Studies on accuracy of the contributions from NNLO hadronic and leptonic corrections to Bhabha scattering in Babayaga MC generator

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Colaboration

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Motivation

- The comparision of Standard Model predictions with precise experimental data is an invaluable tool to test the theory at the quantum level.
- Today precision tests of SM require an inclusion of higher order effects.
- Precise calculations of higher order corrections for the process of Bhabha scaterring ($e^+e^- \to e^+e^-$) are necessary for determine colliders luminosity with high accuracy.

$$L_{tot} = \frac{N}{\sigma_{theory}} \tag{1}$$

N-number of experimental events, σ_{theory} - theoretical cross section

- High accuracy of luminosity in low energy region is necessary to research low energy hadron cross section from $e+e-$ anihilation process.

$$\sigma_{had} = \frac{N_{had}}{L_{tot}} \tag{2}$$

Motivation

- The anomalous magnetic moment of the muon a_μ^{had} is mainly determined by low energy hadrons.

$$a_\mu^{had} = \frac{1}{4\pi^3}\int_{m_\Pi^2}^{\infty} ds K(s)\sigma_{had} \tag{3}$$

- The hadronic part of the shift of fine structure constant.

$$\Delta\alpha^{had}(M_z) = -\frac{\alpha M_z^2}{3\pi} Re \int_{m_\Pi^2}^{\infty} ds \frac{R(s)}{s(s - M_z^2 - i\epsilon)} \tag{4}$$

The role of high accuracy calculations

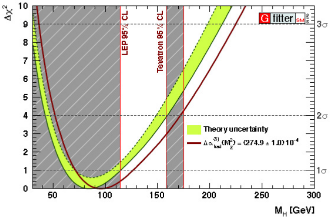
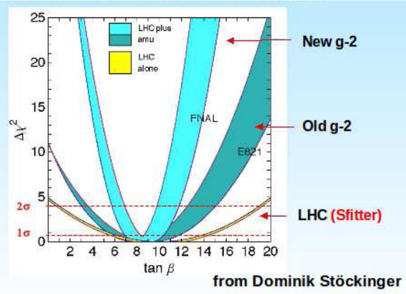


FIG. 10: Standard Gfitter electroweak fit result [55] (green shaded band) and the result obtained for the new evaluation of $\Delta\alpha_{\rm had}(M_Z^2)$. (The plot displays the corresponding five-quark contribution, $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$, where the top term of $-0.72\cdot 10^{-4}$ is excluded). A shift in the most probably M_H value of $+12$ GeV is observed.



left print:Reevaluation of the Hadronic Contributions to the Muon g-2 and to alpha(MZ). Michel Davier, Andreas Hoecker, Bogdan Malaescu, Zhiqing Zhang Published in Eur.Phys.J.C71:1515,2011.

right print: The Physics case for the new muon (g-2) experiment. David W. Hertzog, (Illinois U., Urbana) , James P. Miller, (Boston U.) , Eduardo de Rafael, (Marseille, CPT) , B. Lee Roberts, (Boston U.) , Dominik Stockinger, (Glasgow U.) . May 2007. e-Print: arXiv:0705.4617 [hep-ph]

Luminosity accuracy in the measurments of $\sigma(e+e- \to \Pi+\Pi-(\gamma))$

Table of systematic errors on $a_\mu^{\pi\pi}(0.1\text{-}0.85\ GeV^2)$:

Reconstruction Filter	negligible
Background	0.5%
$f_0+\rho\pi$	0.4%
Ω cut	0.2%
Trackmass	0.5%
p/e-ID and TCA	negligible
Tracking	0.3%
Trigger	0.2%
Acceptance	0.5%
Unfolding	negligible
Software Trigger	0.1%
Luminosity$(0.1_{th} \oplus 0.3_{exp})$%	0.3%

experimental fractional error on a_μ = 1.0 %

FSR treatment	0.8%
Radiator H	0.5%
Vacuum polarization	0.1%

KLOE measurement of $\sigma(e+e- \to \Pi+\Pi-(\gamma))$ with Initial State Radiation and the $\Pi\Pi$ contribution to the muon anomaly, Graziano Venanzoni, EPS HEP 2011,

Aim of the work

- calculation of the virtual (determined by the package bha_nnlo_hf) and real corrections (Monte Carlo generators EKHARA, BHAGHEN–1PH+... and HELAC–PHEGAS) at NNLO for Bhabha scattering
- discussion of the numerical results for energies and with realistic cuts used at the Φ factory Dafne, at the B factories PEP-II and at KEK and at the charm/τ factory BEPC II, Beijing
- comparison complete calculations with approximate ones realized in the MC generator BabaYaga

"NNLO leptonic and hadronic corrections to Bhabha scattering and luminosity monitoring at meson factories."
C. Carloni Calame, (Southampton U.) , H. Czyz, J. Gluza, M. Gunia, (Silesia U.) , G. Montagna, (Pavia U. & INFN, Pavia) , O. Nicrosini, F. Piccinini, (INFN, Pavia) , T. Riemann, (DESY, Zeuthen) , M. Worek, (Wuppertal U.)

Published in JHEP 1107:126,2011

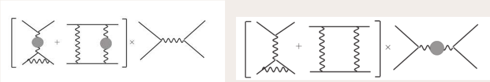
The massive NNLO corrections

The complete NNLO $N_f = 1, 2$ corrections to Bhabha scattering consist of three parts:

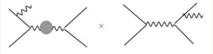
$$\frac{d\sigma^{\rm NNLO}_{N_f}}{d\Omega} = \frac{d\sigma^{\rm NNLO}_{virt}}{d\Omega}^{1} + \frac{d\sigma^{\rm NLO}_{\gamma}}{d\Omega}^{2} + \frac{d\sigma^{\rm LO}_{realpairs}}{d\Omega}^{3}$$
$$= \frac{d\sigma_{e^+e^-}}{d\Omega} + \frac{d\sigma_{\mu^+\mu^-}}{d\Omega} + \frac{d\sigma_{\tau^+\tau^-}}{d\Omega} + \frac{d\sigma_{had}}{d\Omega}. \qquad (5)$$

1 - bha_nnlo_hf
2 - BHAGHEN–1PH+...,bha_nnlo_hf
3 - HELAC–PHEGAS,EKHARA

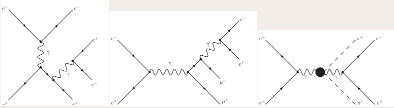
- the $\sigma^{\text{NNLO}}_{virt}$ consists of virtual two-loop corrections $\sigma^{\text{NNLO}}_{2L}$ and loop-by-loop corrections $\sigma^{\text{NNLO}}_{1L1L}$



- contributions with real photon emission $\sigma^{\text{NLO}}_{\gamma} = \sigma^{\text{NLO}}_{\gamma,soft}(\omega) + \sigma^{\text{NLO}}_{\gamma,hard}(\omega)$



- contributions with real pair or hadron emission
$\sigma^{\text{LO}}_{real} = \sigma^{\text{LO}}_{e^+e^-(e^+e^-)} + \sigma^{\text{LO}}_{e^+e^-(f^+f^-)} + \sigma^{\text{LO}}_{e^+e^-(hadrons)}$

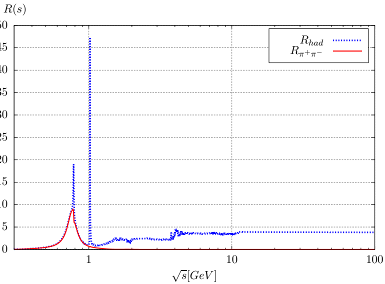


- In the calculations there were used reference realistic event selection. However to **confirm stabilty of error** additional points with different accolinearities and angular cuts were used.
- Calculations for processes $e^+e^- \to e^+e^-$ plus hadrons don't allow the computation of the real hadron emission with the exception of charged pion pairs, beceause there exist any MC generator that allows thsi kind of corrections. However the pions are the lightest hadrons produced and that the highest energy of the meson factories is about 10 GeV, is that they should dominated.

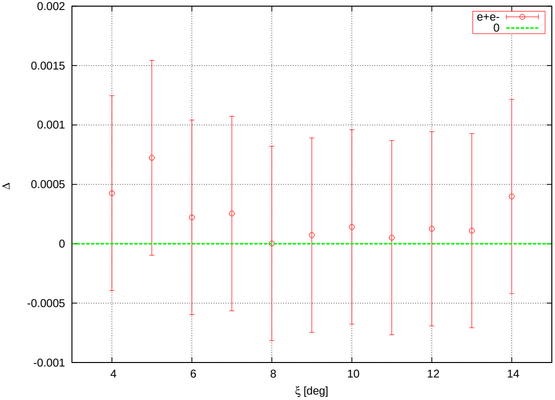
The vacuum polarisation function:

$$\Pi(q^2) = \frac{\alpha q^2}{3\pi} \int_{m^2_{\Pi 0}}^{\text{inf}} \frac{dz}{z} \frac{R(z)}{q^2 - z + i\epsilon} \qquad (6)$$

For leptons VP analitical expresions were used.
For pions VP numerical calculations of the integral were used.
For hadrons program VPHLMNT (T.Teubner et all.) was used.



$\Delta = \frac{|\sigma_h - \sigma_{hBY}|}{\sigma_h}$ - comparision between hard parts, example for KLOE



KLOE: σ_{BY} **= 455.71(5) nb** - all σ_{BY} without vacuum polarisation, $\sqrt{s} = 1.02$ GeV, $E_{min} = 0.4$ GeV, $55^\circ < \theta\pm < 125^\circ$, $\zeta_{max}=9^\circ$

particles	σ_h $[nb]$	σ_{v+s} $[nb]$	σ_{pairs} $[nb]$	sum $[nb]$
NNLO				
electron	9.5021(2)	-11.5666	0.2712(15)	-1.793(2)
muon	1.49406(3)	-1.7356(2)	0.246(7)*10^{-7}	-0.2415(2)
tau	0.0201637(4)	-0.023412(2)	0	-0.003248(2)
leptons sum: σ_{lep}^{NNLO}				**-2.038(2)**
hadrons	1.5248(6)	-1.062(8)	0	0.463(8)
BabaYaga NNLO				
electron	9.5022(8)	-11.0721(4)	-	-1.5699(9)
muon	1.4942(2)	-1.7441(2)	-	-0.2499(3)
tau	0.020166(3)	-0.023704(2)	-	-0.003538(4)
leptons sum: σ_{BYlep}^{NNLO}				**-1.823(1)**
hadrons	1.5247(5)	-1.126(2)	-	0.399(2)
leptons difference: $\sigma_{lep}^{NNLO} - \sigma_{BYlep}^{NNLO}$				-0.214(3)
hadrons difference: $\sigma^{NNLO} - \sigma_{BY}^{NNLO}$				0.064(8)
leptons relative difference: $\lvert\sigma_{lep}^{NNLO} - \sigma_{BYlep}^{NNLO}\rvert/\sigma_{BY}$				**0.471(4)‰**
hadrons relative difference: $\lvert\sigma^{NNLO} - \sigma_{BY}^{NNLO}\rvert/\sigma_{BY}$				**0.14(2)‰**

Attention: $\sigma_{BY}^{20<\theta\pm<160,\zeta_{max}=9} = 6114.9(6)$ nb

BaBar: $\sigma_{BY} = 5.195(2)$ nb

$\sqrt{s} = 10.56$ GeV, $|\vec{p}_+|/E_{beam} > 0.75$ and $|\vec{p}_-|/E_{beam} > 0.50$ or $|\vec{p}_-|/E_{beam} > 0.75$ and $|\vec{p}_+|/E_{beam} > 0.50$, $|cos(\theta\pm)| < 0.65$ and $|cos(\theta+)| < 0.60$ or $|cos(\theta-)| < 0.60$, $\zeta^{3d}_{max} = 30°$

articles	σ_h $[nb]$	σ_{v+s} $[nb]$	σ_{pairs} $[nb]$	sum $[nb]$
NNLO				
electron	0.202439(7)	-0.223667	0.01355(8)	-0.00768(8)
muon	0.075789(2)	-0.079231(2)	0.000451(2)	-0.002991(3)
tau	0.0138398(4)	-0.0144654(2)	$0.120(3)*10^{-8}$	-0.0006257(5)
leptons sum: σ^{NNLO}_{lep}				**-0.01130(8)**
hadrons	0.17995(2)	-0.1888(4)	0.000029(3)	-0.0088(4)
BabaYaga NNLO				
electron	0.20244(2)	-0.20971(5)	-	-0.00727(5)
muon	0.07580(1)	-0.07872(2)	-	-0.00292(2)
tau	0.013847(4)	-0.014541(4)	-	-0.000694(6)
leptons sum: σ^{NNLO}_{BY}				**-0.01088(5)**
hadrons	0.17984(2)	-0.18760(4)	-	-0.00776(5)
leptons difference: $\sigma^{NNLO}_{lep} - \sigma^{NNLO}_{BYlep}$				-0.00042(9)
hadrons difference: $\sigma^{NNLO} - \sigma^{NNLO}_{BY}$				-0.0004(4)
leptons relative difference: $|\sigma^{NNLO}_{lep} - \sigma^{NNLO}_{BYlep}|/\sigma_{BY}$				**0.08(2)‰**
hadrons relative difference: $|\sigma^{NNLO} - \sigma^{NNLO}_{BY}|/\sigma_{BY}$				**0.23(8)‰**

BELLE: $\sigma_{BY} = 5.501(5)$ **nb**

$\sqrt{s}$ =10.58 GeV, $50.5^\circ < \theta_\pm < (180-50.5)^\circ$, ζ_{max}=10°

articles	σ_h [nb]	σ_{v+s} [nb]	σ_{pairs} [nb]	sum [nb]
NNLO				
electron	0.21572(7)	-0.25596	0.01310(5)	-0.02714(9)
muon	0.080377(8)	-0.09009(1)	0.000759(1)	-0.00895(2)
tau	0.014428(4)	-0.01602(1)	0.0000321(1)	-0.00156(1)
leptons sum: σ_{lep}^{NNLO}				**-0.03765(9)**
hadrons	0.18969(1)	-0.2124(5)	0.00015(1)	-0.0226(5)
BabaYaga NNLO				
electron	0.21563(2)	-0.23994(2)	-	-0.02431(3)
muon	0.080376(6)	-0.08948(2)	-	-0.009104(2)
tau	0.014423(1)	-0.016091(7)	-	-0.001668(7)
leptons sum: σ_{BYlep}^{NNLO}				**-0.03508(3)**
hadrons	0.18964(3)	-0.21089(5)	-	-0.02125(6)
leptons difference: $\sigma_{lep}^{NNLO} - \sigma_{BYlep}^{NNLO}$				-0.00257
hadrons difference: $\sigma^{NNLO} - \sigma_{BY}^{NNLO}$				-0.0013(5)
leptons relative difference: $\lvert\sigma_{lep}^{NNLO} - \sigma_{BYlep}^{NNLO}\rvert/\sigma_{BY}$				**0.47(2)‰**
hadrons relative difference: $\lvert\sigma^{NNLO} - \sigma_{BY}^{NNLO}\rvert/\sigma_{BY}$				**0.27(9)‰**

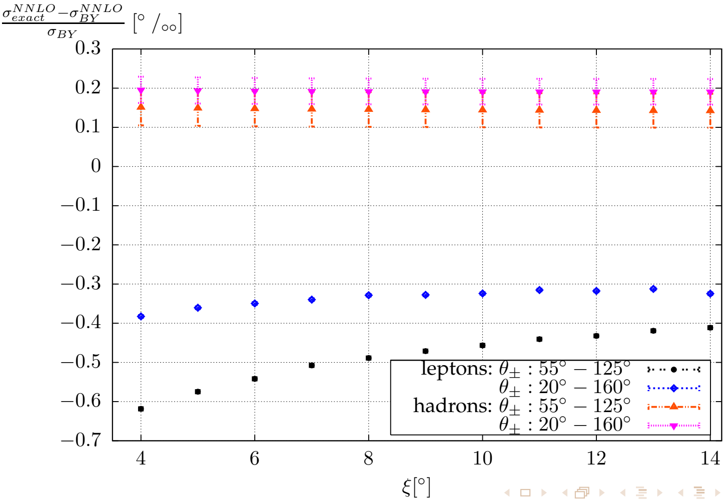
BES $\sigma_{BY} = 116.41(2)$ nb

$\sqrt{s} = 3.65$ GeV, $|\cos\theta| < 0.8$

articles	σ_h $[nb]$	σ_{v+s} $[nb]$	σ_{pairs} $[nb]$	sum $[nb]$
NNLO				
electron	3.19544(9)	-3.55544	0.188856(997)	-0.171(1)
muon	0.83245(2)	-0.88149(1)	0.002003(6)	-0.04704(1)
tau	0.058674(2)	-0.0633(1)	0	-0.0046(1)
leptons sum: σ_{lep}^{NNLO}				**-0.223(1)**
hadrons	1.66065(8)	-1.81(1)	0.000539(7)	-0.15(1)
BabaYaga NNLO				
electron	3.1960(3)	-3.3730(2)	-	-0.1770(4)
muon	0.83252(7)	-0.88041(9)	-	-0.0479(1)
tau	0.058679(7)	-0.06323(2)	-	-0.00455(2)
leptons sum: σ_{BY}^{NNLO}				**-0.2295(4)**
hadrons	1.6613(3)	-1.7860(2)	-	-0.1247(4)
leptons difference: $\sigma_{lep}^{NNLO} - \sigma_{BYlep}^{NNLO}$				0.006(1)
hadrons difference: $\sigma^{NNLO} - \sigma_{BY}^{NNLO}$				-0.03(1)
leptons relative difference: $\lvert\sigma_{lep}^{NNLO} - \sigma_{BYlep}^{NNLO}\rvert/\sigma_{BY}$				**0.057(9)‰**
hadrons relative difference: $\lvert\sigma^{NNLO} - \sigma_{BY}^{NNLO}\rvert/\sigma_{BY}$				**0.21(9)‰**

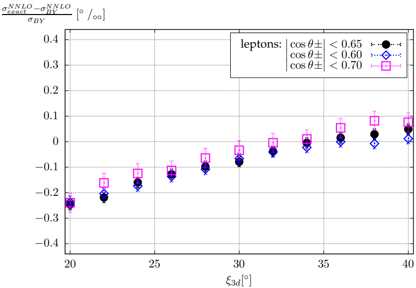
KLOE

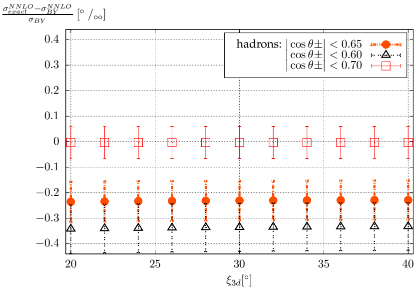
$\Delta = \frac{\sigma^{NNLO} - \sigma^{NNLO}_{BY}}{\sigma_{BY}} 10^3$ - relative difference ‰



BaBar

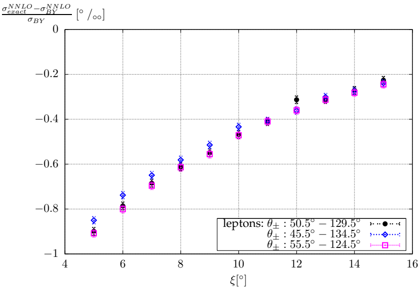
$\Delta = \frac{\sigma^{NNLO} - \sigma_{BY}^{NNLO}}{\sigma_{BY}} 10^3$ - relative difference ‰

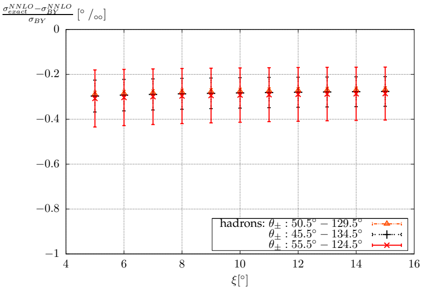




Belle

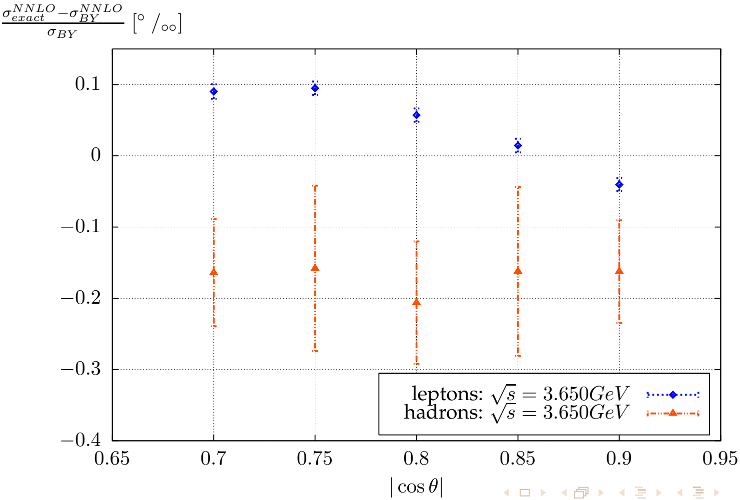
$\Delta = \frac{\sigma^{NNLO} - \sigma^{NNLO}_{BY}}{\sigma_{BY}} 10^3$ - relative difference ‰





BES III

$\Delta = \frac{\sigma^{NNLO} - \sigma^{NNLO}_{BY}}{\sigma_{BY}} 10^3$ - relative difference ‰



Conclusion

- Exact calculations of NNLO massive corrections to Bhabha scattering were presented.
- The theoretical accuracy of the generator BabaYaga@NLO was tested. For reference realistic event selections the maximum observed difference is at the level of 0.07%. When cuts are varied the sum of the missing pieces can reach 0.1%, but for very tight acollinearity cuts only.
- Stability of the results with changing of the event selections was examined - there aren't dramatical changes of errors between points with experimental cuts and their neighbors. NNLO massive corrections are relevant for precision luminosity measurements with 10^{-3} accuracy.
- The electron pair contribution is the largely dominant part of the correction .The muon pair and hadronic correction are the next-to-important effect and quantitatively on the same grounds. The tau pair contribution is negligible for the energies of meson factories.

Narrow resonances - problems to be studied

For experiments performed on top of a narrow resonance (BESIII with $\sqrt{s}$=3.097J/Ψ,3.686 Ψ(2S) GeV), this resonance cannot be treated as a mere correction and more detailed studies have to be performed.

	$\sqrt{s}$		σ_{BY}	$S_{e^+e^-}$‰	S_{lep}‰	S_{had}‰	S_{tot}‰
BES	3.097	BabaYaga	158.23	-2.019(3)	-2.548(3)	558.7(7)	556.1(7)
BES	3.650	BabaYaga	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BES	3.686	BabaYaga	114.27	-1.502(4)	-1.947(4)	-59.42(1)	-61.36(1)

Precise studies in narrow resonances region require using of beam spread.

ADDITIONAL SLIDES

Cuts dependence study for different experiments

1.Φ factories KLOE/DAΦNE (Frascati)

(a) $\sqrt{s} = 1.02$ GeV
(b) $E_{min} = 0.4$ GeV
(c) For $\theta\pm$ two selections have to be checked
i. tighter selection $55^\circ < \theta\pm < 125\ ^\circ$
ii. wider selection $20^\circ < \theta\pm < 160\ ^\circ$
(d) ζ_{max}=4,5,6,7,8,...,14°, with reference value ζ_{max}=9°

2. B-factories BABAR/PEP-II (SLAC) & BELLE/KEKB (KEK)

(a) $\sqrt{s} = 10.56$ GeV
(b) $|\vec{p}_+|/E_{beam} > 0.75$ and $|\vec{p}_-|/E_{beam} > 0.50$
or $|\vec{p}_-|/E_{beam} > 0.75$ and $|\vec{p}_+|/E_{beam} > 0.50$
(c) For $|cos(\theta\pm)|$ the following selections have to be checked
i. $|cos(\theta\pm)| < 0.65$ and $|cos(\theta+)| < 0.60$ or $|cos(\theta-)| < 0.60$
ii. $|cos(\theta\pm)| < 0.70$ and $|cos(\theta+)| < 0.65$ or $|cos(\theta-)| < 0.65$
iii. $|cos(\theta\pm)| < 0.60$ and $|cos(\theta+)| < 0.55$ or $|cos(\theta-)| < 0.55$
(d) ζ^{3d}_{max}= 20,22,24,...,40°, with reference value ζ^{3d}_{max}=30 °

Cuts dependence study for different experiments

3. BES-III experiment at BEPCII (Beijing)

(a) $\sqrt{s} = 3.686$ GeV, 3.65 GeV and 3.097 GeV
(b) $|\cos\theta| < 0.8$, where θ is the polar angle of the electron or positron in the lab system, this corresponds to the barrel region of BES-III detector. Since in BEPC, e^+ and e^- beams are colliding with equal energy but at a 22mrad crossing angle, the lab system is slightly different from the CoM system.
(c) $E_{e^+} > 1.0\ \mathrm{GeV}$ and $E_{e^-} > 1.0\ \mathrm{GeV}$, where E is the energy deposited in the electromagnetic calorimeter (EMC).
(d) E/p for one track greater than 0.5 and the other track greater than 0.8, here E is the energy deposited in the electromagnetic calorimeter (EMC), and p is the track momentum measured in the Main Drift Chamber.
To see how the NNLO corrections depend on the event selection we obtained results also for $|\cos\theta| < 0.7,\ 0.75,\ 0.85$ and 0.9

4. *B*-factory Belle (KEK)- the reference event selection

Belle runs at an asymmetric e^+e^- collider, but all criteria are expressed in the CoM fram (a) $\sqrt{s}$ =10.58 GeV
(b) For $\theta\pm$ two selections have to be checked
i. $50.5^\circ < \theta_\pm < (180 - 50.5)^\circ$
ii. $45.5^\circ < \theta_\pm < (180 - 45.5)^\circ$
iii. $55.5^\circ < \theta_\pm < (180 - 55.5)^\circ$
(c) Two charged tracks have momentum > 2.645 GeV
(d) The track with maximum deposited energy in EMC greater than 2 GeV,
(e) The sum of the deposited energies of all tracks in EMC is greater than 4 GeV (both charged and neutral particle can deposite energy in EMC and it is not checked if the particle is charged or neutral)
(f) Acollinearity angle (2D) ζ_{max}=5,6,7,8,...,15°, with reference value ζ_{max}=10°
(g) Transverse momentum of any observed charged particle greater than 0.1 GeV

Where:

$$\zeta = |\theta_+ + \theta_- - 180| \quad (7)$$

$$\zeta^{3d} = |\arccos s(\vec{p}_+\vec{p}_-/|\vec{p}_+|/|\vec{p}_-|)180/\pi - 180| \quad (8)$$

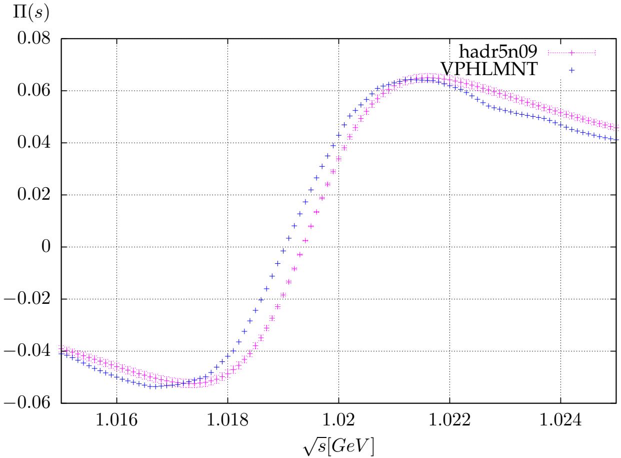
VP codes comparision

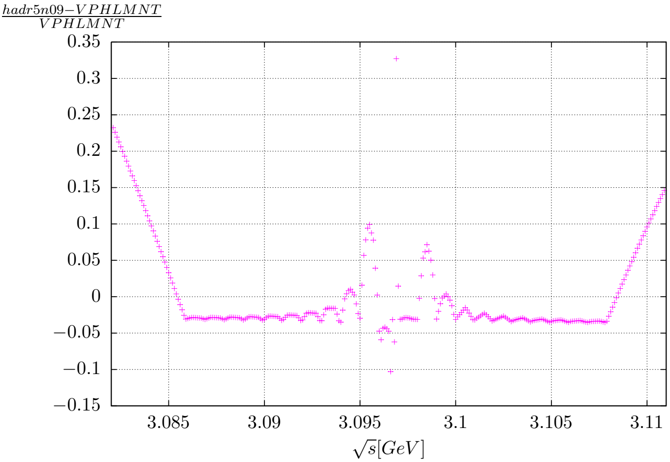
BES:$\sqrt{s} = 3.686$ GeV, $|\cos\theta| < 0.8$
σ_{BY} **= 114.27(2) nb**

vacpol	σ_h	σ_{v+s}	sum:
VPHLMNT2.0(2010)	10.006(4)	-16.80(1)	-6.79(1)
hadr5n09	9.60(1)	-16.28(2)	-6.68(3)
relative difference: $\left\lvert \frac{\sigma^{NNLO}_{VPHLMNT} - \sigma^{NNLO}_{hadr5n09}}{\sigma_{BY}} \right\rvert$			0.96‰

BES:$\sqrt{s} = 3.097$ GeV, $|\cos\theta| < 0.9$
σ_{BY} **= 378.48(5) nb**

vacpol	σ_h	σ_{v+s}	sum:
VPHLMNT2.0(2010)	-116.50(6)	287.7(3)	171.2(3)
hadr5n09	-119.1(2)	291.9(3)	172.8(4)
relative difference: $\left\lvert \frac{\sigma^{NNLO}_{VPHLMNT} - \sigma^{NNLO}_{hadr5n09}}{\sigma_{BY}} \right\rvert$			4.227‰


Π(s)
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
1.016
1.018
1.02
1.022
1.024
√s[GeV]
hadr5n09
VPHLMNT


$\frac{hadr5n09-VPHLMNT}{VPHLMNT}$
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
3.085
3.09
3.095
3.1
3.105
3.11
$\sqrt{s}[GeV]$