Precision Corrections to the Weak Radiative B Decay

Michał Poradziński

University of Warsaw

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Outline

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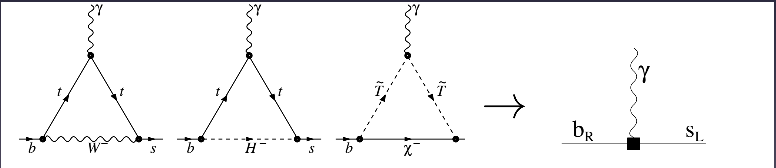
- Tree-level $s\gamma q\bar{q}$ final state contributions
- NNLO BLM corrections
- Summary

Inclusive $\bar{B} \to X_s\gamma$

In inclusive $\bar{B} \to X_s\gamma$, X_s stands for any hadronic state with strangeness equal to -1 and without charmed hadrons.

$b \in \bar{B} \equiv (\bar{B}^0 or B^-)$
Weak radiative decay of the B meson is induced at one loop level. Some new particles may be circulating in the loops.

Information about the electroweak interactions is encoded in low-energy effective theory couplings.



The current experimental world average is

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{exp}^{E_0>1.6GeV} = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}\,.$$

The Standard Model prediction for the inclusive branching ratio reads

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{SM}^{E_0>1.6GeV} = (3.15 \pm 0.23) \times 10^{-4}\,.$$

- Agreement at the 1.2σ level.
- Both uncertainties are at the $\pm$**7**$\%$ level.

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Thus, there is not much room left for new effects. Therefore: $\bar{B} \to X_s\gamma$ **is used to put constraints on New Physics rather than to find deviations from the SM.**

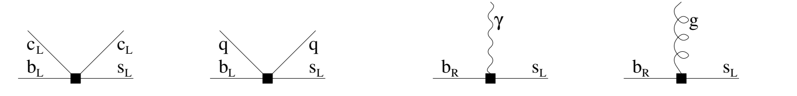
Calculations of the inclusive rate are based on the relation

$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^p\gamma)_{E_\gamma > E_0} + \left[\begin{array}{c} \text{non-perturbative} \\ \text{effects} \sim 5\% \end{array} \right]$$

where

- $\Gamma(b \to X_s^p\gamma)$ is evaluated perturbatively,
- X_s^p stands for s, sg, sgg, $sq\bar{q}$, etc.

Information on the electroweak-scale physics in the $b \to s\gamma$ transition is encoded in effective low-energy interactions after integrating out $W^{\pm}$, Z and any other heavy particles:



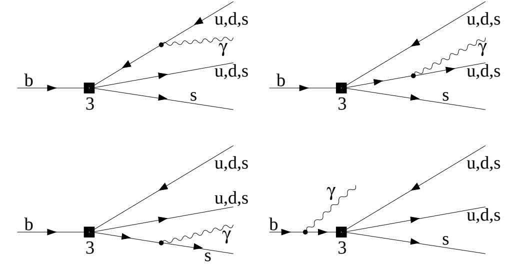
$Q_{1,2}$ $\quad Q_{3,4,5,6}$ $\quad Q_7$ $\quad Q_8$

Instead of explicit electroweak interactions, dimension-five and -six local flavour-changing operators arise

$$Q_{1,2} = (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b)\,, \qquad Q_{3,4,5,6} = (\bar{s}\Gamma_i b)\sum\nolimits_q(\bar{q}\Gamma'_i q)\,,$$
$$Q_7 = \frac{em_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}\,, \qquad Q_8 = \frac{gm_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}b_R G_{\mu\nu}\,.$$

Q_7 – Q_7 and Q_7 – $Q_{1,2}$ interferences give dominant contributions.

Tree-level $s\gamma q\bar{q}$ final state contributions



Presence of the collinear logarithms $\ln \frac{m_b^2}{m_q^2}$, where m_q is the light quark mass.
Power corrections $\left(\frac{m_q^2}{m_b^2}\right)^n$ are neglected.

Difficulties with the tree-level calculation:

- collinear logarithms
- integration over the 4-particle partly massive phase space

An example of a function present in the final result

$$\begin{aligned} T_1(\delta) &= \left(-\frac{5}{3}\delta - \frac{1}{3}\delta^2 + \frac{4}{9}\delta^3 - \frac{1}{2}\delta^4 - \frac{5}{3}\ln(1-\delta)\right)\ln\frac{\delta\, m_b^2}{m_q^2} \\ &+ \frac{109}{18}\delta + \frac{17}{18}\delta^2 - \frac{191}{108}\delta^3 + \frac{23}{16}\delta^4 + \frac{79}{18}\ln(1-\delta) - \frac{5}{3}\mathrm{Li}_2(\delta)\,, \end{aligned}$$

where $E^{\gamma}_{min} \equiv \frac{m_b}{2}(1-\delta)$.

The Wilson coefficients $C_1 - C_8$ play the role of coupling constants at the effective interactions $Q_1 - Q_8$

$$\mathcal{L}_{\text{weak}} \sim \sum_i C_i(\mu) Q_i,$$

Their evaluation at $\mu = \mu_b \sim m_b/2$ up to $O(\alpha_s^2)$ was completed a few years ago
The partonic decay rate is evaluated according to the formula

$$\Gamma(b \to X_s^p \gamma)_{E_\gamma > E_0} = N \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b),$$

where $N = |V_{ts}^\star V_{tb}|^2 \, (G_F^2 m_b^5 \alpha_{\text{em}})/(32\pi^4)$.

$$G_{ij} = G_{ij}^{(0)} + \alpha_s G_{ij}^{(1)} + \alpha_s^2 G_{ij}^{(2)} + ...$$

The LO contribution $G_{ij}^{(0)}$ and the NLO one $G_{ij}^{(1)}$ are known since a long time.
At the NNLO, it is sufficient to restrict our attention to $i, j \in \{1, 2, 7, 8\}$ because the penguin operators have very small Wilson coefficients ($|C_{3,5,6}(\mu_b)| < |C_4(\mu_b)| \sim \alpha_s(\mu_b)/\pi$).

If we treat the two similar operators Q_1 and Q_2 as a single one (represented by Q_2), there are six independent cases of the NNLO contributions to $G_{ij}^{(2)}$

- Q_7 : $G_{77}^{(2)}, G_{78}^{(2)}, G_{27}^{(2)}$
- remaining: $G_{22}^{(2)}, G_{28}^{(2)}, G_{88}^{(2)}$.

Contributions involving Q_7:

- $G_{77}^{(2)}$ was found several years ago
- $G_{78}^{(2)}$ was finalized recently, see arXiv:1005.5587 & arXiv:0805.3911
- $G_{27}^{(2)}$ is currently being calculated by M. Czakon, R. N. Lee, M. Misiak, A. V. Smirnov, V. A. Smirnov and M. Steinhauser

For the remaining cases ($G_{22}^{(2)}, G_{28}^{(2)}$ and $G_{88}^{(2)}$), contributions from the two-body final states are known, while the $(n \geq 3)$-body final states give small effects.

NNLO BLM corrections

In the Brodsky-Lepage-Mackenzie (BLM) approximation, we split $G_{ij}^{(2)}$ into the β_0-parts $G_{ij}^{(2)\beta_0}$ and the remaining parts $G_{ij}^{(2)\mathrm{rem}}$

$$G_{ij}^{(2)} \;=\; A_{ij}\, n_l + B_{ij} \;=\; G_{ij}^{(2)\beta_0} + G_{ij}^{(2)\mathrm{rem}},$$

where n_l stands for the number of massless flavours in the effective theory and

$$G_{ij}^{(2)\beta_0} \equiv -\frac{3}{2}\beta_0 A_{ij} = -\frac{3}{2}\left(11 - \frac{2}{3}(n_l + 2)\right) A_{ij},$$

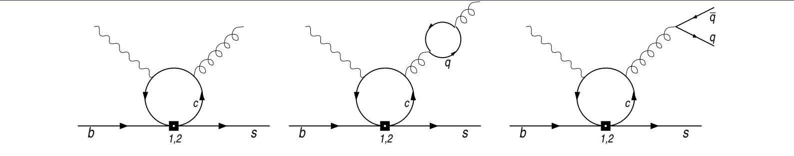
$$G_{ij}^{(2)\mathrm{rem}} \equiv \frac{33}{2} A_{ij} + B_{ij}.$$

The BLM parts are calculated first because:

- calculation of the BLM corrections is usually simpler to perform than of the non-BLM ones
- experience shows that the BLM parts give dominant contributions to complete corrections. Hence, they can be used to estimate them.

At the order $\mathcal{O}(\alpha_s^2)$, we found the BLM contributions $G_{28}^{(2)\beta_0}$, $G_{27}^{(2)\beta_0}$ and $G_{22}^{(2)\beta_0}$ [M. Misiak, M. Poradziński, Phys. Rev. D83 (2011) 014024] .

The calculation of $G_{28}^{(2)\beta_0}$ has been performed for the first time, while the other cases are confirmations of the already known results.



Diagrams with $Q_{1,2}$ contributing to $G_{28}^{(2)\beta_0}$, $G_{27}^{(2)\beta_0}$ and $G_{22}^{(2)\beta_0}$.

We followed the method introduced by Smith and Voloshin in 1994. The two-loop calculation is reduced to a one-loop calculation but with an off-shell gluon. Integration over the gluon q^2 is performed afterwards according to

$$\Gamma^{\alpha_s^2}_{\beta_0} = -\frac{\alpha_s}{4\pi}\beta_0 \left[\int_0^\delta \frac{\Gamma^{\alpha_s}(v^2) - \Gamma^{\alpha_s}(0)}{v^2} dv^2 + \Gamma^{\alpha_s}(0) \left(\ln \frac{\delta m_b^2}{\mu^2} - \frac{5}{3} \right) \right]$$

where $v^2 = q^2/m_b^2$, q^μ is the gluon four-momentum, μ is the renormalization scale and $\delta = 1 - \frac{2E_0}{m_b}$ parametrizes the photon energy cut.

The result for $G_{88}^{(2)\beta_0}$ reads:

$$G_{88}^{(2)\beta_0} = 4\beta_0 \left[\phi_{88}^{(1)}(\delta) \ln \frac{\mu_b^2}{m_b^2} + h_{88}^{(2)}(\delta) \right],$$

$$\begin{aligned} h_{88}^{(2)}(\delta) &= \tfrac{4}{27} \Big\{ \Big[\big(1 + \tfrac{1}{2}\delta\big)\, \delta \ln \delta - 6 \ln(1-\delta) - 2\mathrm{Li}_2(1-\delta) \\ &+ \tfrac{1}{3}\pi^2 - \tfrac{16}{3}\delta - \tfrac{5}{3}\delta^2 + \tfrac{1}{9}\delta^3 \Big] \ln \frac{m_b}{m_s} - 2\mathrm{Li}_3(\delta) \\ &+ (5 - 2\ln\delta) \left[\mathrm{Li}_2(1-\delta) - \tfrac{1}{6}\pi^2 \right] - \tfrac{1}{12}\pi^2 \delta\, (2+\delta) \\ &+ \left[\tfrac{1}{2}\delta + \tfrac{1}{4}\delta^2 - \ln(1-\delta) \right] \ln^2 \delta + \big(\tfrac{151}{18} - \tfrac{1}{3}\pi^2 \big) \times \\ &\times \ln(1-\delta) + \big(-\tfrac{53}{12} - \tfrac{19}{12}\delta + \tfrac{2}{9}\delta^2 \big)\, \delta \ln \delta \\ &+ \tfrac{787}{72}\delta + \tfrac{227}{72}\delta^2 - \tfrac{41}{72}\delta^3 \Big\}. \end{aligned}$$

Summary

- Radiative $B \to X_s \gamma$ decay provides constraints on the SM extensions
- NLO i NNLO calculations are almost complete
- There is an urge to reduce theory uncertainty to meet the experimental accuracy