Automatic calculation of NLO splitting functions with loops for exclusive parton shower Monte-Carlo

Oleksandr Gituliar
oleksandr.gituliar@ifj.edu.pl

Institute of Nuclear Physics
Polish Academy of Sciences,
Cracow, Poland
http://www.ifj.edu.pl

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Motivation:
Construct (first ever) NLO parton shower for LHC

NLO parton shower Monte-Carlo for QCD does not exist.

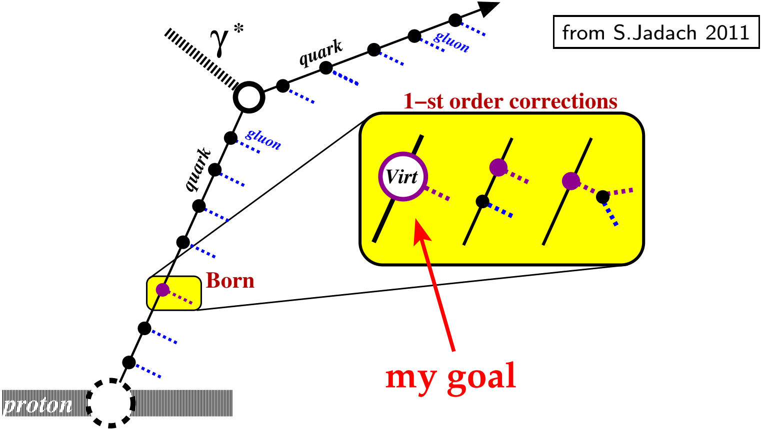
Brief history of Monte-Carlo for QCD

- LO Hard Process + LO Parton Shower – Pythia, Herwig (1980s)
- NLO Hard Process + LO Parton Shower – MC@NLO, PowHEG (2000s)
- NLO Hard Process + NLO Parton Shower – **KrKMC** (ongoing)

Aim of this work is to calculate **exclusive** NLO splitting functions for KrKMC project (S.Jadach et al.)

NLO corrections to the LO parton shower

Splitting functions describe a probabilistic rate for incoming parton (quark or gluon) to emit outgoing partons.



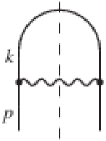
Splitting Functions

Inclusive

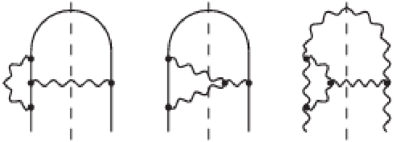
- depend on x (see below)
- defined in [EGMPR79]
- calculated to NNLO
- used to
  - solve DGLAP equation
  - build LO Parton Shower MC

Exclusive

- depend on full momenta
- require extension of collinear factorization theorem
- needed to build NLO Parton Shower

LO: 

$$P^{(0)}(\alpha_s, x) = \frac{\alpha_s}{2\pi}\frac{1+x^2}{1-x}, \quad x = \frac{k\cdot n}{p\cdot n} = \frac{k_0 + k_3}{p_0 + p_3}$$

NLO:  . . .

Axiloop package for Mathematica

Designed to calculate:

- inclusive and exclusive splitting kernels up to NLO order
  - for singlet (incoming gluon) and non-singlet (incoming quark) cases
  - for one (plus loop) and two particles in final state
  - with geometrical cut-off for real emissions in 4 dimensions
  - for various evolution times
- corresponding hard processes
- all results in analytical form

Tools we use:

- Wolfram Mathematica 9
- Wolfram Workbench 2
- Git and GitHub (`https://github.com/gituliar/axiloop.git`)

Architecture of Axiloop

Key calculation steps, based on [CFP80] [Hei98]:

1. **Calculate trace** in n dimensions
2. **Regularize** infra-red singularities
3. **Regularize** spurious singularities
4. **Integrate over loop momenta**
5. **Renormalize** ultra-violet singularities
6. **Integrate over final state** (for different evolution times)

Calculation framework

Axial gauge (massless QCD) [EGMPR79]

- pros:
  - nice factorization properties (two-particle irreducible diagrams)
  - on internal lines only physical states survive
  - suitable for exclusive parton shower Monte-Carlo
- cons:
  - has more difficult analytical structure than unphysical gauges
  - introduces spurious poles (regulated with some prescription, e.g. Principal Value (PV), Mandelstam-Leibbrant (ML), or other)

Principal Value prescription [CFP80] [Hei98]

- pros:
  - doesn't introduce unphysical states (ghosts)
  - much simpler than ML prescription
- cons:
  - is based on heuristic rules
  - has no formal proof

Regularization of UV, IR, and spurious singularities

- UV poles
  - live in $m = 4 - 2\epsilon_{\rm uv}$ dimensions, $\epsilon_{\rm uv} > 0$
  - are defined for off-shell momenta

E.g. $$\int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2 (l-p)^2} = i(4\pi)^{-2+\epsilon_{\rm uv}} \frac{\Gamma(1+\epsilon_{\rm uv})}{\epsilon_{\rm uv}} \frac{\beta(1-\epsilon_{\rm uv}, 1-\epsilon_{\rm uv})}{(p^2)^{\epsilon_{\rm uv}}}$$

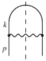
After renormalization

- IR poles
  - live in $m = 4 + 2\epsilon_{\rm ir}$ dimensions, $\epsilon_{\rm ir} > 0$
  - appears for on-shell momenta

- Spurious poles
  - Principal Value prescription: $\frac{1}{l \cdot n} \to \frac{l \cdot n}{(l \cdot n)^2 + \delta^2 (P \cdot n)^2}$

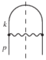
Results: UV renormalization constant

$$Z(\alpha_s, x, \delta) = \lim_{\epsilon \to 0} \frac{\mathrm{Res}_{\epsilon_{\rm uv}} \left[\text{diagram} \right]_B}{\text{diagram}}$$



Z for C_f color structure

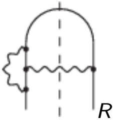
$$Z_{C_f}(\alpha_s, x, \delta) = \lim_{\epsilon \to 0} \frac{\mathrm{Res}_{\epsilon_{\rm uv}} \left(\left[\text{diagram}\right]_B + \left[\text{diagram}\right]_B \right)}{\text{diagram}} =$$

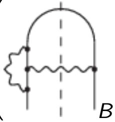


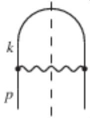
$$= \frac{\alpha_s}{4\pi} C_f \left(-4 \ln \delta + 4 \ln x - 3 \right) + \frac{\alpha_s}{4\pi} C_f \left(4 \ln \delta - 2 \ln x + 3 \right) = \frac{\alpha_s}{4\pi} C_f \; 2 \ln x$$

NOTE, δ **disappears** for gauge-invariant quantities.

Results: splitting function (inclusive)

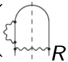
$P^{(1)}(\alpha_s, x, \xi, \delta) = \mathrm{Res}_{\epsilon_{\rm ir}}$ 

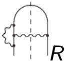
$= \mathrm{Res}_{\epsilon_{\rm ir}} \Bigg($ 

$- Z_{C_f}$ 

$\Bigg)$

$P^{(1)}$ for C_f^2 color structure

$P^{(1)}_{C_f^2}(\alpha_s, x, \xi, \delta) = \mathrm{Res}_{\epsilon_{\rm ir}} \Big($ 

$+$ 

$\Big) =$

$$= \left(\frac{\alpha_s}{2\pi}\right)^2 C_f^2 \left(x - 2(1-x)\ln x - \frac{1+x^2}{1-x}\left(2\xi \ln x \ln(1-x) - \mathrm{Li}_2(1-x)\right)\right)$$

Evolution time

- transverse momentum for $\xi = 0$
- virtuality for $\xi = 1$ (in agreement with [CFP80])
- rapidity for $\xi = 2$

NOTE, again δ **disappears** for gauge-invariant quantities.

Summary

Done:

- Axiloop – a complete package for calculating
  - inclusive splitting functions
  - exclusive splitting functions
  - UV renormalization constant
  - loop integrals in axial gauge with PV prescription
  - final-state integrals
- C_F^2 color structure calculated

In progress:

- complete singlet and non-singlet splitting kernels
- define exclusive splitting kernels in *4 dimensions* for parton shower
- integration for two final states
- analysis of IR singularities at exclusive level

References

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