Automation in fixed-order calculations

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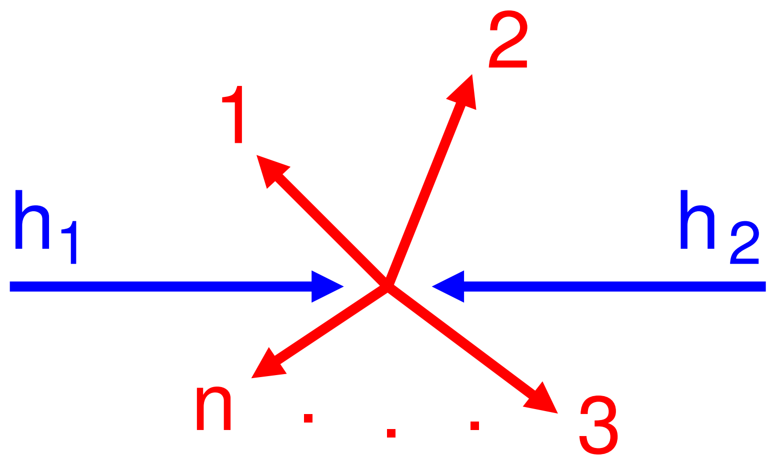
Outline

- factorization
- numerical evaluation of amplitudes
- automated LO programs
- merging with parton shower
- ingredients to NLO calculations
- automated NLO programs
- matching to parton shower
- summary

Hard scattering cross sections within collinear factorization



PDFs are related to the structure of the hadrons, universal to the scattering process

$$\sigma_{h_1,h_2\to n}(p_1,p_2) = \sum_{a,b}\int dx_1 dx_2\, \boxed{f_a(x_1,\mu)\, f_b(x_2,\mu)}\; \hat{\sigma}_{a,b\to n}(x_1p_1,x_2p_2;\mu)$$

$$\hat{\sigma}_{a,b\to n}(p_a,p_b;\mu) = \boxed{\int d\Phi(p_a,p_b\to\{p\}_n)}\,\boxed{|\mathcal{M}_{a,b\to n}(p_a,p_b\to\{p\}_n;\mu)|^2}\,\boxed{\mathcal{O}(p_a,p_b,\{p\}_n)}$$

Phase space (includes spin/color summation) governs the kinematics

Matrix element (squared) contains model parameters, governs the dynamics

Observable, imposes phase space cuts

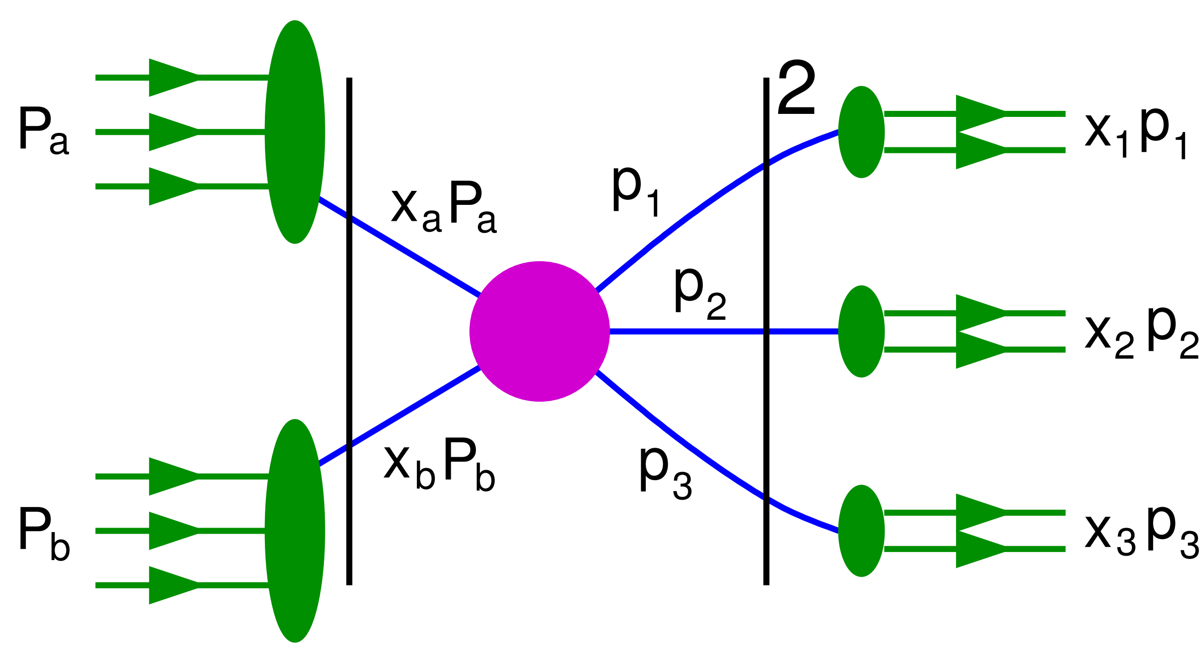
Collinear factorization

Naïve parton model for hadron scattering

$$d\sigma(P_a, P_b \to \{P_i\}) = \int \prod_{j=a,b,1,\dots} dx_j f_j(x_j)\ \ d\hat{\sigma}(p_a, p_b \to \{p_i\})$$

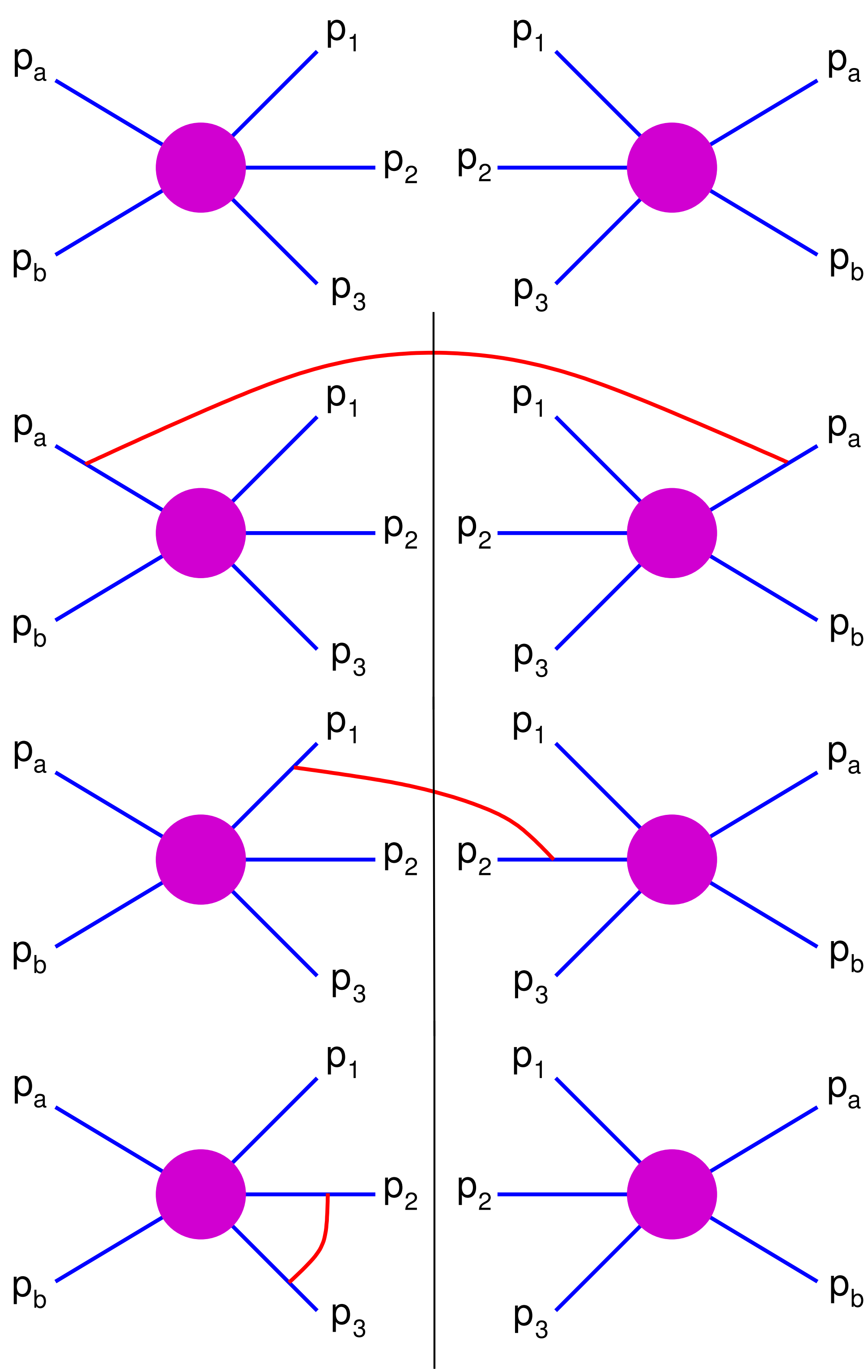
- the parton densities f_a and fragmentation function f_i describe physics of long time scales
  - not calculable within perturbation theory
  - universal to the hard scattering process
  - to be extracted from experiments

$$p_{a/b} = x_{a/b} P_{a/b} \qquad x_i p_i = P_i$$



- the partonic scattering cross section $\hat{\sigma}$ describes physics of short time scales, and should be calculable within perturbative QCD
  - Asymptotic freedom: small coupling for high energy

Perturbative calculations in QCD



for the squared scattering amplitude $|\mathcal{M}|^2$

- blue lines represent identified partons
- mirrored graphs represent $\mathcal{M}$ and $\mathcal{M}^*$
- first square represents leading order
- higher orders by adding one coupling, that is two 3-point vertices connected by gluon integrated over its phase space
- gluons crossing the cut are real
  - are on-shell
  - participate in momentum conservation
- gluons not crossing the cut are virtual, are off-shell

Trouble with divergencies

Integrating over the phase space of the extra, unobserved gluons, one encounters IR divergences stemming form non-integratable singularities:

soft singularities: $\displaystyle\int_0 \frac{d|k|}{|k|}$

appear because all components of integration momentum k may become arbitrarily small

collinear singularities: $\displaystyle\int_0 \frac{d\theta}{1-\cos\theta}$

appear because integration momentum k may become arbitrarily collinear with massless parton momentum p_i

- all soft singularities cancel with each other, as predicted by the Kinoshita-Lee-Nauenberg theorem
- collinear singularities do not all cancel

R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer, G.G. Ross, 1979:
Non-cancelling collinear divergences can be indentified with the external partons, and can be factorized, to all orders in QCD.

Factorization following EGMPR

"Renormalized" formula for hard scattering cross section

$$d\sigma(P_a, P_b \to \{P_i\}) = \int \prod_{j=a,b,1,\ldots} dy_j \tilde{f}_j(y_j;\mu) \;\; d\tilde{\sigma}(p_a, p_b \to \{p_i\};\mu)$$

$$p_{a/b} = x_{a/b} P_{a/b} \quad P_i = x_i p_i$$

- $d\sigma$ does not depend on μ, while the $\tilde{f}_j$ and $\tilde{\sigma}$ do
- μ may be put equal to renormalization scale
- dependence of $\tilde{f}_j$ on μ is known, so QCD evolution can be applied to them
- $d\tilde{\sigma}$ may be sensitive logarithms of ratios of various scales in the process, which are remnants from the cancellations
- these may need to be resummed to all orders in the coupling in certain kinematical regions (*eg* via parton shower)
- higher fixed order terms may be needed to reduce scale dependence

Numerical evaluation of amplitudes

- performing a calculation means obtaining a number, or a plot
- for scattering cross sections, this implies performing phase space integrals

$$\hat{\sigma}_{a,b\to n} = \int d\Phi(p_a, p_b \to \{p\}_n) \left|\mathcal{M}_{a,b\to n}(p_a, p_b \to \{p\}_n)\right|^2 \mathcal{O}(p_a, p_b, \{p\}_n)$$

  - over several dimensions
  - are restricted by even more physical cuts

$\Longrightarrow$ The phase space integrals have to be performed numerically.
$\Longrightarrow$ $\left|\mathcal{M}_{a,b\to n}(p_a, p_b \to \{p\}_n)\right|^2$ has to be efficiently evaluated numerically.

- the evaluation of $\left|\mathcal{M}_{a,b\to n}(p_a, p_b \to \{p\}_n)\right|^2$ involves a sum over helicities

$$\left|\mathcal{M}_{a,b\to n}(p_a, p_b \to \{p\}_n)\right|^2 = \sum_{\lambda_a, \lambda_b, \{\lambda\}_n} \left|\mathcal{M}_{a,b\to n}(p_a, \lambda_a, p_b, \lambda_b \to \{p, \lambda\}_n)\right|^2$$

Perform this sum explicitly
$\Longrightarrow$ $\mathcal{M}_{a,b\to n}(p_a, \lambda_a, p_b, \lambda_b \to \{p, \lambda\}_n)$ has to be efficiently evaluated numerically.

From expressions to algorithms

For specific tree-level helicity amplitudes, very elegant and compact expressions exist.

$$\mathcal{M}_{ng}(\{p,\lambda,a\}_n) = i\,g^{n-2}\sum_{\text{perm}}\mathrm{Tr}\big(T^{a_1}T^{a_2}\cdots T^{a_n}\big)\mathcal{A}_{ng}\big(p_1^{\lambda_1},p_2^{\lambda_2},\ldots,p_n^{\lambda_n}\big)$$

$$\mathcal{A}_{ng}(p_1^-,p_j^-,p_{\text{rest}}^+) = \frac{\langle p_1\,p_j\rangle^4}{\langle p_1\,p_2\rangle\langle p_2\,p_3\rangle\cdots\langle p_{n-1}\,p_n\rangle\langle p_n\,p_1\rangle}$$

The expressions can be proven with the Berends-Giele (1988) recursive relations

$$J^\mu_{i.j} = \frac{-i}{p_{i,j}^2}\left[\sum_{k=i}^{j-1}V^\mu_{\nu\sigma}(p_{i,k},p_{k+1,j})\,J^\nu_{i,k}\,J^\sigma_{k+1,j} + \sum_{k=i}^{j-2}\sum_{l=k+1}^{j-1}W^\mu_{\nu\sigma\rho}\,J^\nu_{i,k}\,J^\sigma_{k+1,l}\,J^\rho_{l+1,j}\right]$$

$$p_{i,j} = p_i + p_{i+1} + \ldots + p_j \qquad J^\mu_{i,j} = J^\mu\big(p_i^{\lambda_i},p_{i+1}^{\lambda_{i+1}},\ldots,p_j^{\lambda_j}\big)$$

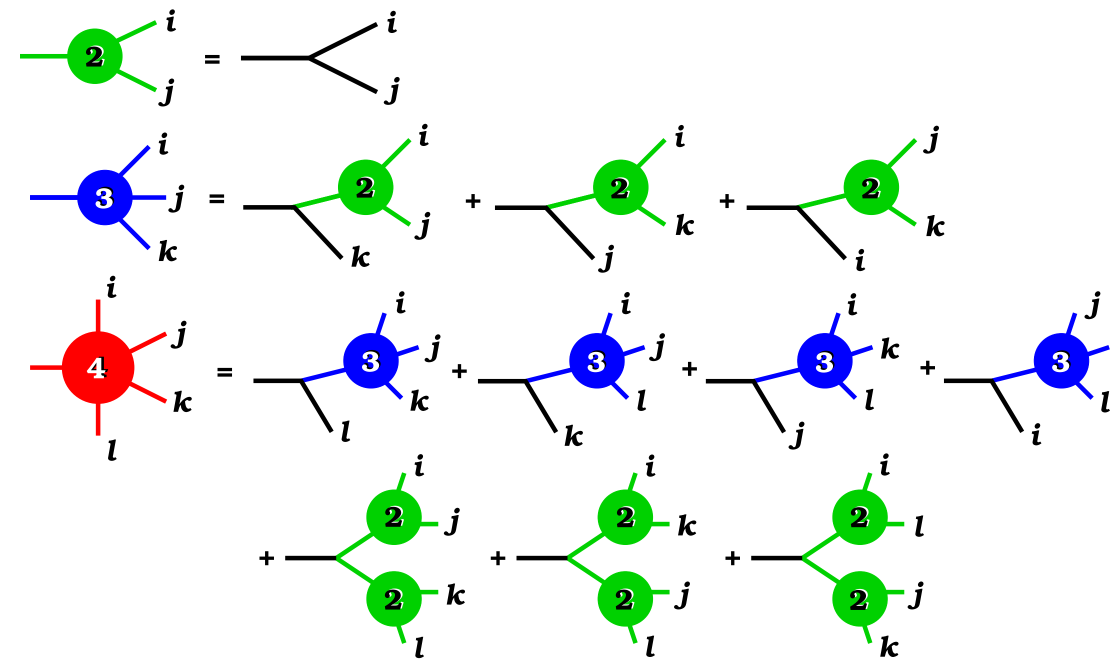
$$V^\mu_{\nu\sigma}(p,q) = \frac{ig^2}{\sqrt{2}}[(p-q)^\mu g_{\nu\sigma} + 2g^\mu_\sigma q_\nu - 2g^\mu_\nu p_\sigma] \quad W^\mu_{\nu\sigma\rho} = \frac{ig^2}{2}[2g^\mu_\sigma g_{\nu\rho} - g^\mu_\nu g_{\sigma\rho} - g^\mu_\sigma g_{\nu\rho}]$$

These relations are numerically very efficient, and can be used instead of the expression to evaluate the amplitude.

Dyson–Schwinger at tree–level

Algorithm can be generalized for arbitrary field theories (Caravaglios, Moretti 1995)

- completely numerical
- avoids Feynman graphs
- asymptotic computational complexity exponential instead of factorial



multi-gluon amplitudes Draggiotis, Kleiss, Papadopoulos 1998

multi-jet processes Caravaglios, Mangano, Moretti, Pittau 1998

ALPGEN Mangano, Moretti, Piccinini, Pittau, Polosa 2002

HELAC-PHEGAS Kanaki, Papadopoulos 2000 Cafarella, Papadopoulos, Worek 2007

O'MEGA-WHIZARD Moretti, Ohl, Reuter 2001 Kilian, Ohl, Reuter 2007

COMIX Gleisberg, Hoeche 2008

CAMORRA Kleiss, van den Oord 2011

Using Feynman graphs:

COMPHEP 1989-2009 Boos, Bunichev, Dubinin, Dudko, Edneral, Ilyin, Kryukov, Savrin, Semenov, Sherstnev

MADGRAPH Stelzer, Long 1994

MADEVENT Maltoni, Stelzer 2002

AMEGIC++ Krauss, Kuhn, Soff 2001

CARLOMAT Kolodziej 2007

Also non-trivial phase space integration

Multi–jet processes

- LO-only calculations can only provide inclusive observables, which depend strongly on the scale of phase space cuts, up to IR disasters for small scale values.
- LL-only parton shower programs cannot accurately describe processes with many hard jets.

Improve both by merging parton showers and exact tree-level matrix elements

1. generate parton-level n-jet event from exact matrix element.
   n is chosen following the parton-level total cross sections
2. apply jet-algorithm repeatedly, to associate a branching tree with the parton-level momenta and obtain splitting scales

3. apply rejection on event with probabilities including effects of running coupling constants and Sudakov form factors
4. shower from each of the n partons, reject events that produce more than n jets

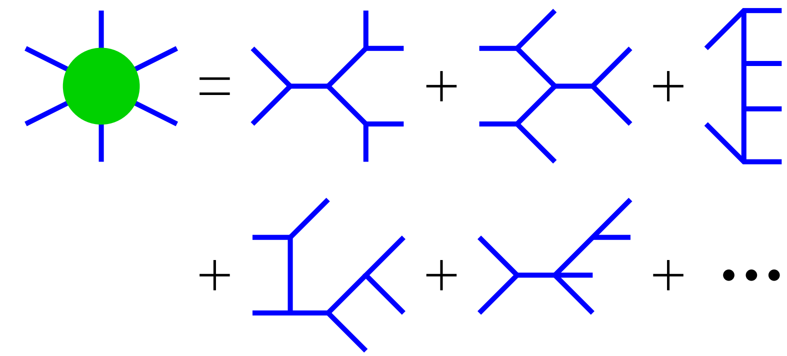
Catani, Krauss, Kuhn, Webber 2001

3. apply rejection on event with probabilities including effects of running coupling constants
4. shower from each of the n partons, reject events that produce more than n jets, and only accept event when each jet can be identified with a parton

M.L. Mangano

Multi–jet processes at NLO QCD

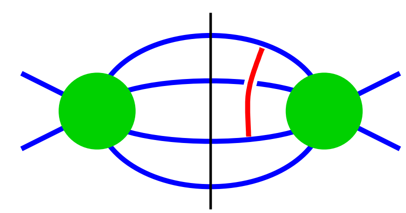
- Reduce dependence on unphysical scales. Reduce uncertainty on normalization to around 20%.
- Shape of distributions changes, there is no global K-factor.
- Include structure of jets.



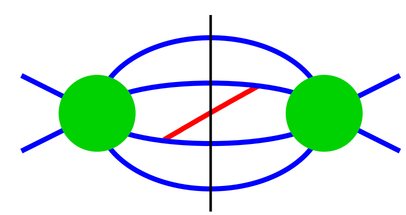
Each term represents a formula with a tree-topology.

Next-to-Leading Order cross section:

$$\hat{\sigma}^{\mathrm{NLO}}_{a,b\to n} = \int d\Phi_n\, 2\Re\Big(\mathcal{M}^{(0)}_{a,b\to n}\,\mathcal{M}^{(1)}_{a,b\to n}\Big)\,\mathcal{O}^{\mathrm{LO}}_n$$



$$+ \int d\Phi_{n+1}\,\big|\mathcal{M}^{(0)}_{a,b\to n+1}\big|^2\,\mathcal{O}^{\mathrm{NLO}}_{n+1}$$



- The one-loop amplitude $\mathcal{M}^{(1)}_{a,b\to n}$ is much more complicated than $\mathcal{M}^{(0)}_{a,b\to n}$.
- Both $\mathcal{M}^{(1)}_{a,b\to n}$ and the integral over Φ_{n+1} are IR divergent.

$\Longrightarrow$ the formula above cannot straightforwardly be attacked numerically

Multi–jet processes at NLO QCD

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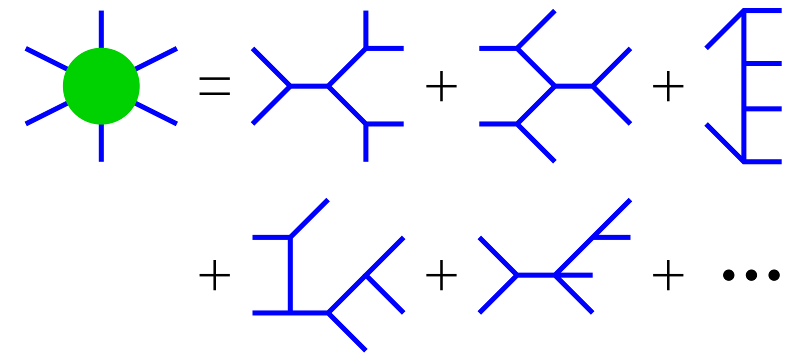


Each term represents a formula with a tree-topology.

Next-to-Leading Order cross section:

$$\hat{\sigma}^{\mathrm{NLO}}_{a,b\to n} = \int d\Phi_n \, 2\Re\left(\mathcal{M}^{(0)}_{a,b\to n}\,\mathcal{M}^{(1)}_{a,b\to n}\right)\mathcal{O}^{\mathrm{LO}}_n$$

$$+ \int d\Phi_{n+1} \left|\mathcal{M}^{(0)}_{a,b\to n+1}\right|^2 \mathcal{O}^{\mathrm{NLO}}_{n+1}$$

$$\hat{\sigma}^{\mathrm{NLO}}_{a,b\to n} = \int d\Phi_n \left[2\Re\left(\mathcal{M}^{(0)}_{a,b\to n}\,\mathcal{M}^{(1)}_{a,b\to n}\right) + \int d\Phi_1 \, D_{n+1}\right]\mathcal{O}^{\mathrm{LO}}_n$$
$$+ \int d\Phi_{n+1} \left[\left|\mathcal{M}^{(0)}_{a,b\to n+1}\right|^2 \mathcal{O}^{\mathrm{NLO}}_{n+1} - D_{n+1}\,\mathcal{O}^{\mathrm{LO}}_n\right]$$

Subtraction methods

Construct a list of

- phase space restricting functions $\theta^{(j)}_{n+1}$
- phase space mappings $T^{(j)}_{n\to n+1}$ and $T^{(j)}_{n\leftarrow n+1}$
- subtraction functions $D^{(j)}_{n+1}$

such that

$$\int d\Phi_{n+1}\left[\left|\mathcal{M}^{(0)}_{n+1}\right|^2 \mathcal{O}^{\mathrm{NLO}}_{n+1} - \sum_j D^{(j)}_{n+1}\,\theta^{(j)}_{n+1}\,\mathcal{O}^{\mathrm{LO}}_{n} \circ T^{(j)}_{n\leftarrow n+1}\right]$$

can be integrated numerically, and

$$\int d\phi_1\, D^{(j)}_{n+1} \circ T^{(j)}_{n\to n+1}\,\theta^{(j)}_{n+1} \circ T^{(j)}_{n\to n+1}\,\mathcal{O}^{\mathrm{LO}}_{n}$$

can be performed analytically, within dimensional regularization.

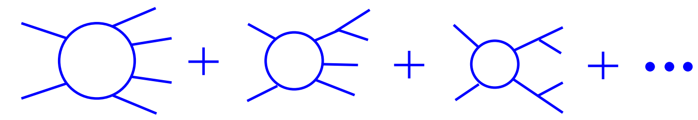
Catani, Seymour 1996:

- $\theta^{(j)}_{n+1} = 1$
- mapping for $2 \leftarrow 3$ partons
- IR singularities spread over many terms

Frixione, Kunszt, Signer 1995:

- $\sum_j \theta^{(j)}_{n+1} = 1$
- $\theta^{(j)}_{n+1}$ contains one, and vanishes at any other, collinear singularity
- $\theta^{(j)}_{n+1}$ contributes to one soft singularity

One–loop amplitudes



perform loop integral numerically

subtraction

Nagy Soper 2003
Becker Reuschle Weinzierl 2010

expand amplitude in terms of universal integrated one–loop functions

tensor integrals

Passarino Veltman 1978 Denner Dittmaier 2005
Binoth Guillet Heinrich 1999
del Aguila Pittau 2004 Giele Glover 2004
Diakonidis Fleischer Riemann Tausk 2009

construct expression for coefficients

"traditional approach"

evaluate coefficients numerically
via recursion

AvH 2009 Cascioli Maierhofer Pozzorini 2011
Actis Denner Hofer Scharf Uccirati 2012

scalar integrals

construct expression for coefficients
(unitarity)

Bern Dixon Dunbar Kosower 1994
Britto Cachazo Feng 2004
Anastasiou Britto Feng Kunszt Mastrolia 2006
Forde 2007 Badger 2008

evaluate coefficients numerically
by solving linear system (OPP)

Ossola Papadopoulos Pittau 2007
Ellis Giele Kunszt Melnikov 2008

Recent multi–leg NLO calculations

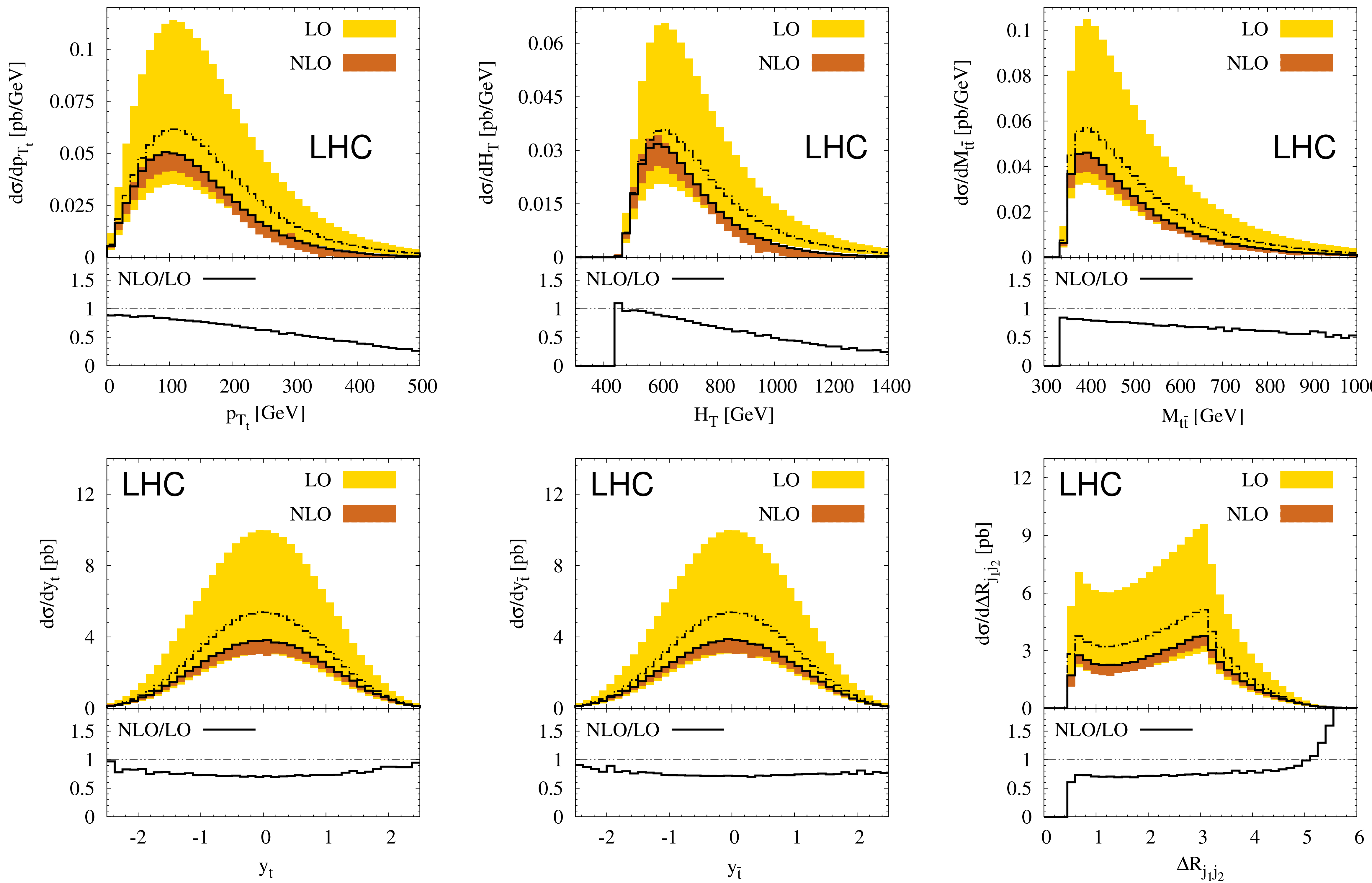
proton-proton scattering, involving at least one-loop six-point functions:

$t\bar{t}b\bar{b}$	Bredenstein, Denner, Dittmaier, Pozzorini 2008, 2010
$t\bar{t}b\bar{b}$	Bevilacqua, Czakon, Papadopoulos, Pittau, Worek 2009
$Wjjj$	Ellis, Melnikov, Zanderighi 2009
$Wjjj$	Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre 2009
$b\bar{b}b\bar{b}$	Binoth, Greiner, Guffanti, Reuter, Guillet, Reiter 2010
W^+W^+jj	Melia, Melnikov, Rontsch, Zanderighi 2010
$Wjjjj$	Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre 2011
$W^+W^-b\bar{b}$	Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek 2011
$W^+W^-b\bar{b}$	Denner, Dittmaier, Kallweit, Pozzorini 2011, 2012
W^+W^-jj	Melia, Melnikow, Rontsch, Zanderighi 2011
$t\bar{t}jj$	Bevilacqua, Czakon, Papadopoulos, Worek 2011
$Zjjjj$	Ita, Bern, Dixon, Febres Cordero, Kosower, Maitre 2012
$jjjj$	Bern, Diana, Dixon, Febres Cordero, Hoeche, Kosower, Ita, Maitre, Ozeren 2012
$jjjj$	Badger, Biedermann, Uwer, Yundin 2012
$t\bar{t}t\bar{t}$	Bevilacqua, Worek 2012

$e^+e^- \to 7j$ Becker, Goetz, Reuschle, Schwan, Weinzierl 2012

$pp(p\bar{p}) \to t\bar{t}\,jj$ at NLO QCD

Bevilacqua, Czakon, Papadopoulos, Worek



Programs for NLO calculations

Complete packages:

MCFM Campbell, Ellis, Williams
NLOJet++ Nagy
VBFNLO Arnold, Bellm, Bozzi, Brieg, Campanario, Englert, Feigl, Frank, Figy, Geyer, Hackstein, Hankele, Jäger, Kerner, Kubocz, Oleari, Palmer, Plätzer, Rauch, Rzehak, Schissler, Schlimpert, Spannowsky, Worek, Zeppenfeld
Helac-NLO Bevilacqua, Czakon, Garzelli, van Hameren, Kardos, Papadopoulos, Pittau, Worek
GRACE Bredenstein, Fujimoto, Igarashi, Ishikawa, Jimbo, Kaneko, Kato, Kawabata, Kawamura, Kon, Kurihara, Kuroda, Lafage, Munehisa, Nakazawa, Sasaki, Shimizu, Tanaka, Tobimatsu, Uematsu, Watanabe, Yuasa, Yasui
BlackHat/SHERPA Berger, Bern, Diana, Dixon, Febres Cordero, Forde, Hoeche, Gleisberg, Ita, Kosower, Maitre, Ozeren
MadGraph Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau, Stelzer

One-loop amplitudes:

FormCalc Agrawal, Hahn, Mirabella, Perez-Victoria
GoSam Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano
Ngluon Badger, Biederman, Uwer

Matching to parton shower

following Hoeche, Krauss, Schonnher, Siegert 2012

$$\mathcal{O}^{\mathrm{LO+NLO}} = \int d\Phi_n \left[B_n + V_n + \sum_j I_S^{(j)} + \sum_j \int d\Phi_1 \left(A_{n+1}^{(j)} \circ T_{n\to n+1}^{(j)} - S_{n+1}^{(j)} \circ T_{n\to n+1}^{(j)} \right) \right] \mathcal{O}_n^{\mathrm{LO}}$$
$$+ \int d\Phi_{n+1} \sum_j A_{n+1}^{(j)} \left[\mathcal{O}_{n+1}^{\mathrm{NLO}} - \mathcal{O}_n^{\mathrm{LO}} \circ T_{n\leftarrow n+1}^{(j)} \right]$$
$$+ \int d\Phi_{n+1} \left[R_{n+1} - \sum_j A_{n+1}^{(j)} \right] \mathcal{O}_{n+1}^{\mathrm{NLO}}$$

NLO matching: replace first two lines by

$$\int d\Phi_n \bar{B}_n \left[\Delta^{(A)}(t_0)\mathcal{O}_n^{\mathrm{LO}} + \sum_j \int_{t_0} d\phi_1 \frac{A_{n+1}^{(j)} \circ T_{n\to n+1}^{(j)}}{B_n} \Delta^{(A)}(t)\, \theta(t(\Phi_1) - t_0)\, \mathcal{O}_{n+1}^{\mathrm{NLO}} \circ T_{n\to n+1}^{(j)} \right]$$

- generate event following first line or third line
- just keep event if third line is chosen (real-emission kinematics)
- if first line is chosen (Born-like kinematics) perform one-step Sudakov branching.

Matching to parton shower

Methods:

MC@NLO Frixione, Webber 2002

POWHEG Nason 2004, Frixione, Nason, Oleari 2007

Programs:

POWHEG BOX Alioli, Nason, Oleari, Re 2010

MC@NLO IN SHERPA Hoeche, Krauss, Schonherr, Siegert 2012

aMC@NLO Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli 2012

Recent calculations:

$pp \to t\bar{t}j$ Kardos, Papadopoulos, Trocsanyi 2012

$pp \to t\bar{t}Z$ Garzelli, Kardos, Papadopoulos, Trocsanyi 2012

$pp \to Wjj$ Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli 2012

$gg \to Hjj$ Campbell, ELlis, Frederix, Nason, Oleari, Williams 2012

$pp \to Zjj$ Jäger, Schneider, Zanderighi 2012

$pp \to Wjjj$ Hoeche, Krauss, Schonherr, Siegert 2012

Matching and merging:

Gehrmann Hoeche, Krauss, Schonherr, Siegert 2012

Frederix, Frixione 2012

Full NLO parton shower Jadach, Kusina, Placzek, Skrzypek, Slawinska 2011