

Time-dependent CPV and mixing at B-factories

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KM unitarity triangle and CPV parameter convention

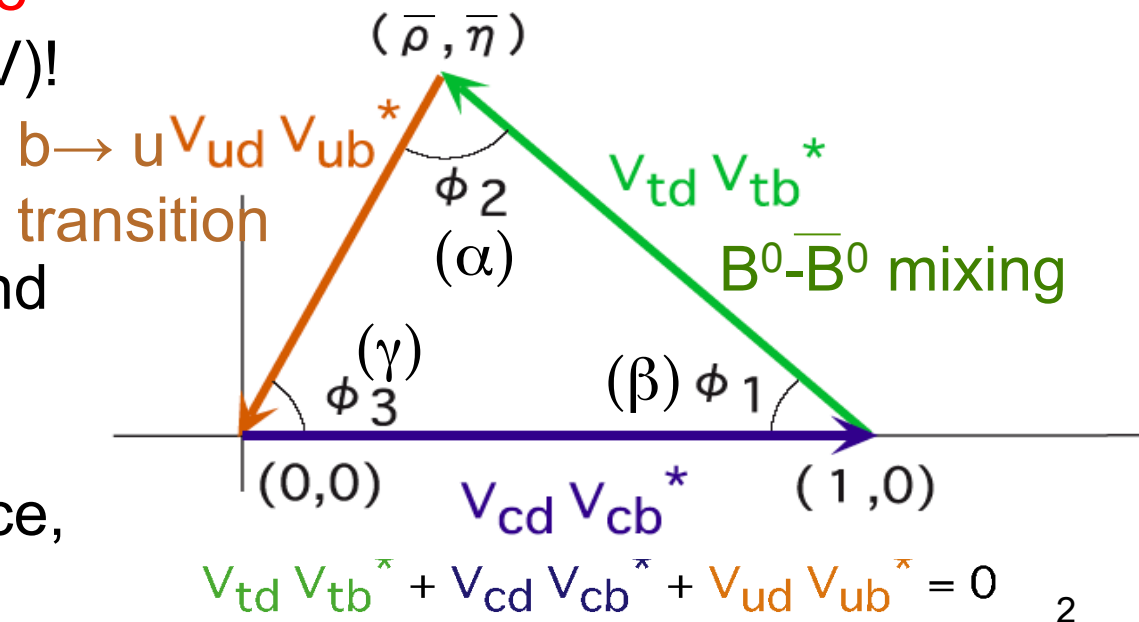
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

by Wolfenstein parametrization

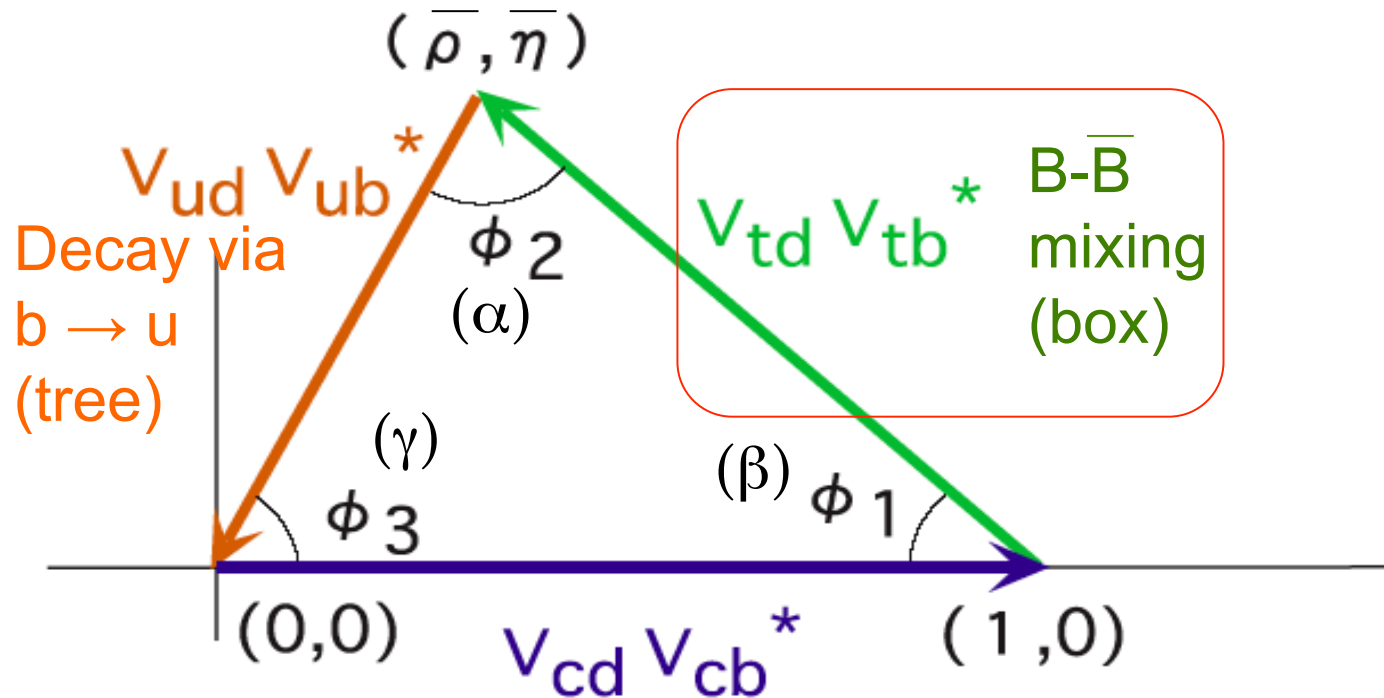
Irreducible complex phase
causes CP Violation (CPV)!

Comprehensive test;
measure all the angles and
sides.

B system : very good place,
all the angles are $O(0.1)$!



Angle measurements and mixing



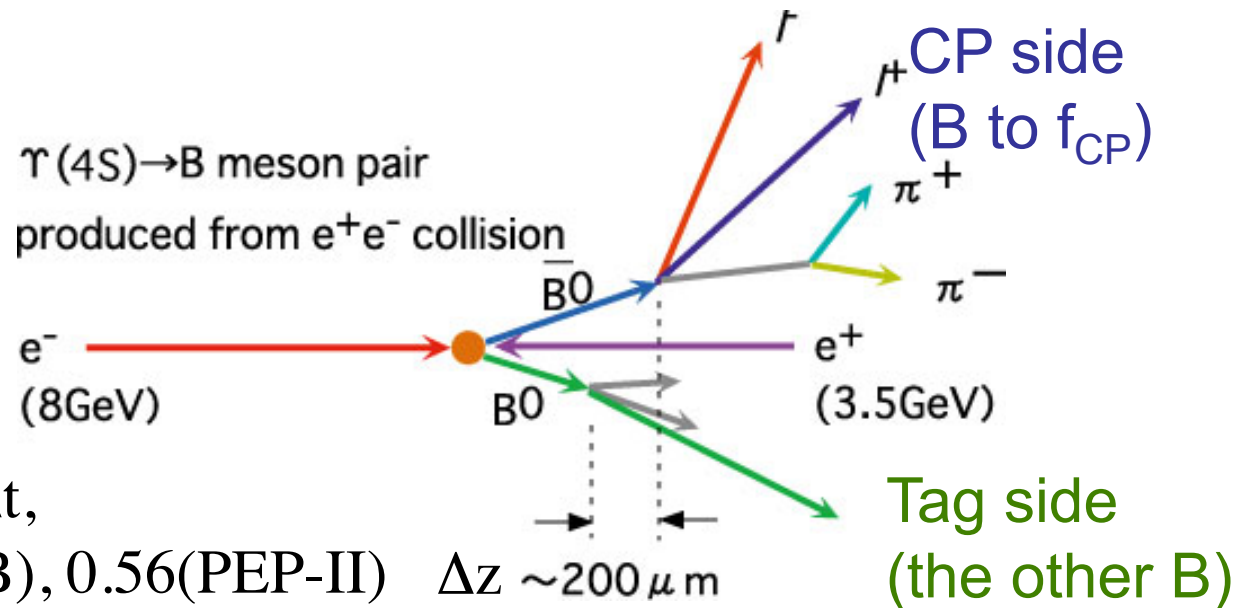
Decay via $b \rightarrow c$ (tree)

How about $b \rightarrow s$ (penguin)??

$b \rightarrow d$ (penguin) is also participating in some cases
 \rightarrow direct CPV.

Time-dependent CPV

In order to see CPV by interference between decay and mixing.



$$\Delta z = \beta \gamma c \Delta t,$$

$$\beta \gamma = 0.425 (\text{KEKB}), 0.56 (\text{PEP-II}) \quad \Delta z \sim 200 \mu m$$

$$A_{CP}(\Delta t) = \frac{\Gamma(\bar{B}^0(\Delta t) \rightarrow f_{CP}) - \Gamma(B^0(\Delta t) \rightarrow f_{CP})}{\Gamma(\bar{B}^0(\Delta t) \rightarrow f_{CP}) + \Gamma(B^0(\Delta t) \rightarrow f_{CP})} = S_{f_{CP}} \sin(\Delta m \Delta t) + A_{f_{CP}} \cos(\Delta m \Delta t)$$

$$S_{f_{CP}} = \frac{2 \operatorname{Im}(\lambda)}{|\lambda|^2 + 1}$$

$$A_{f_{CP}} = \frac{|\lambda|^2 - 1}{|\lambda|^2 + 1}$$

$$\lambda = \frac{q}{p} \frac{\bar{A}(f_{CP})}{A(f_{CP})}$$

$$-C_{f_{CP}} = A_{f_{CP}}$$

$$|\lambda| = 1 \text{ if no DCPV}$$

In order to perform such studies

	PDG2011	Charmonium modes	
Γ_{156}	$\eta_c K^0$	$(8.9 \pm 1.6) \times 10^{-4}$	
Γ_{157}	$\eta_c K^*(892)^0$	$(6.1 \pm 1.0) \times 10^{-4}$	
Γ_{158}	$\eta_c(2S) K^{*0}$	$< 3.9 \times 10^{-4}$	CL=90%
Γ_{159}	$h_c(1P) K^{*0}$	$< 4 \times 10^{-4}$	CL=90%
Γ_{160}	$J/\psi(1S) K^0$	$(8.71 \pm 0.32) \times 10^{-4}$	
Γ_{161}	$J/\psi(1S) K^+ \pi^-$	$(1.2 \pm 0.6) \times 10^{-3}$	
Γ_{162}	$J/\psi(1S) K^*(892)^0$	$(1.33 \pm 0.06) \times 10^{-3}$	
Γ_{163}	$J/\psi(1S) \eta K_S^0$	$(8 \pm 4) \times 10^{-5}$	
Γ_{164}	$J/\psi(1S) \eta' K_S^0$	$< 2.5 \times 10^{-5}$	CL=90%

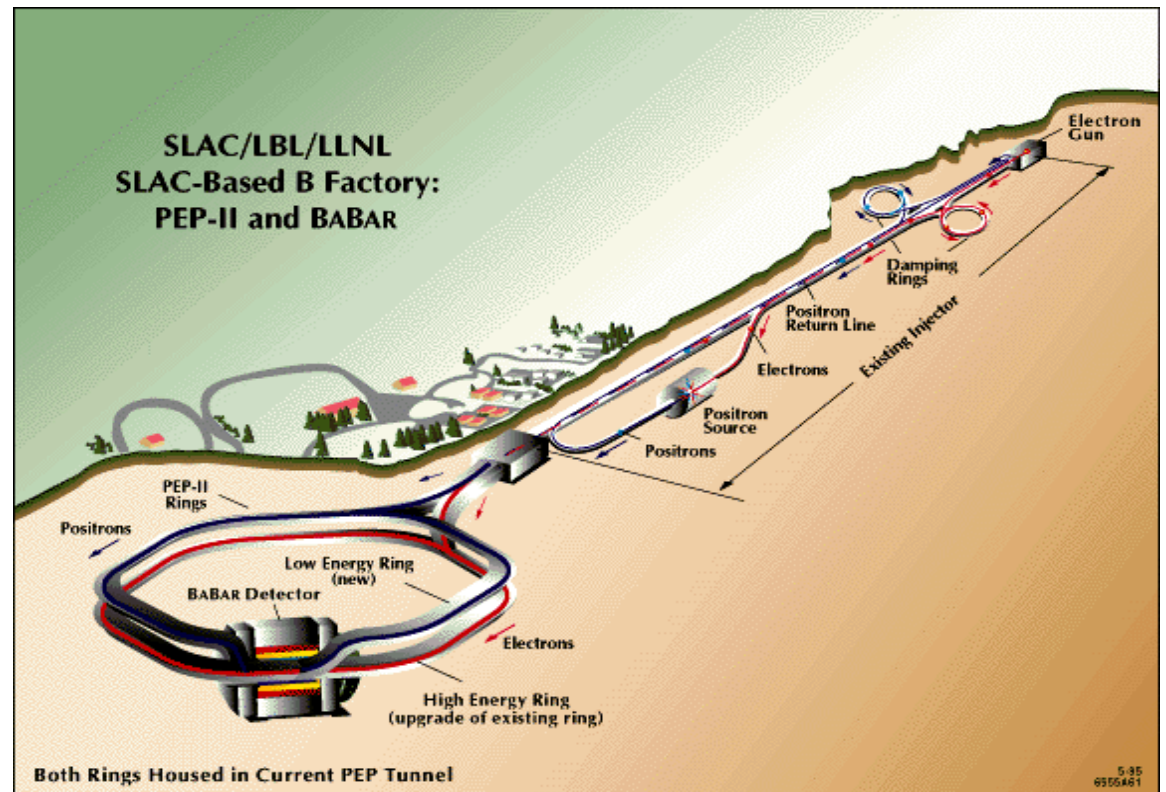
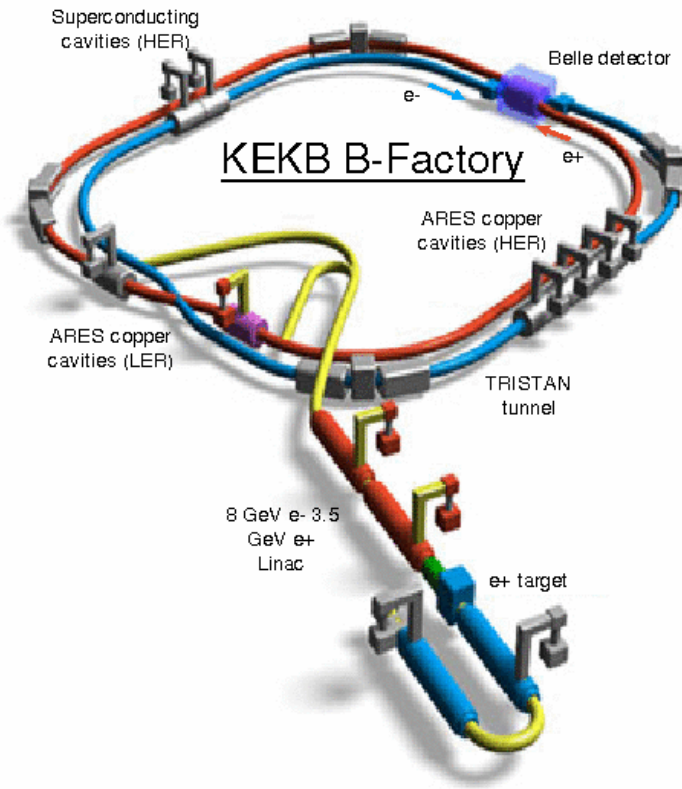
B meson is so heavy that many decay modes are available.

Branching fraction to the modes usable for CPV is limited.

→ Huge ($O(10^8)$) amount of B mesons is necessary.

→ Measurement of time evolution of B meson pair is required.

Two B-factories at KEK and SLAC



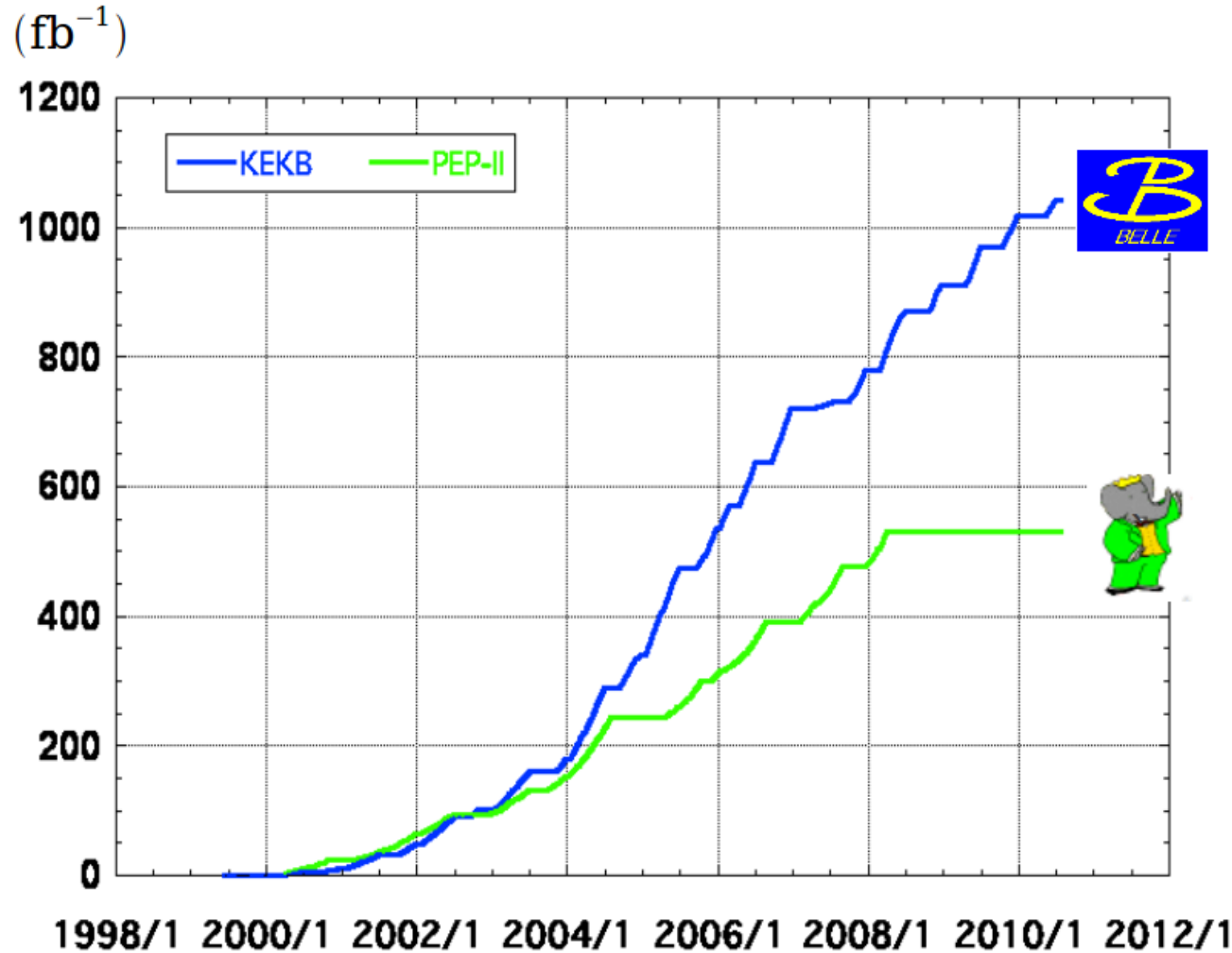
$$8\text{GeV}(e^-) \times 3.5\text{GeV}(e^+),$$

$$L_{\text{max}} = 2.1 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$$

$$9\text{GeV}(e^-) \times 3.1\text{GeV}(e^+),$$

$$L_{\text{max}} = 1.2 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$$

Integrated luminosity of B factories

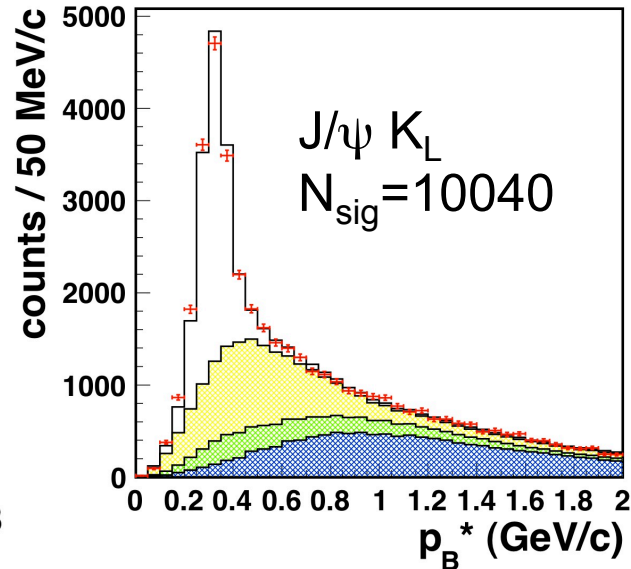
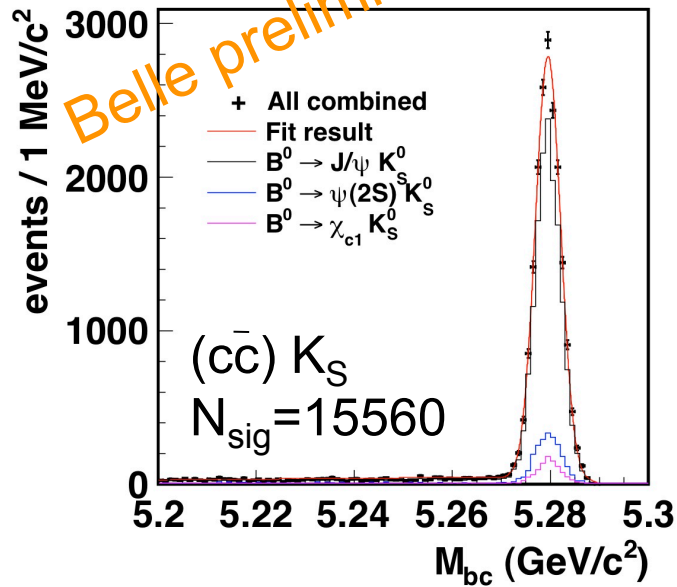


> 1 ab⁻¹
On resonance:
 $\Upsilon(5S)$: 121 fb^{-1}
 $\Upsilon(4S)$: 711 fb^{-1} 772M $\overline{B}B$
 $\Upsilon(3S)$: 3 fb^{-1}
 $\Upsilon(2S)$: 25 fb^{-1}
 $\Upsilon(1S)$: 6 fb^{-1}
Off reson./scan:
 $\sim 100 \text{ fb}^{-1}$

$\sim 550 \text{ fb}^{-1}$
On resonance:
 $\Upsilon(4S)$: 433 fb^{-1} 470M $\overline{B}B$
 $\Upsilon(3S)$: 30 fb^{-1}
 $\Upsilon(2S)$: 14 fb^{-1}
Off resonance:
 $\sim 54 \text{ fb}^{-1}$

sin2 ϕ_1 at Belle

(772M $B\bar{B}$, final sample)

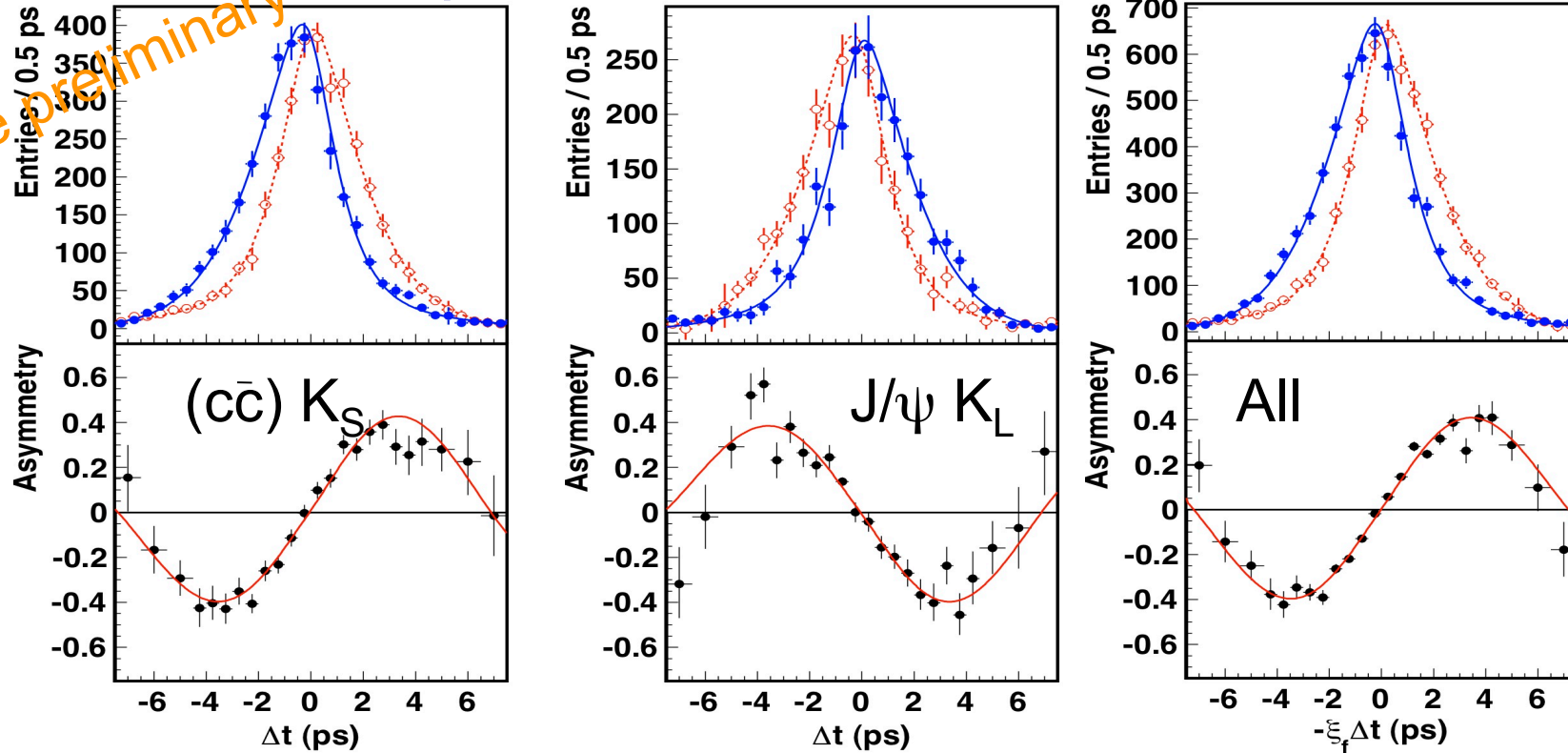


Signal yield increased more than $N_{B\bar{B}}$ compared to the previous publication (PRL98,031802), thanks to the data reprocessing with improved tracking.

	$J/\psi K_S$	$J/\psi K_L$	$\psi(2S) K_S$	$\chi_{c1} K_S$	$N_{B\bar{B}}$
N_{sig}	12727 ± 115	10087 ± 154	1981 ± 46	943 ± 33	772 M
Purity(%)	97	63	93	89	
N_{sig} (prev.)	7484 ± 87	6512 ± 123	N/A	N/A	535 M
Purity(%) (prev.)	97	59			

$\sin 2\phi_1$ at Belle (772M $B\bar{B}$)

Belle preliminary

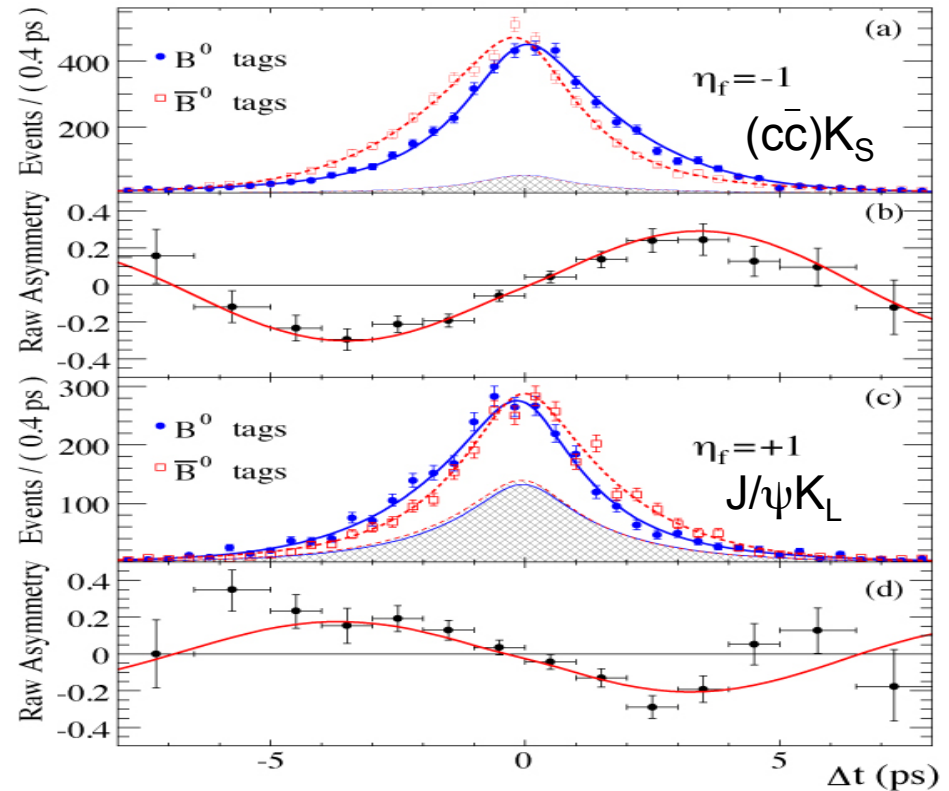
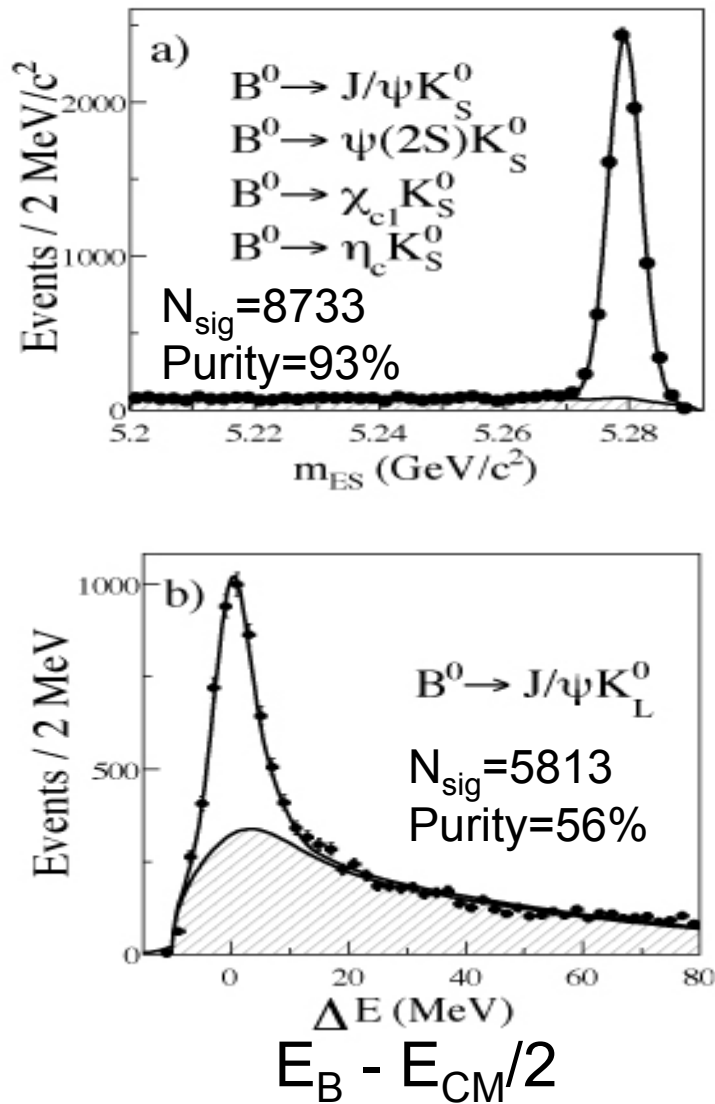


B decay mode	S_f	A_f
$J/\psi K_S^0$	0.671 ± 0.029	-0.014 ± 0.021
$\psi(2S) K_S^0$	0.739 ± 0.079	0.103 ± 0.055
$\chi_{c1} K_S^0$	0.636 ± 0.117	-0.023 ± 0.083
$J/\psi K_L^0$	-0.641 ± 0.047	0.019 ± 0.026

$$\sin 2\phi_1 = 0.668 \pm 0.023 \pm 0.013$$

$$A_{fCP} = 0.007 \pm 0.016 \pm 0.013$$

$\sin 2\phi_1 (= \sin 2\beta)$ at BaBar (465M $B\bar{B}$)



$$\sin 2\phi_1 = 0.687 \pm 0.028 \pm 0.012$$

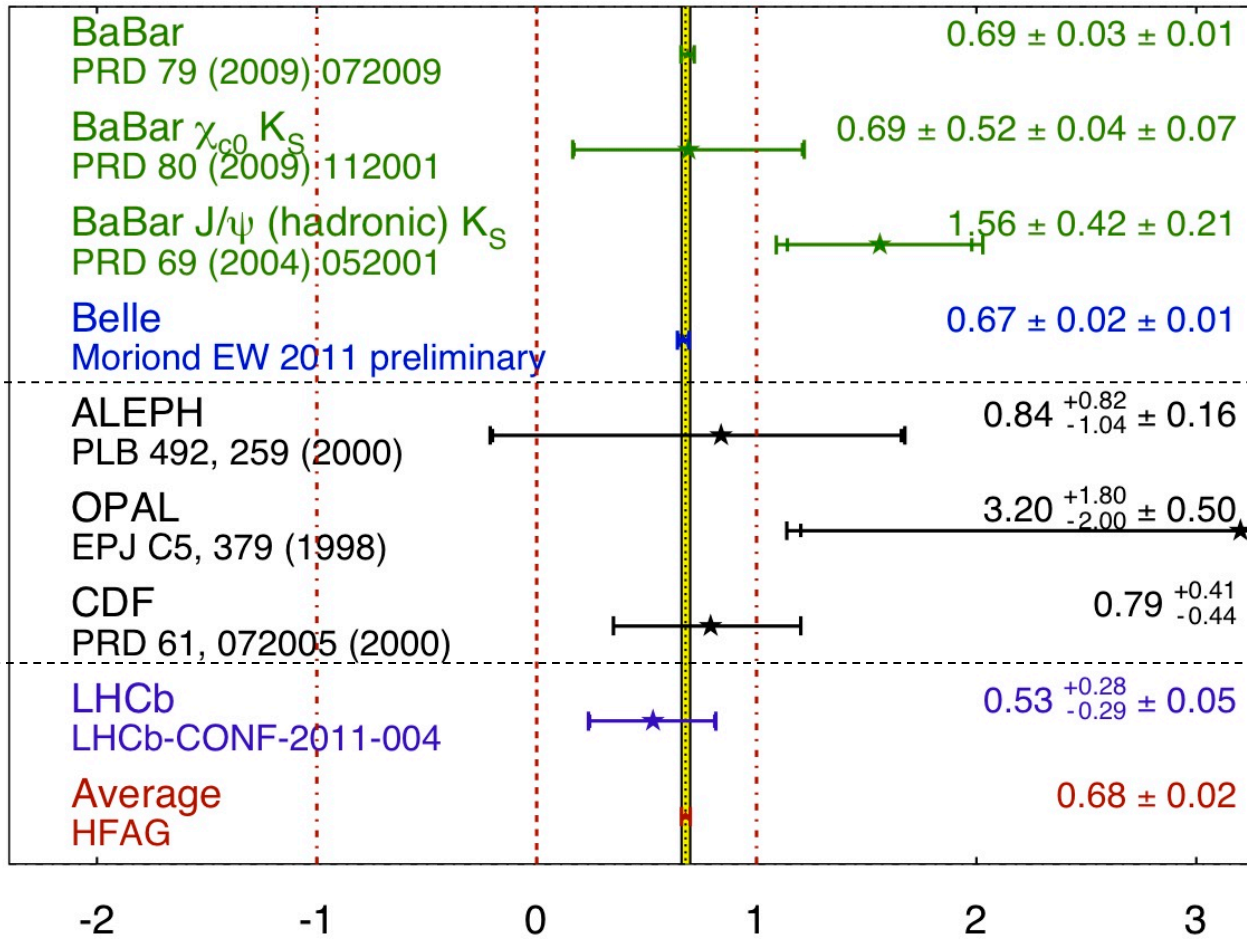
$$A_{\text{fCP}} = -0.024 \pm 0.020 \pm 0.016$$

PRD79,072009(2009)

Now it is a firm SM reference!

$$\sin(2\beta) \equiv \sin(2\phi_1)$$

HFAG
Beauty 2011
PRELIMINARY

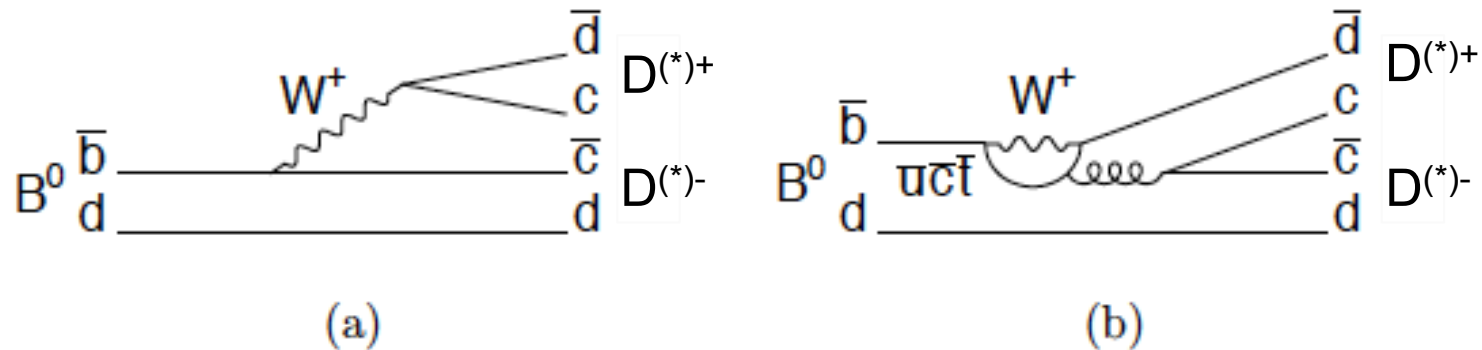


Measurements by
B-factories

Measurements
before B-factories

Newcomer, LHCb
(35pb^{-1} , $A_{\text{fCP}}=0$ assumed.)

$b \rightarrow c\bar{c}d$ process is pursuit of



In $B^0 \rightarrow D^+ D^-$ case,

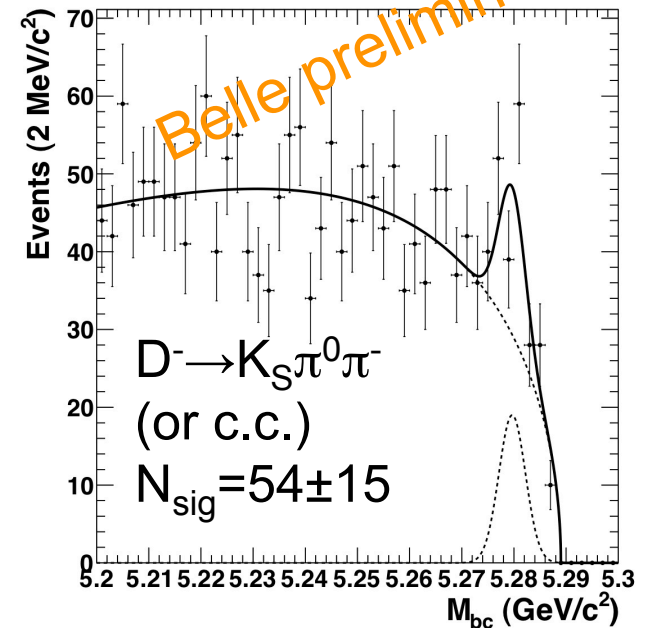
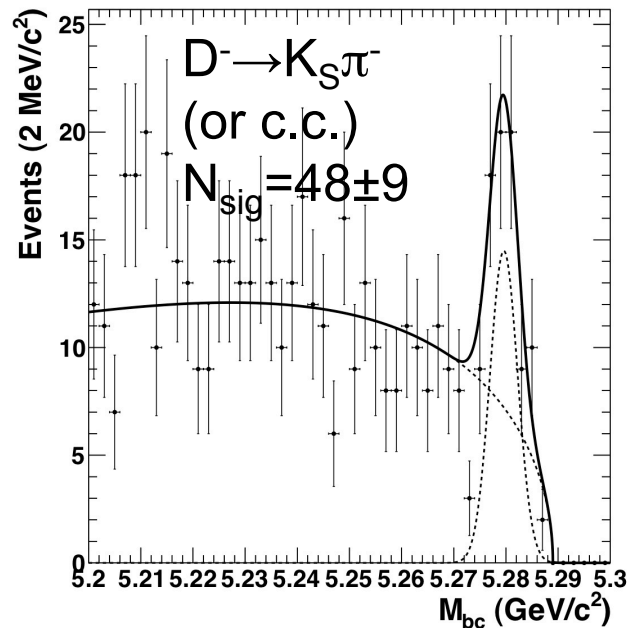
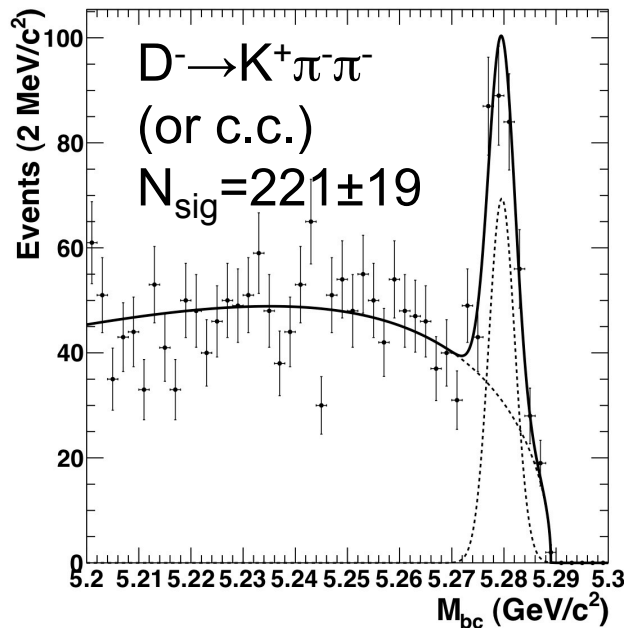
If tree (a) dominant, $S_{fCP} \rightarrow -\sin 2\phi_1$, $A_{fCP} \rightarrow 0$,

while if penguin (b) is substantial, complex phase due to V_{td} may cause Direct CPV.

Since $B^0 \rightarrow D^{*+} D^{*-}$ is a $B \rightarrow VV$ mode, the admixture of CP even/odd eigenstates must be determined before measuring CP violation.

$B^0 \rightarrow D^+ D^-$ reconstruction

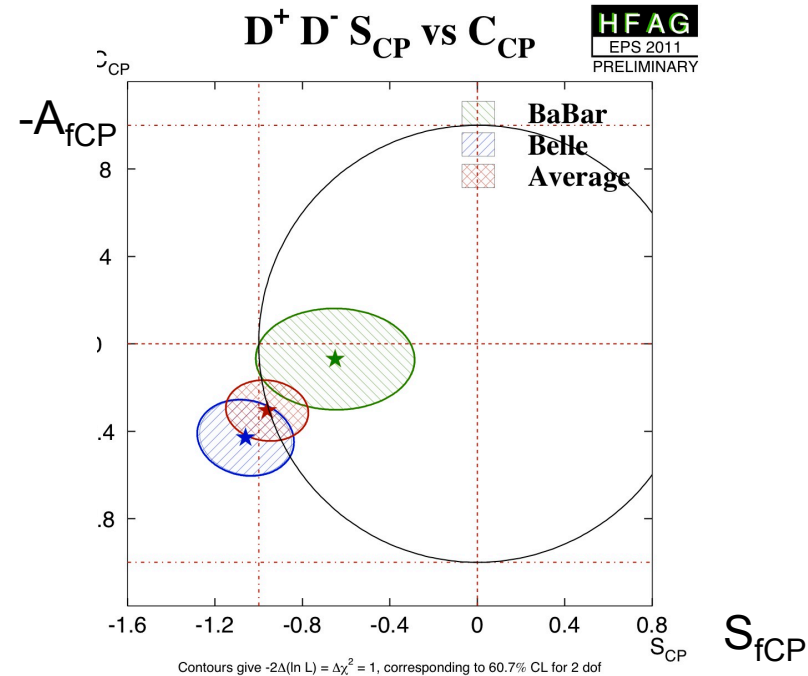
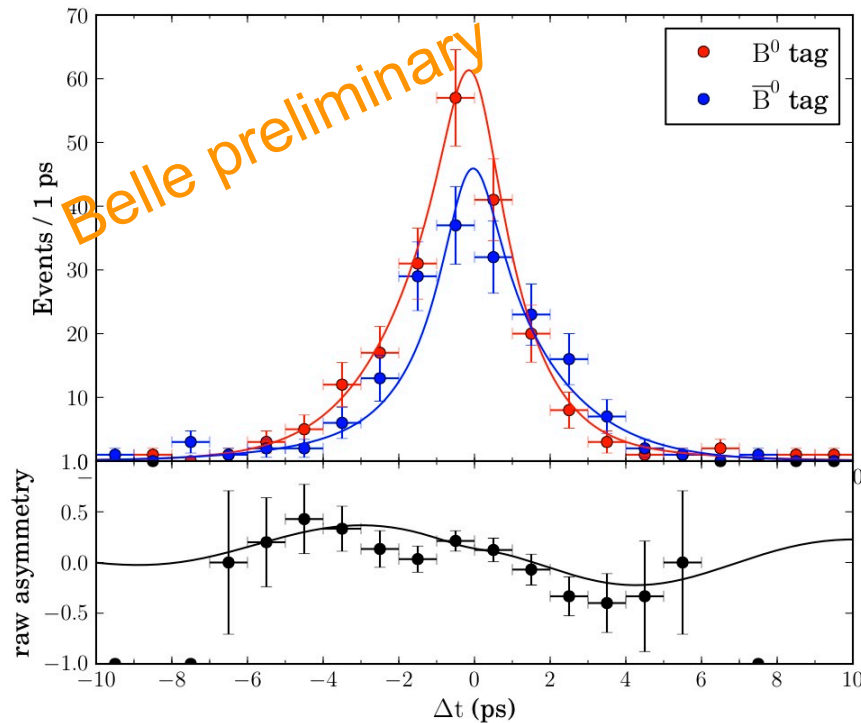
One $D^+ \rightarrow K^- \pi^+ \pi^+$ (or c.c.), three modes for other D.



$$\text{Br}(B^0 \rightarrow D^+ D^-) = (2.09 \pm 0.15 \pm 0.18) \times 10^{-4}$$

cf. Previous result (PRL98,221802) based on 535M $B\bar{B}$,
 $N_{\text{sig}} = 150 \pm 15$ ($D^- \rightarrow K_S \pi^0 \pi^-$ not used), improvement in N_{sig} by data reprocessing is more significant than $(c\bar{c}) K^0$ because of the larger track multiplicity.

$B^0 \rightarrow D^+ D^-$ CP violation



$$S_{fCP} = -1.06 \pm 0.21 \pm 0.07$$

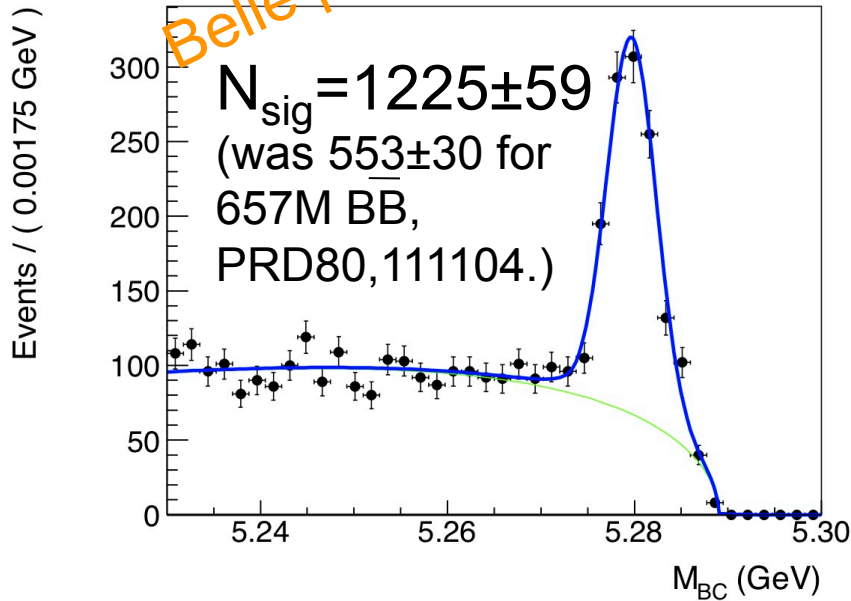
$$A_{fCP} = +0.43 \pm 0.17 \pm 0.04 \quad (D^- \rightarrow K_S \pi^0 \pi^- \text{ not used because of background forming a peak at same position as signal.})$$

S_{fCP} is similar. However A_{fCP} has decreased compared to previous publication with 535M $B\bar{B}$ (PRL98,221802).

$$(S_{fCP} = -1.13 \pm 0.37 \pm 0.09, A_{fCP} = +0.91 \pm 0.23 \pm 0.06)$$

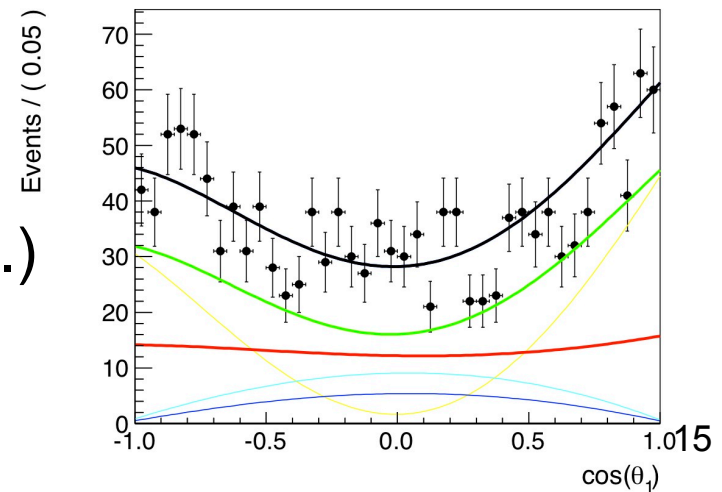
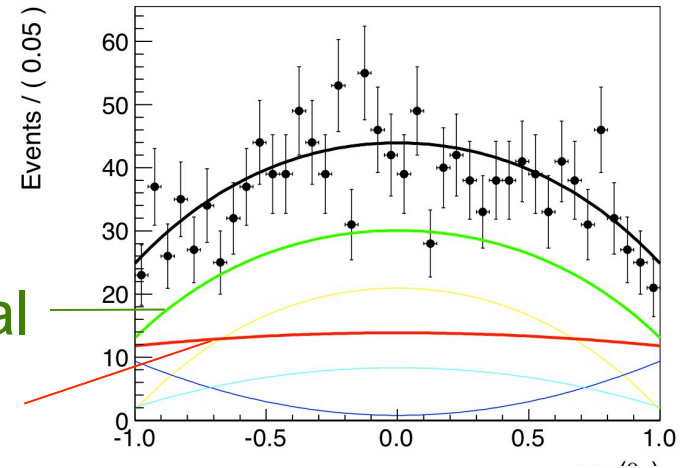
$B^0 \rightarrow D^{*+}D^{*-}$ branching and polarization

Belle preliminary



Signal

B.G.

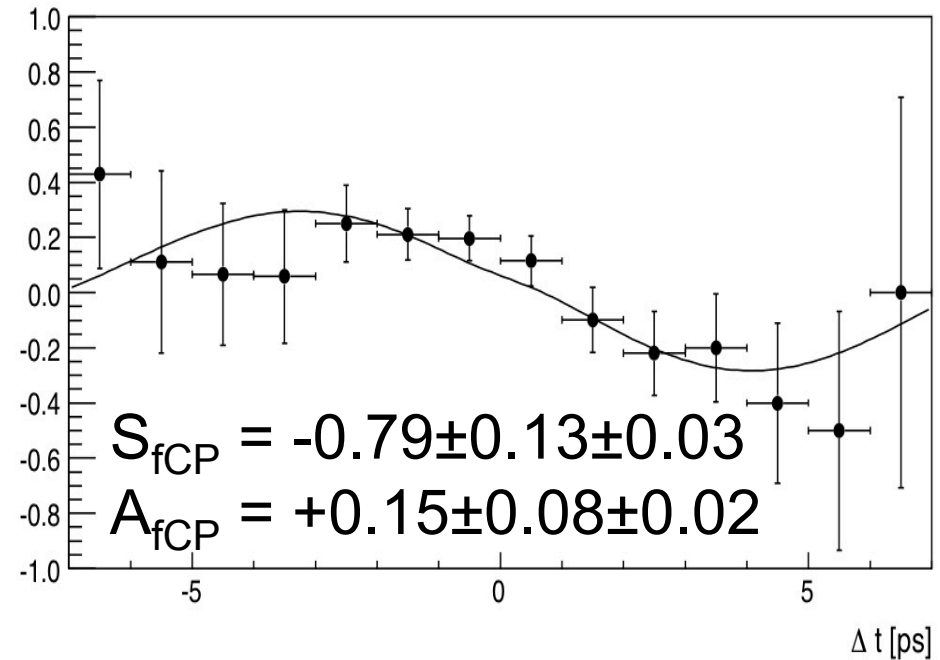
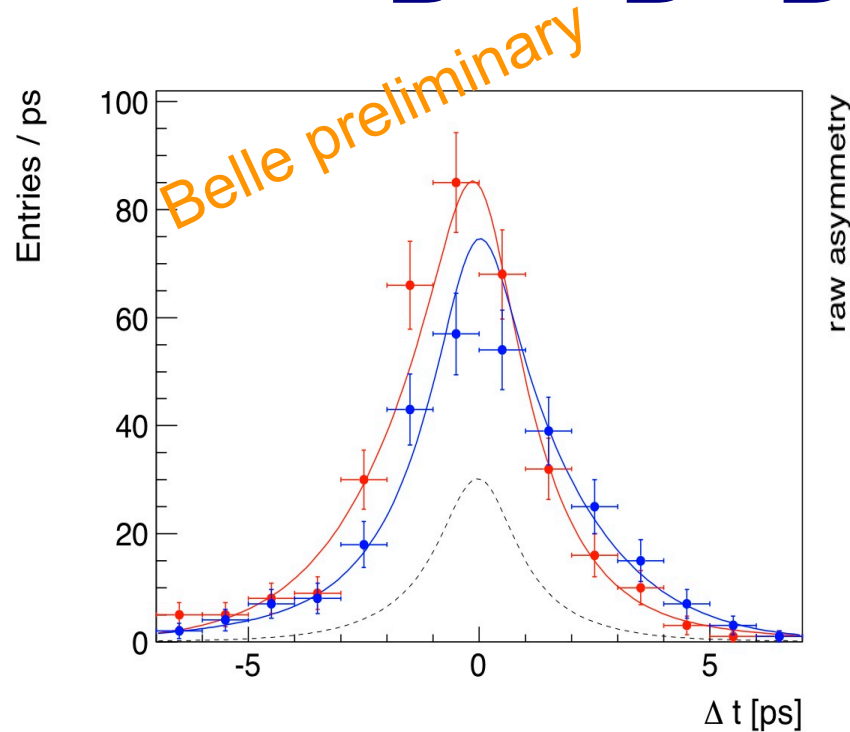


$$\text{Br}(B^0 \rightarrow D^{*+}D^{*-}) = (7.82 \pm 0.38 \pm 0.60) \times 10^{-4}$$

$$R_0 = 0.62 \pm 0.03 \pm 0.01 \text{ (longitudinal pol.)}$$

$$R_{\text{perp}} = 0.14 \pm 0.02 \pm 0.01 \text{ (CP-odd)}$$

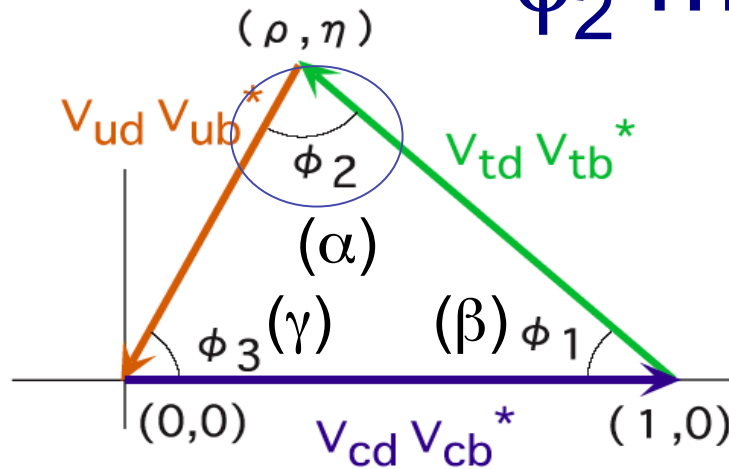
$B^0 \rightarrow D^{*+} D^{*-}$ CP violation



As a result of data reprocessing, signal yield from 772M $B\bar{B}$ pairs is $\times 2.2$ larger than the yield with the 657M $B\bar{B}$ sample used for the previous result (PRD80,111104).

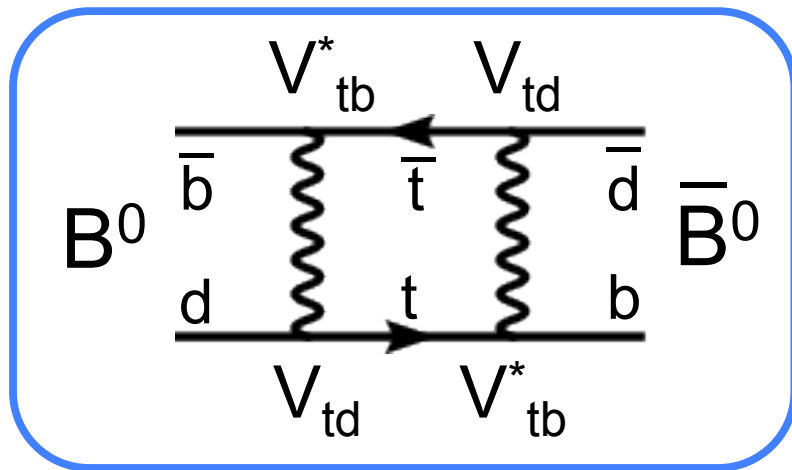
→ significant improvement (S_{fCP} and A_{fCP} errors down to 60%)!

ϕ_2 measurement

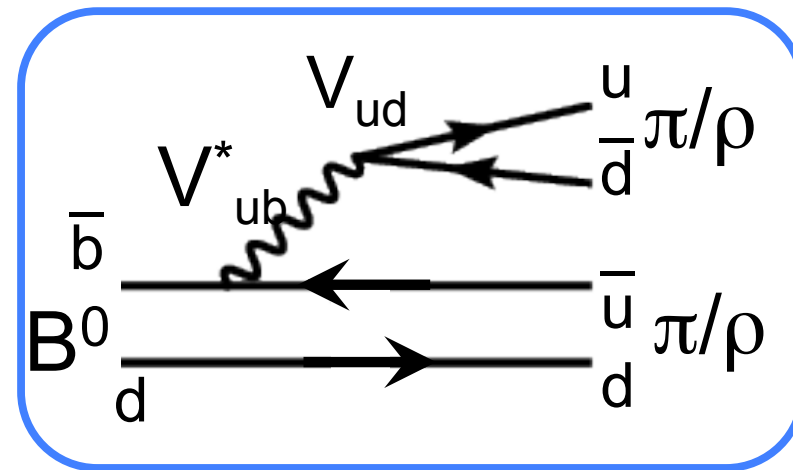


If tree only, S_f is directly connected to $\sin 2\phi_2$ and $A_f=0$.

Interference with $b \rightarrow d$ penguin can be solved by isospin analysis.



Mixing diagram

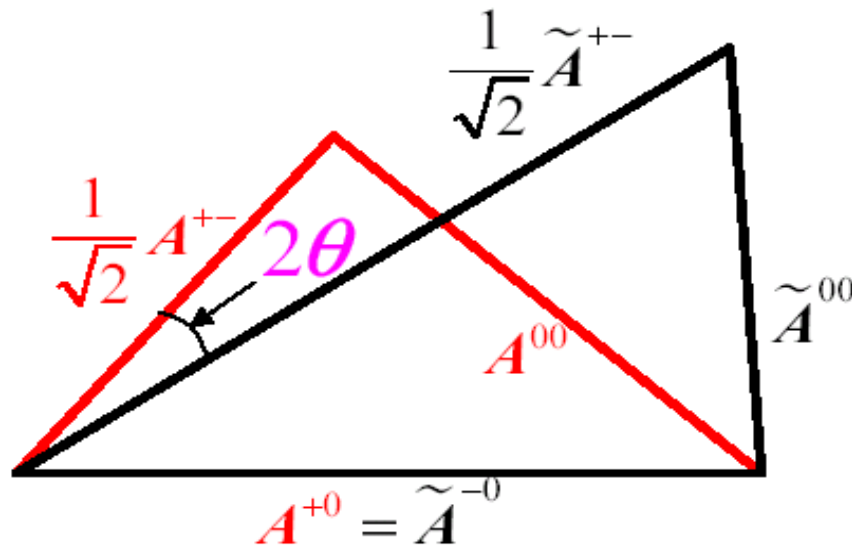


Decay diagram (tree)

There are 3 modes; $\pi\pi$, $\rho\rho$, $\rho\pi$. In addition $a_1\pi$.

Extract ϕ_2 ; isospin analysis

M. Gronau and D. London, PRL 65, 3381 (1990)



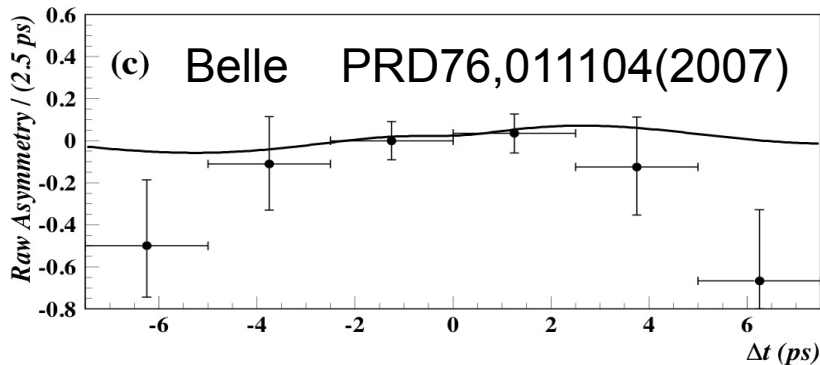
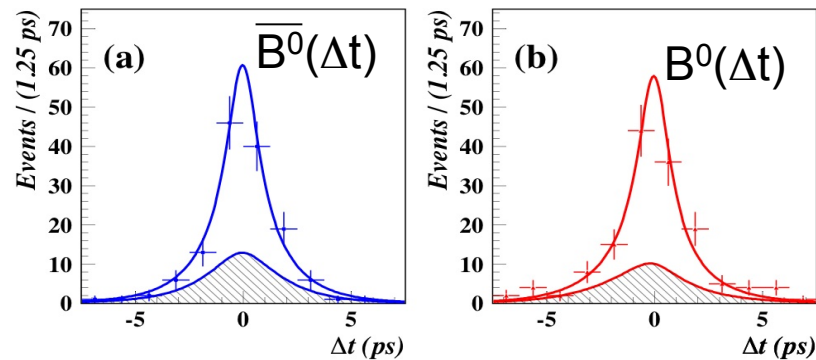
	<i>Amplitude for</i>
$A^{+-}(\bar{A}^{+-})$	$B^0(\bar{B}^0) \rightarrow \pi^+\pi^-$
$A^{00}(\bar{A}^{00})$	$B^0(\bar{B}^0) \rightarrow \pi^0\pi^0$
$A^{+0}(\bar{A}^{-0})$	$B^+(B^-) \rightarrow \pi^+\pi^0(\pi^-\pi^0)$

$$\tilde{A}^{ij} = e^{2\phi_j} \bar{A}^{ij}$$

$B^0 \rightarrow \pi^+\pi^-$, $\pi^0\pi^0$, $B^\pm \rightarrow \pi^\pm\pi^0$ branching fractions,
 and $B^0 \rightarrow \pi^0\pi^0$ Direct CPV are used as inputs to solve this relation.
 The correction from SU(2) breaking effect is still much smaller than
 measurements' errors.

$B^0 \rightarrow \rho^+ \rho^-$

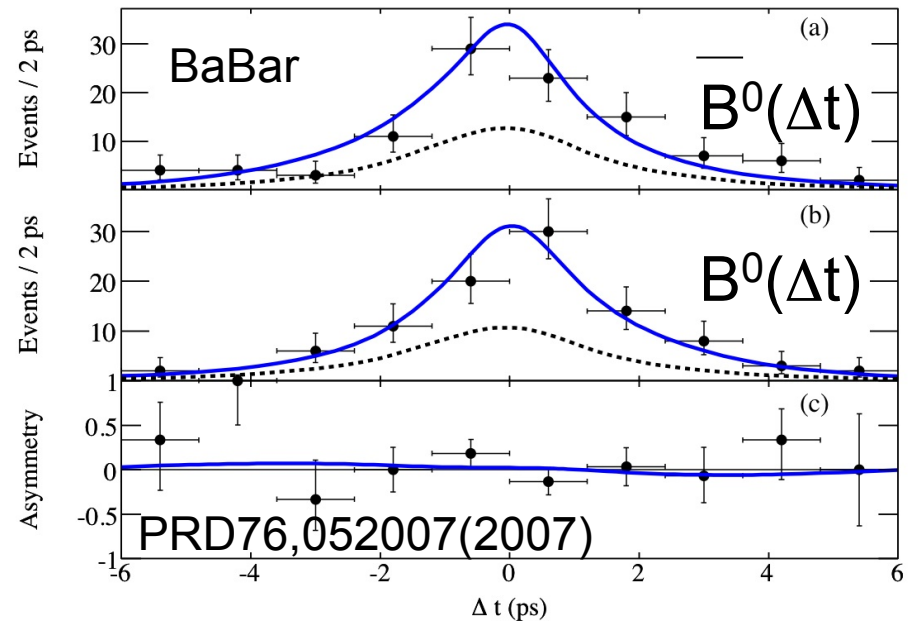
$B \rightarrow VV$, almost purely longitudinally polarized = CP eigenstate.
 Small $\text{Br}(B^0 \rightarrow \rho^0 \rho^0)$, i.e. small penguin pollution.



$$f_L = 0.941 \pm 0.034 / -0.040 \pm 0.030$$

$$S_f = 0.19 \pm 0.30 \pm 0.07$$

$$A_f = 0.16 \pm 0.21 \pm 0.07$$

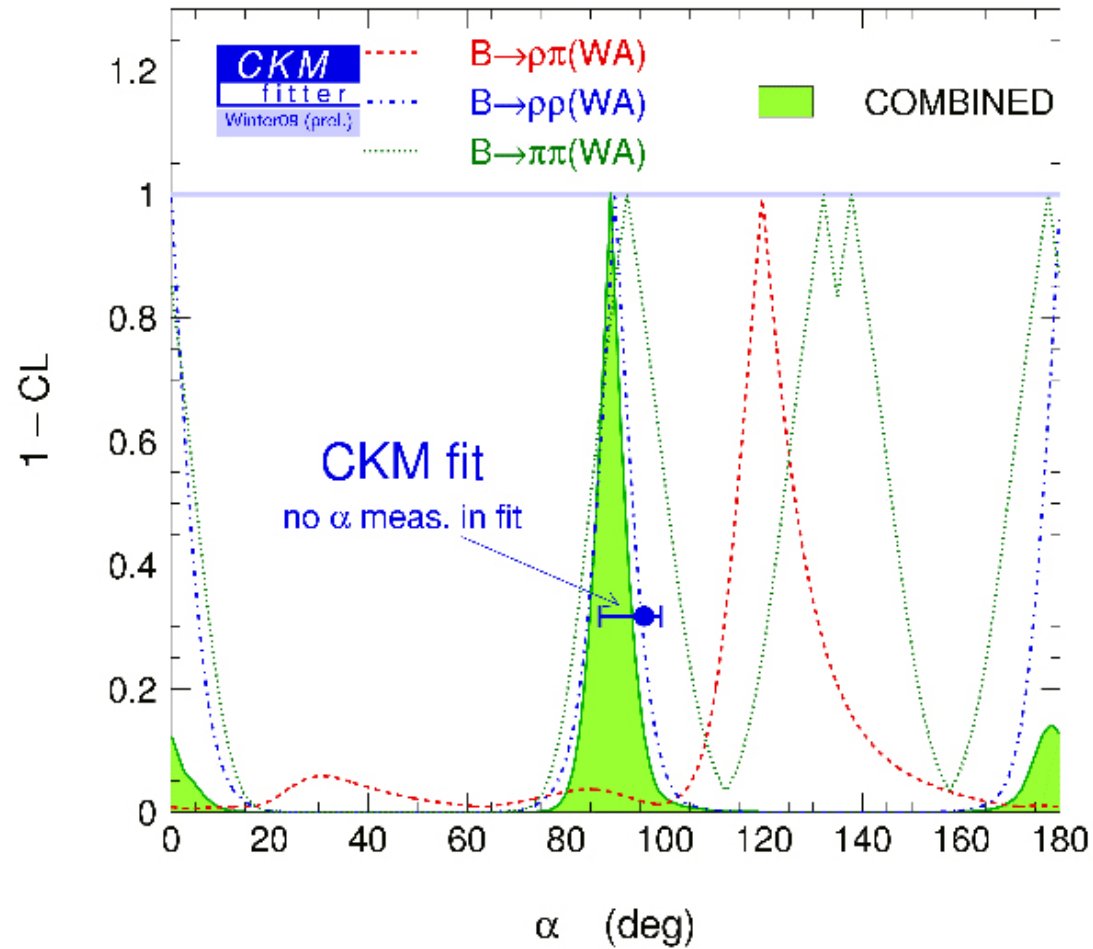


$$f_L = 0.992 \pm 0.024 + 0.026 / -0.013$$

$$S_f = 0.19 \pm 0.30 \pm 0.07$$

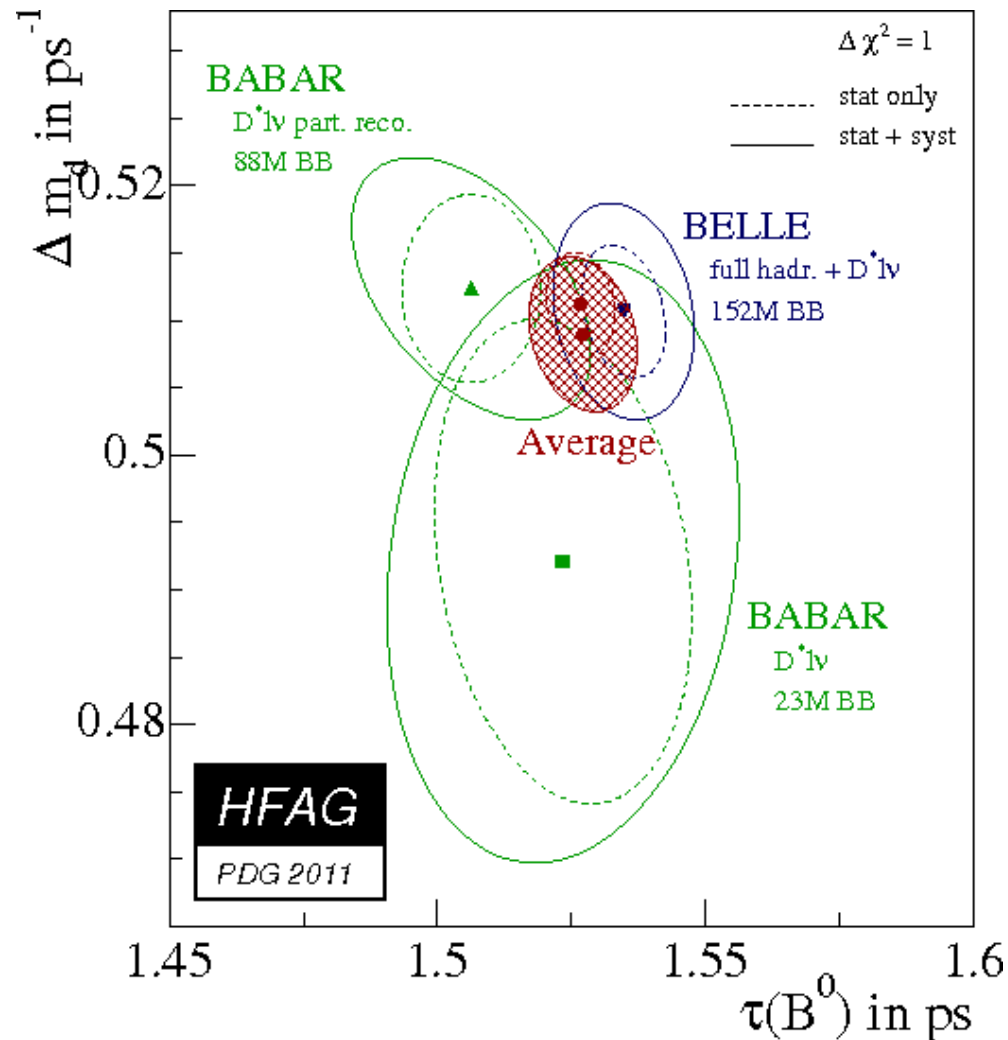
$$A_f = 0.16 \pm 0.21 \pm 0.07$$

Constraint on ϕ_2



$$\phi_2 = 89.0 + 4.4 / -4.2 \text{ deg.}$$

As for Δm_d measurement

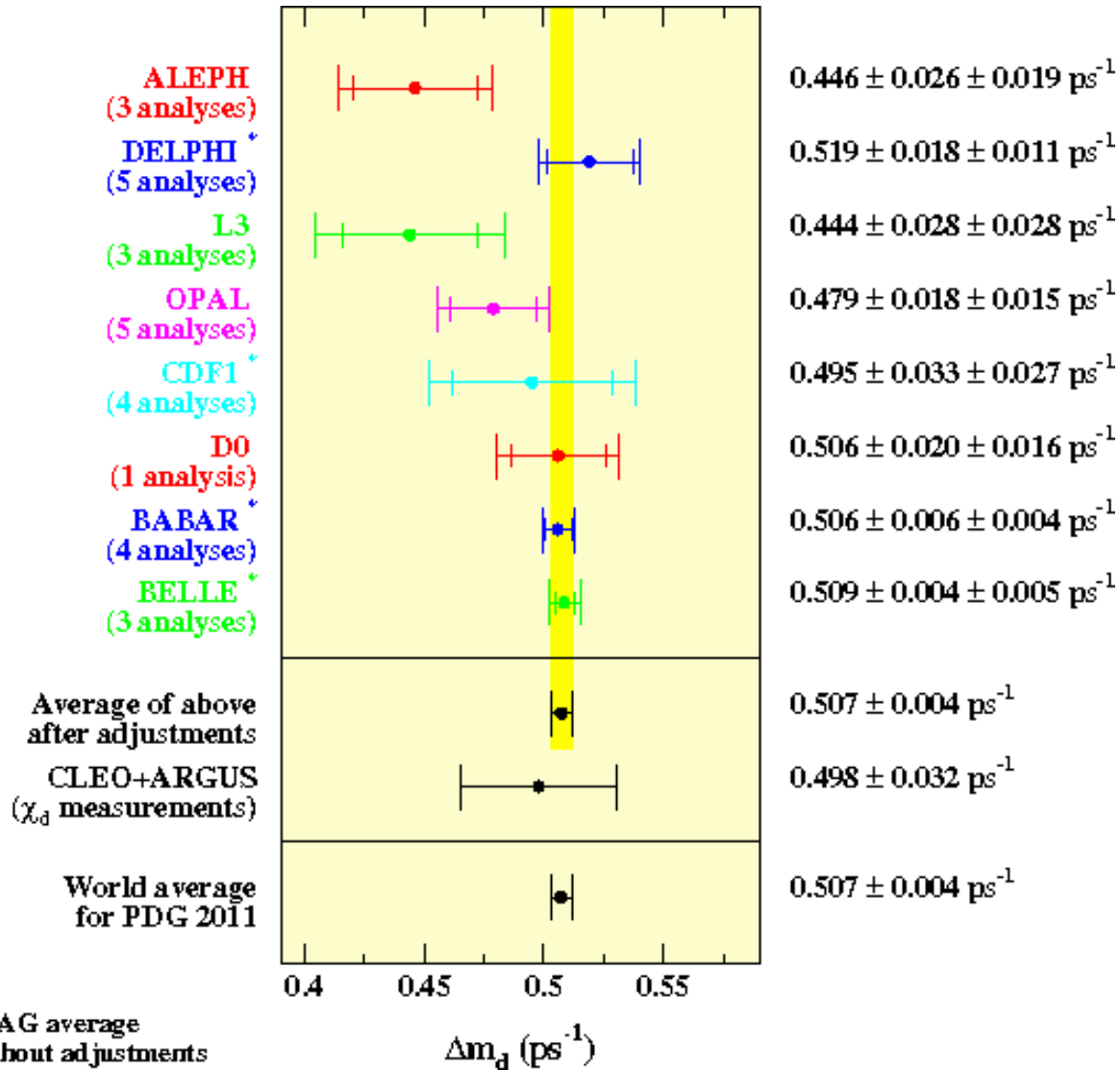


BaBar: $D^*l\nu$ partial recon., opposite side B is tagged by high momentum lepton.

Belle: $D^*l\nu$ and $D^{(*)}X$ hadronic modes full recon., opposite side B tagging is the one for time-dependent CPV.

Δm_d and B lifetime are obtained simultaneously. With $\sim 20\%$ of entire $\Upsilon(4S)$ data, but systematic dominant.

Δm_d without/with B-factories

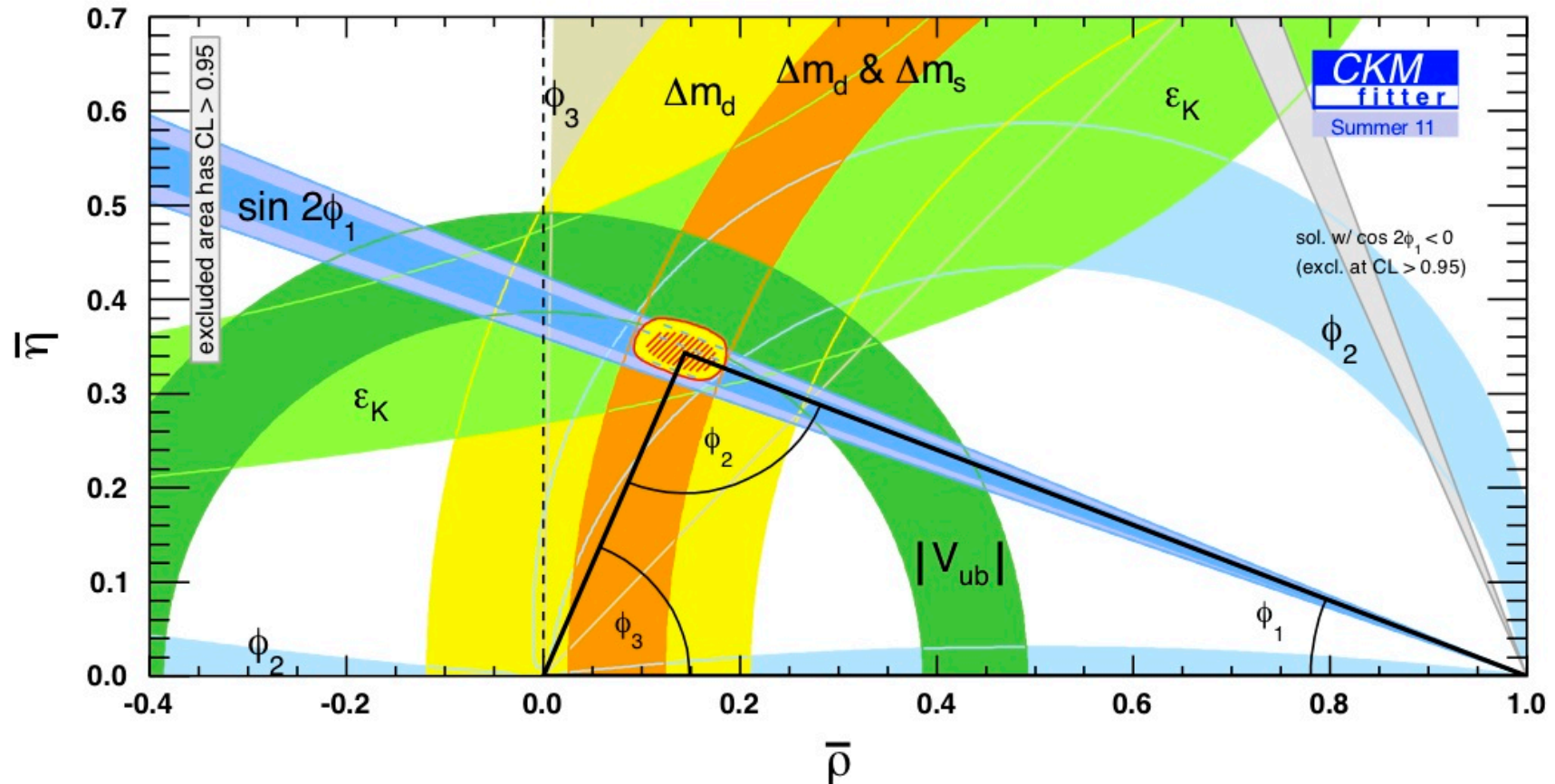


BaBar and Belle results 4-5 times more precise than LEP and Tevatron experiments.

Now 1% precision has been achieved. This gives another reference point to constrain unitarity triangle, i.e. $|V_{td}|$ in the SM framework.

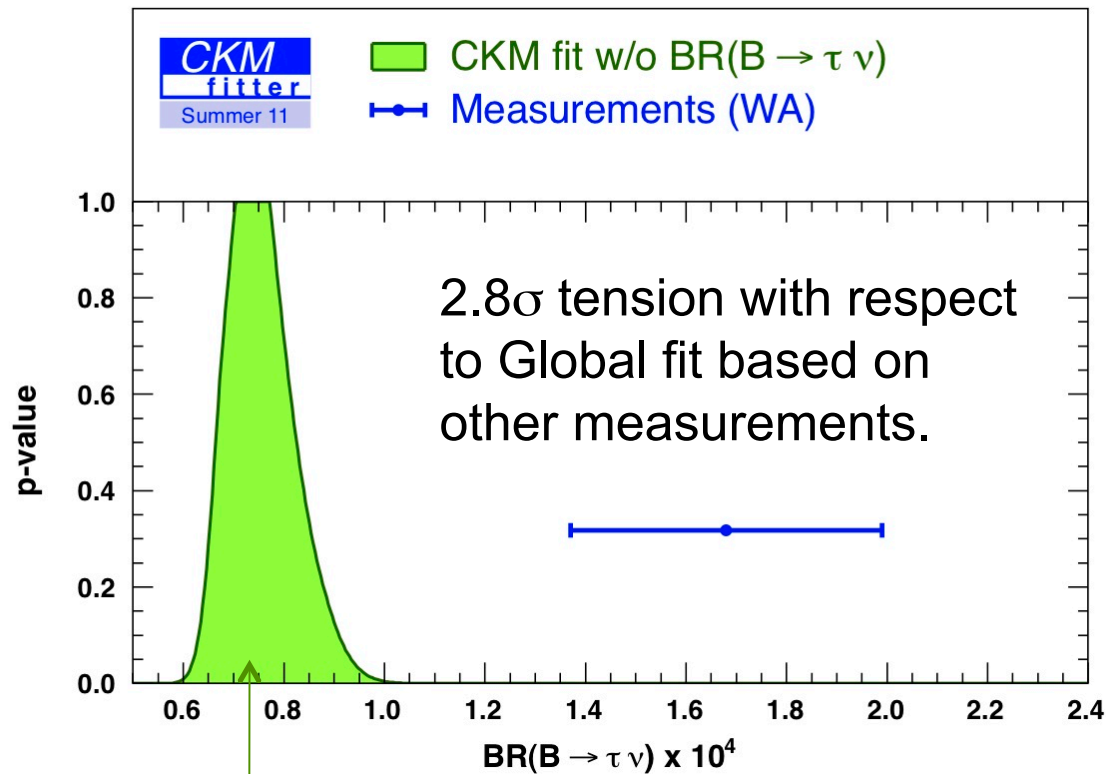
* HFAG average without adjustments

KM scheme has been tested.



ϕ_3 precision improved, $\sigma(\phi_3) \sim 10^\circ$ (See Y.Horii's talk).
Is the unitarity triangle a right triangle?

However, tension with $\text{Br}(B^+ \rightarrow \tau^+ \nu)$

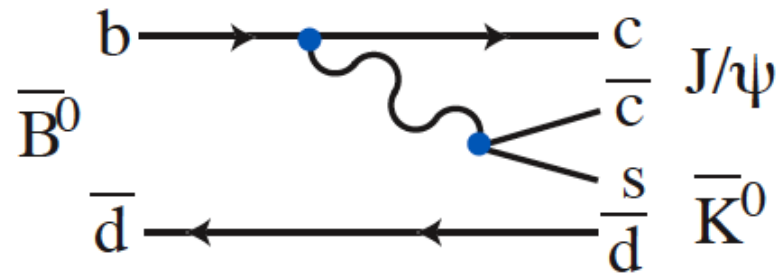


$\sin 2\phi_1$ measurement gives a stringent constraint.

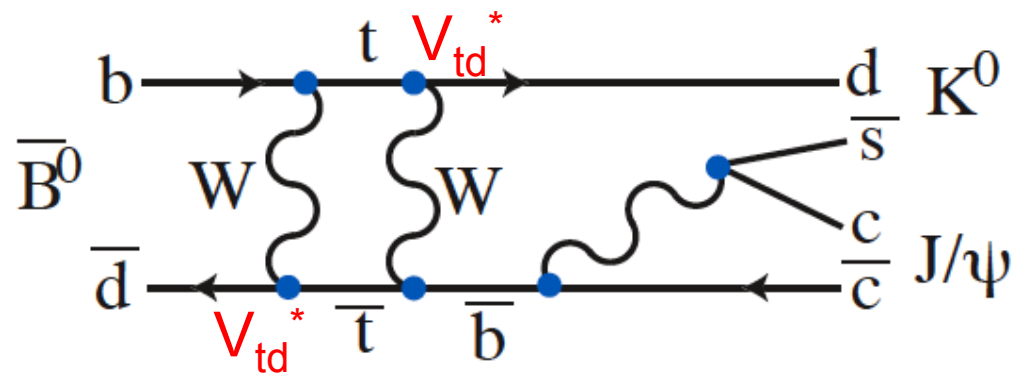
Of course, new physics (NP) could be present in $B^+ \rightarrow \tau^+ \nu$. We need an update of this measurement. But no other place?

$S_{f_{CP}}$ and $\sin 2\phi_1$ SM relation

(1) Decay



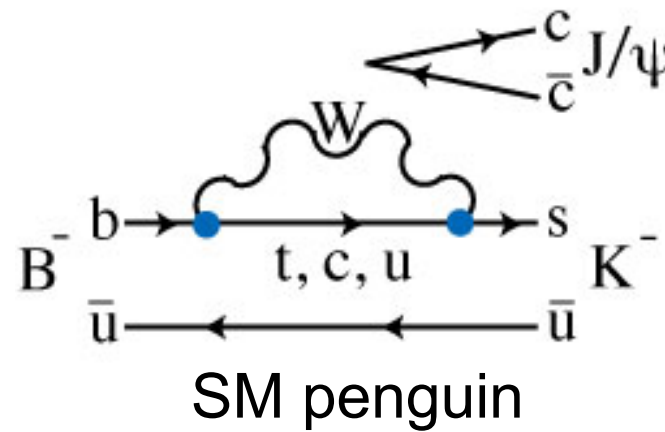
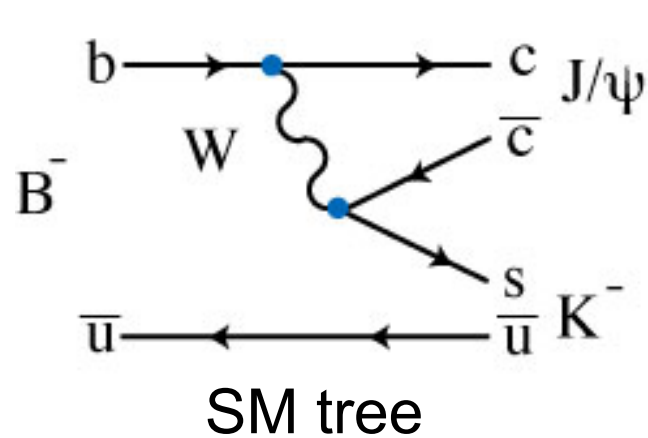
(2) Decay with mixing



Interference between (1) and (2) results in CP violation.

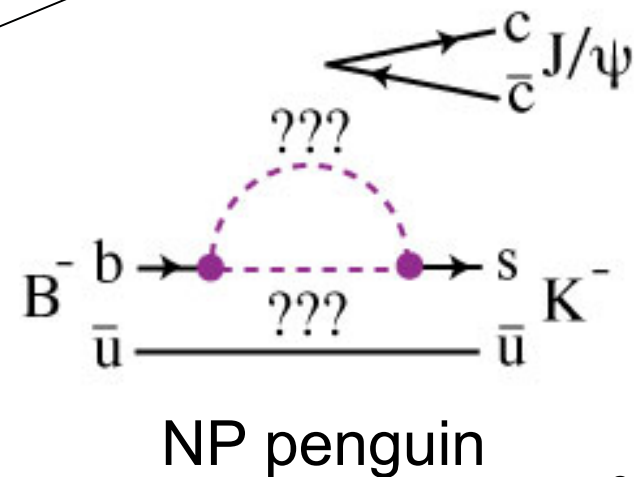
$S_{f_{CP}} = -\xi_{CP} \sin 2\phi_1$, $\xi_{CP} = -1$ (CP-odd), $+1$ (CP-even), $A_{f_{CP}} = 0$.
Is there room to accommodate new physics (NP)?

NP room is unlikely in $b \rightarrow c\bar{c}s$ decays

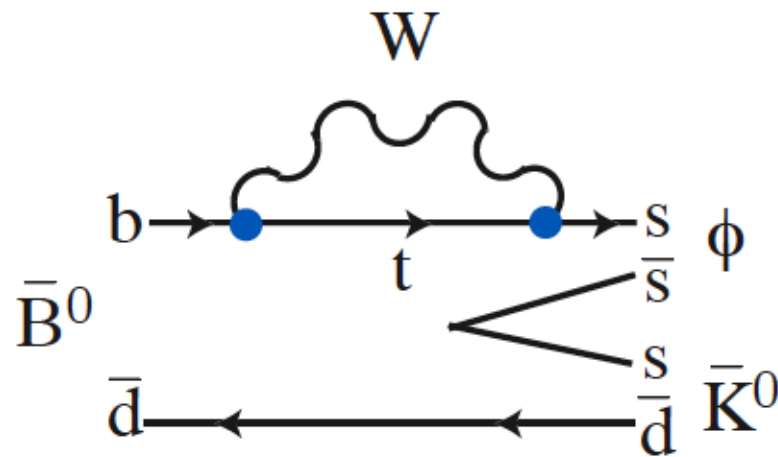


Same weak phase

If NP penguin is substantial and has different phase, it causes Direct CPV in $B^\pm \rightarrow J/\psi K^\pm$.
 No direct CP violation has been observed so far.
 $-0.76 \pm 0.50 \pm 0.22\%$ at Belle (PRD82,091104),
 $(1 \pm 7) \times 10^{-3}$ in PDG2011.
 However, there is room for NP in B - \bar{B} mixing.



Effective ϕ_1 in penguin decays



SM penguin;
 No complex phase in decay.

as well as

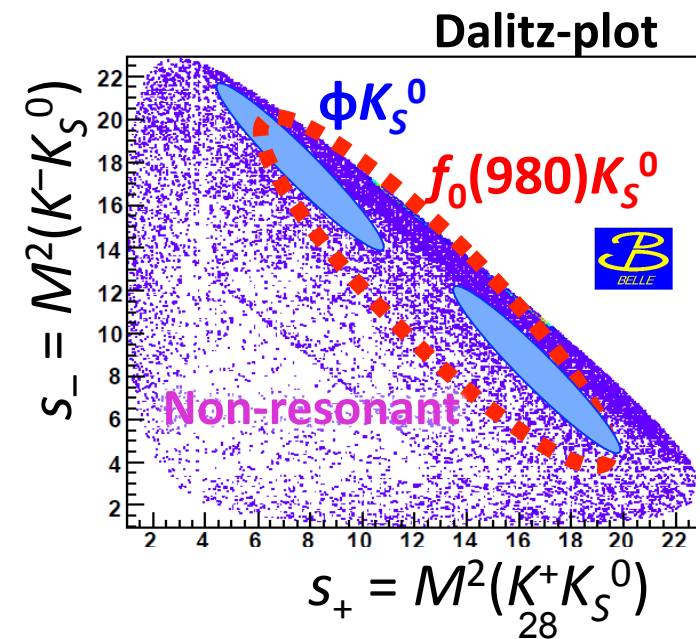
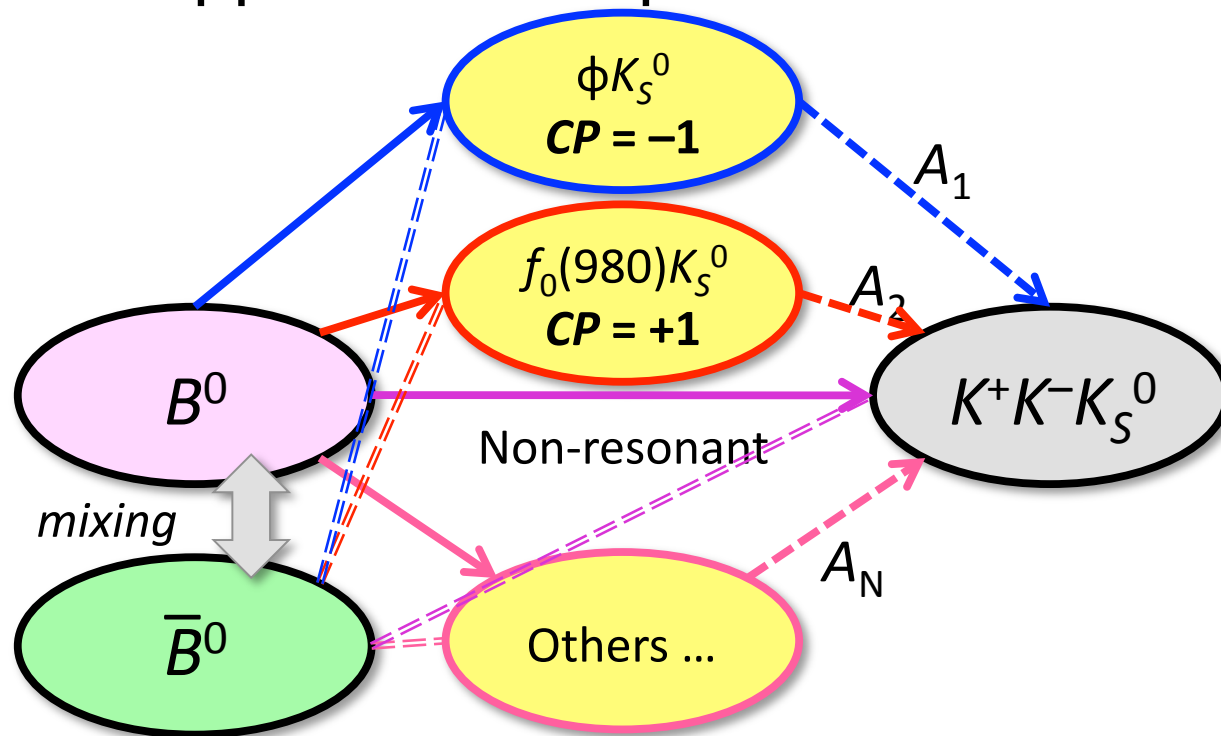
Diagram illustrating a New Physics penguin decay: $\bar{B}^0 \rightarrow \phi + \bar{K}^0$. The process involves a b quark and an anti- d quark. The b quark emits a W boson and becomes an s quark. The W boson then decays into a top quark and an anti- s quark. The top quark and anti- d quark form the \bar{K}^0 meson, while the s quark and anti- s quark form the ϕ meson.

New Physics in the loop;
 may have a different weak phase.
 CPV deviation from $J/\psi K^0$?

Many two-body and quasi-two body analyses have been done. Since $\phi \rightarrow K^+K^-$, $f_0 \rightarrow K^+K^-$ and non-resonant contributions overlap in invariant mass (as do $\rho^0 \rightarrow \pi^+\pi^-$ and $f_0 \rightarrow \pi^+\pi^-$), recently time-dependent Dalitz analyses have been performed in three-body decays such as $B^0 \rightarrow (K^+K^-)K_S$ and $B^0 \rightarrow (\pi^+\pi^-)K_S$.

Several contributions are overlapping

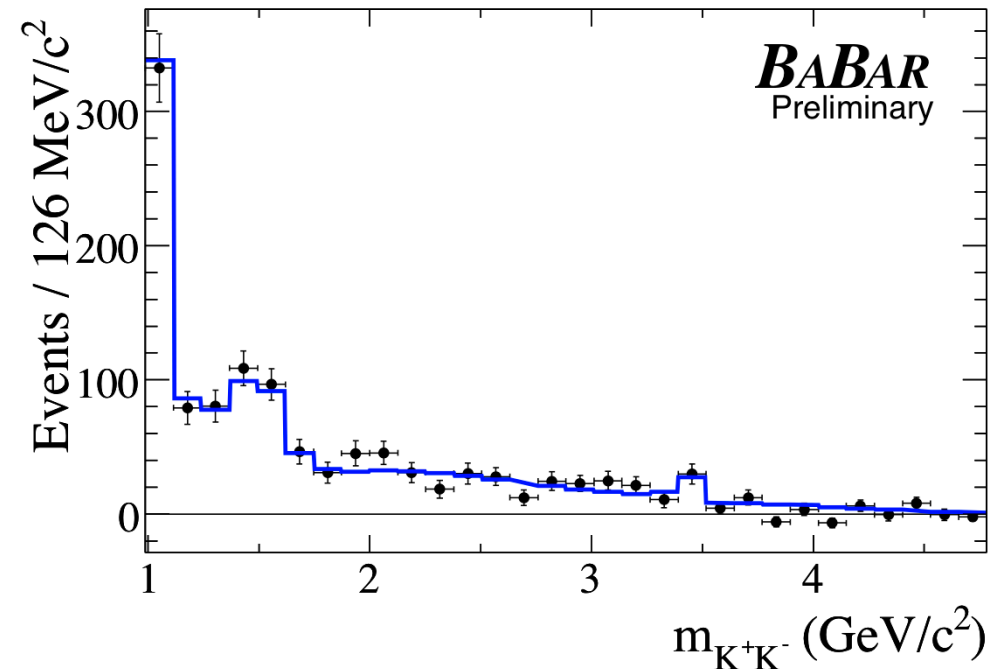
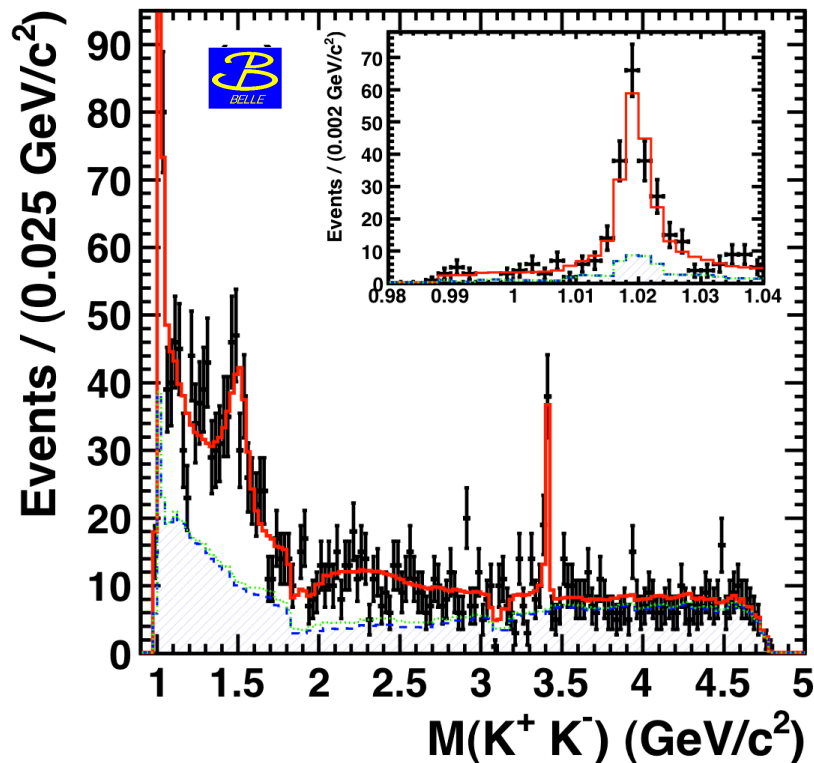
- For example, $B^0 \rightarrow K^+K^-K_S^0$ final state has several different paths.
- Resolve them by fitting the Dalitz distribution. Same approach is required for $B^0 \rightarrow \pi^+\pi^-K_S^0$.



$$s_+ = M^2(K^+K_S^0)$$

28

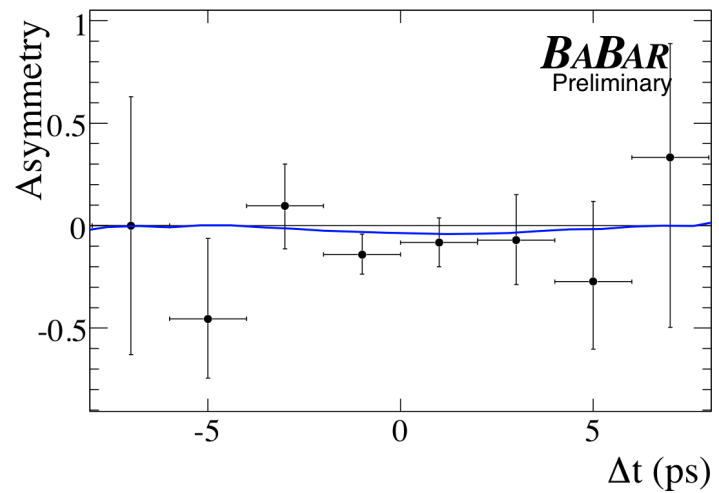
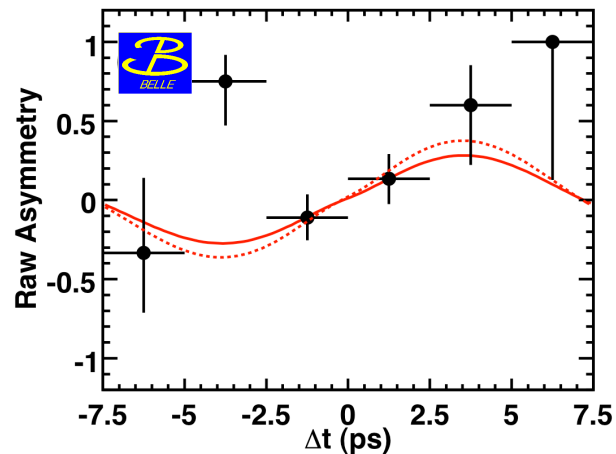
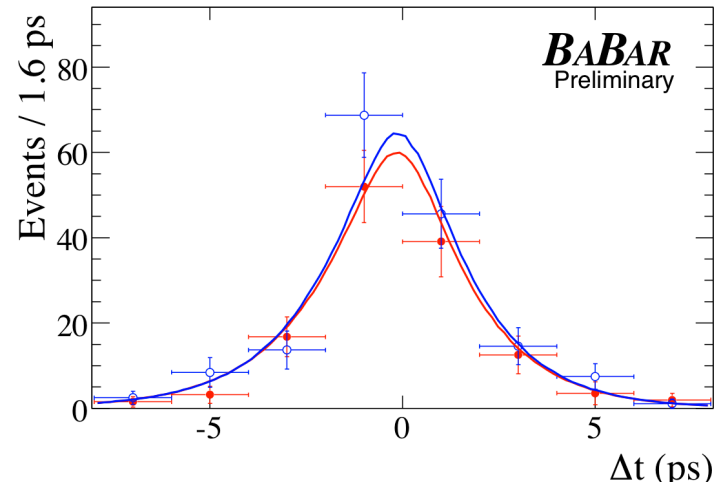
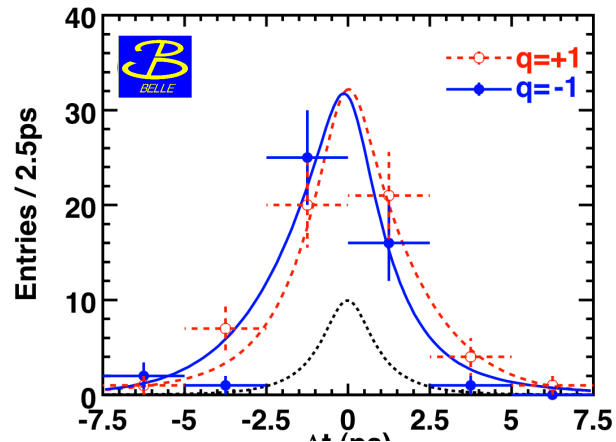
Projections of Dalitz distribution ($M_{K^+K^-}$)



Peak around 1 GeV/c² : $\phi(1020)$ and $f_0(980)$, at 1.5 GeV/c² : f_X ,
 at 3.4 GeV/c² : χ_{c0}

There are multiple solutions (Belle found 4, BaBar found 2). 29

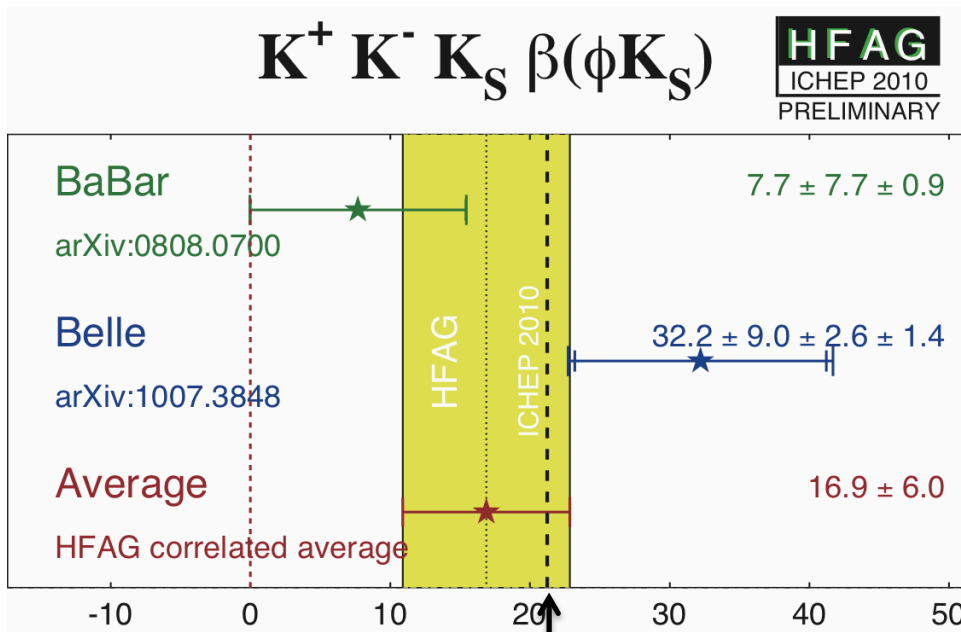
Δt distribution in ϕ mass region



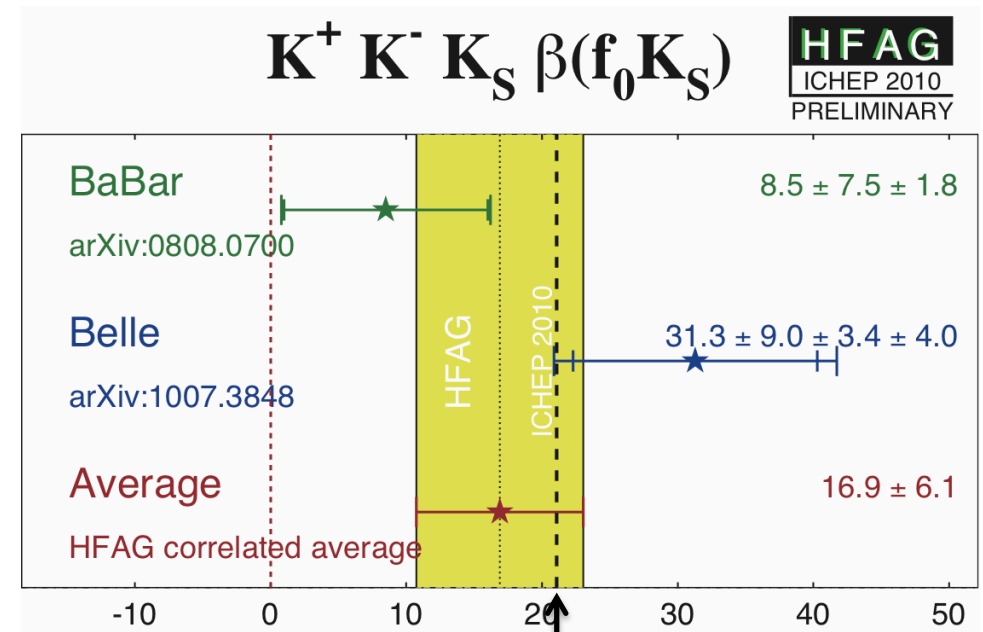
PRD82,073011(2010)

arXiv:0808.0700

Effective ϕ_1 of “solution 1”



ϕ_1 by $(c\bar{c})K^0$



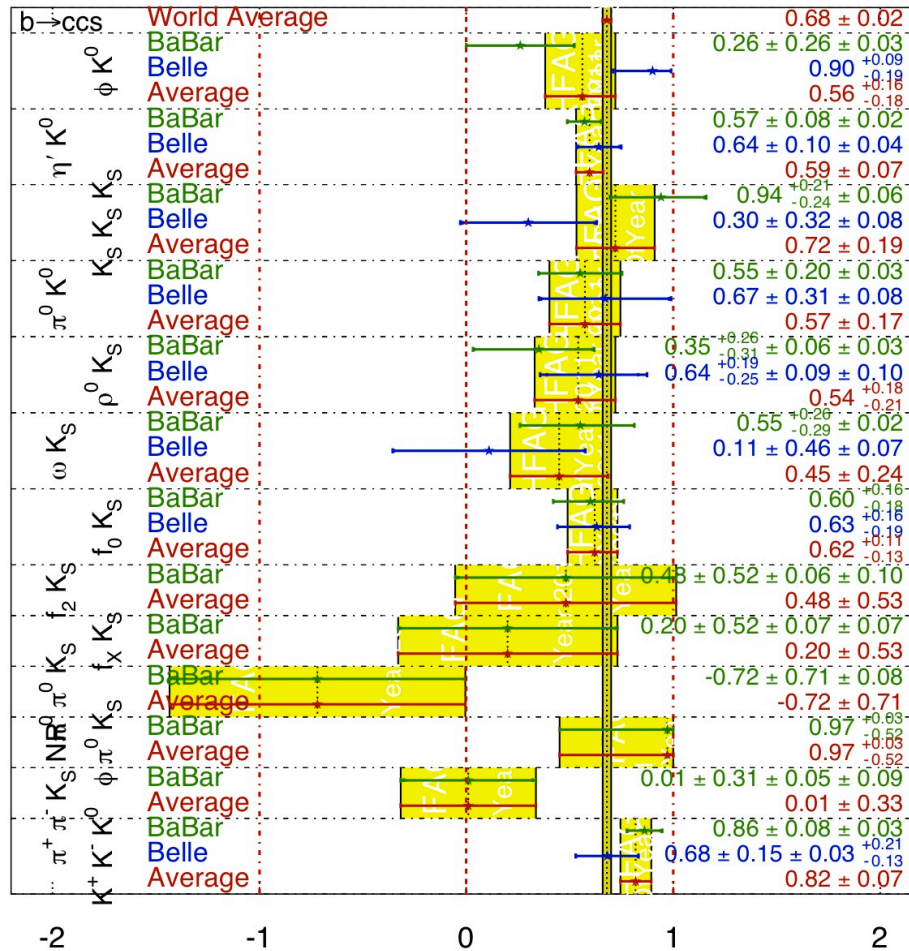
ϕ_1 by $(c\bar{c})K^0$

With current statistics, we could not distinguish multiple solutions by the likelihood alone. The preferred solution is shown.

No significant deviation from measurements with $B^0 \rightarrow (c\bar{c}) K^0$. 31

Compilation of effective $\sin 2\phi_1$

$$\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}}) \quad \text{HFAG} \\ \text{EndOfYear 2011} \\ \text{PRELIMINARY}$$

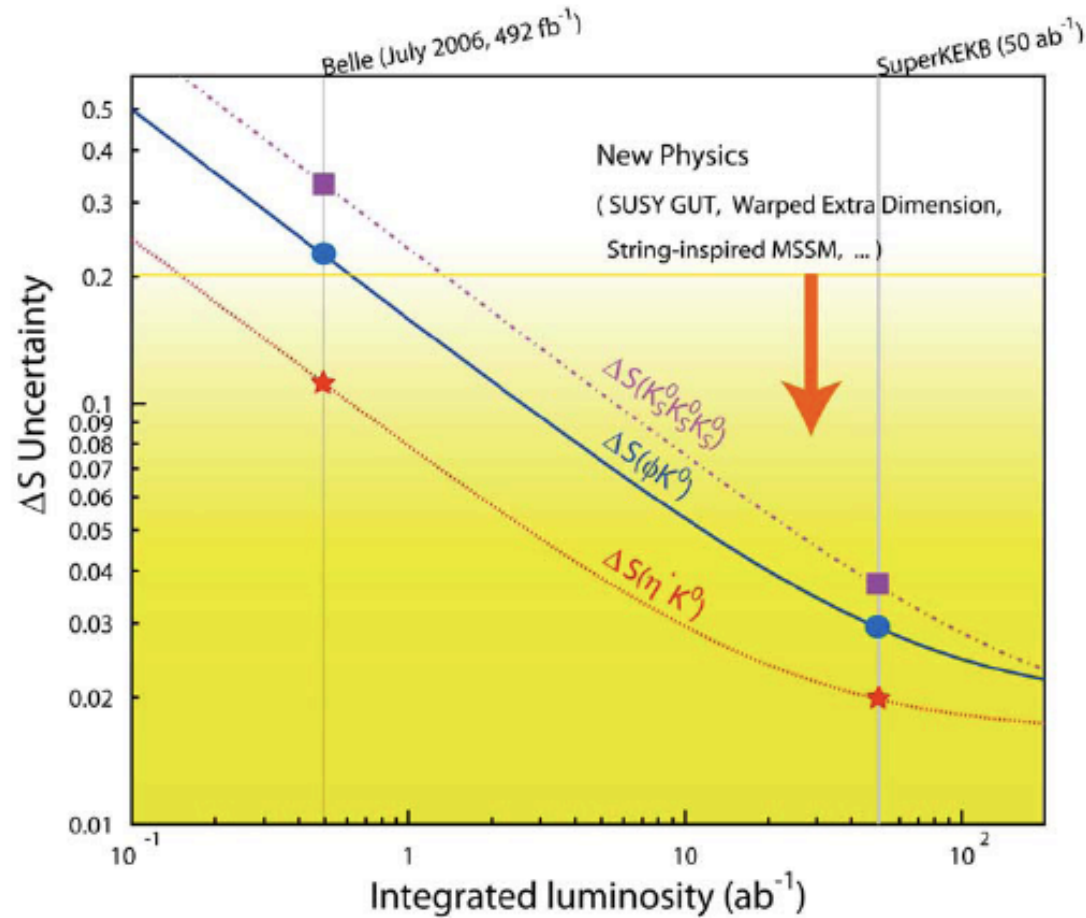


Still precision is statistically dominated.



To obtain sensitivity in effective $\sin 2\phi_1$ of $O(10^{-2})$, we need $O(10 \text{ ab}^{-1})$ integrated luminosity.

Future sensitivity



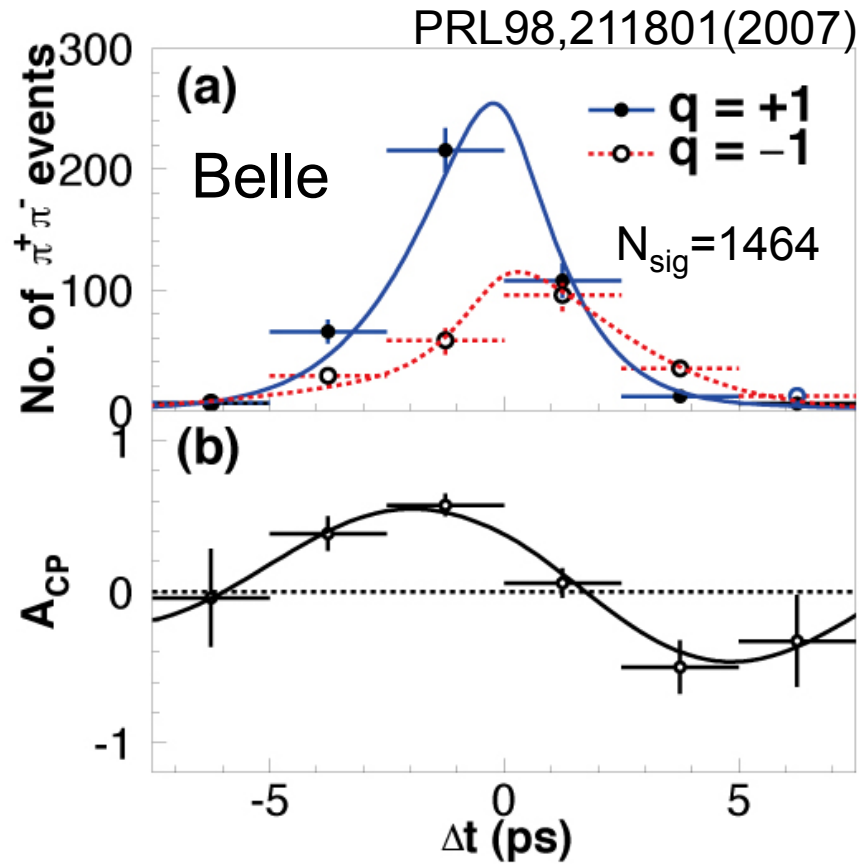
Error of effective $\sin 2\phi_1$ would be $0.03(\eta' K^0)$ - $0.1(K_S K_S K_S)$.

Summary

- $\sin 2\phi_1 = 0.68 \pm 0.02$ in World Average
 - It is a firm SM reference point.
- Constraint on ϕ_2 : $89.0 +4.4/-4.2$ deg.
 - The unitarity triangle appears to be a right triangle.
- Δm_d is precisely determined by B-factories.
 - Now 1% precision has been achieved, giving a firm reference.
- Tension around $\text{Br}(B^+ \rightarrow \tau^+ \nu)$
 - Need an update of measurement.
 - Comparing to $\sin 2\phi_1$ measurement, expect mixing has room for NP.
- CPV in $b \rightarrow s$ penguin modes
 - Reach $O(10^{-2})$ sensitivity with Super B-factories.

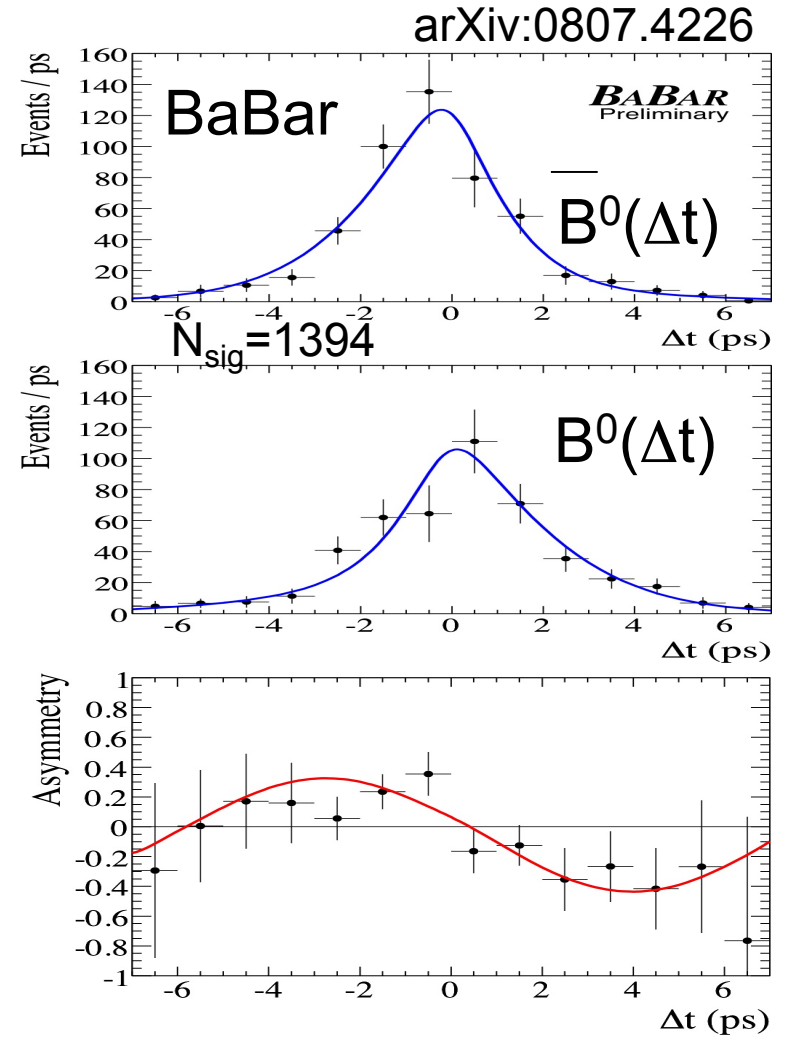
Backup slides

$B^0 \rightarrow \pi^+ \pi^-$



$$S_f = -0.61 \pm 0.10 \pm 0.04$$

$$A_f = 0.55 \pm 0.08 \pm 0.05$$

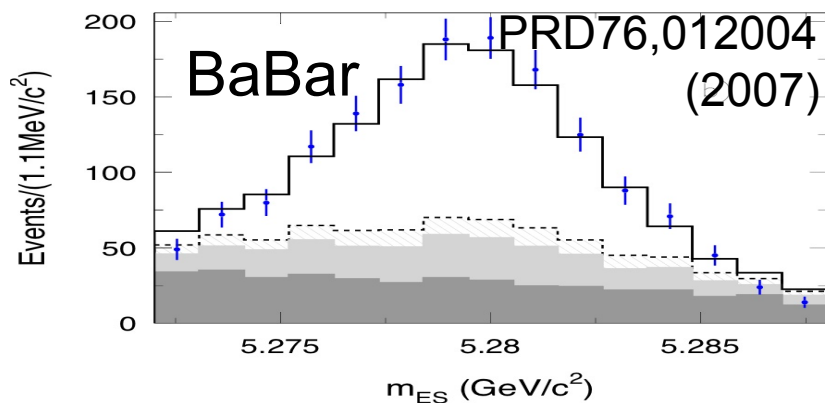
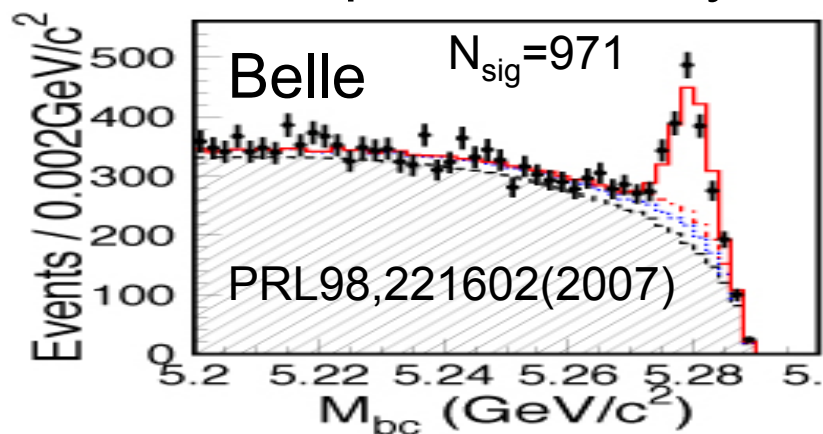


$$S_f = -0.68 \pm 0.10 \pm 0.03$$

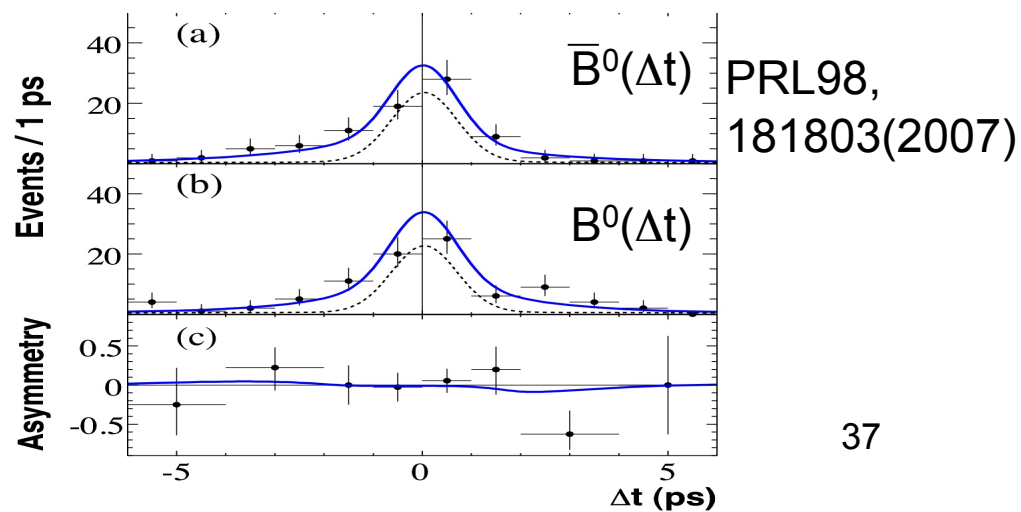
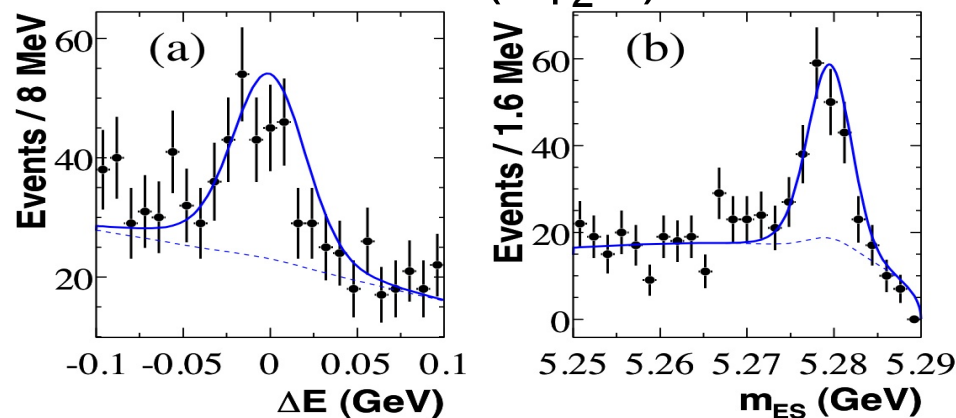
$$A_f = 0.25 \pm 0.08 \pm 0.02$$

$B^0 \rightarrow (\rho\pi)^0, B^0 \rightarrow a_1^\pm \pi^\mp$

$B^0 \rightarrow \rho^+\pi^-, \rho^-\pi^+, \rho^0\pi^0$ interfere
 \rightarrow time-dep. Dalitz analysis



BaBar (384M BB) $a_1^\pm \pi^\mp$;
 obtained $\alpha^{\text{eff}} (= \phi_2^{\text{eff}})$



Coefficients of Dalitz plot functions are interrupted to CPV parameters of quasi-2-body decays, $B \rightarrow \rho^+ \pi^-$ and $B \rightarrow \rho^0 \pi^0$

$$c^+ = \frac{U_+^-}{U_+^+}, \quad c^- = \frac{U_-^-}{U_-^+}, \quad s^+ = \frac{2I_+}{U_+^+}, \quad s^- = \frac{2I_-}{U_-^+}, \quad A_{\rho\pi}^{CP} = \frac{U_+^+ - U_-^+}{U_+^+ + U_-^+}$$

$$c \equiv \frac{c^+ + c^-}{2}, \quad \Delta c \equiv \frac{c^+ - c^-}{2}, \quad s \equiv \frac{s^+ + s^-}{2}, \quad \Delta s \equiv \frac{s^+ - s^-}{2}$$

$$A_{\rho^0\pi^0} = -\frac{U_0^-}{U_0^+}, \quad \text{and} \quad S_{\rho^0\pi^0} = \frac{2I_0}{U_0^+}$$

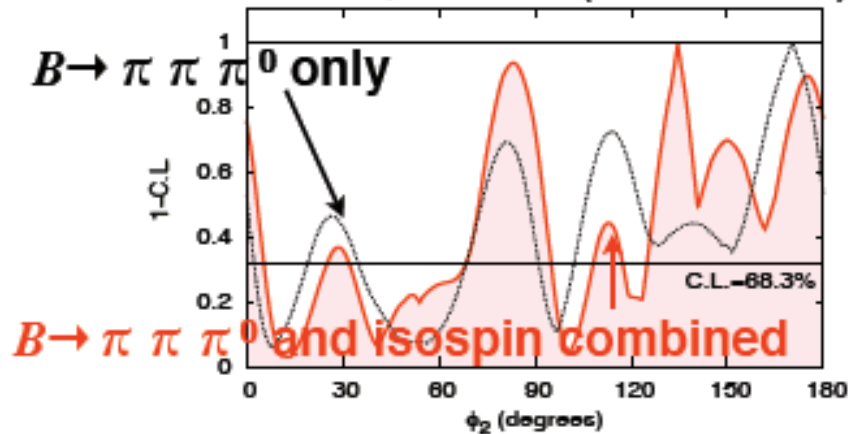
Belle 449M $B\bar{B}$ (PRL98 221602)

$$\begin{aligned} A_{\rho\pi}^{CP} &= -0.12 \pm 0.05 \pm 0.04 \\ C &= -0.13 \pm 0.09 \pm 0.05 \\ \Delta C &= +0.36 \pm 0.10 \pm 0.05 \\ S &= +0.06 \pm 0.13 \pm 0.05 \\ \Delta S &= -0.08 \pm 0.13 \pm 0.05 \\ A_{\rho^0\pi^0} &= -0.49 \pm 0.36 \pm 0.28 \\ S_{\rho^0\pi^0} &= +0.17 \pm 0.57 \pm 0.35 \end{aligned}$$

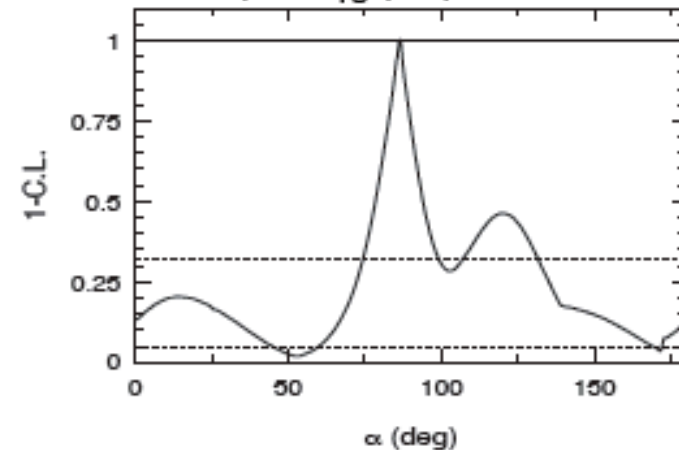
BABAR 375M $B\bar{B}$ (PRD76 012004)

$$\begin{aligned} A_{\rho\pi} &= -0.14 \pm 0.05 \pm 0.02 \\ C &= 0.15 \pm 0.09 \pm 0.05 \\ S &= -0.03 \pm 0.11 \pm 0.04 \\ \Delta C &= 0.39 \pm 0.09 \pm 0.09 \\ \Delta S &= -0.01 \pm 0.14 \pm 0.06 \\ C_{00} = \frac{U_0^-}{U_0^+} &= -0.10 \pm 0.40 \pm 0.53 \\ S_{00} = \frac{2I_0}{U_0^+} &= 0.04 \pm 0.44 \pm 0.18 \end{aligned}$$

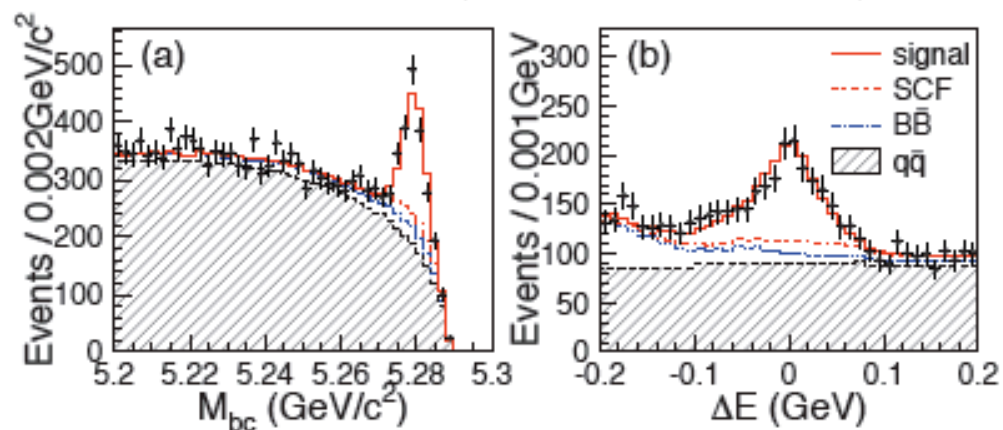
$68^\circ < \phi_2 < 95^\circ$ (68.3% C.L.)



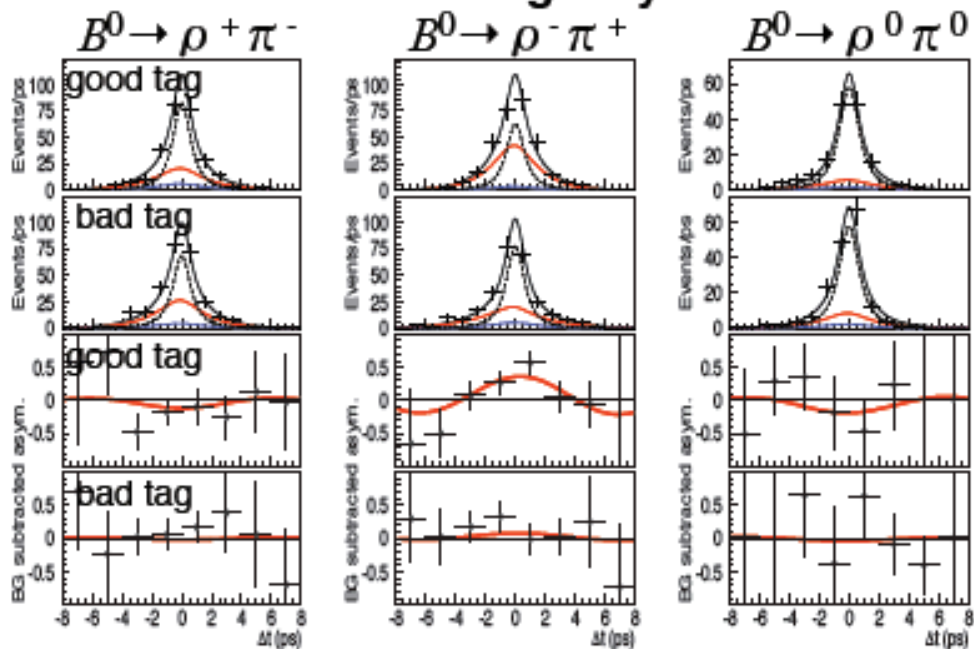
$\phi_2 = (87^{+45}_{-13})^\circ$ (68.3% C.L.)



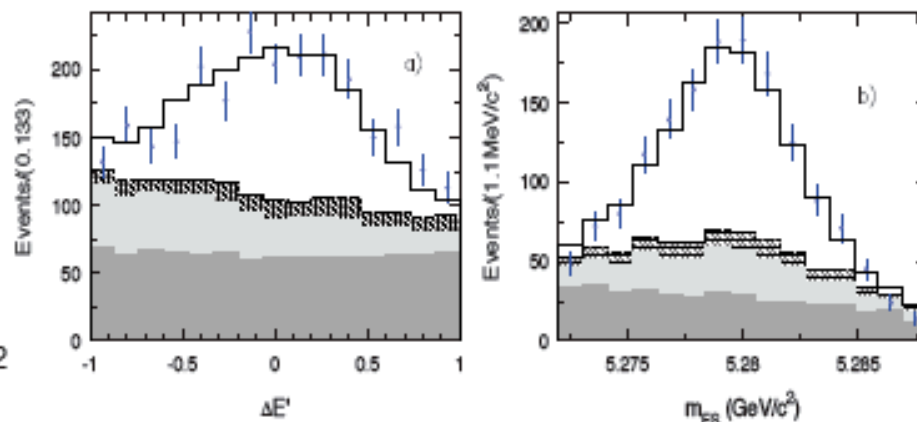
Belle 449M $B\bar{B}$ (PRL98 221602)



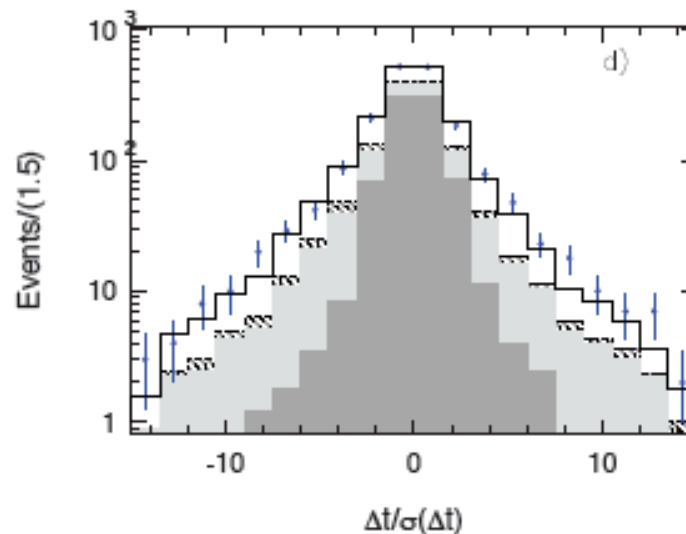
971±41 signal yields



BABAR 375M $B\bar{B}$ (PRD76 012004)



2067±86 signal yields



Multiple solutions

Belle found 4 solutions

	Solution 1	Solution 2	Solution 3	Solution 4
$\mathcal{A}_{CP}(f_0(980)K_S^0)$	$-0.30 \pm 0.29 \pm 0.11 \pm 0.09$	$-0.20 \pm 0.15 \pm 0.08 \pm 0.05$	$+0.02 \pm 0.21 \pm 0.09 \pm 0.09$	$-0.18 \pm 0.14 \pm 0.08 \pm 0.06$
$\phi_1^{\text{eff}}(f_0(980)K_S^0)$	$(31.3 \pm 9.0 \pm 3.4 \pm 4.0)^\circ$	$(26.1 \pm 7.0 \pm 2.4 \pm 2.5)^\circ$	$(25.6 \pm 7.6 \pm 2.9 \pm 0.8)^\circ$	$(26.3 \pm 5.7 \pm 2.4 \pm 5.8)^\circ$
$\mathcal{A}_{CP}(\phi(1020)K_S^0)$	$+0.04 \pm 0.20 \pm 0.10 \pm 0.02$	$+0.08 \pm 0.18 \pm 0.10 \pm 0.03$	$-0.01 \pm 0.20 \pm 0.11 \pm 0.02$	$+0.21 \pm 0.18 \pm 0.11 \pm 0.05$
$\phi_1^{\text{eff}}(\phi(1020)K_S^0)$	$(32.2 \pm 9.0 \pm 2.6 \pm 1.4)^\circ$	$(26.2 \pm 8.8 \pm 2.7 \pm 1.2)^\circ$	$(27.3 \pm 8.6 \pm 2.8 \pm 1.3)^\circ$	$(24.3 \pm 8.0 \pm 2.9 \pm 5.2)^\circ$
$\mathcal{A}_{CP}(\text{others})$	$-0.14 \pm 0.11 \pm 0.08 \pm 0.03$	$-0.06 \pm 0.15 \pm 0.08 \pm 0.04$	$-0.03 \pm 0.09 \pm 0.08 \pm 0.03$	$+0.04 \pm 0.11 \pm 0.08 \pm 0.02$
$\phi_1^{\text{eff}}(\text{others})$	$(24.9 \pm 6.4 \pm 2.1 \pm 2.5)^\circ$	$(29.8 \pm 6.6 \pm 2.1 \pm 1.1)^\circ$	$(26.2 \pm 5.9 \pm 2.3 \pm 1.5)^\circ$	$(23.8 \pm 5.5 \pm 1.9 \pm 6.4)^\circ$

The preferred solution can not be selected by the fit likelihood value alone. With external information, solution 1 is preferred.

BaBar found 2 solutions in low-mass fit, (1) is chosen as nominal.

Name	Solution (1)	Solution (2)	Correlation			
			1	2	3	4
1 $\mathcal{A}_{CP}(\phi K_S^0)$	$0.14 \pm 0.19 \pm 0.02$	0.13 ± 0.18	1.0	-0.09	-0.28	0.09
2 $\beta_{\text{eff}}(\phi K_S^0)$	$0.13 \pm 0.13 \pm 0.02$	0.14 ± 0.14		1.0	0.54	0.65
3 $\mathcal{A}_{CP}(f_0 K_S^0)$	$0.01 \pm 0.26 \pm 0.07$	-0.49 ± 0.25			1.0	0.25
4 $\beta_{\text{eff}}(f_0 K_S^0)$	$0.15 \pm 0.13 \pm 0.03$	3.44 ± 0.19				1.0

Again multiple solutions

Belle found 4 solutions. After ensemble test checks and by using external information, two of them are chosen as possible physical solutions. Solution 1 is preferred ($K^*_0(1430)\pi$ fraction and $K_S\pi$ mass spectrum). (PRD79,072004(2009))

Parameter	Solution 1 ($-2\ln L=18472.5$)	Solution 2 ($-2\ln L=18465.5$)
$A(f_0K_S)$	$0.08\pm 0.19\pm 0.03\pm 0.04$	$0.23\pm 0.19\pm 0.03\pm 0.04$
$\beta(f_0K_S)=\phi_1(f_0K_S)$	$(36.0\pm 9.8\pm 2.1\pm 2.1)^\circ$	$(56.2\pm 10.4\pm 2.1\pm 2.1)^\circ$
$A(\rho^0K_S)$	$-0.05\pm 0.26\pm 0.10\pm 0.03$	$-0.14\pm 0.26\pm 0.10\pm 0.03$
$\beta(\rho^0K_S)=\phi_1(\rho^0K_S)$	$(10.2\pm 8.9\pm 3.0\pm 1.9)^\circ$	$(33.4\pm 10.4\pm 3.0\pm 1.9)^\circ$

Again multiple solutions

Parameter	Solution 1	Solution 2
$C(f_0K_S)=-A(f_0K_S)$	$0.08\pm 0.19\pm 0.03\pm 0.04$	$0.23\pm 0.19\pm 0.03\pm 0.04$
$\beta(f_0K_S)=\phi_1(f_0K_S)$	$(36.0\pm 9.8\pm 2.1\pm 2.1)^\circ$	$(56.2\pm 10.4\pm 2.1\pm 2.1)^\circ$
$C(\rho^0K_S)=-A(\rho^0K_S)$	$-0.05\pm 0.26\pm 0.10\pm 0.03$	$-0.14\pm 0.26\pm 0.10\pm 0.03$
$\beta(\rho^0K_S)=\phi_1(\rho^0K_S)$	$(10.2\pm 8.9\pm 3.0\pm 1.9)^\circ$	$(33.4\pm 10.4\pm 3.0\pm 1.9)^\circ$

BaBar found 2 solutions.
(PRD80,112001(2009))