Rare B decays

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1. $B_s \to \mu^+\mu^-$ — the highlight of 2011/2012

2. Low-energy effective Lagrangian

3. More on $B_s \to l^+l^-$ and $B^0 \to l^+l^-$

4. $\bar{B} \to X_s\gamma$: the photon spectrum and CP asymmetry

5. Processes generated by the quark-level $b \to sl^+l^-$ transition

6. Summary

Not covered:
Purely hadronic decays, $b \to d$ transitions, $b \to s\nu\bar{\nu}$.

$B_s \to \mu^+\mu^-$ — the highlight of 2011/2012

- **Strongly suppressed loop-generated process in the SM**

$$\mathcal{B}_{\mathrm{SM}} = (3.34 \pm 0.21) \times 10^{-9}.$$

- **Very sensitive to new physics even in models with Minimal Flavour Violation (MFV). Enhancements by orders of magnitude possible, even when constraints from all the other measurements are taken into account.**

- **Clear experimental signature: PEAK in the dimuon invariant mass.**

- **CDF bound (arXiv:1107.2304):** $\mathcal{B} < 40 \times 10^{-9}$ @ 95% **CL.**

  If interpreted as observation: $\mathcal{B} = \left(18^{+11}_{-9}\right) \times 10^{-9}.$

- **LHCb & CMS combination at EPS-2011:** $\mathcal{B} < 10.8 \times 10^{-9}$ @ 95% **CL. Since then, the recorded data samples have increased by factors of around three (LHCb) and five (CMS). Updates? ATLAS?**

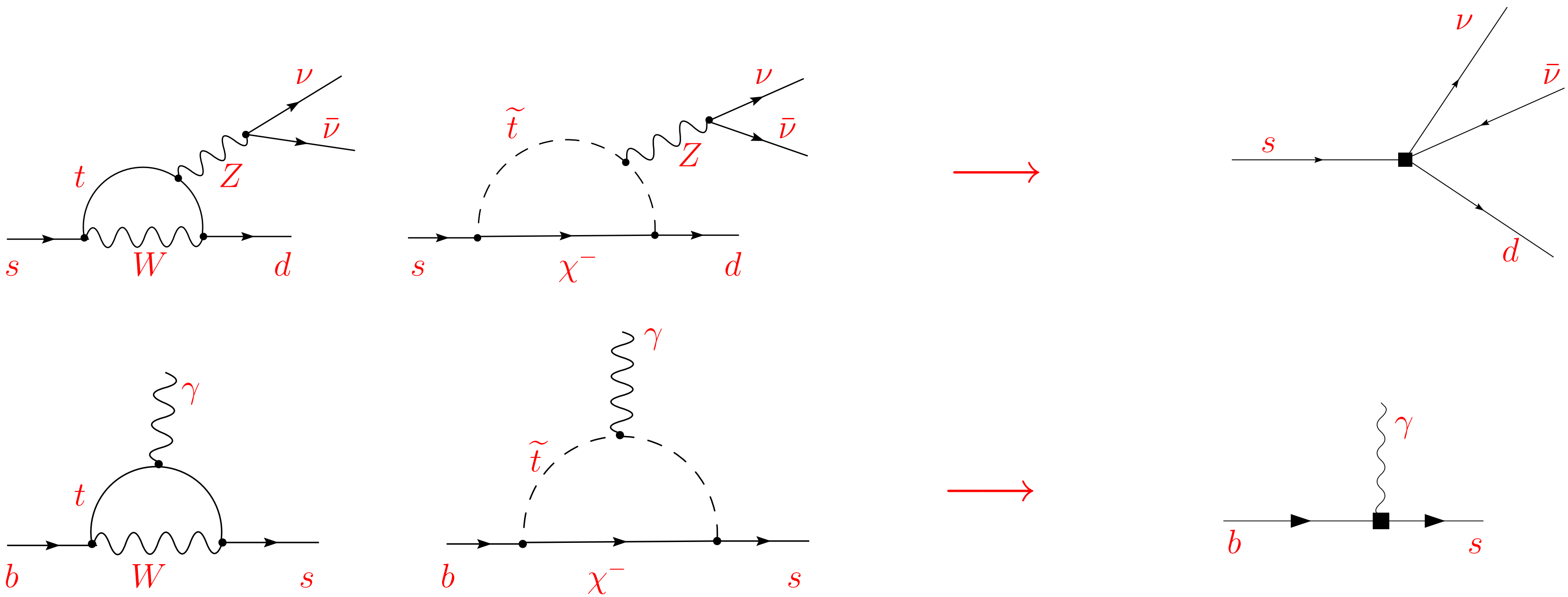
B-meson or Kaon decays occur at low energies, at scales $\mu \ll M_W$.

We pass from the full theory of electroweak interactions to an effective theory by removing the high-energy degrees of freedom, i.e. integrating out the W-boson and all the other particles with $m \sim M_W$.

$$\mathcal{L}_{\text{(full EW}\times\text{QCD)}} \longrightarrow \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}\times\text{QCD}}\left(\begin{array}{l}\text{quarks} \neq t\\ \text{\& leptons}\end{array}\right) + N \sum_n C_n(\mu)\, Q_n$$

Q_n – local interaction terms (operators), C_n – coupling constants (Wilson coefficients)

Information on the electroweak-scale physics is encoded in the values of $C_i(\mu)$, e.g.,



This is a modern version of the Fermi theory for weak interactions. It is "nonrenormalizable" in the traditional sense but actually renormalizable. It is also predictive because all the C_i are calculable, and only a finite number of them is necessary at each given order in the (external momenta)/M_W expansion.

Advantages: **Resummation of $\left(\alpha_s \ln \frac{M_W^2}{\mu^2}\right)^n$ using renormalization group, easier account for symmetries.**

Operators (dim 6) that matter for $B_s \to \mu^+\mu^-$ read

$$Q_A = \left(\bar{b}\gamma^\alpha\gamma_5 s\right)\left(\bar{\mu}\gamma_\alpha\gamma_5\mu\right)$$

$$Q_{S(P)} = \left(\bar{b}\gamma_5 s\right)\left(\bar{\mu}(\gamma_5)\mu\right) \;=\; \frac{i\left(\bar{b}\gamma^\alpha\gamma_5 s\right)\partial_\alpha\left(\bar{\mu}(\gamma_5)\mu\right)}{m_b+m_s} + \underbrace{\boxed{E}}_{\text{vanishing by EOM}} + \underbrace{\boxed{T}}_{\text{total derivative}}$$

Necessary non-perturbative input: $\langle 0|\bar{b}\gamma^\alpha\gamma_5 s|B_s(p)\rangle \;=\; ip^\alpha f_{B_s}$

Recent lattice determinations of the B_s-meson decay constant:

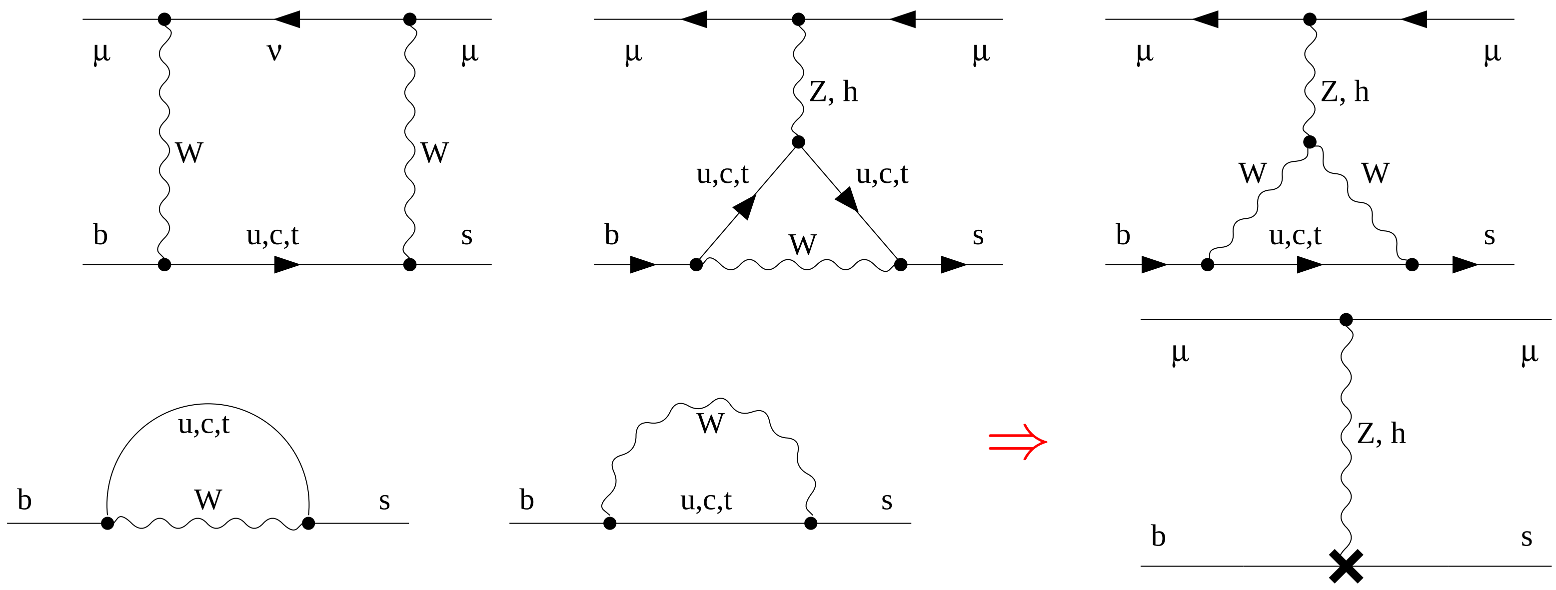
$$f_{B_s} = \begin{cases} 225.0(4.0)\ \text{MeV}, & \text{HPQCD}, \quad \text{arXiv:1110.4150} \\ 242.0(9.5)\ \text{MeV}, & \text{FNAL/MILC}, \quad \text{arXiv:1112.3051} \end{cases}$$

Branching ratio:

$$\mathcal{B}(B_s \to \mu^+\mu^-) = \frac{|N|^2}{8\pi} M_{B_s}^3 f_{B_s}^2 \tau_{B_s}\left[\left|rC_A - uC_P\right|^2 + \left|uC_S\right|^2\left(1-r^2\right)\right]\sqrt{1-r^2}$$

where $\quad N = \frac{V_{tb}^* V_{ts}\, G_F^2 M_W^2}{\pi^2}, \qquad r = \frac{2m_\mu}{M_{B_s}} \quad$ and $\quad u = \frac{M_{B_s}}{m_b+m_s}.$

Evaluation of the Wilson coefficients in the SM:



$$C_A = \frac{1}{2} Y_0\left(m_t^2/M_W^2\right), \qquad Y_0(x) = \frac{3x^2}{8(x-1)^2}\ln x + \frac{x^2-4x}{8(x-1)},$$

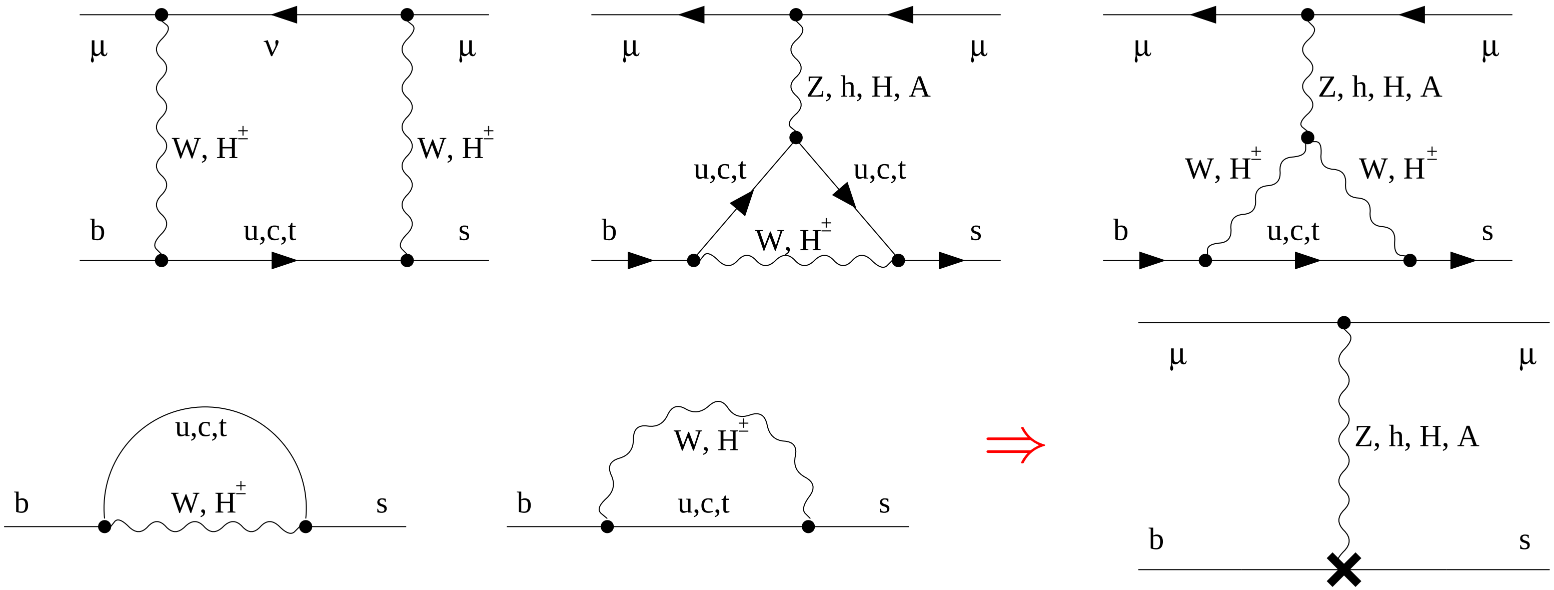
$$C_{S,P} = \mathcal{O}\left(\frac{m_\mu}{M_W}\right).$$

Effects of $C_{S,P}$ are on the branching ratio are suppressed by $M_{B_s}^2/M_W^2$ $\Rightarrow$ negligible.

Thus, only C_A matters in the SM.

Evaluation of the Wilson coefficients beyond the SM.

Example 1: the Two-Higgs-Doublet Model II



$$\tan\beta = v_2/v_1, \qquad z = M_{H^\pm}^2/m_t^2,$$

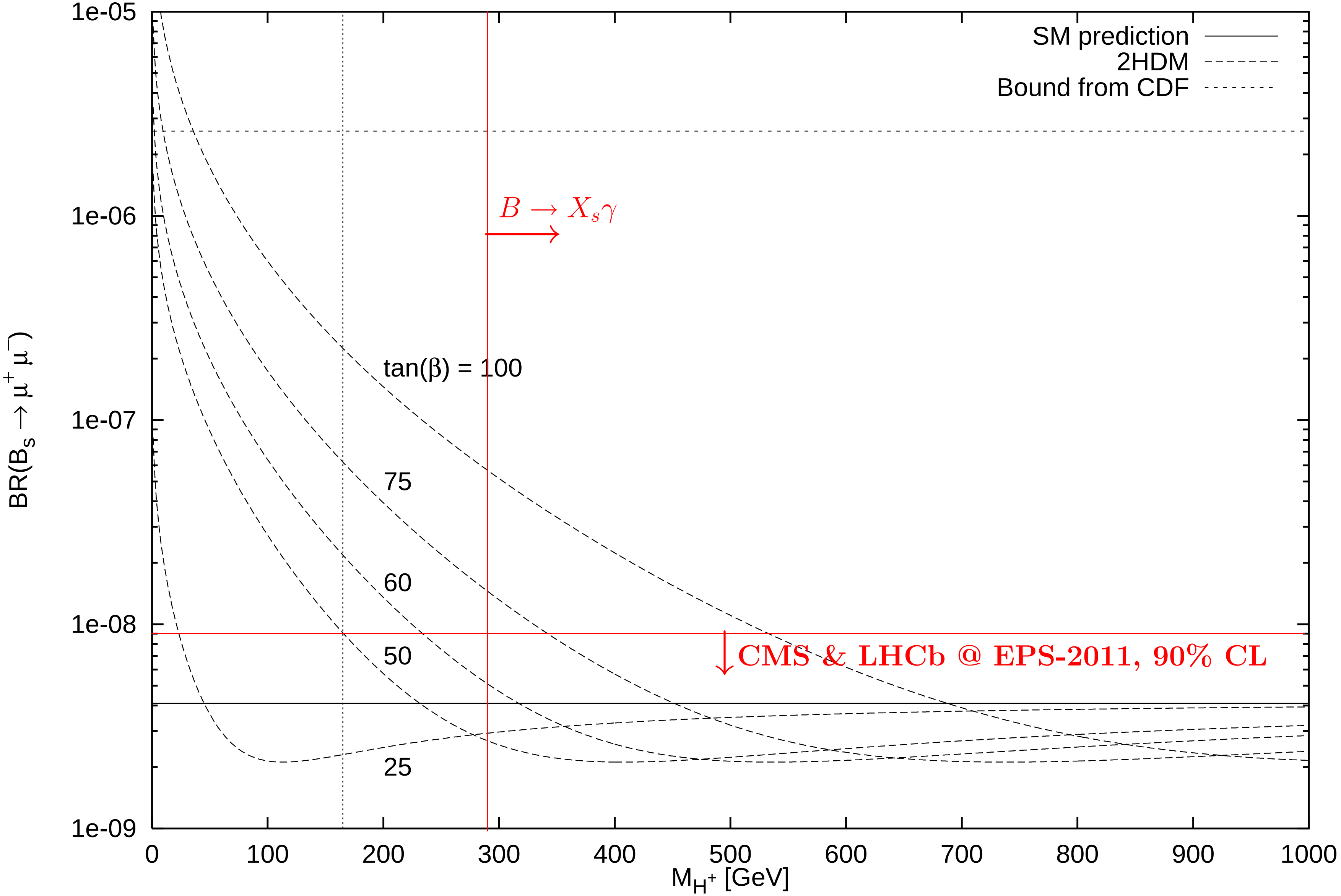
$$C_S \simeq C_P \simeq \frac{m_\mu m_b \tan^2\beta}{4M_W^2}\,\frac{\ln z}{z-1} > 0,$$

H.E. Logan and U. Nierste, NPB 586 (2000) 39 ($\mathcal{O}(\tan\beta)$ neglected)

$$\mathcal{B}(B_s \to \mu^+\mu^-) \simeq (\text{const.}) \left[\left| \frac{2m_\mu}{M_{B_s}} C_A - C_P \right|^2 + |C_S|^2 \right]$$

$$C_A = \underbrace{C_A^{\rm SM}}_{\text{positive}} + \underbrace{\Delta C_A}_{\text{small}} \quad \Rightarrow \begin{cases} \text{suppression for moderate } C_{S,P} \\ \text{enhancement for huge } \tan\beta \text{ only} \end{cases}$$

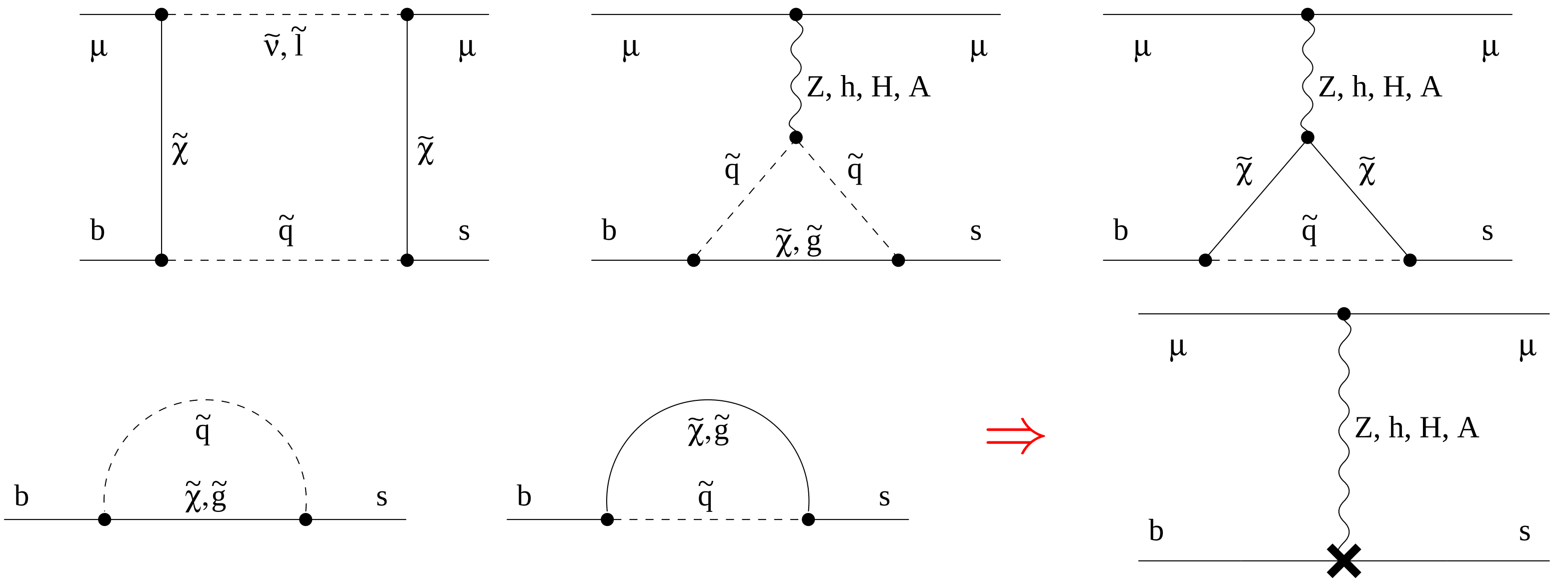
Fig. 3 of Logan & Nierste, hep-ph/0004139:



For $M_{H^\pm} = 500 GeV$ and $\tan\beta = 50$: suppression by a factor of ~ 2.
Enhancement possible only for $\tan\beta > 60$.

Evaluation of the Wilson coefficients beyond the SM.

Example 2: the MSSM.



For large $\tan\beta$: $$\mathcal{B}\left(B_s \to \mu^+\mu^-\right) \sim \frac{m_b^2 m_\mu^2}{M_A^4} \tan^6\beta$$

K. S. Babu and C. F. Kolda, Phys. Rev. Lett. 84 (2000) 228.

Example of constraints on the MSSM parameter space:

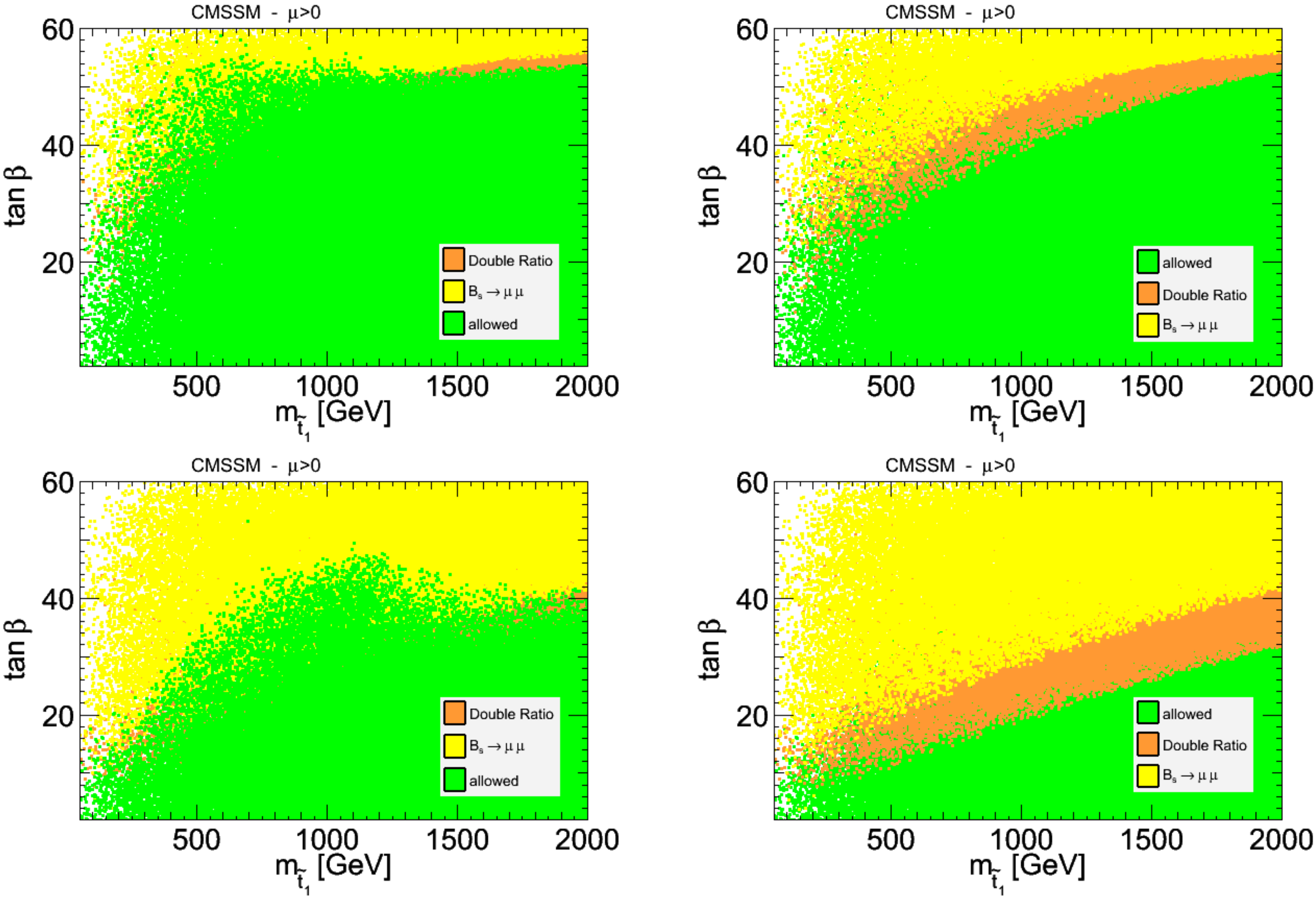


Fig. 17 from A.G. Akeroyd, F. Mahmoudi, D. Martínez Santos, arXiv:1108.3018 (Aug 15th, 2011). Constraints from $\mathcal{B}(B_s \to \mu^+\mu^-)$ in the CMSSM parameter planes $(m_{\tilde{t}_1}, \tan\beta)$ with the current LHC limits (upper plots) and for a hypothetical SM-like measurement $(3.5 \pm 0.3) \times 10^{-9}$ (lower plots). The allowed (green) points are in the background (left plots) or foreground (right plots).

Error budget for $B_s \to \mu^+\mu^-$ in the SM

$$\mathcal{B}(B_s \to \mu^+\mu^-)_{\rm SM} = \frac{G_F^4 M_W^4 m_\mu^2 M_{B_s}}{8\pi^5} \times$$

$$\times \underbrace{|V_{tb}^* V_{ts}|^2}_{\pm 3.5\%} \underbrace{\tau_{B_s}}_{\pm 1.8\%} \left\{ \underbrace{f_{B_s}^2}_{\pm(3.6 \div 7.9)\%} \left[\underbrace{Y_0\left(\frac{m_t^2}{M_W^2}\right) + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha}_{\pm 1.6\%(\text{for } \overline{m}_t(\overline{m}_t) = 165(1)\ \text{GeV})} \underbrace{{}_{\rm em}) + \mathcal{O}(\alpha_s^2)}_{\pm 3\%(\text{unknown})} \right]^2 + \mathcal{O}(\alpha_{\rm em}) \right\}$$

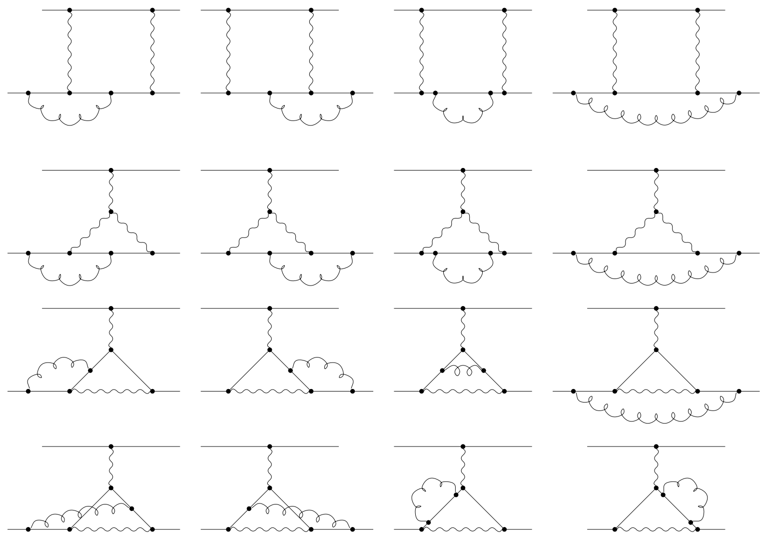
$$= \begin{cases} (3.34 \pm \underbrace{0.21}_{6.3\%}) \times 10^{-9} & \text{for } f_{B_s} = 225.0(4.0)\ \text{MeV} \quad \text{[HPQCD, arXiv:1110.4150]} \\ (3.86 \pm \underbrace{0.36}_{9.3\%}) \times 10^{-9} & \text{for } f_{B_s} = 242.0(9.5)\ \text{MeV} \quad \text{[FNAL/MILC, arXiv:1112.3051]} \end{cases}$$

The $\mathcal{O}(\alpha_s)$ corrections enhance the branching ratio by around $+2.2\%$ when $\overline{m}_t(\overline{m}_t)$ is used at the leading order.

G. Buchalla and A.J. Buras, Nucl. Phys. B 400 (1993) 225,
MM and J. Urban, Phys. Lett. B 451 (1999) 161,
G. Buchalla and A.J. Buras, Nucl. Phys. B 548 (1999) 309.

Logarithmically $\left(\ln\left(m_t^2/m_b^2\right)\right)$ enhanced electromagnetic corrections and the known electroweak corrections suppress the branching ratio by around -1.7%.

G. Buchalla, A. J. Buras, Phys. Rev. D 57 (1998) 216,
C. Bobeth, P. Gambino, M. Gorbahn, U. Haisch, JHEP 0404 (2004) 071,
T. Huber, E. Lunghi, MM, D. Wyler, Nucl. Phys. B 740 (2006) 105.



Another observable:

(with different NP sensitivity)

$$\frac{\mathcal{B}(B_s \to \mu^+\mu^-)}{\Delta M_{B_s} \tau_{B_s}}$$

A. J. Buras, Phys. Lett. B 566 (2003) 115..

Summary of SM predictions for all the $B_q \to l^+ l^-$ decays:

[arXiv:0801.1833, WG2 report, "Flavor in the Era of the LHC"]

$$\mathcal{B}\left(B_s \to \tau^+\tau^-\right) = (8.20 \pm 0.31) \cdot 10^{-7} \times \frac{\tau_{B_s}}{1.527\,\mathbf{ps}} \left[\frac{|V_{ts}|}{0.0408}\right]^2 \left[\frac{f_{B_s}}{240\text{ MeV}}\right]^2$$

$$\mathcal{B}\left(B_s \to \mu^+\mu^-\right) = (3.86 \pm 0.15) \cdot 10^{-9} \times \frac{\tau_{B_s}}{1.527\,\mathbf{ps}} \left[\frac{|V_{ts}|}{0.0408}\right]^2 \left[\frac{f_{B_s}}{240\text{ MeV}}\right]^2$$

$$\mathcal{B}\left(B_s \to e^+e^-\right) = (9.05 \pm 0.34) \cdot 10^{-14} \times \frac{\tau_{B_s}}{1.527\,\mathbf{ps}} \left[\frac{|V_{ts}|}{0.0408}\right]^2 \left[\frac{f_{B_s}}{240\text{ MeV}}\right]^2$$

$$\mathcal{B}\left(B_d \to \tau^+\tau^-\right) = (2.23 \pm 0.08) \cdot 10^{-8} \times \frac{\tau_{B_d}}{1.527\,\mathbf{ps}} \left[\frac{|V_{td}|}{0.0082}\right]^2 \left[\frac{f_{B_d}}{200\text{ MeV}}\right]^2$$

$$\mathcal{B}\left(B_d \to \mu^+\mu^-\right) = (1.06 \pm 0.04) \cdot 10^{-10} \times \frac{\tau_{B_d}}{1.527\,\mathbf{ps}} \left[\frac{|V_{td}|}{0.0082}\right]^2 \left[\frac{f_{B_d}}{200\text{ MeV}}\right]^2$$

$$\mathcal{B}\left(B_d \to e^+e^-\right) = (2.49 \pm 0.09) \cdot 10^{-15} \times \frac{\tau_{B_d}}{1.527\,\mathbf{ps}} \left[\frac{|V_{td}|}{0.0082}\right]^2 \left[\frac{f_{B_d}}{200\text{ MeV}}\right]^2$$

The exp. 90%CL bounds are above the SM predictions by factors of
2.6, **35**, $\mathcal{O}(10^6)$, $\mathcal{O}(10^7)$ **and** $\mathcal{O}(10^5)$ **for**
$B_s \to \mu^+\mu^-$, $B_d \to \mu^+\mu^-$, $B_s \to e^+e^-$, $B_d \to e^+e^-$ **and** $B_{s,d} \to \tau^+\tau^-$,
respectively.

What other rare B decays are interesting?

Scenario "A" (Attractive or Arbitrary)

Generic Flavour Violation (GFV) in interactions of new particles with the SM ones.

- **Large deviations from the SM values of the Wilson coefficients possible.**
- **Observable new physics effects despite QCD-induced theory uncertainties in many FCNC decays of the B meson (penguin-induced exclusive hadronic decays, $B \to K^*\gamma$, $B \to K^{(*)} l^+ l^-$, etc.).**
- **Interesting constraints from branching ratios, angular distributions and various asymmetries.**

Scenario "B" (Boring or Beautiful)

Quite heavy new particles and Minimal Flavour Violation (MFV)

- **Only mild deviations of the Wilson coefficients from their SM values expected.**
- **CP-asymmetries unaffected.**
- **Precise measurements needed. $\Rightarrow$ Not too small rates welcome. $\Rightarrow$ $b \to s$ preferred over $b \to d$.**
- **Precise TH predictions in the SM case needed $\Rightarrow$ Inclusive rather than exclusive hadronic final states welcome.**
- **Suppression in the SM due to parameters other than CKM angles welcome.**
- **Apart from $B \to l^+ l^-$, the inclusive decay $\bar{B} \to X_s \gamma$ is of main interest. Other inclusive decays like $\bar{B} \to X_s \nu \bar{\nu}$, $\bar{B} \to X_s l^+ l^-$ are unsuppressed in the SM, but still deserve consideration.**
- **Exclusive observables (asymmetries) may still be useful to resolve discrete ambiguities (e.g. sign of the $b \to s\gamma$ amplitude).**

4. The $\bar{B} \to X_s\gamma$ photon spectrum and CP asymmetry

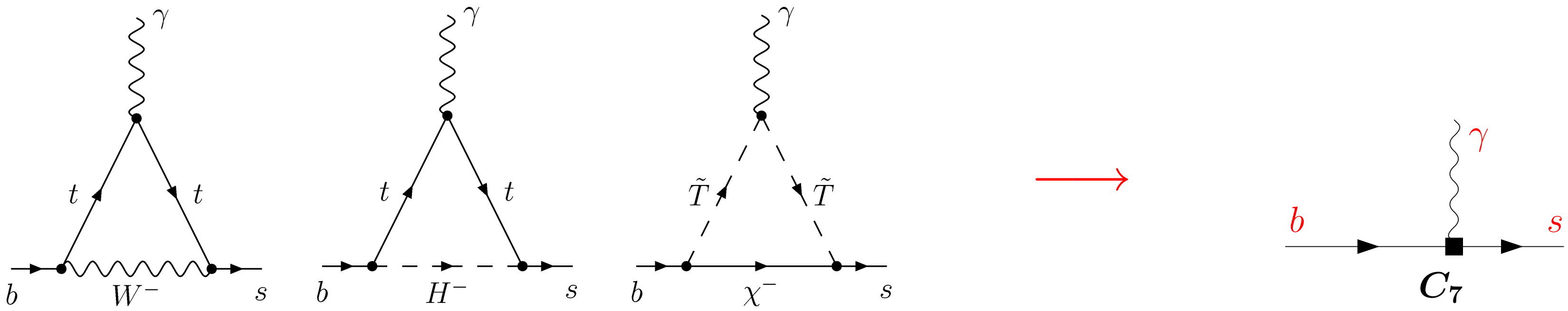
4a. Photon spectrum and cuts

4b. Non-perturbative uncertainties in the decay rate

4c. Isospin asymmetry

4d. Direct CP asymmetry

Information on electroweak-scale physics in the $b \to s\gamma$ transition is encoded in an effective low-energy local interaction:



$$b \in \bar{B} \equiv (\bar{B}^0 \text{ or } B^-)$$

The inclusive $\bar{B} \to X_s\gamma$ decay rate is well approximated by the corresponding perturbative decay rate of the b-quark:

$$\Gamma(\bar{B} \to X_s\,\gamma)_{E_\gamma > E_0} \;=\; \Gamma(b \to X_s^p\,\gamma)_{E_\gamma > E_0} \;+\; \left(\begin{array}{c} \text{non-perturbative effects} \\ (2 \pm 5)\% \\ \text{Benzke et al., arXiv:1003.5012} \end{array} \right)$$

provided E_0 is large $(E_0 \sim m_b/2)$
but not too close to the endpoint $(m_b - 2E_0 \gg \Lambda_{\rm QCD})$.

Conventionally, $E_0 = 1.6\,\text{GeV} \simeq m_b/3$ is chosen.

Results of the SM calculations:

$$\mathcal{B}(\bar{B} \to X_s \gamma)_{E_\gamma > 1.6\ \mathrm{GeV}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, & \text{MM \textit{et al.}, hep-ph/0609232, using the 1S scheme.} \\ (3.26 \pm 0.24) \times 10^{-4}, & \text{following the kinetic scheme analysis of P. Gambino and P. Giordano in arXiv:0805.0271.} \end{cases}$$

Contributions to the total TH uncertainty (summed in quadrature):

5% **non-perturbative,** **3%** m_c**-interpolation ambiguity at the NNLO** **(to be reduced soon),**

3% **higher order** $\mathcal{O}(\alpha_s^3)$**,** **3%** **parametric** $\left(\alpha_s(M_Z),\ \mathcal{B}^{\mathrm{exp}}_{\mathrm{semileptonic}},\ m_c\ \&\ C,\ \ldots\right)$.

2.0% **1.6%** **1.1% (1S)** **2.5% (kin)**

Experimental world averages:

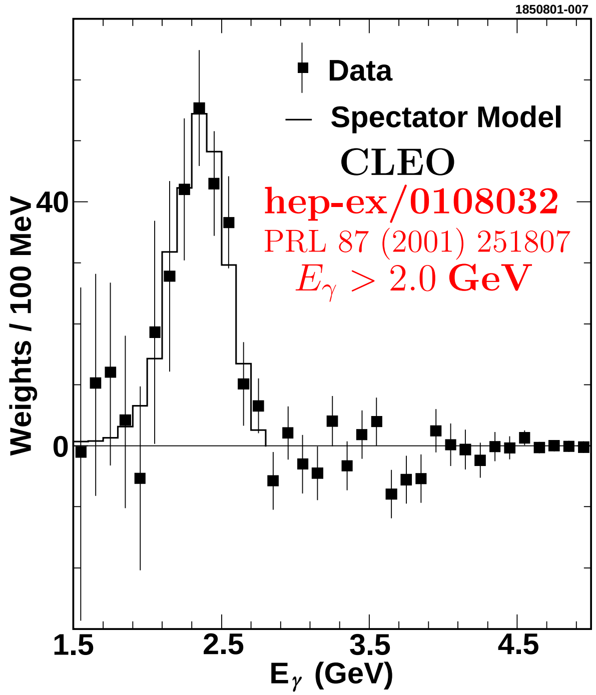
$$\mathcal{B}(\bar{B} \to X_s \gamma)^{\mathrm{EXP}}_{E_\gamma > 1.6\ \mathrm{GeV}} = \begin{cases} (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}, & \text{[HFAG, arXiv:1010.1589],} \\ (3.50 \pm 0.17) \times 10^{-4}, & \text{[Artuso, Barberio, Stone, arXiv:0902.3743].} \end{cases}$$

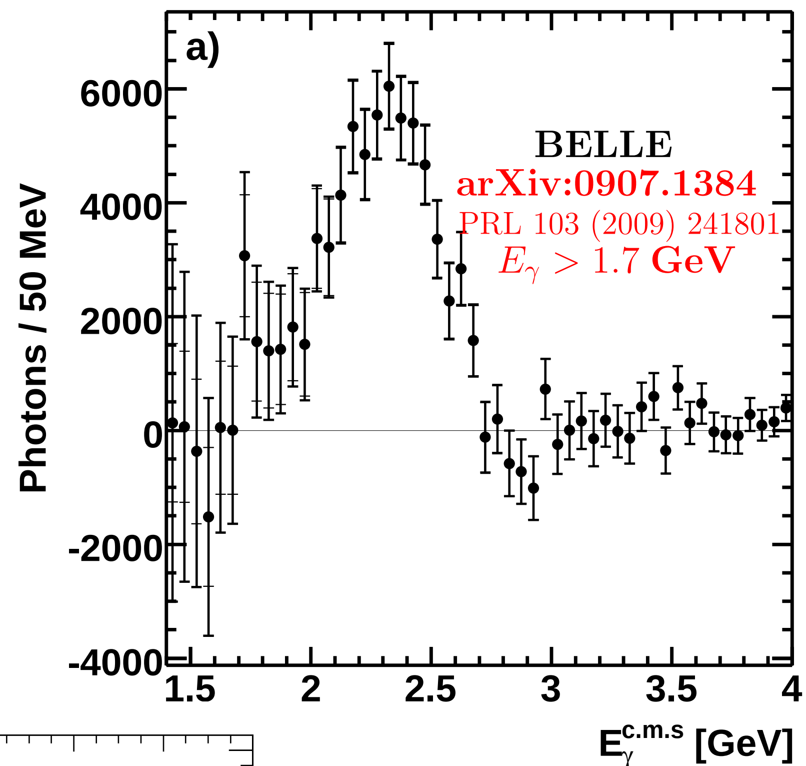
Experiment agrees with the SM at the $\sim 1.2\sigma$ **level. Uncertainties:** **TH** $\sim 7\%$, **EXP** $\sim 7\%$.

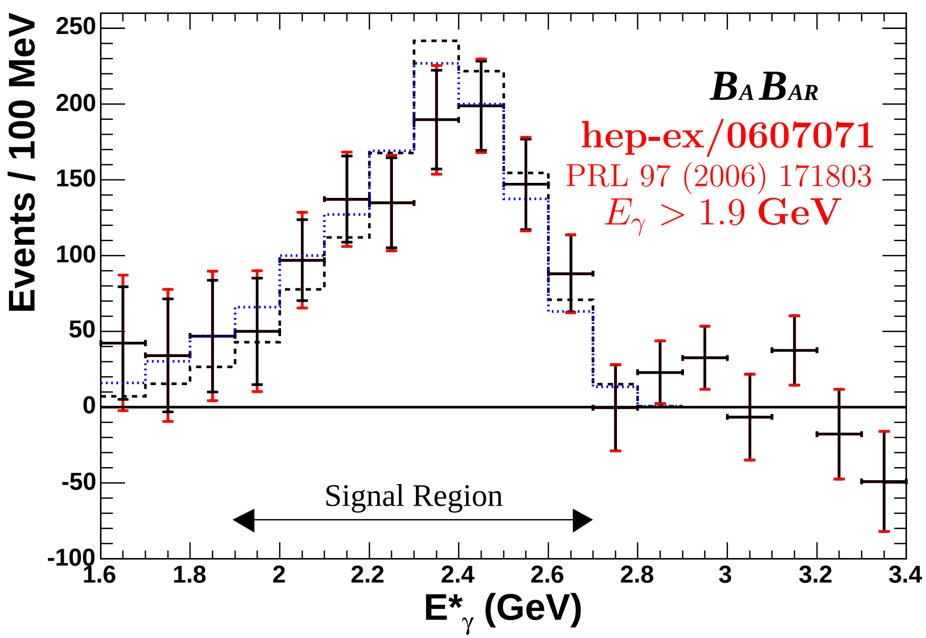
The HFAG average includes the following measurements:

Reference	Method	# of $B\bar{B}$	E_0 [GeV]	$\mathcal{B} \times 10^4$ at E_0
CLEO [PRL 87 (2001) 251807]	inclusive	9.70×10^6	2.0	$2.94 \pm 0.41 \pm 0.26$
BABAR [PRL 97 (2006) 171803]	inclusive	8.85×10^7	1.9 2.0 2.1 2.2	$3.67 \pm 0.29 \pm 0.34 \pm 0.29$ $3.41 \pm 0.27 \pm 0.29 \pm 0.23$ $2.97 \pm 0.24 \pm 0.25 \pm 0.17$ $2.42 \pm 0.21 \pm 0.20 \pm 0.13$
BELLE [PRL 103 (2009) 241801]	inclusive	6.57×10^8	1.7 1.8 1.9 2.0	$3.45 \pm 0.15 \pm 0.40$ $3.36 \pm 0.13 \pm 0.25$ $3.21 \pm 0.11 \pm 0.16$ $3.02 \pm 0.10 \pm 0.11$
BABAR [PRD 77 (2008) 051103]	inclusive with a hadronic tag (hadronic decay of the recoiling B ($\bar{B}$))	2.32×10^8, which gives 6.8×10^5 tagged events	1.9 2.0 2.1 2.2 2.3	$3.66 \pm 0.85 \pm 0.60$ $3.39 \pm 0.64 \pm 0.47$ $2.78 \pm 0.48 \pm 0.35$ $2.48 \pm 0.38 \pm 0.27$ $2.07 \pm 0.30 \pm 0.20$
BABAR [PRD 72 (2005) 052004]	semi-inclusive	8.89×10^7	1.9	$3.27 \pm 0.18^{+0.55+0.04}_{-0.40-0.09}$
BELLE [PLB 511 (2001) 151]	semi-inclusive	6.07×10^6	?	$3.36 \pm 0.53 \pm 0.42^{+0.50}_{-0.54}$

The "raw" photon energy spectra in the inclusive measurements







The peaks are centered around

$$\tfrac{1}{2} m_b \simeq 2.35 \text{ GeV}$$

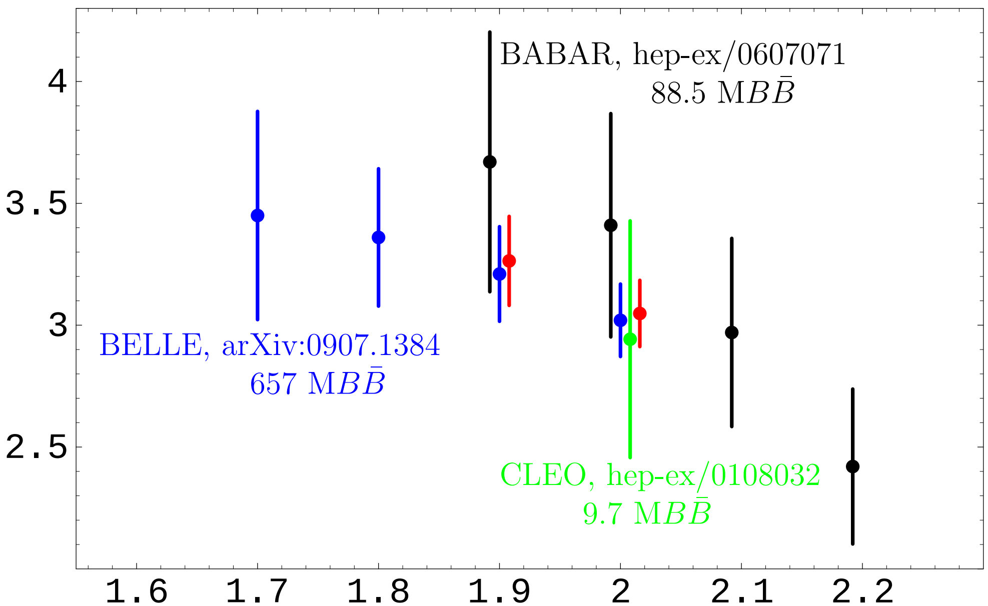
which corresponds to a two-body $b \to s\gamma$ decay.

Broadening is due to (mainly):

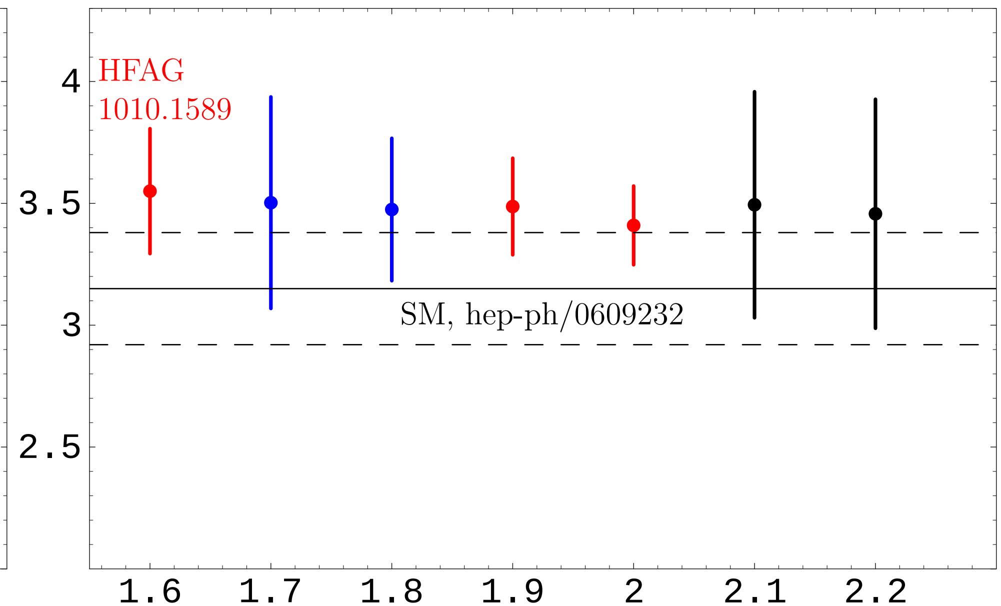
- perturbative gluon bremsstrahlung,
- motion of the b quark inside the $\bar{B}$ meson,
- motion of the $\bar{B}$ meson in the $\Upsilon(4S)$ frame.

Comparison of the inclusive measurements of $\mathcal{B}(\bar{B} \to X_s\gamma)$ by CLEO, BELLE and BABAR for each E_0 separately

$\mathcal{B} \times 10^4$ for each E_0 [GeV]



Averages for each E_0 extrapolated to $E_0 = 1.6$ GeV using the HFAG factors



The HFAG factors

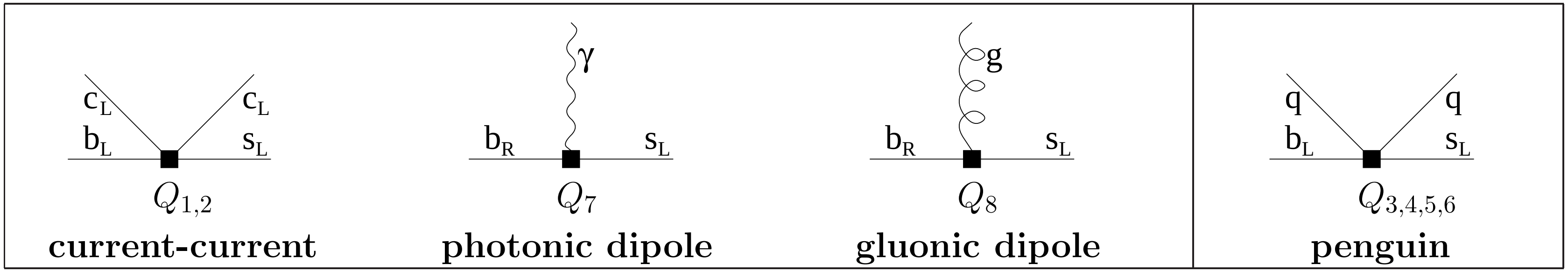
Scheme	$E_\gamma < 1.7$	$E_\gamma < 1.8$	$E_\gamma < 1.9$	$E_\gamma < 2.0$	$E_\gamma < 2.242$
Kinetic	0.986 ± 0.001	0.968 ± 0.002	0.939 ± 0.005	0.903 ± 0.009	0.656 ± 0.031
Neubert SF	0.982 ± 0.002	0.962 ± 0.004	0.930 ± 0.008	0.888 ± 0.014	0.665 ± 0.035
Kagan-Neubert	0.988 ± 0.002	0.970 ± 0.005	0.940 ± 0.009	0.892 ± 0.014	0.643 ± 0.033
Average	0.985 ± 0.004	0.967 ± 0.006	0.936 ± 0.010	0.894 ± 0.016	0.655 ± 0.037

- **Why do we need to extrapolate to lower E_0?**
- **Are the HFAG factors trustworthy?**

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak interaction Lagrangian:

$$L_{\text{weak}} \sim \sum C_i(\mu_b)\, Q_i$$

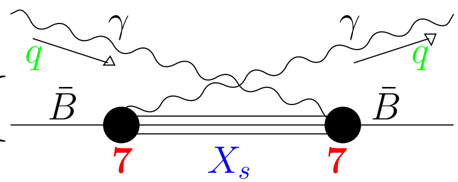
8 operators matter in the SM when the higher-order EW and/or CKM-suppressed effects are neglected:



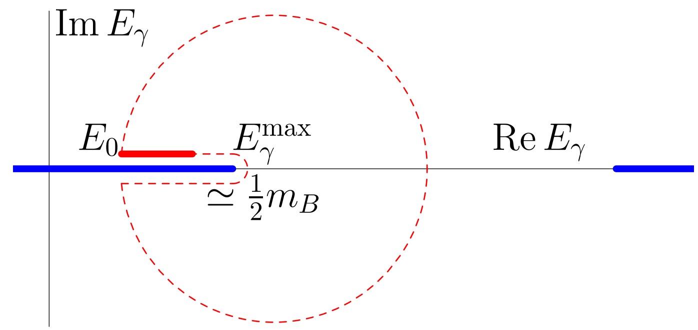
$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = |C_7|^2\, \Gamma_{77}(E_0) + (\text{other})$$

Optical theorem:

$$\frac{d\Gamma_{77}}{dE_\gamma} \sim \mathrm{Im}\{\quad\} \equiv \mathrm{Im}\, A$$



Integrating the amplitude A over E_γ:



OPE on the ring $\Rightarrow$ Non-perturbative corrections to $\Gamma_{77}(E_0)$ form a series in $\frac{\Lambda_{\text{QCD}}}{m_b}$ and α_s that begins with

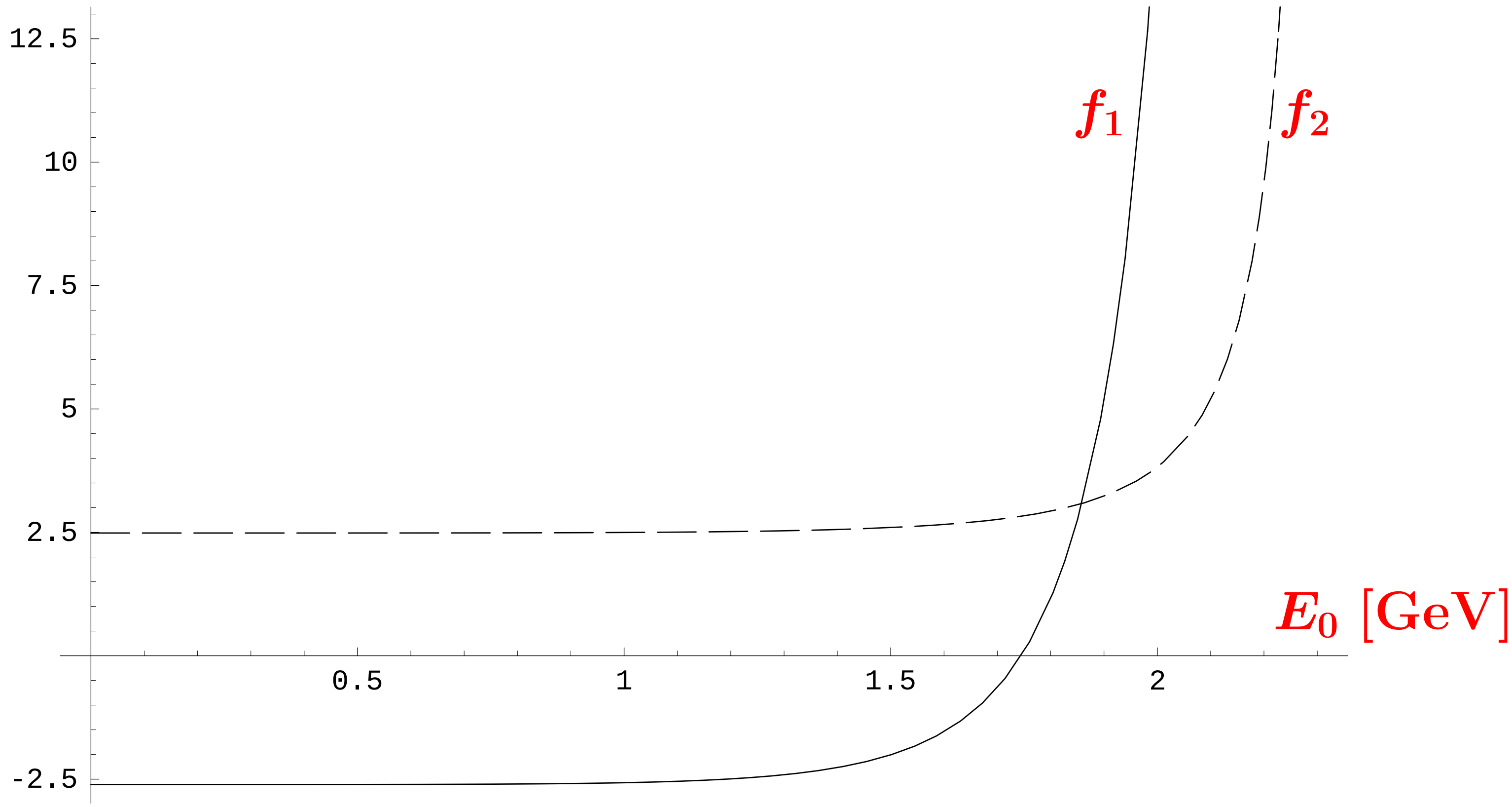
$$\frac{\mu_\pi^2}{m_b^2},\ \frac{\mu_G^2}{m_b^2},\ \frac{\rho_D^3}{m_b^3},\ \frac{\rho_{LS}^3}{m_b^3},\ldots;\ \frac{\alpha_s\mu_\pi^2}{(m_b-2E_0)^2},\ \frac{\alpha_s\mu_G^2}{m_b(m_b-2E_0)};\ldots,$$

where $\mu_\pi, \mu_G, \rho_D, \rho_{LS} = \mathcal{O}(\Lambda_{\text{QCD}})$ are extracted from the semileptonic $\bar{B} \to X_c e\bar{\nu}$ spectra and the B–$B^\star$ mass difference.

The $\mathcal{O}\left(\frac{\alpha_s \mu_\pi^2}{(m_b-2E_0)^2}\right)$ and $\mathcal{O}\left(\frac{\alpha_s \mu_G^2}{m_b(m_b-2E_0)}\right)$ corrections

[T. Ewerth, P. Gambino and S. Nandi, arXiv:0911.2175]

$$\Gamma_{77}(E_0) = \Gamma_{77}^{\text{tree}} \Big\{ 1 + (\text{pert. corrections}) - \frac{\mu_\pi^2}{2m_b^2}\left[1 + \frac{\alpha_s}{\pi}\left(f_1(E_0) - \frac{4}{3}\ln\frac{\mu}{m_b}\right)\right] - \frac{3\mu_G^2(\mu)}{2m_b^2}\left[1 + \frac{\alpha_s}{\pi}\left(f_2(E_0) + \frac{1}{6}\ln\frac{\mu}{m_b}\right)\right]\Big\}$$



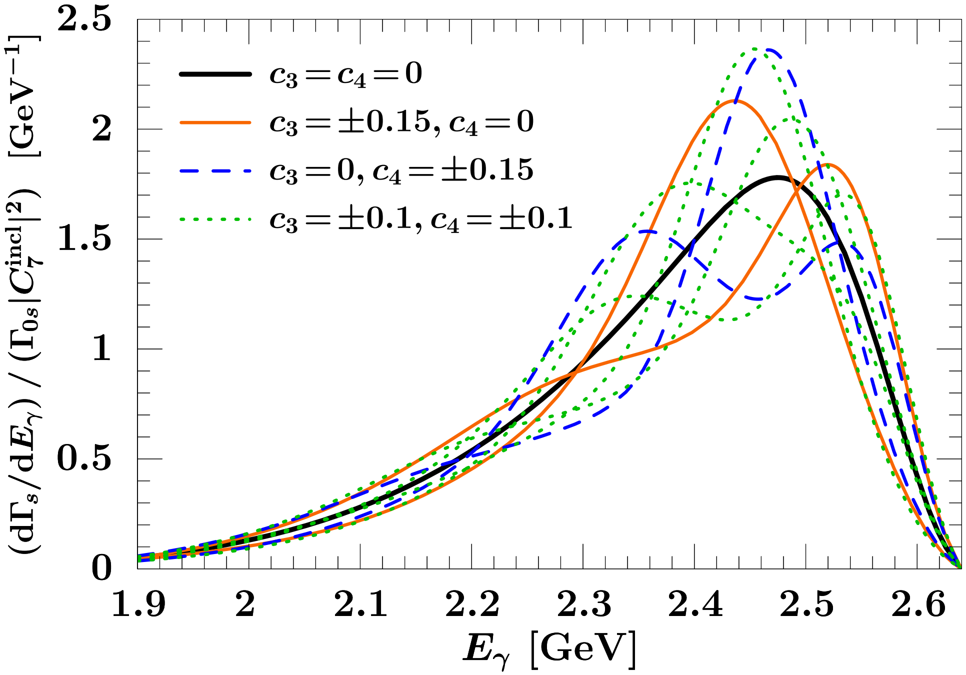
When $(m_b - 2E_0) \sim \Lambda \equiv \Lambda_{\rm QCD}$, no OPE can be applied.

Local operators $\longrightarrow$ Non-local operators

Non-perturbative parameters $\longrightarrow$ Non-perturbative functions

$$\frac{d}{dE_\gamma}\Gamma_{77} = N \underbrace{H(E_\gamma)}_{\text{pert.}} \int_0^{M_B-2E_\gamma} dk \; \underbrace{P(M_B - 2E_\gamma - k)}_{\text{pert.}} \; \underbrace{F(k)}_{\text{non-pert.}} + \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$$

Photon spectra from models of $F(k)$ [Ligeti, Stewart, Tackmann, arXiv:0807.1926]



The function $F(k)$ is:

- perturbatively related to the standard shape function $S(\omega)$,
- exponentially suppressed for $k \gg \Lambda$,
- positive definite,
- constrained by measured moments of the $\bar{B} \to X_c e \bar{\nu}$ spectrum (local OPE),
- constrained by measured properties of the $\bar{B} \to X_u e \bar{\nu}$ and $\bar{B} \to X_s \gamma$ spectra (not imposed in the plot).

Upgrading the HFAG factors by fitting $F(k)$ to data:

- The SIMBA Collaboration [arXiv:1101.3310] (work in progress)

$$F(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n f_n\left(\frac{k}{\lambda}\right)\right]^2, \qquad f_n \text{ – basis functions. Truncate and fit.}$$

- Another way: $F(k) = A(k)B(k)$ and use the SIMBA approach for $B(k)$.

perfect fit

Why do we need to upgrade the HFAG factors?

- The old models (Kagan-Neubert 1998, ...) are not generic enough (too few parameters).
- Inclusion of $\mathcal{O}\left(\frac{\Lambda}{m_b}\right)$ effects and and taking other operators $(Q_i \neq Q_7)$ into account is necessary [Benzke, Lee, Neubert, Paz, arXiv:1003.5012].

What about just fitting C_7 without extrapolation to any particular E_0?

- Fine, but measurements at low E_0 (even less precise) are still going to be crucial for constraining the parameter space.
- The fits are going to give the extrapolation factors anyway. Publishing them is necessary for cross-checks/upgrades by other groups.

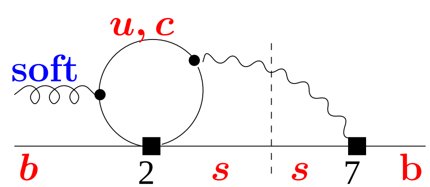
Non-perturbative effects in the presence of other operators $(Q_i \neq Q_7)$

[Benzke, Lee, Neubert, Paz, arXiv:1003.5012].

$$\frac{d}{dE_\gamma}\Gamma(\bar{B} \to X_s\gamma) = (\Gamma_{77}\text{-like term}) \;+\; \tilde{N} E_\gamma^3 \sum_{i \leq j} \mathrm{Re}\left(C_i^* C_j\right) F_{ij}(E_\gamma).$$

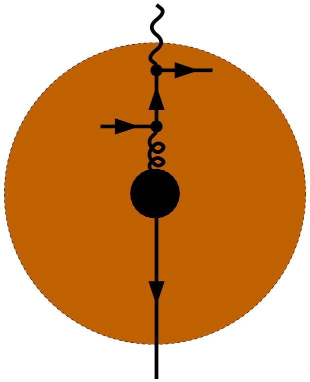
Remarks:

- The SCET approach is valid for large E_γ only. It is fine for $E_\gamma > E_0 \sim \frac{1}{3} m_b \simeq 1.6$ GeV. Lower cutoffs are academic anyway.
- For such E_0, non-perturbative effects in the integrated decay rate are estimated to remain within 5%. They scale like:



  - $\frac{\Lambda^2}{m_b^2}$, $\frac{\Lambda^2}{m_c^2}$ (known),
  - $\frac{\Lambda}{m_b}\frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}}$ (negligible),
  - $\frac{\Lambda}{m_b}$, $\frac{\Lambda^2}{m_b^2}$, $\alpha_s \frac{\Lambda}{m_b}$ but suppressed by tails of subleading shape functions ("27"),
  - $\alpha_s \frac{\Lambda}{m_b}$ to be constrained by future measurements of the isospin asymmetry ("78"),
  - $\alpha_s \frac{\Lambda}{m_b}$ but suppressed by $Q_d^2 = \frac{1}{9}$ ("88").
- Extrapolation factors? Tails of subleading functions are less important for them.

Importance of the isospin asymmetry



A hard gluon scatters on the valence quark or a "sea" quark and produces an energetic photon. The quark that undergoes this Compton-like scattering is assumed to remain soft in the $\bar{B}$-meson rest frame to ensure effective interference with the leading "hard" amplitude. Without interference the contribution would be negligible ($\mathcal{O}(\alpha_s^2 \Lambda^2/m_b^2)$).

Suppression by Λ can be understood as originating from dilution of the target (size of the $\bar{B}$-meson $\sim \Lambda^{-1}$).

A rough estimate using vacuum insertion approximation gives

$$\Delta\Gamma/\Gamma \in [-2.8\%, -0.3\%] \qquad (\mathcal{O}(\alpha_s \Lambda/m_b)).$$

[Lee, Neubert, Paz, hep-ph/0609224]
[Benzke, Lee, Neubert, Paz, arXiv:1003.5012]

However:

1. Contribution to the interference from scattering on the "sea" quarks vanishes in the $SU(3)_{\rm flavour}$ limit because $Q_u + Q_d + Q_s = 0$.

2. If the valence quark dominates, then the isospin-averaged $\Delta\Gamma/\Gamma$ is given by:

$$\frac{\Delta\Gamma}{\Gamma} \simeq \frac{Q_d + Q_u}{Q_d - Q_u} \Delta_{0-} = \left(+0.2 \pm 1.9_{\rm stat} \pm 0.3_{\rm sys} \pm 0.8_{\rm ident}\right)\%,$$

using the BABAR semi-inclusive measurement (hep-ex/0508004) of the isospin asymmetry

$$\Delta_{0-} = [\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)]/[\Gamma(\bar{B}^0 \to X_s\gamma) + \Gamma(B^- \to X_s\gamma)],$$

for $E_\gamma > 1.9$ GeV.

Quark-to-photon conversion gives a soft s-quark and poorly interferes with the "hard" $b \to s\gamma g$ amplitude.

The direct CP asymmetry

$$A_{X_s\gamma} = \frac{\Gamma(\bar{B}\to X_s\gamma) - \Gamma(B\to X_{\bar{s}}\gamma)}{\Gamma(\bar{B}\to X_s\gamma) + \Gamma(B\to X_{\bar{s}}\gamma)}$$

Semi inclusive measurements $\Rightarrow$ $A^{\rm exp}_{X_s\gamma} = -(1.2 \pm 2.8)\%$ (HFAG average)

SM estimate [Benzke, Lee, Neubert, Paz, arXiv:1012.3167]:

$$A^{\rm SM}_{X_s\gamma} \simeq {\rm Im}\left(\frac{V^*_{us}V_{ub}}{V^*_{ts}V_{tb}}\right)\pi\left|\frac{C_1^{\rm their}}{C_7}\right|\left[\frac{\tilde{\Lambda}^u_{17}-\tilde{\Lambda}^c_{17}}{m_b} + \frac{40\alpha_s}{9\pi}\frac{m_c^2}{m_b^2}\left(1-\frac{2}{5}\ln\frac{m_b}{m_c}+\frac{4}{5}\ln^2\frac{m_b}{m_c}-\frac{\pi^2}{15}\right)\right]$$

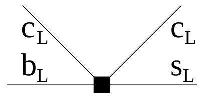
$$\simeq \left(1.15\,\frac{\tilde{\Lambda}^u_{17}-\tilde{\Lambda}^c_{17}}{300\ {\rm MeV}} + 0.71\right)\% \in [-0.6\%, +2.8\%] \quad \text{using} \begin{cases} -330\ {\rm MeV} < \tilde{\Lambda}^u_{17} < +525\ {\rm MeV} \\ -9\ {\rm MeV} < \tilde{\Lambda}^u_{17} < +11\ {\rm MeV} \end{cases}$$

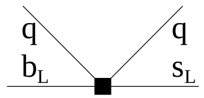
Despite the uncertainties, $A_{X_s\gamma}$ provides constraints on models with non-minimal flavour violation. Such models are also constrained by:

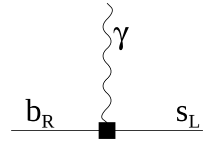
$$A_{X_{(s+d)}\gamma} = \frac{\Gamma(\bar{B}\to X_{(s+d)}\gamma) - \Gamma(B\to X_{(\bar{s}+\bar{d})}\gamma)}{\Gamma(\bar{B}\to X_{(s+d)}\gamma) + \Gamma(B\to X_{(\bar{s}+\bar{d})}\gamma)} \qquad (A^{\rm SM}_{X_{(s+d)}\gamma} \simeq 0)$$

The following vertices Q_i matter for $b \to s\gamma$ and $b \to sl^+l^-$:

(SM – only the red ones)

$Q_{1,2}$ =


$Q_{3,4,5,6}$ =


Q_7 =


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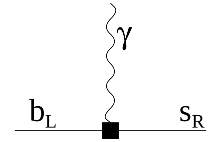
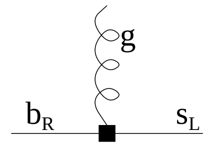
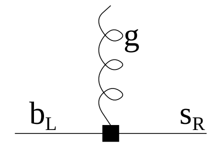
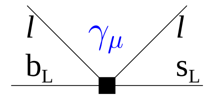
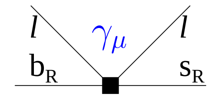
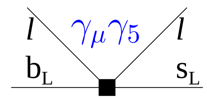


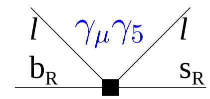
Q_8 =


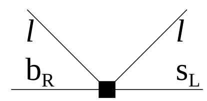
Q'_8 =


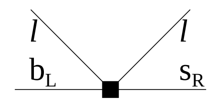
Q_9 =


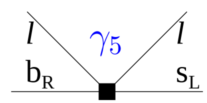
Q'_9 =


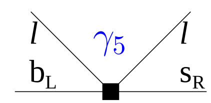
Q_{10} =


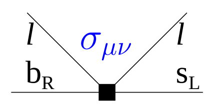
Q'_{10} =


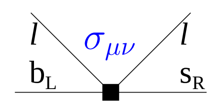
Q_S =


Q'_S =


Q_P =


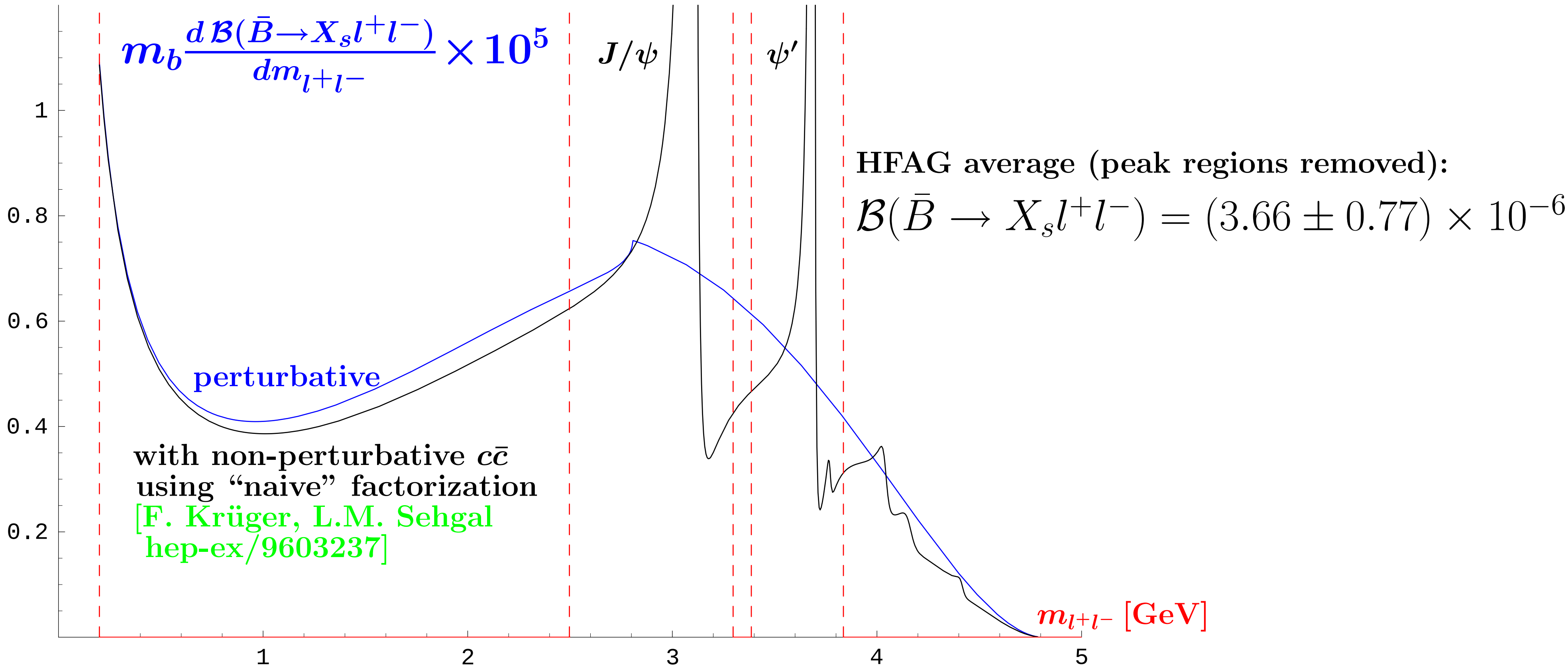
Q'_P =


Q_T =


Q'_T =


Assumption: no relevant NP effects in the 4-quark operators.

Dilepton mass spectrum in $\bar{B} \to X_s l^+ l^-$ $(l = e \text{ or } \mu)$



Problem:

$$\mathcal{B}(B \to J/\psi\ X) \simeq 1.1 \times 10^{-2}$$

$$\mathcal{B}(J/\psi \to l^+ l^-) \simeq 5.9 \times 10^{-2}$$

$$\mathcal{B}(J/\psi \to l^+ l^- X) < ?.? \times 10^{-?}$$ ← Even $\mathcal{O}\left(10^{-4}\right)$ matters!

$$\mathcal{B}(J/\psi \to \gamma X) \simeq 8.8 \times 10^{-2}$$ (CLEO'08, with and error 5×10^{-3})

Could we get any experimental hints from exclusive decays at the LHCb?

For instance, $B^+ \to K^+ J/\psi$ followed by $J/\psi \to l^+ l^- (\pi^0 \text{ or } \pi^+\pi^-)$?

$\mathcal{B} = (1.014 \pm 0.034) \times 10^{-3}$

Inclusive decay rates and the sign of C_7

$$\left(\hat{s} = \frac{q^2_{l^+l^-}}{m_b^2}\right)$$

$$\frac{d\Gamma(\bar{B} \to X_s l^+ l^-)}{d\hat{\mathbf{s}}} = \frac{G_F^2 m_{b,\text{pole}}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\pi^3} \left(\frac{\alpha_{\text{em}}}{4\pi}\right)^2 (1-\hat{\mathbf{s}})^2 \times$$

$$\left\{(1+2\hat{\mathbf{s}})\left(|\mathbf{C}_9^{\text{eff}}(\hat{\mathbf{s}})|^2 + |\mathbf{C}_{10}^{\text{eff}}(\hat{\mathbf{s}})|^2\right) + \left(4+\frac{8}{\hat{\mathbf{s}}}\right)|\mathbf{C}_7^{\text{eff}}(\hat{\mathbf{s}})|^2 + 12\,\text{Re}\left(\mathbf{C}_7^{\text{eff}}(\hat{\mathbf{s}})\mathbf{C}_9^{\text{eff}*}(\hat{\mathbf{s}})\right)\right\} + \mathbf{R_1},$$

$$\Gamma(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_{b,\text{pole}}^5 \left|V_{ts}^* V_{tb}\right|^2}{8\pi^3} \frac{\alpha_{\text{em}}}{4\pi} |\mathbf{C}_7^{\text{eff}}(\hat{\mathbf{s}} = \mathbf{0})|^2 + \mathbf{R_2}$$

are conveniently expressed in terms of the so-called effective coefficients

$$C_i^{\text{eff}}(\hat{\mathbf{s}}) = C_i(\mu_\mathbf{b}) + \textbf{(loop corrections)}(\hat{\mathbf{s}}).$$

The quantities R_i stand for small bremsstrahlung contributions and for the non-perturbative corrections.

$$\text{s}gn\,\mathbf{C_7}(\mu_b) = \textbf{(“sign of the } b \to s\gamma \textbf{ amplitude”).}$$

This sign matters for the $\bar{B} \to X_s l^+ l^-$ rate and (even more) for the forward-backward asymmetry:

$$A_{\text{FB}} = \int_{-1}^{1} dy \frac{d^2\Gamma(\bar{B} \to X_s l^+ l^-)}{d\hat{\mathbf{s}}\, dy}\, \text{s}gn\, y \sim (1-\hat{\mathbf{s}})^2\, \text{Re}\left[\mathbf{C}_{10}^{\text{eff}*}(\hat{\mathbf{s}})\left(\hat{\mathbf{s}}\mathbf{C}_9^{\text{eff}}(\hat{\mathbf{s}}) + 2\mathbf{C}_7^{\text{eff}}(\hat{\mathbf{s}})\right)\right] + \mathbf{R_3},$$

where $y = \cos\theta_l$ and θ_l is the angle between the momenta of $\bar{B}$ and l^+ in the dilepton rest frame. Forward-backward asymmetries for the exclusive $\bar{B} \to K^{(\star)} l^+ l^-$ modes are defined analogously.

Recent "global fits" to exclusive and inclusive $b \to s$ observables

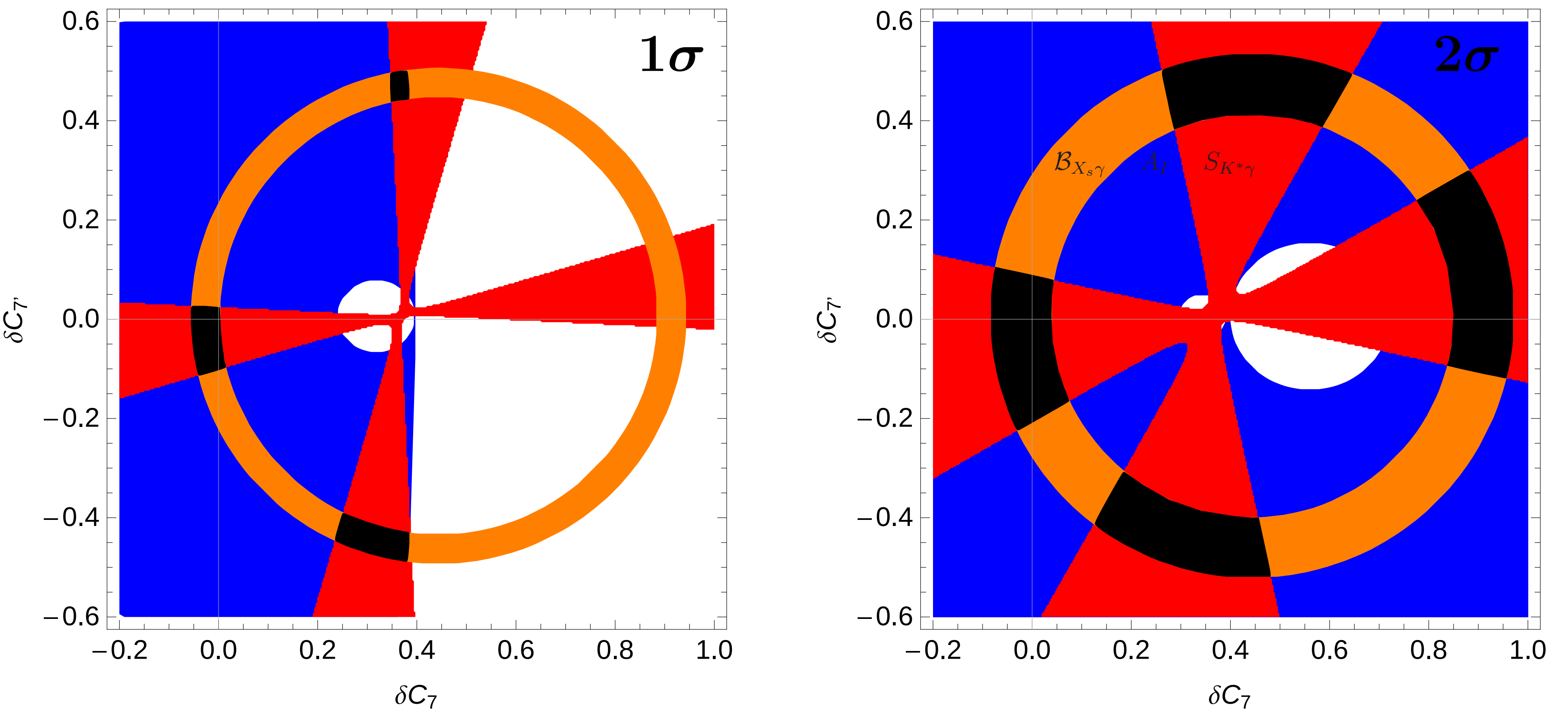
[1] S. Descotes-Genon, D. Ghosh, J. Matias and M. Ramon, JHEP 1106 (2011) 099 [arXiv:1104.3342]
[2] W. Altmannshofer, P. Paradisi, D. M. Straub, arXiv:1111.1257

Constraints from $\mathcal{B}(\bar{B} \to X_s\gamma)$, $\mathcal{B}(\bar{B} \to X_s l^+ l^-)$ [1,2], $\mathcal{B}(B_s \to \mu^+\mu^-)$ [2] and

$\bar{B} \to K^*\gamma$: $S_{K^*\gamma}$ (mixing-induced CP asymmetry) [1,2], A_I (isospin asymmetry) [1],

$\bar{B} \to K^* l^+ l^-$: angular observables $\{A_{FB}, F_L\}$ [1,2], $A_T^{(2)}$ [1], $\{S_k, A_k\}$ [2].

Sample bounds on the Wilson coefficients [1]:



Summary

- Experimental bounds on $\mathcal{B}(B_s \to \mu^+\mu^-)$ are quickly approaching the SM value. Given the new-physics sensitivity of this mode, it is going to remain the highlight of rare B decays for a while.

- For the $\bar{B} \to X_s\gamma$ branching ratio and moments of the photon spectrum, measurements at all the photon energy cutoffs $E_0 \in [1.6, 2.1]$ GeV are relevant (with correlation matrices) for getting constraints on C_7. A coordinated effort of theorists and experimentalists can lead to significant reduction of TH/EXP errors and making them reliable.

- The direct CP asymmetry $A_{X_s\gamma}$ in the SM is likely to be dominated by unknown non-perturbative contributions. Nevertheless, it can still provide constraints on non-MFV models, in parallel to $A_{X_{(s+d)}\gamma}$.

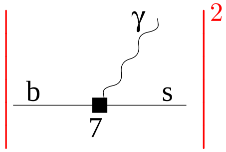
- Processes generated by the quark-level $b \to s l^+ l^-$ transition provide us with information on a large number of Wilson coefficients, and can be used to resolve discrete ambiguities.

BACKUP SLIDES

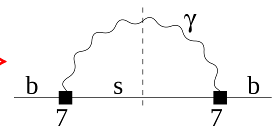
Perturbative evaluation of $\Gamma(b \to X_s^{\rm p}\gamma)$ at $\mu_b \sim \frac{m_b}{2}$.

$$\Gamma(b \to X_s^{\rm p}\gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{\rm em}}{32\pi^4} \left|V_{ts}^* V_{tb}\right|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

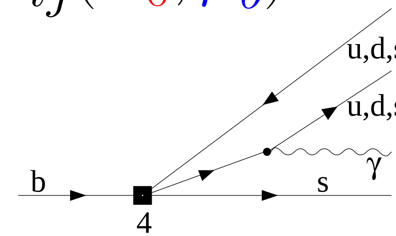
LO: $G_{77} = 1$



$\Leftrightarrow$



Other LO are small, e.g.,:



[Kamiński, Poradziński, MM, in preparation]

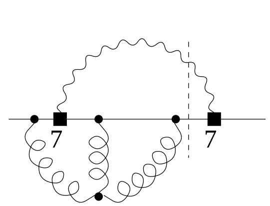
NLO: **1996: Quasi-complete** G_{ij} {[Greub, Hurth, Wyler, 1996] [Ali, Greub, 1991-1995]

2002: Complete$^{(*)}$ G_{ij} {[Buras, Czarnecki, Urban, MM, 2002] [Pott, 1995]

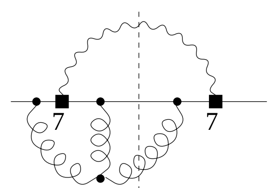
$^{(*)}$ **Up to** $b \to sq\bar{q}\gamma$ **channel contributions involving diagrams similar to the above LO one.**
They get suppressed by $\alpha_s C_{3,4,5,6}$ **and phase-space for** $E_0 \sim m_b/3$.

NNLO: **We are still on the way to the quasi-complete case:**

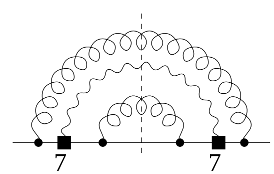
G_{77} **is fully known:**



$+$



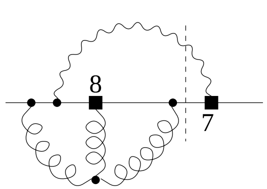
$+$



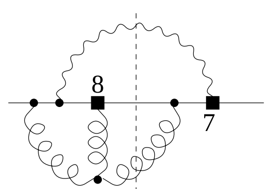
$+ \ldots$

{[Blokland et al., 2005] [Melnikov, Mitov, 2005] [Asatrian et al., 2006-2007]

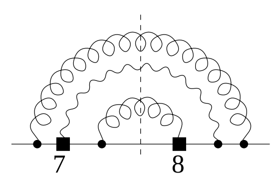
G_{78} **is fully known:**



$+$



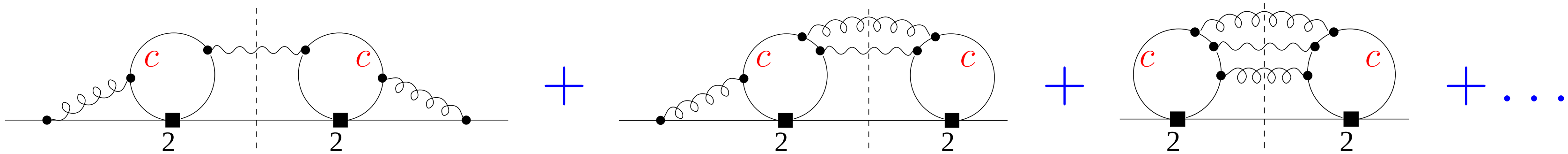
$+$



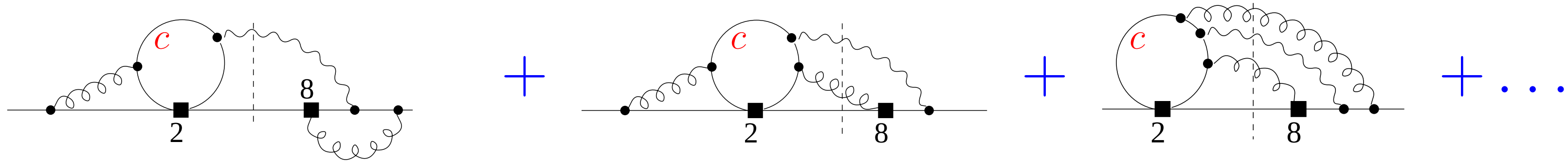
$+ \ldots$

[Asatrian et al., arXiv:1005.5587]

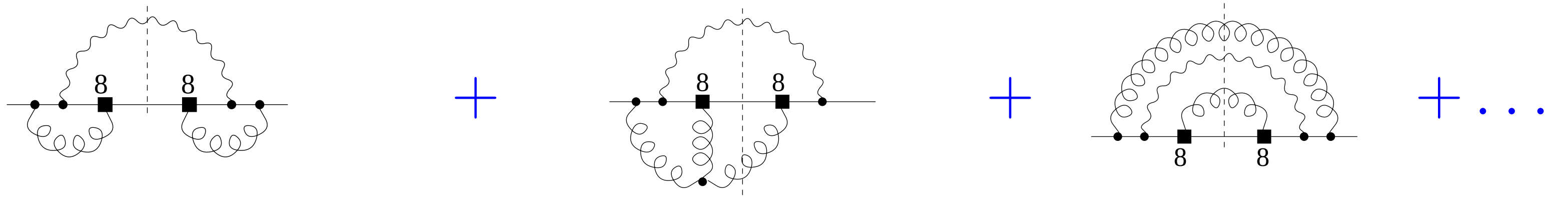
G_{22}**:**
(and analogous G_{11} & G_{12})



G_{28}**:**
(and analogous G_{18})



G_{88}**:**



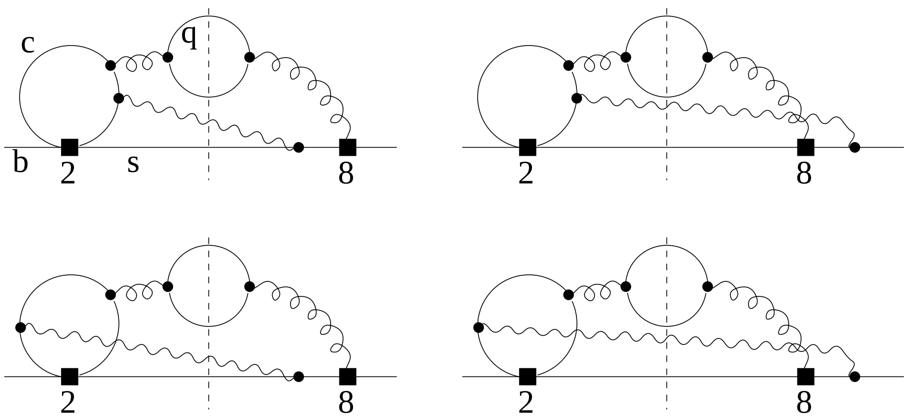
Two-particle cuts are known (just $|\mathrm{NLO}|^2$).

Three- and four-particle cuts are known in the BLM approximation only: [Ligeti, Luke, Manohar, Wise, 1999], [Ferroglia, Haisch, arXiv:1009.2144], [Poradziński, MM, arXiv:1009.5685].

NLO+(NNLO BLM) corrections are not big (+3.8%).

Example:

Evaluation of the ($n > 2$)-particle cut contributions to G_{28} in the Brodsky-Lepage-Mackienzie (BLM) approximation ("naive nonabelianization", large-β_0 approximation) [Poradziński, MM, arXiv:1009.5685]:



q – **massless quark,**

N_q – **number of massless flavours (equals to 3 in practice because masses of u, d, s are neglected).**

Replacement in the final result:

$$-\frac{2}{3}N_q \quad \longrightarrow \quad \beta_0 = 11 - \frac{2}{3}(N_q + 2).$$

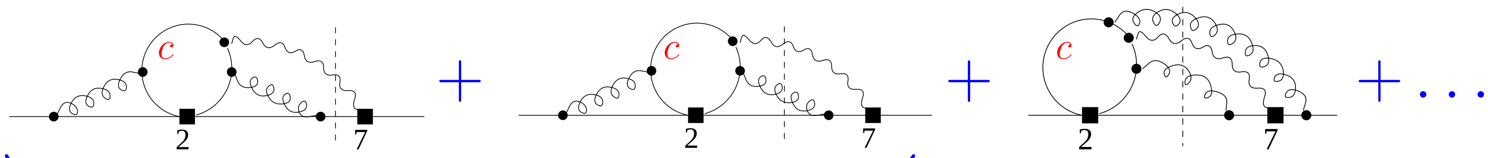
The diagrams have been evaluated using the method of Smith and Voloshin [hep-ph/9405204].

Non-BLM contributions to G_{ij} from quark loops on the gluon lines are quasi-completely known.

[Boughezal, Czakon, Schutzmeier, 2007], [Asatrian, Ewerth, Gabrielyan, Greub, 2007], [Ewerth, 2008].

The only important but still missing NNLO contribution to G_{ij}:

G_{27}:
(and analogous G_{17})



$m_c = 0$: [Boughezal, Czakon, Schutzmeier, to be published]
[T. Schutzmeier, Ph.D. thesis, 2010]
$\mathcal{O}(200)$ massive 4-loop on-shell master integrals.

$m_c = 0$: [Czakon, Huber, Schutzmeier, Fiedler]
in progress...

The $m_c \gg m_b/2$ limit is known [Steinhauser, MM, 2006].

The BLM approximation is known for arbitrary m_c: [Bieri, Greub, Steinhauser, 2003], [Ligeti, Luke, Manohar, Wise, 1999].

The non-BLM correction to G_{27} has been interpolated in m_c assuming BLM in Γ at $m_c = 0$.

Towards G_{27} at the NNLO for arbitrary m_c.

[M. Czakon, R.N. Lee, M. Steinhauser, A.V. Smirnov, V.A. Smirnov, MM] **in progress.**

1. **Generation of diagrams and performing the Dirac algebra to express everything in terms of four-loop two-scale scalar integrals with unitarity cuts.**

2. **Reduction to master integrals with the help of Integration By Parts (IBP).**

   Available C++ codes:

FIRE	[A.V. Smirnov, arXiv:0807.3243] (public in the *Mathematica* version only),
REDUZE	[C. Studerus, arXiv:0912.2546],
DiaGen/IdSolver	[M. Czakon, unpublished (2004)].

   The IBP for 2-particle cuts has just been completed with the help of FIRE: ~ 0.5 TB RAM has been used ~ 1 month at CERN and KIT.

   Number of master integrals: around 500.

3. **Extending the set of master integrals I_n so that it closes under differentiation with respect to $z = m_c^2/m_b^2$. This way one obtains a system of differential equations**

$$\frac{d}{dz} I_n = \Sigma_k \, w_{nk}(z, \epsilon) \, I_k, \qquad (*)$$

where w_{nk} are rational functions of their arguments.

4. **Calculating boundary conditions for $(*)$ using automatized asymptotic expansions at $m_c \gg m_b$.**

5. **Calculating three-loop single-scale master integrals for the boundary conditions using dimensional recurrence relations** [R.N. Lee, arXiv:0911.0252].

6. **Solving the system $(*)$ numerically** [A.C. Hindmarsch, `http://www.netlib.org/odepack`] **along an ellipse in the complex z plane. Doing so along several different ellipses allows us to estimate the numerical error.**

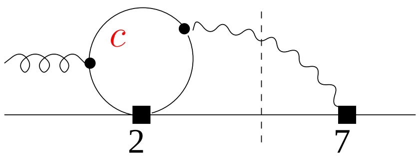
This algorithm has already been successfully applied for diagrams with (massless and massive) quark loops on the gluon lines where $18 + 47 + 38 = 103$ master integrals were present.

[R. Boughezal, M. Czakon, T. Schutzmeier, arXiv:0707.3090]

Non-perturbative contributions from the photonic dipole operator alone ("77" term) are well controlled for $E_0 = 1.6$ GeV:

$\mathcal{O}\left(\frac{\alpha_s^n \Lambda}{m_b}\right)_{n=0,1,2,\ldots}$ vanish, $\mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right)$ [Bigi, Blok, Shifman, Uraltsev, Vainshtein, 1992], [Falk, Luke, Savage, 1993], $\mathcal{O}\left(\frac{\Lambda^3}{m_b^3}\right)$ [Bauer, 1997], $\mathcal{O}\left(\frac{\alpha_s \Lambda^2}{m_b^2}\right)$ [Ewerth, Gambino, Nandi, 2009].

The dominant non-perturbative uncertainty originates from the "27" interference term:



$$\frac{\Delta\mathcal{B}}{\mathcal{B}} = -\frac{6C_2 - C_1}{54 C_7}\left[\frac{\lambda_2}{m_c^2} + \sum_n b_n \mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\left(\frac{m_b \Lambda}{m_c^2}\right)^n\right)\right]$$

$\lambda_2 \simeq 0.12\,\mathrm{GeV}^2$

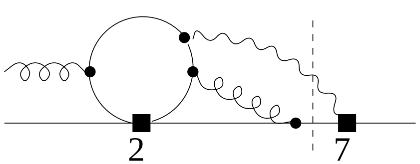
from B–B^* mass splitting

The coefficients b_n decrease fast with n.

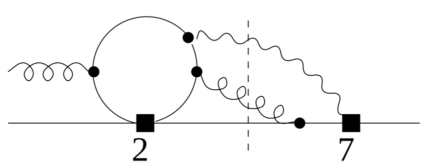
[Voloshin, 1996], [Khodjamirian, Rückl, Stoll, Wyler, 1997]
[Grant, Morgan, Nussinov, Peccei, 1997]
[Ligeti, Randall, Wise, 1997], [Buchalla, Isidori, Rey, 1997]

New claims by Benzke, Lee, Neubert and Paz in arXiv:1003.5012:

One cannot really expand in $m_b\Lambda/m_c^2$. All such corrections should be treated as Λ/m_b ones and estimated using models of subleading shape functions. Dominant contributions to the estimated $\pm 5\%$ non-perturbative uncertainty in $\mathcal{B}$ are found this way, with the help of alternating-sign shape functions that undergo weaker suppression at large gluon momenta.



correction to the above



phase-space suppressed

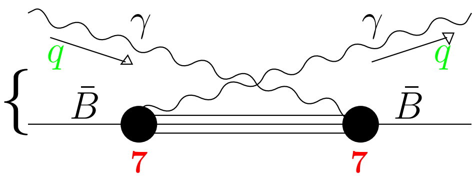
$\mathcal{O}\left(\frac{\alpha_s \Lambda}{m_b}\right)$ **Main worry in hep-ph/0609232, and reason for the $\pm 5\%$ non-perturbative uncertainty.**

The “hard” contribution to $\bar{B} \to X_s\gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b)\langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + \ldots \right|^2$

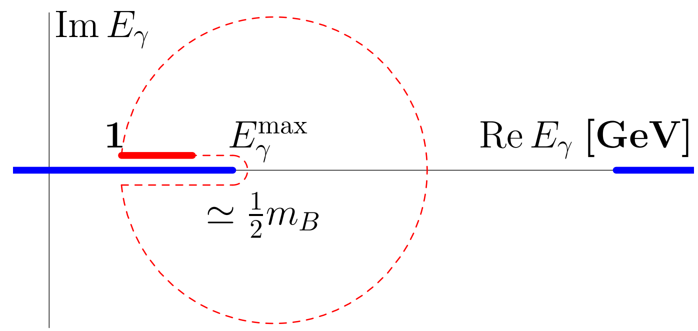
The “77” term in this sum is purely “hard”. It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$**:**

$\mathrm{Im}\{$  $\} \equiv \mathrm{Im}\, A$

When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \Rightarrow$ **Short-distance dominance** $\Rightarrow$ **OPE.**
However, the $\bar{B} \to X_s\gamma$ **photon spectrum is dominated by hard photons** $E_\gamma \sim m_b/2$**.**

Once $A(E_\gamma)$ **is considered as a function of arbitrary complex** E_γ**,** $\mathrm{Im}A$ **turns out to be proportional to the discontinuity of** A **at the physical cut. Consequently,**

$$\int_{1\ \mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma\ \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma\ A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ **is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives**

$$A(E_\gamma)|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1-2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0)|Q^{(j)}_{\mathrm{local\ operator}}|\bar{B}(\vec{p}=0)\rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b.

At $(\Lambda/m_b)^0$**:** $\quad \langle \bar{B}(\vec{p})|\bar{b}\gamma^\mu b|\bar{B}(\vec{p})\rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s\gamma) = \Gamma(b \to X_s^{\mathrm{parton}}\gamma) + \mathcal{O}(\Lambda/m_b).$

At $(\Lambda/m_b)^1$**:** $\quad$ **Nothing! All the possible operators vanish by the equations of motion.**

At $(\Lambda/m_b)^2$**:** $\quad \langle \bar{B}(\vec{p})|\bar{h}D^\mu D_\mu h|\bar{B}(\vec{p})\rangle = -2m_B\lambda_1, \qquad \lambda_1 = (-0.27 \pm 0.04)\mathrm{GeV}^2$ **from** $\bar{B} \to X\ell^-\nu$ **spectrum.**

$\quad \langle \bar{B}(\vec{p})|\bar{h}\sigma^{\mu\nu}G_{\mu\nu}h|\bar{B}(\vec{p})\rangle = 6m_B\lambda_2, \qquad \lambda_2 \simeq \frac{1}{4}\left(m_{B^*}^2 - m_B^2\right) \simeq 0.12\,\mathrm{GeV}^2.$

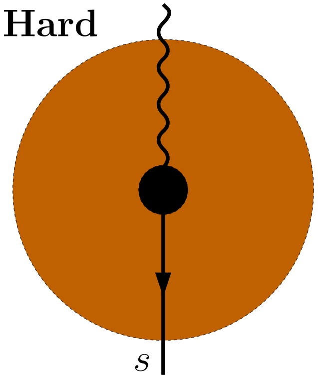
The HQET heavy-quark field $h(x)$ **is defined by** $h(x) = \frac{1}{2}(1+\not{v})b(x)\exp(im_b\, v\cdot x)$ **with** $v = p/m_B$**.**

Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\mathrm{GeV})$ [see MM, arXiv:0911.1651]

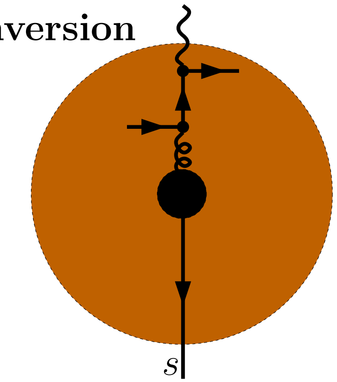
A. Without long-distance charm loops:

1. **Hard**



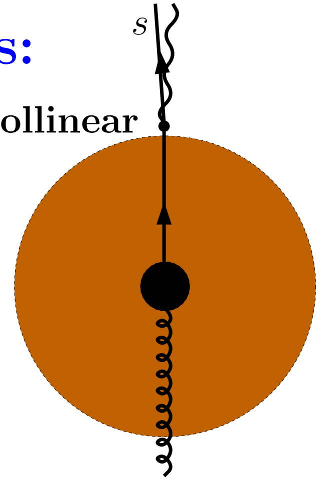
Dominant, well-controlled.

2. **Conversion**



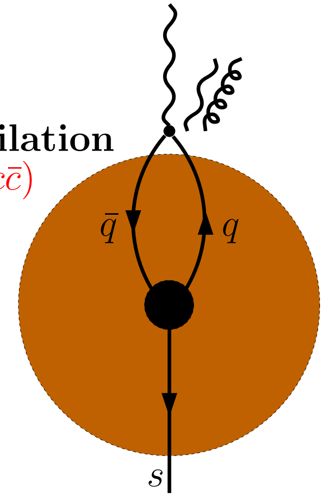
$\mathcal{O}(\alpha_s \Lambda / m_b)$, $(-1.6 \pm 1.2)\%$.
[Benzke, Lee, Neubert, Paz, 2010]

3. **Collinear**



$\sim -0.2\%$ or $(+0.8 \pm 1.1)\%$.
[Kapustin,Ligeti,Politzer, 1995]
[Benzke, Lee, Neubert, Paz, 2010]

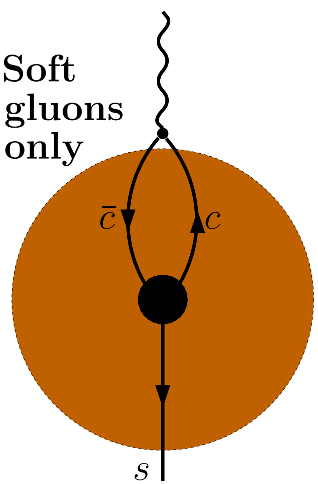
4. **Annihilation** $(q\bar{q} \neq c\bar{c})$



Exp. π^0, η, η', ω subtracted.
Perturbatively $\sim 0.1\%$.

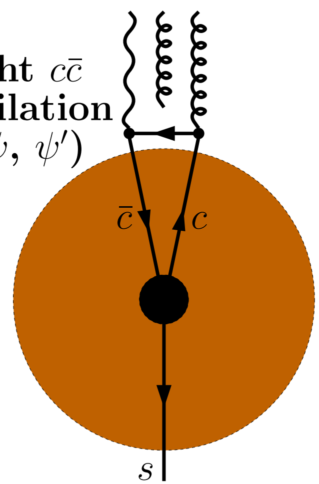
B. With long-distance charm loops:

5. **Soft gluons only**



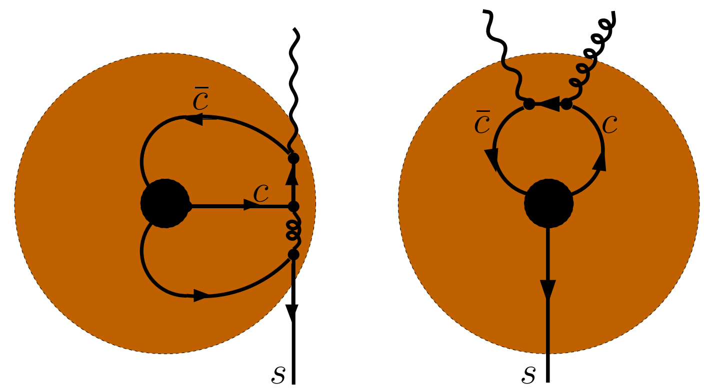
$\mathcal{O}(\Lambda^2/m_c^2)$, $\sim +3.1\%$.
[Voloshin, 1996], [...],
[Buchalla, Isidori, Rey, 1997]
[Benzke, Lee, Neubert, Paz, 2010]: add $(+1.1 \pm 2.9)\%$

6. **Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')**



Exp. J/ψ subtracted ($< 1\%$).
Perturbatively (including hard): $\sim +3.6\%$.

7. **Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state**



$\mathcal{O}(\alpha_s(\Lambda/M)^2)$ $\mathcal{O}(\alpha_s \Lambda/M)$

$M \sim 2m_c, 2E_\gamma, m_b$.

e.g. $\mathcal{B}[B^- \to D_{sJ}(2457)^- \, D^*(2007)^0 \,] \simeq 1.2\%$,
$\mathcal{B}[B^0 \to D^*(2010)^+ \, \bar{D}^*(2007)^0 K^-] \simeq 1.2\%$.