100
1000
10000
0
1
2
3
4
5
6
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11