Final state QED bremsstrahlung in resonance decays and detector level universality: Phenomenological precision tools.

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- There is large difference between phenomenlologically interesting quantities such as particles masses/withdths coupling constant and distributions used for the measurements. For example:

- detector response to τ lepton looks very different for its decay to $e\nu\bar{\nu}$, $\mu\nu\bar{\nu}$ or $\pi^-\pi^0\nu$.

- bremsstrahlung photon close to electron requires different approach than bremsstrahlung close to muon: background, efficiency, precision

- this is especially important when detector callibration is still to be performed ....

Introduction

● **Q:** Control parts of physics process in experiments **or** use big MC prepared by theorists with knobs to turn?

Adjust knobs of MC to measure its parameters (couplings masses etc), or get involved in theoretical issues.

● Decision depends on complexity of detector response and theoretical system. The higher precision the more details need to be understood on theoretical AND experimental sides including interplays.

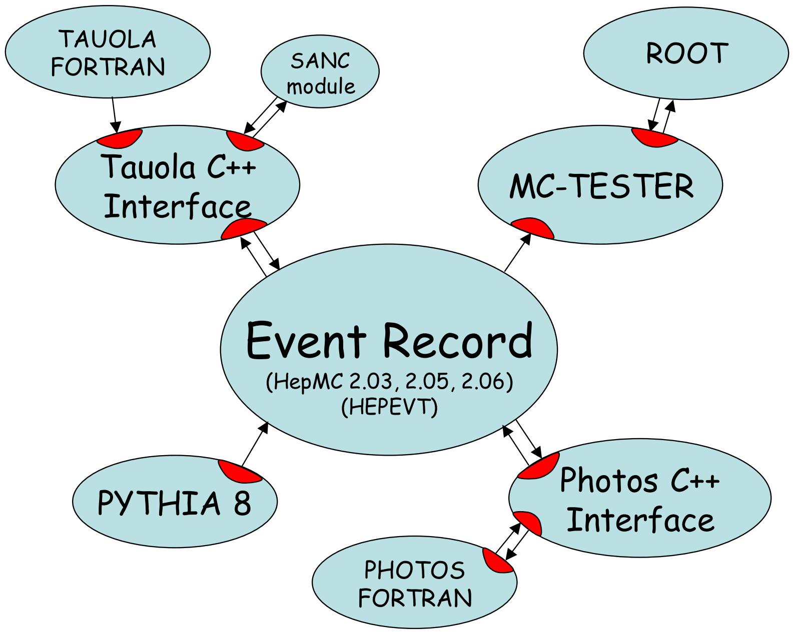
● Details are important:

(-) *Technical points:* event record type HEPEVT, HepMC; weighted events for fits, internal variables of generation to be used in tuning detector response parameters.

(-) *Principles.* Tests. From heuristic/intuitive pictures to high precision and detector effects at the same time. Observables idealized $\longrightarrow$ realistic. Matrix element calculations. Factorization schemes.

Introduction

Simulation parts communicate through event record:



- Parts:

- hard process: (Born, weak, new physics),
- parton shower,
- τ decays
- QED bremsstrahlung

- High precision achieved

- Detector studies: acceptance, resolution lepton with or wihout photon.

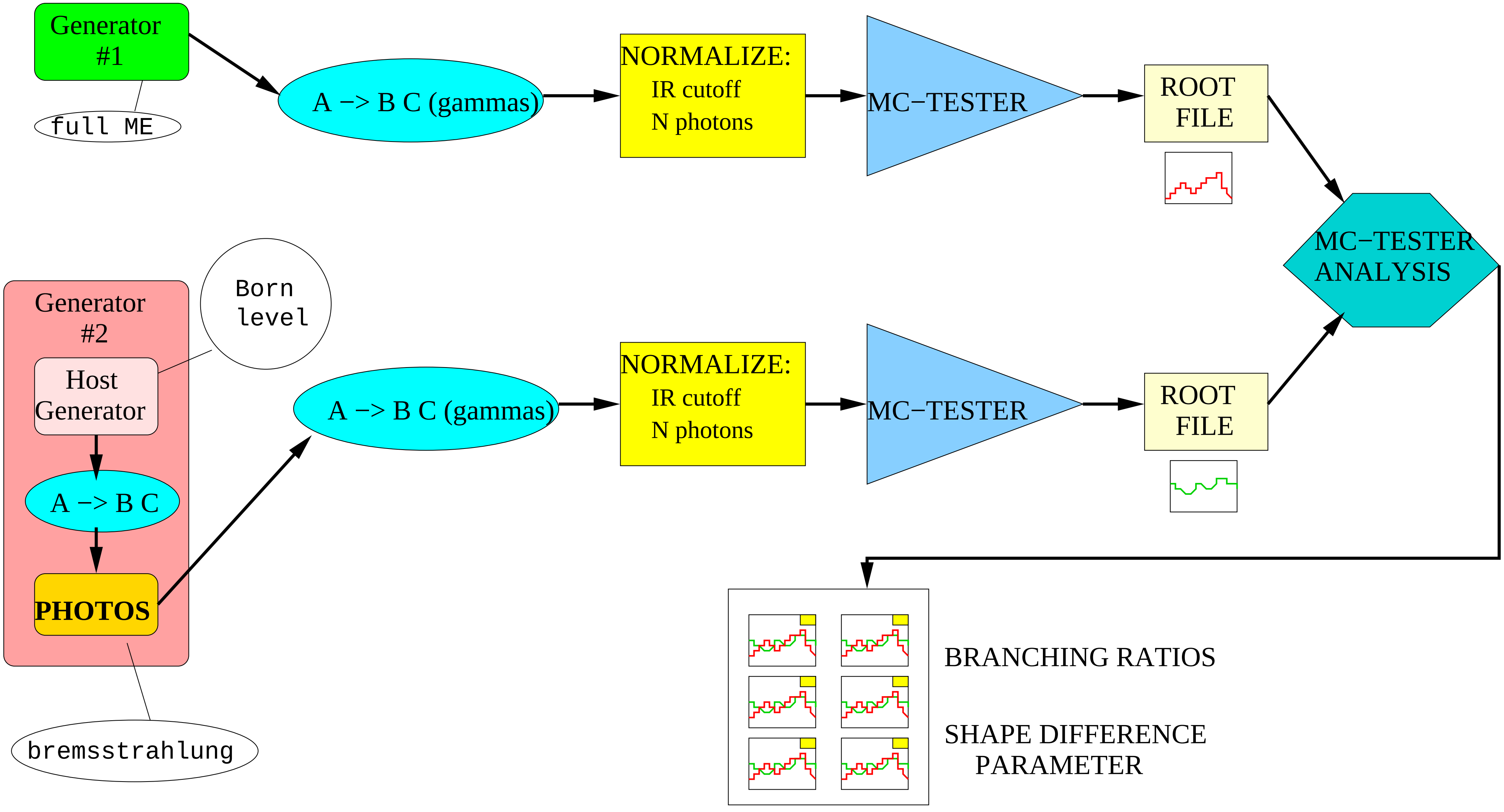
Such organization requires:

- Good control of factorization (theory)
- Good understanding of tools on user side.

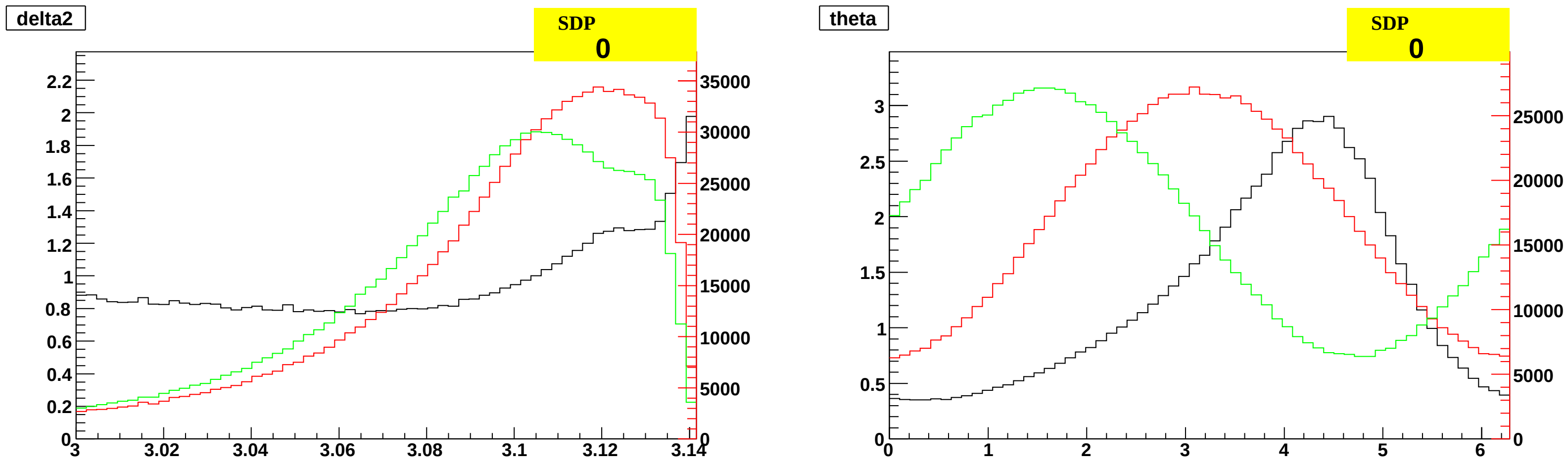
Projects in collaboration with: N. Davidson, Piotr Golonka, G. Nanava, T. Przedzinski, E. Richter-Was, Q. Xu, O. Shekhovtsova, P. Roig.

Thanks for discussions with LCG/Genser, ATLAS, CMS, CDF, Belle members

MC-TESTER to test PHOTOS/TAUOLA



Example: Distribution for Higgs parity

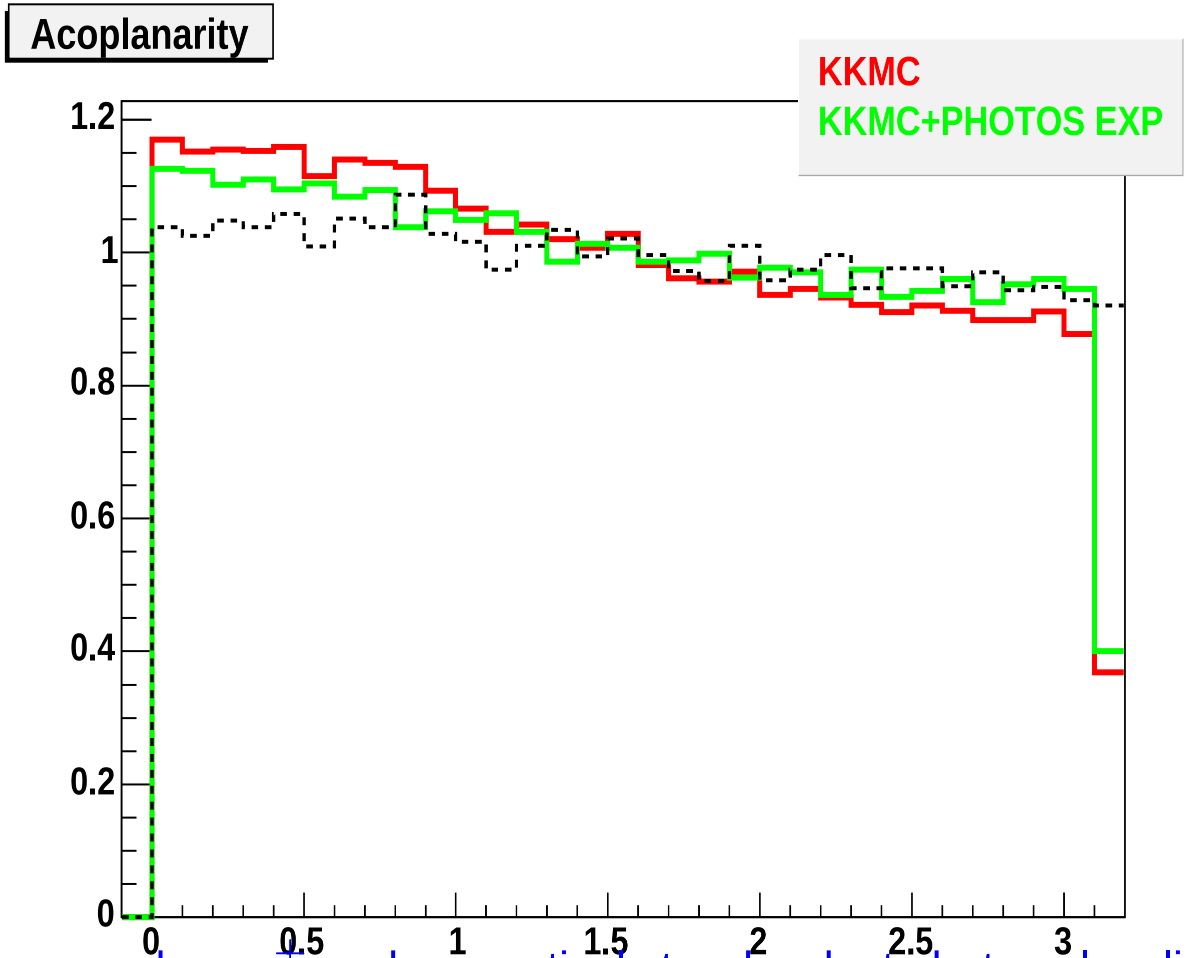


(a) $\pi^+\pi^-$ acollinearity distribution ($\approx \pi$))

(b) $\pi^+\pi^-$ acoplanarity distribution

Figure 1: Transverse spin observables for the H boson for $\tau^\pm \longrightarrow \pi^\pm \nu_\tau$. Distributions are shown for scalar higgs (red), scalar-pseudoscalar higgs with mixing angle $\frac{\pi}{4}$ (green) and the ratio between the two (black).

Acoplanarity distribution – Looks good



Two plane spanned on μ^+ and respectively two hardest photons localized in the same hemisphere as μ^+. In exlusive exponentiation this asymmetry appears with second order matrix element only.

References for MC-TESTER

- *MC-Tester v.2* Nadia Davidson, Piotr Golonka, Tomasz Przedzinski, Zbigniew Was, preprint CERN-LCGAPP-2008-02, http://arxiv.org/abs/0812.3215, CPC in print,
  http://mc-tester.web.cern.ch/MC-TESTER/,
  LCG Generator Services Subproject
  http://lcgapp.cern.ch/project/simu/generator/
- Works in FORTRAN and C++ environments,
- prepared for monitoring programs installations,
- it is useful in work on our programs.

General formalism for semileptonic decays

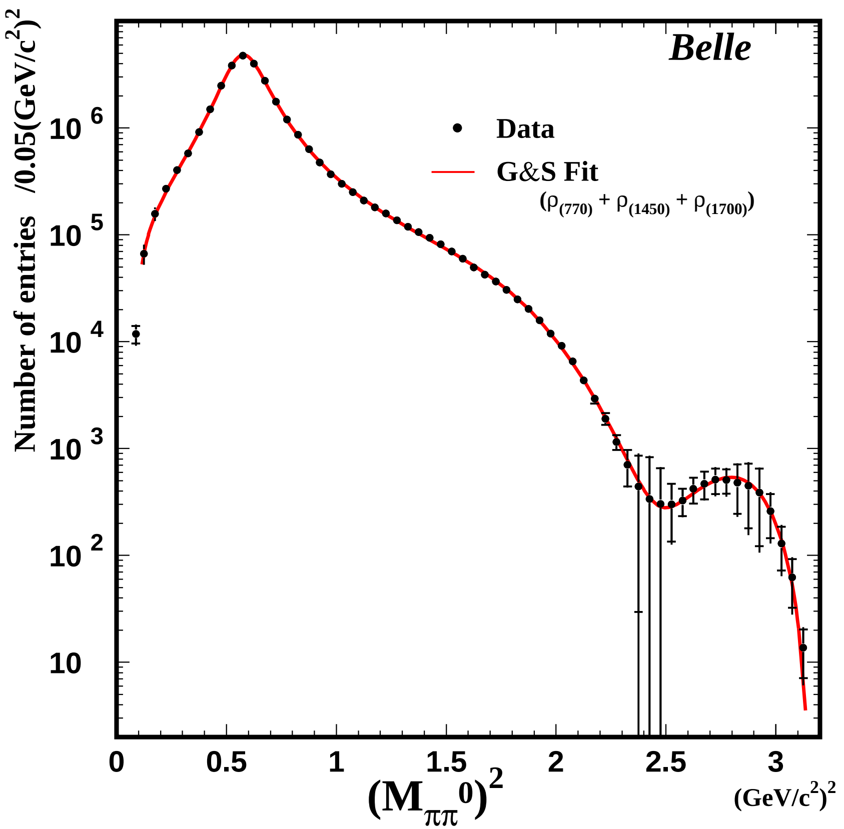
- TAUOLA's Matrix element (+exact phase space)

$$\tau(P,s) \to \nu_\tau(N)X$$
$$\mathcal{M} = \frac{G}{\sqrt{2}}\bar{u}(N)\gamma^\mu(v+a\gamma_5)u(P)J_\mu$$

- J_μ the current depends on the momenta of all hadrons

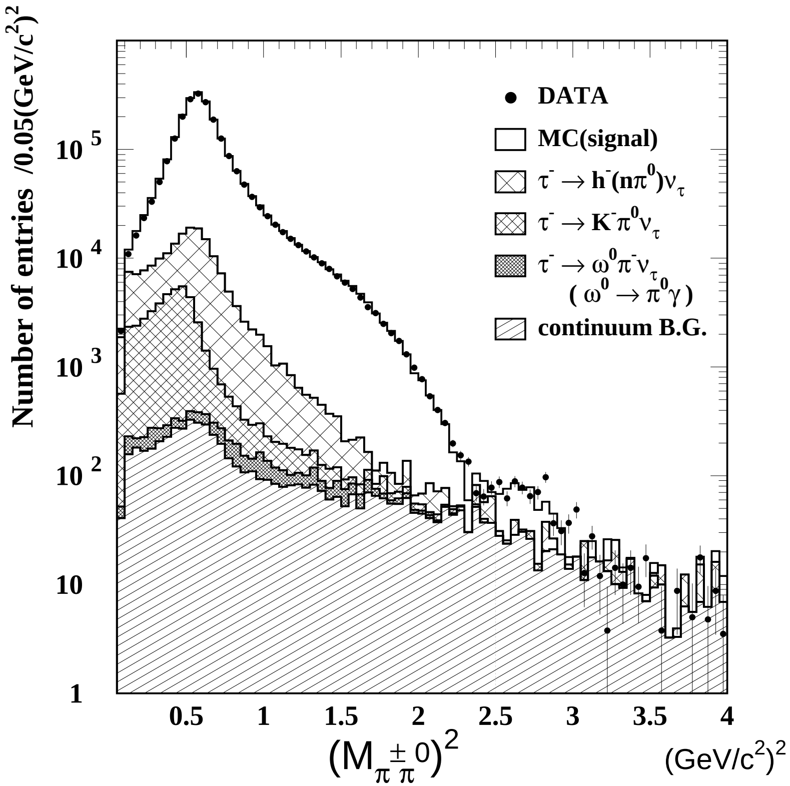
$$|\mathcal{M}|^2 = G^2\frac{v^2+a^2}{2}(\omega + H_\mu s^\mu)$$
$$\omega = P^\mu(\Pi_\mu - \gamma_{va}\Pi^5_\mu)$$
$$H_\mu = \frac{1}{M}(M^2\delta^\nu_\mu - P_\mu P^\nu)(\Pi^5_\nu - \gamma_{va}\Pi_\nu)$$
$$\Pi_\mu = 2[(J^*\cdot N)J_\mu + (J\cdot N)J^*_\mu - (J^*\cdot J)N_\mu]$$
$$\Pi^{5\mu} = 2\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma}J^*_\nu J_\rho N_\sigma$$
$$\gamma_{va} = -\frac{2va}{v^2+a^2}$$
$$\hat{\omega} = 2\frac{v^2-a^2}{v^2+a^2}m_\nu M(J^*\cdot J)$$
$$\hat{H}^\mu = -2\frac{v^2-a^2}{v^2+a^2}m_\nu\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma}J^*_\nu J_\rho P_\sigma$$

TAOLA - generator of τ decays



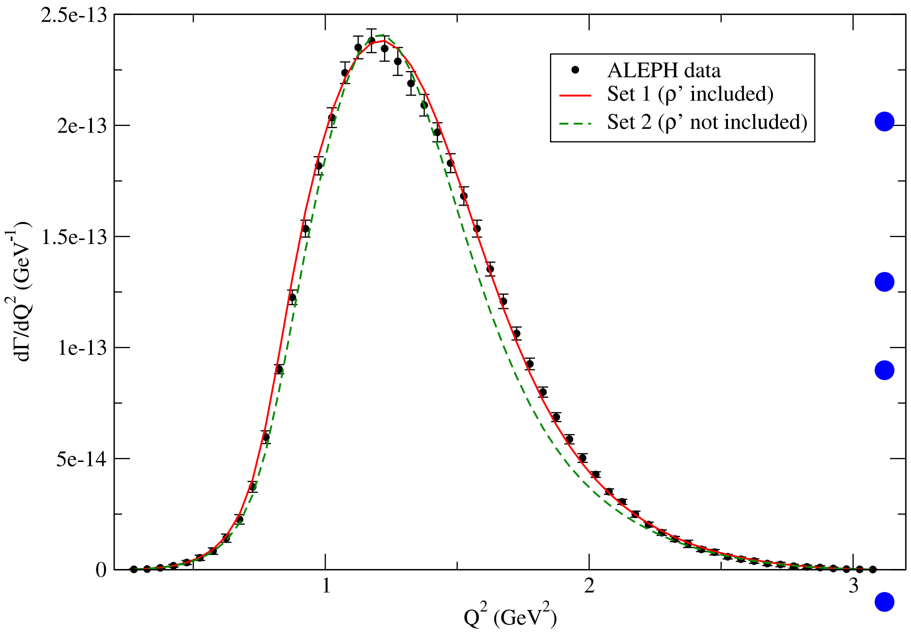
- Publicly available TAUOLA hadronic current are based on data up to 1997.
- Quite in contrary, the internal Belle collaboration parametrization used in TAUOLA is making perfect match for invariant mass of $\pi^+\pi^0$-pair in $\tau \to \pi^+\pi^0\nu$ decay channel.
- Single channel improvement does not bring real progress?
- Up to the date simulation of τ decays for LHC/LC use requires effort by phenomenologists and experimental physicists (Belle/BaBar)

TAUOLA - generator of τ decays



- Measured (Belle) distribution in interesting range has to be disentangled from background.
- At higher end of the spectrum background dominates over $\pi^+\pi^0\nu_\tau$.
- The same should be expected from LHC detectors.
- Correct simulation of τ decays for LHC applications and for Belle BaBar backgrounds as well!
- But, do it for all channels simultaneously!
- Who should play dominant role in validating final choices: model builders? MC authors?Experiments?
- Man power and coordination issues are essential too.

Installation of new current from P. Roig



- The 3π decay mode is important contributor to τ decays. We install new currents into TAUOLA. We start from 0911.4436 [hep-ph].
- There are 4 complex functions of 2 variables in the game.
- We attempt 0.05 % technical precision.
- Methods of porting the code to Belle BaBar to be discussed in context of precision fits are important.
- Only when this step is completed we can announce real progress.
- Other decay channels must be woked outr as well.

Installation of new current from P. Roig

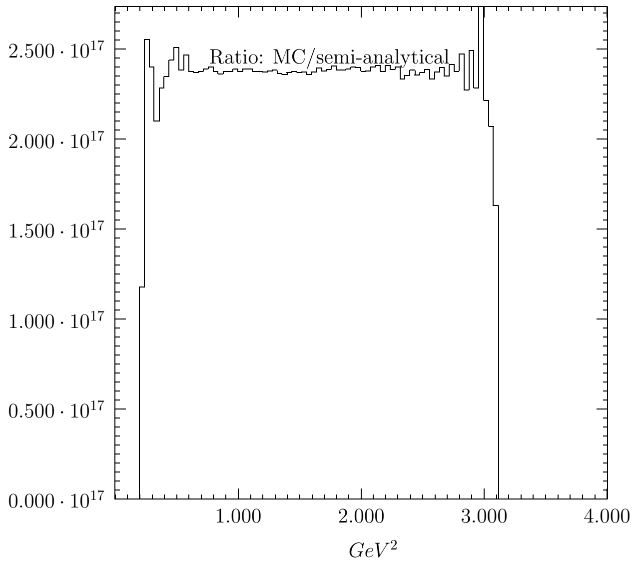


Figure 3: First attempt for comparison of Monte Carlo result with numerical calculation for spectrum of hadronic system invariant mass sqared. Ratio of the two is shown. Statistical sample of 2.5M evts was used and semi-realistic initialization as explained in the section. Reasonable agreement between Monte Carlo and numerical integration is found.

It is promising but no final plot. Following is missing On Monte Carlo side: 0.07 % events are overweighted, maximum weight and/ot presampler parameters have to be tuned, size of the owerweighting need monitoring. Plot need normalization.

5

- Ratio of Monte Carlo obtained $\frac{d\Gamma}{dQ^2}$ and semi analytical formula is shown.
- Agreement looks perfect.
- Physics precision is not as good, also $\frac{d\Gamma}{dQ^2}$ represents an input to the model parametrization.
- Differential distributions and full data need to be confronted.
- How to parametrize differences between model and data if it is needed?

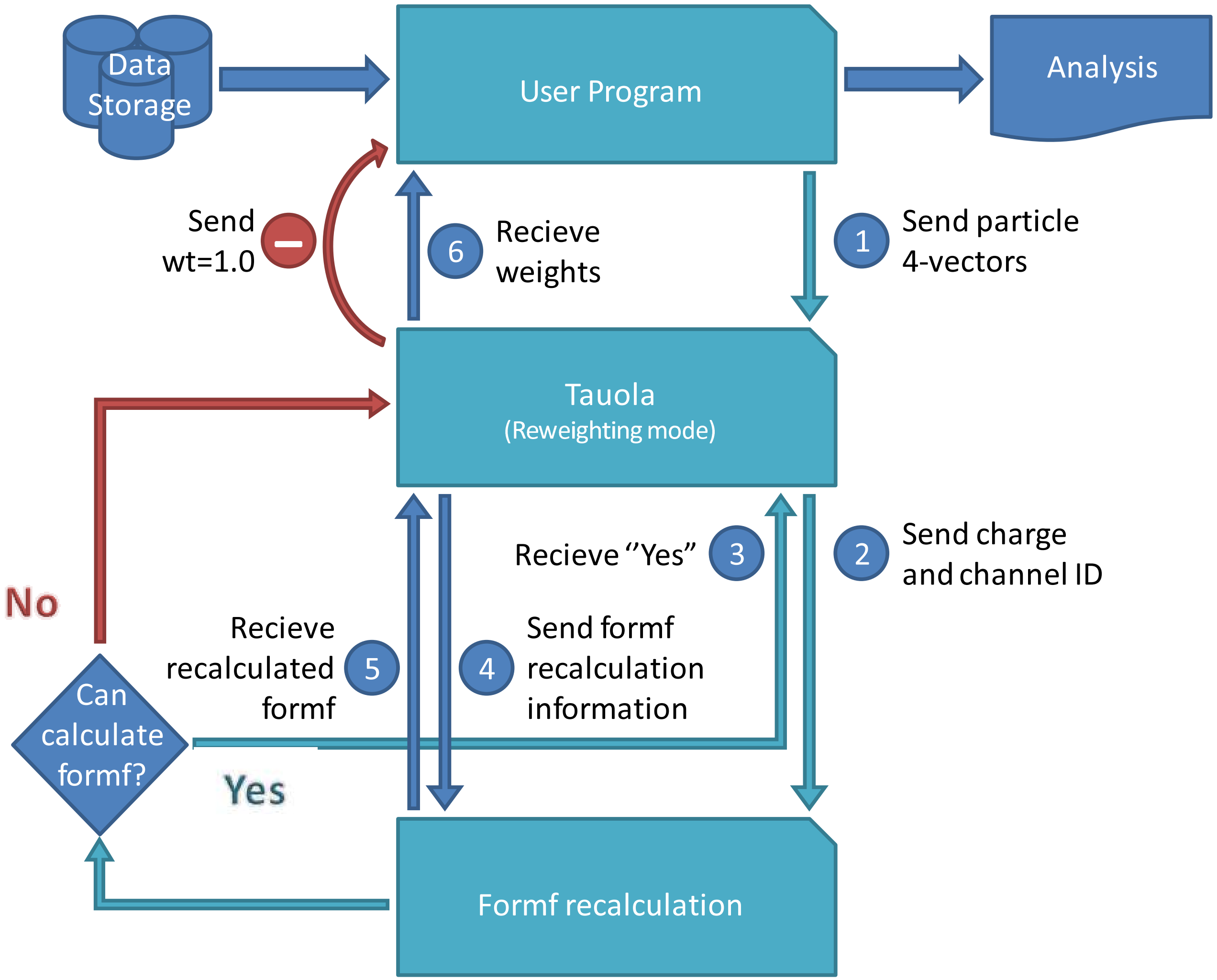
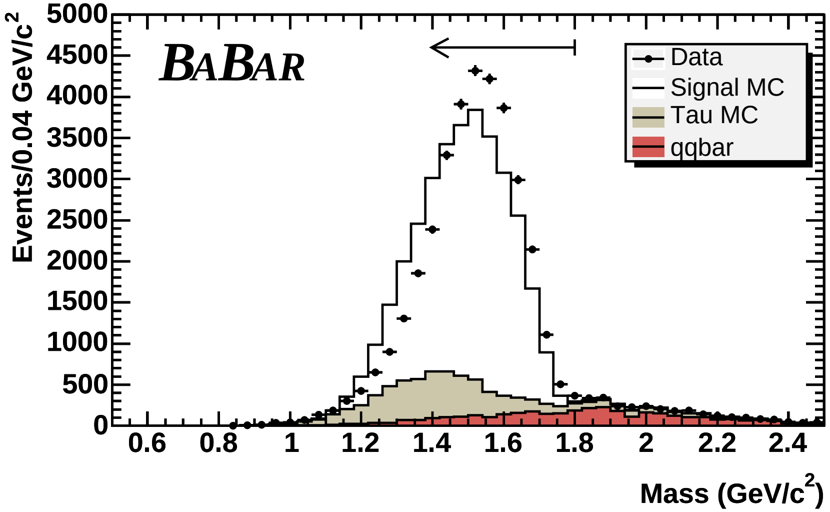


Figure 2: Flow chart for fifo communication. Verified to be compatible with Belle and BaBar software.



The invariant mass of five charged particles for $\tau^- \rightarrow 3h^- 2h^+ \nu_\tau$ at BaBar. How to improve in systematic way?

- For multi-scalar final states challenge: simultaneous fits of many complex form-factors of many variables into massively multi-dimensional distributions. Theoretical constrains apply (or not)
- I hope that this challenge will be adressed by Belle and BaBar.
- But it is not going to be easy.
- On the technical side, that is the reason why parts of TAUOLA will remain in FORTRAN until this work is finished.
- We have prepared some software which may be helpful. Let me explain how it works.

TAUOLA interface

1. Internal τ decay dynamic is still of secondary interest at LHC. It is challenging for low energy precision measurements: see hep-ph/0912.0749. That is why internal part of TAUOLA project remain in FORTRAN.

2. Event record interface is now also in C++ .

3. Physics quality of that HepMC interface is already better than its FORTRAN predecessor, but tests are less profound.

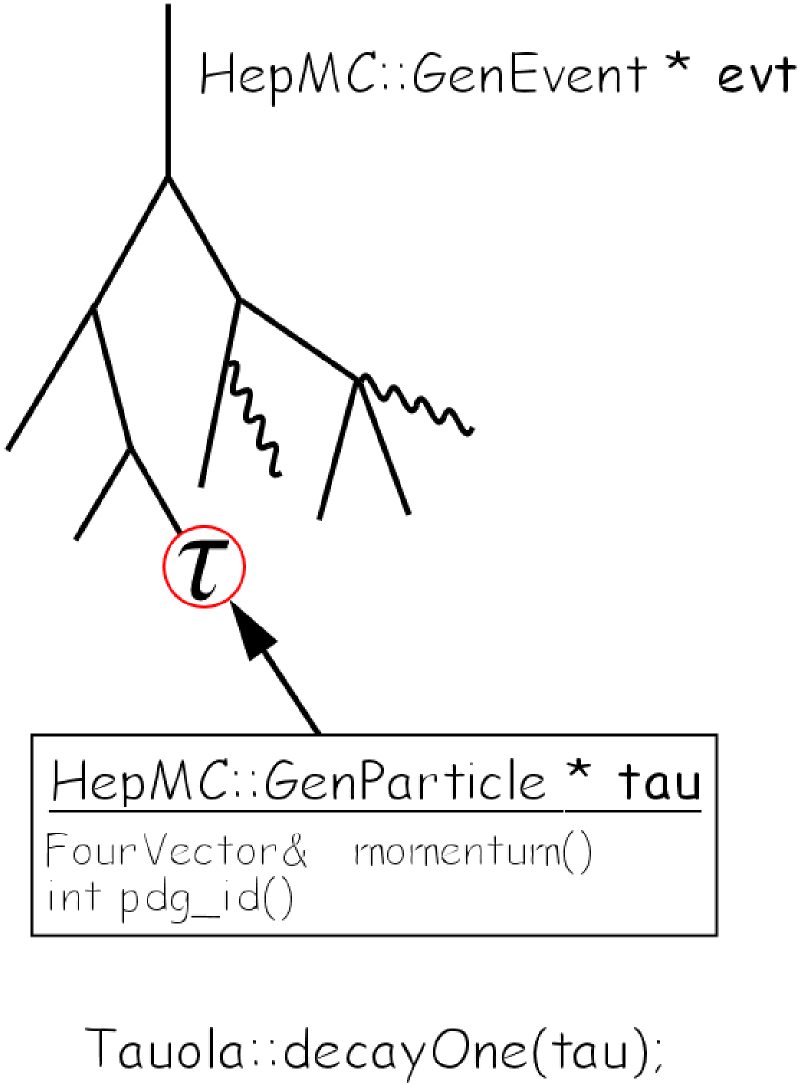
4. Web pages of TAUOLA C++
www.ph.unimelb.edu.au/˜ndavidson/tauola/doxygen/index.html

5. Reference: arXiv:1001.0070 [hep-ph]

6. High precision must be assured. At the same time only information as available in measurements. One does not need to rely on guessing but profound studies of spin amplitudes are necessary (A. van Hameren). The challenge: detector level lepton universality $\longrightarrow$ control backgrounds of $H \rightarrow \tau^+\tau^-$ signatures.

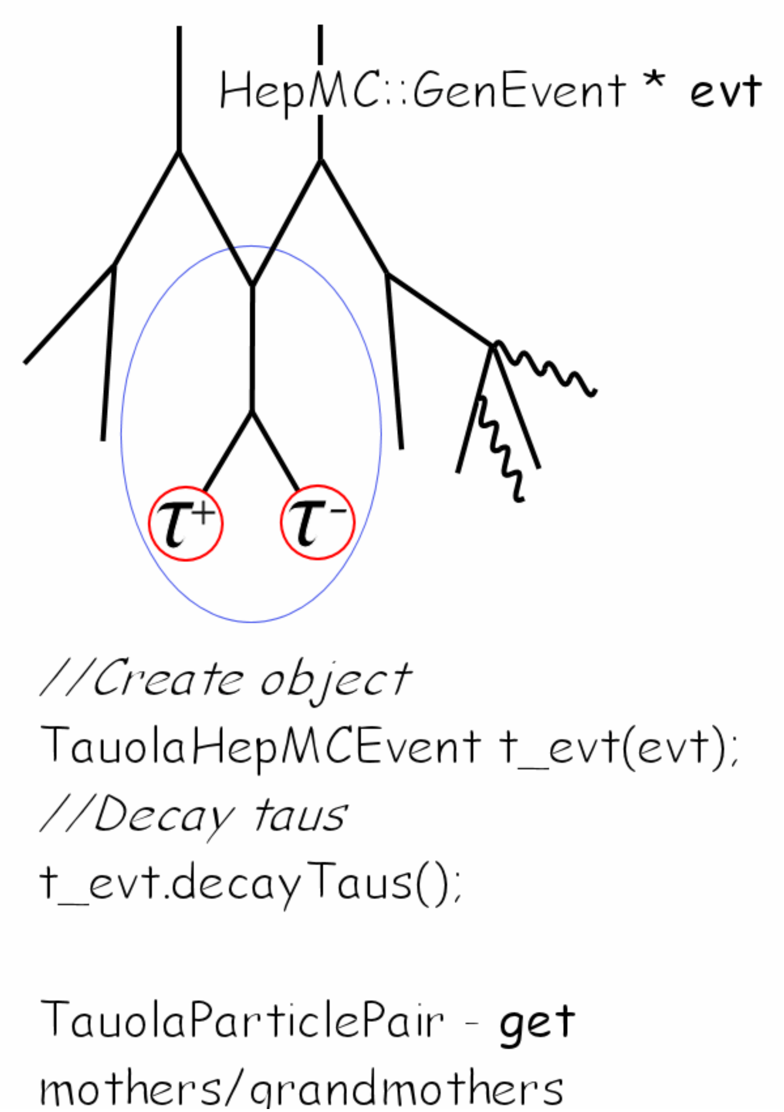
www.ph.unimelb.edu.au/˜ndavidson/photos/doxygen/index.html$\longrightarrow$ PHOTOS C++/HepMC

Single tau decay → NEW



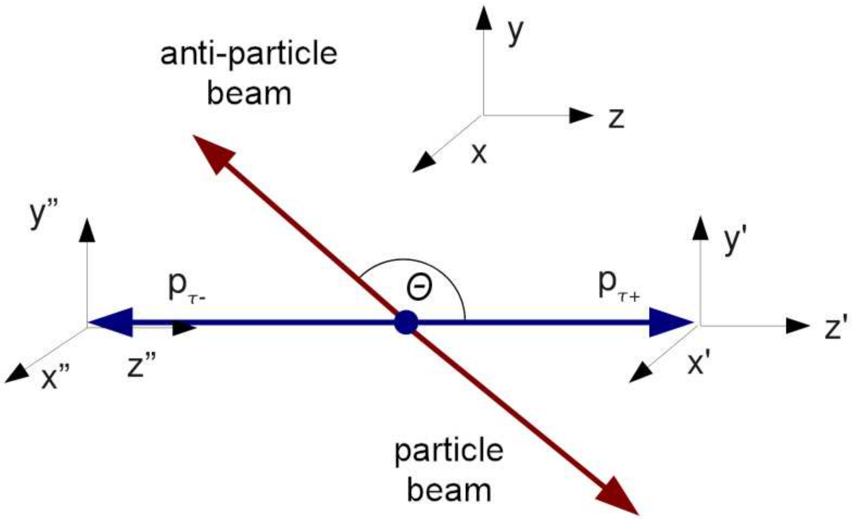
- For the individual τ decay method `Tauola::decayOne()` is provided
- Pointer `tau` to τ in `HepMC` must be known.
- Unpolarized τ decay will be performed, decay products will be transferred to lab. frame using τ 4-momentum. Event record will be updated.
- Tau polarization vector, flag to re-decay already decayed τ and pointer to user defined method for boosting from τ rest-frame to lab frame can be passed as well.
- **Interface is prepared for use in user applications when exact spin effects are required (like in EvtGen** if TAUOLA needed there).

Decay of $\tau^+\tau^-$ $(\tau^\pm\nu_\tau)$ pair.



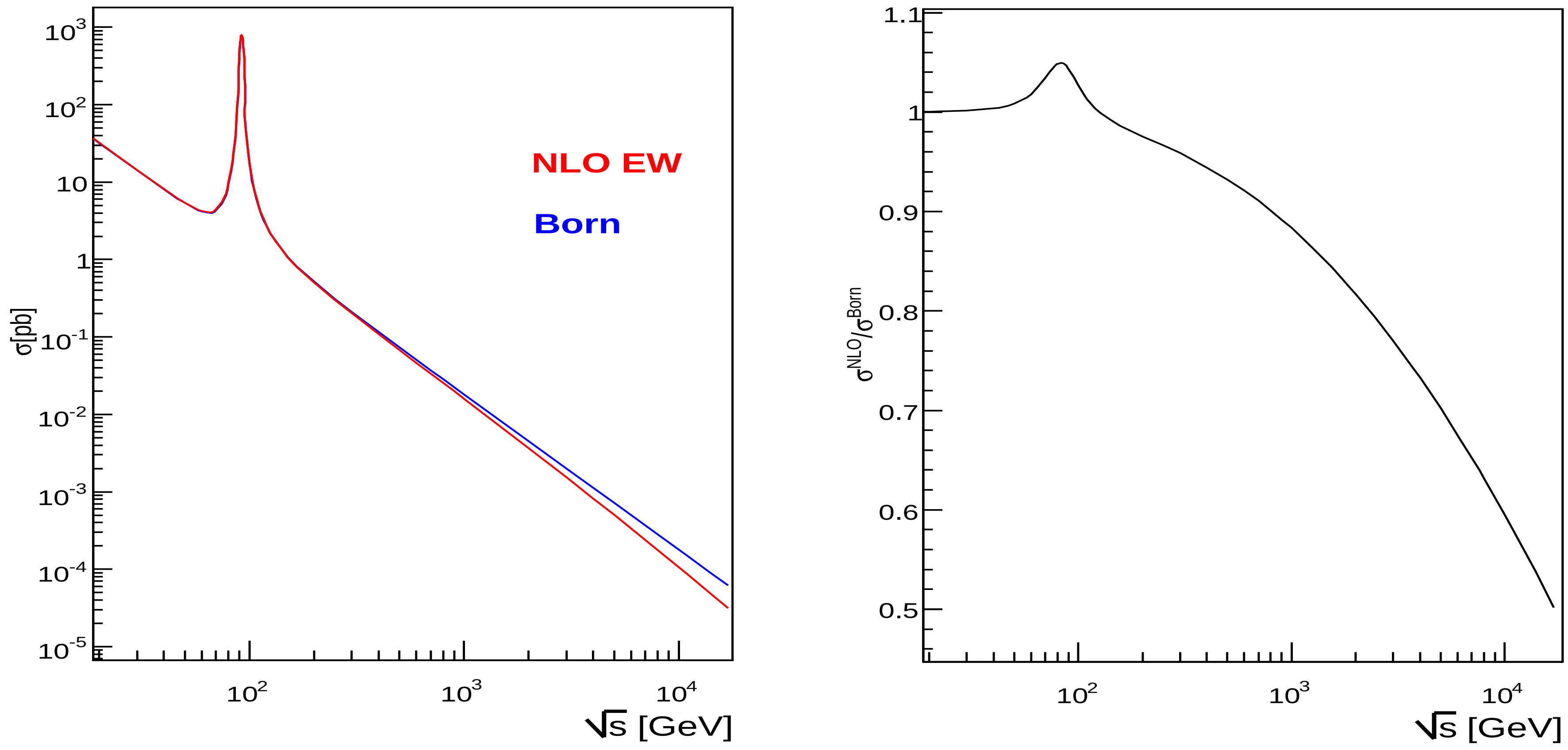
- Create object `t_evt` of class `TauolaHepMCEvent` which inherit from abstract class `TauolaEvent` and use `evt` of `HepMC::GenEvent` class as parameter. Then apply `t_evt.decayTaus()`
- For method `.decayTaus()` event record is searched for elementary processes like $1 \to 2$ (decays) or $2 \to 2$ or $2 \to 1 \to 2$ the s-channel production. For pairs found algorithm of next page is invoked.
- Interface was checked to work well with main processes as produced by PYTHIA 8.1.
- Further testing means checking correctness of HepMC trees.

Decay of $\tau^+\tau^-$ ($\tau^\pm\nu_\tau$) pair.



- Configuration of hard process: flavors and 4-momenta of incoming quarks and outgoing τ's (ν_τ)
- **NEW:** algorithm for spin correlations has no approximation.
- However, method to calculate density matrix from that input usually will impose approximations.
- **NEW:** Density matrix including EW corrections is an option. This arrangement can be used to add Z' or to play with spin correlation component by component.
- **NEW:** Helicity states are attributed at the end (approximation is then used). Useful for some LEP style analyses.

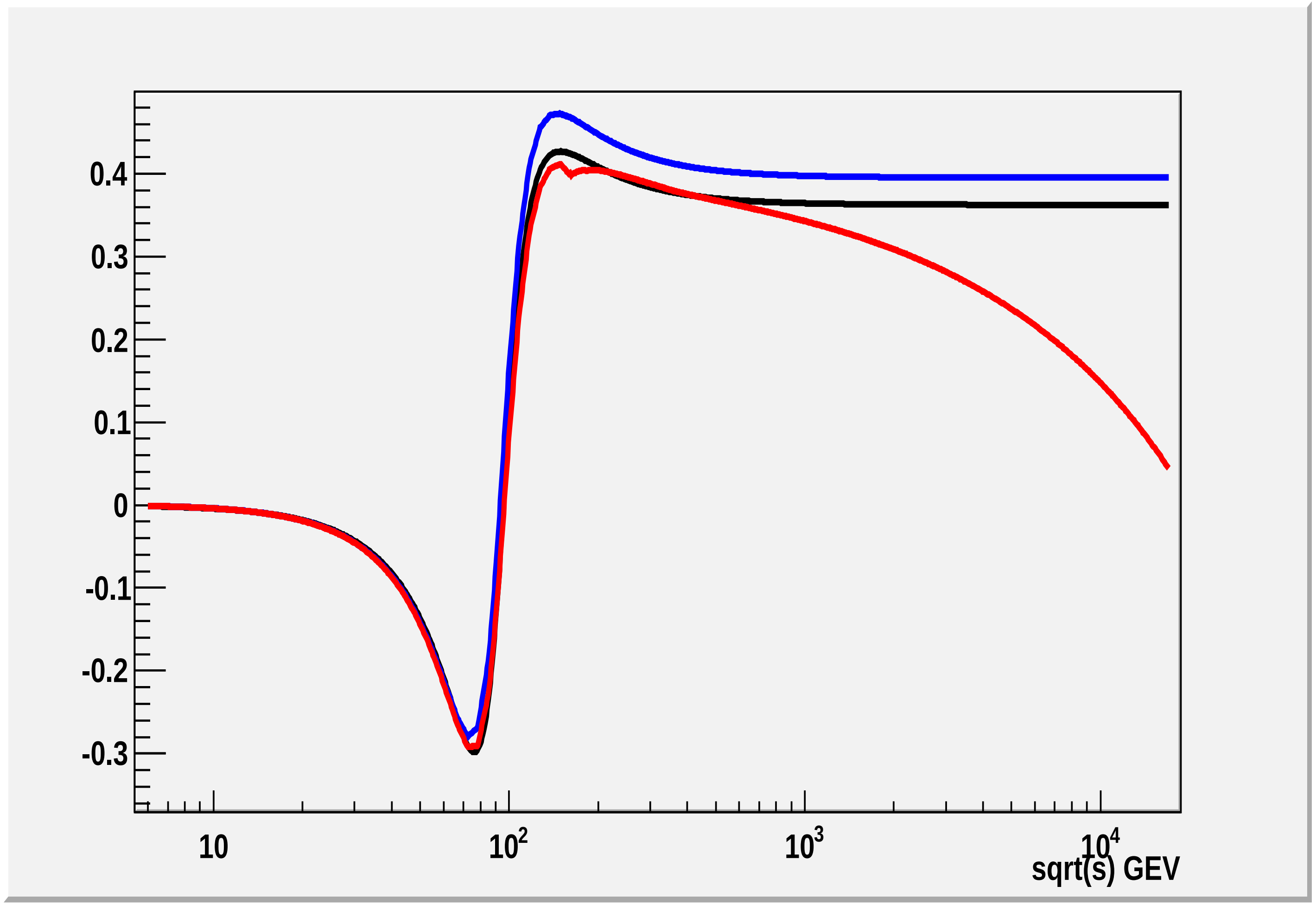
Numerical results, tests.



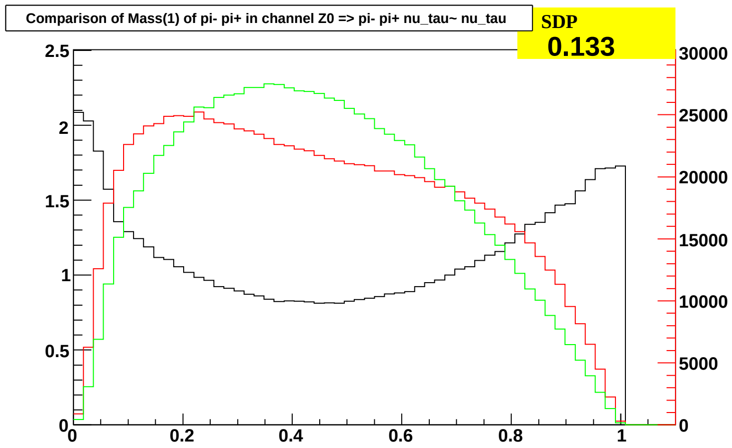
Effect of electroweak corrections on τ-pair production, up quarks, alpha scheme.

Q: What Born parameters are used in PYTHIA?

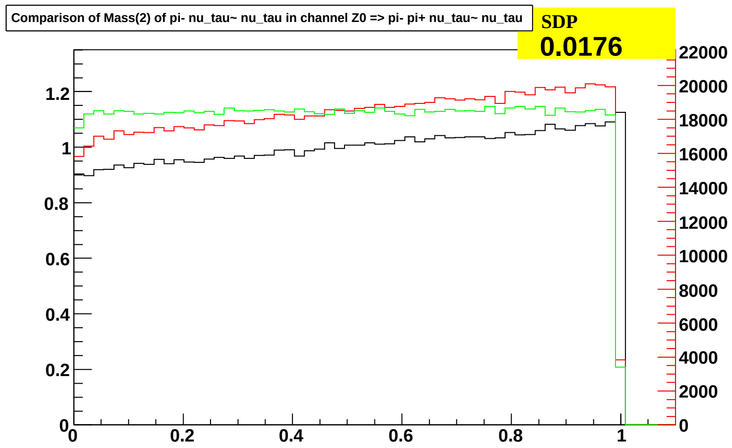
Numerical results, tests.



Effect of electroweak corrections on τ-polarization, up quarks. Red line includes electroweak corrections, Black is TAUOLA standard and blue is Born, alpha scheme. Scattering angle $\cos\theta = -0.2$



(a) $M_{\pi^+\pi^-}$



(b) $1 - 2\frac{E_{\pi^+}}{M_Z}$

Figure 3: Longitudinal spin observables for the Z boson. Distributions are shown for spin effects switched on (red), spin effects switched off (green) and the ratio between spin on and off (black). Left plot show effect of correlation between τ^+ and τ^- decays, right one is for polarization. Figures are obtained with the help of MC-TESTER.

PHOTOS: short presentation

Presentation

- PHOTOS (by E.Barberio, B. van Eijk, Z. W., P.Golonka) is used to simulate the effect of radiatiative corrections in decays, since 1989.
- many citations from experiments $\longrightarrow$ responsability
- Full events combining complicated tree structure of production and subsequent decays are fed into PHOTOS, usually with the help of HEPEVT event record of F77
- PHOTOS version for HepMC event record used in C++ applications is ready for tests now.
- At every event decay branching, PHOTOS intervene. With certain probability extra photon may be added and kinematics of other particles adjusted.

Main References

- E. Barberio, B. van Eijk and Z. Was, Comput. Phys. Commun. **66**, 115 (1991): single emission
- E. Barberio and Z. Was, Comput. Phys. Commun. **79**, 291 (1994). double emission introduced, tests with second order matrix elements
- P. Golonka and Z. Was, EPJC 45 (2006) 97 multiple photon emisson introduced, tests with precioson second order exponentiation MC.
- P. Golonka and Z. Was, EPJC 50 (2007) 53 complete matrix element for Z decay, and further tests
- G. Nanava, Z. Was, Eur.Phys.J.C51:569-583,2007, best description of phase space
- G. Nanava, Z. Was, Q. Xu, arXiv:0906.4052. EPJC in print complete matrix element for W decay
- N. Davidson, T. Przedzinski, Z. Was, IFJPAN-IV-2010-6, Presently main web-page for program C++ version: http://www.ph.unimelb.edu.au/˜ndavidson/photos/doxygen/index.html HepMC interface

Status: practical

- PHOTOS feature complete exact phase space for multiphoton radiation.
- Unique double iteration algorithm: Internal loop is over emitting particles external one over consecutive photons, that is why one can simultaneously benefit from parton shower and exponentiation properties.
- Studies of single/double photon spin amplitudes were essential.
- Comparisons with SANC started by D. Bardin, for Z and W decays. Necessary to understand numerically separation of electroweak corrections into genuine weak and QED.
- TAUOLA features interface to HepMC and electroweak library of SANC can be used to re-weight events with weak corrections + new physics.
- Comparisons with KKMC to confirm technical precision. KKMC is the program used at LEP for 2 MeV precision level measurements of Z. KKMC is based on exclusive exponentiation and features second order matrix element for FSR. Agreement better than 0.2 % in experimental cuts (ATLAS CDF) between PHOTOS and KKMC was found.

Summary

1. I was advocating simulation solutions based on theoretical segments communicating with the help of event record HepMC/HEPEVT
2. As an example I have used bremsstrahlung in decays and τ lepton decays
3. I have stressed question of theoretical uncertainties
4. and equally important benchamrks for installations in collaboration software.
5. Finally: overall precison does not need to be compromised and flexibity in use (special weights, correlated samples etc) can be avalilable.

Extra transparencies explaining spin amplitude role in evaluating theoretical precision of PHOTOS MC follow ...

PHOTOS: phase space and crude distribution

Phase Space: must be exact to discuss matrix elements

Orthodox exact Lorentz-invariant phase space (*Lips*) is in use in PHOTOS!

$$
\begin{aligned}
& dLips_{n+1}(P) = \\
& \frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3} \frac{d^3q}{2q^0(2\pi)^3} (2\pi)^4 \delta^4\Big(P - \sum_1^n k_i - q\Big) \\
= \quad & d^4p\delta^4(P-p-q) \frac{d^3q}{2q^0(2\pi)^3} \frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3} (2\pi)^4 \delta^4\Big(p - \sum_1^n k_i\Big) \\
= \quad & d^4p\delta^4(P-p-q) \frac{d^3q}{2q^0(2\pi)^3} dLips_n(p \to k_1 ... k_n).
\end{aligned}
$$

Integration variables, the four-vector p, compensated with $\delta^4\big(p - \sum_1^n k_i\big)$, and another integration variable M_1 compensated with $\delta\big(p^2 - M_1^2\big)$ are introduced.

PHOTOS: phase space and crude distribution

Phase Space Formula of Photos

$$dLips_{n+1}(P \to k_1...k_n, k_{n+1}) = dLips_n^{+1\ tangent} \times W_n^{n+1},$$

$$dLips_n^{+1\ tangent} = dk_\gamma d\cos\theta d\phi \times dLips_n(P \to \bar{k}_1...\bar{k}_n),$$
$$\{k_1, \ldots, k_{n+1}\} = \mathbf{T}\big(k_\gamma, \theta, \phi, \{\bar{k}_1, \ldots, \bar{k}_n\}\big). \quad (1)$$

1. One can verify that if $dLips_n(P)$ was exact, then this formula lead to exact parametrization of $dLips_{n+1}(P)$
2. Practical implementation: Take completely construced n-body phase space point (event).
3. Reconstruct coordinate variables, any parametrization can be used.
4. Construct new kinematical configuration from those variables and $k_\gamma\theta\phi$.
5. **Forget about temporary** $k_\gamma\theta\phi$. **Now, only weight and new four vectors count.**
6. A lot depend on $\mathbf{T}$. Options depend on matrix element: must tangent at singularities. Simultaneous use of several $\mathbf{T}$ is necessary/convenient if more than one charge is present in final state.

Phase Space: (main formula)

If we choose

$$G_n \ : \ M^2_{2\dots n}, \theta_1, \phi_1, M^2_{3\dots n}, \theta_2, \phi_2, \dots, \theta_{n-1}, \phi_{n-1} \ \longrightarrow \ \bar{k}_1 \dots \bar{k}_n \tag{2}$$

and

$$G_{n+1} \ : \ k_\gamma, \theta, \phi, M^2_{2\dots n}, \theta_1, \phi_1, M^2_{3\dots n}, \theta_2, \phi_2, \dots, \theta_{n-1}, \phi_{n-1} \ \longrightarrow \ k_1 \dots k_n, k_{n+1} \tag{3}$$

then

$$\mathbf{T} = G_{n+1}(k_\gamma, \theta, \phi, G_n^{-1}(\bar{k}_1, \dots, \bar{k}_n)). \tag{4}$$

The ratio of the Jacobians form the phase space weight W_n^{n+1} for the transformation. Such solution is universal and valid for any choice of G's. However, G_{n+1} and G_n has to match matrix element, otherwise algorithm will be inefficient (factor 10^{10} ...).

In case of PHOTOS G_n's

$$W_n^{n+1} = k_\gamma \frac{1}{2(2\pi)^3} \times \frac{\lambda^{1/2}(1, m_1^2/M^2_{1\dots n}, M^2_{2\dots n}/M^2_{1\dots n})}{\lambda^{1/2}(1, m_1^2/M^2, M^2_{2\dots n}/M^2)}, \tag{5}$$

Phase Space: (multiply iterated)

By iteration, we can generalize formula (1) and add l particles:

$$dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l}) = \frac{1}{l!} \prod_{i=1}^{l} \Big[dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W^{n+i}_{n+i-1} \Big]$$
$$\times dLips_n(P \to \bar{k}_1...\bar{k}_n), \quad (6)$$
$$\{k_1, \ldots, k_{n+l}\} = \mathbf{T}\big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}\big(\ldots, \mathbf{T}\big(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \ldots, \bar{k}_n\}\big)\ldots\big).$$

Note that variables $k_{\gamma_m}, \theta_{\gamma_m}, \phi_{\gamma_m}$ are used at a time of the $m-$th step of iteration only, and are not needed elsewhere in construction of the physical phase space; the same is true for invariants and angles $M^2_{2\ldots n}, \theta_1, \phi_1, \ldots, \theta_{n-1}, \phi_{n-1} \;\to\; \bar{k}_1 \ldots \bar{k}_n$ of (2,3), which are also redefined at each step of the iteration. Also intermediate steps require explicit construction of temporary $\bar{k}'_1 \ldots \bar{k}'_n \ldots \bar{k}'_{n+m}$, statistical factor $\frac{1}{l!}$ added.

We have **exact distribution** of **weighted** events over l and $n+l$ body phase spaces.

Crude Distribution for multiple emission

If we add arbitrary factors $f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i})$ and sum over l we obtain:

$$\sum_{l=0} \exp(-F)\frac{1}{l!}\prod_{i=1}^{l} f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dLips_{n+l}(P \to k_1 ... k_n, k_{n+1} ... k_{n+l}) =$$

$$\sum_{l=0} \exp(-F)\frac{1}{l!}\prod_{i=1}^{l}\Big[f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W^{n+i}_{n+i-1}\Big] \times$$

$$dLips_n(P \to \bar{k}_1 ... \bar{k}_n), \tag{7}$$

$$\{k_1, \dots, k_{n+l}\} = \mathbf{T}\Big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}\big(\dots, \mathbf{T}\big(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \dots, \bar{k}_n\}\big) \dots\big)\Big),$$

$$F = \int_{k_{min}}^{k_{max}} dk_\gamma d\cos\theta_\gamma d\phi_\gamma f(k_\gamma, \theta_\gamma, \phi_\gamma).$$

- The Green parts of rhs. alone, give crude distribution over tangent space (orthogonal set of variables k_i, θ_i, ϕ_i).

PHOTOS: phase space and crude distribution

- Factors f (W' ignored) must be integrable over coordinates. Regulators of singularities necessary, but simple.

- If we request from infrared regulators, f and F that

$$\sigma_{tangent} = 1 =$$
$$\sum_{l=0} \exp(-F) \frac{1}{l!} \prod_{i=1}^{l} \Big[f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} \Big]$$

we get Poissonian distribution in l.

- Sum rules originating from perturbative approach (KLM theorem) are necessary to inccorporate dominant part of virtual corrections, into the scheme. We get Monte Carlo solution of PHOTOS type.

- For that to work, real emission and virtual corrections need to be calculated and their factorization properties analyzed. Choice for f and G are fixed from that.

- If such conditions are fulfilled construction of Monte Carlo algorithm is prepared.

- Truncate $\sigma_{tangent}\big|_{\mathcal{O}(\alpha),\mathcal{O}(\alpha^2)}$, $\longrightarrow$ phase space in single/double photon mode.

Important property of fully differential distribution

- Fully differential single photon emission formula in Z decay reads:

$$X_f = \frac{Q'^2 \alpha(1-\Delta)}{4\pi^2 s} s^2 \quad \left\{ \frac{1}{(k'_+ k'_-)} \left[\frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t,u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t',u) \right] \right\}$$

- Variables in use:

$$s = 2p_+ \cdot p_-, \quad s' = 2q_+ \cdot q_-, \quad t = 2p_+ \cdot q_+, \quad t' = 2p_+ \cdot q_-,$$
$$u = 2p_+ \cdot q_-, \quad u' = 2_- \cdot q_+, \quad k'_\pm = q_\pm \cdot k, \quad x_k = 2E_\gamma/\sqrt{s}$$

- The Δ term is responsable for final state mass dependent terms, p_+, p_-, q_+, q_-, k denote four-momenta of incoming positron, electron beams, outcoming muons and bremsstrahlung photon.

- Factorization of first order matrix element and fully differential distribution breaks at the level $\frac{\alpha^2}{\pi^2} \simeq 10^{-4}$

Important property of fully differential distribution

- after trivial manipulation it can be written as:

$$X_f = \frac{Q'^2 \alpha(1-\Delta)}{4\pi^2 s} s^2 \quad \Big\{ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_-} \Big[\frac{d\sigma_B}{d\Omega}(s,t,u') + \frac{d\sigma_B}{d\Omega}(s,t',u) \Big]$$

$$+ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_+} \Big[\frac{d\sigma_B}{d\Omega}(s,t,u') + \frac{d\sigma_B}{d\Omega}(s,t',u) \Big] \Big\}$$

- In PHOTOS the following kernel is used (decay channel, decay particle orientation, independent, (essential: universal interference wt introduced too):

$$X_f^{PHOTOS} = \frac{Q'^2 \alpha(1-\Delta)}{4\pi^2 s} s^2 \Big\{$$

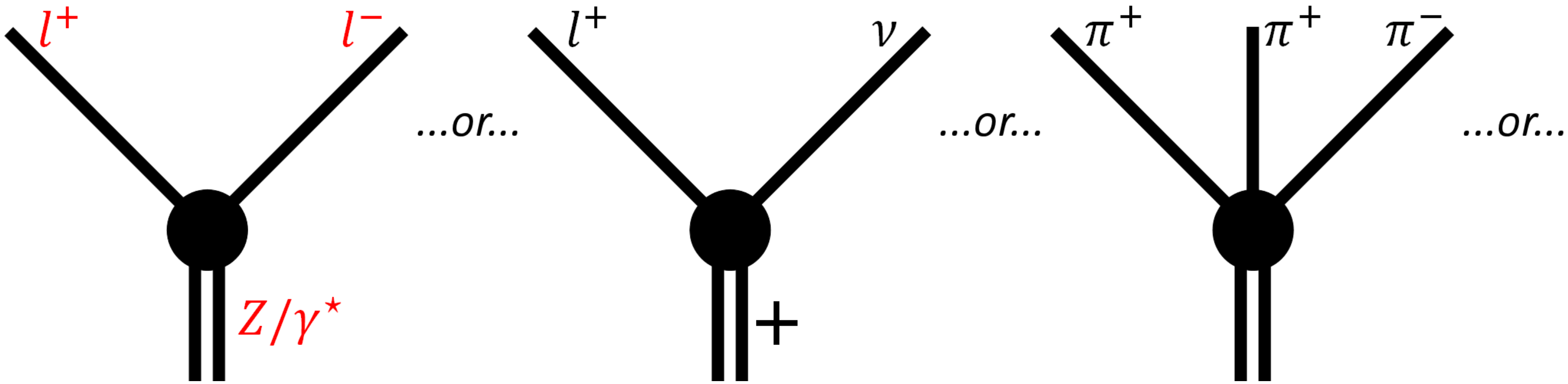
$$\frac{1}{k'_+ + k'_-} \frac{1}{k'_-} \Big[\big(1 + (1-x_k)^2\big) \frac{d\sigma_B}{d\Omega} \Big(s, \frac{s(1-\cos\Theta_+)}{2}, \frac{s(1+\cos\Theta_+)}{2}\Big) \Big] \frac{(1+\beta\cos\Theta_\gamma)}{2}$$

$$+ \frac{1}{k'_+ + k'_-} \frac{1}{k'_+} \Big[\big(1 + (1-x_k)^2\big) \frac{d\sigma_B}{d\Omega} \Big(s, \frac{s(1-\cos\Theta_-)}{2}, \frac{s(1+\cos\Theta_-)}{2}\Big) \Big] \frac{(1-\beta\cos\Theta_\gamma)}{2} \Big\}$$

$$\text{where}: \Theta_+ = \angle(p_+, q_+), \; \Theta_- = \angle(p_-, q_-)$$

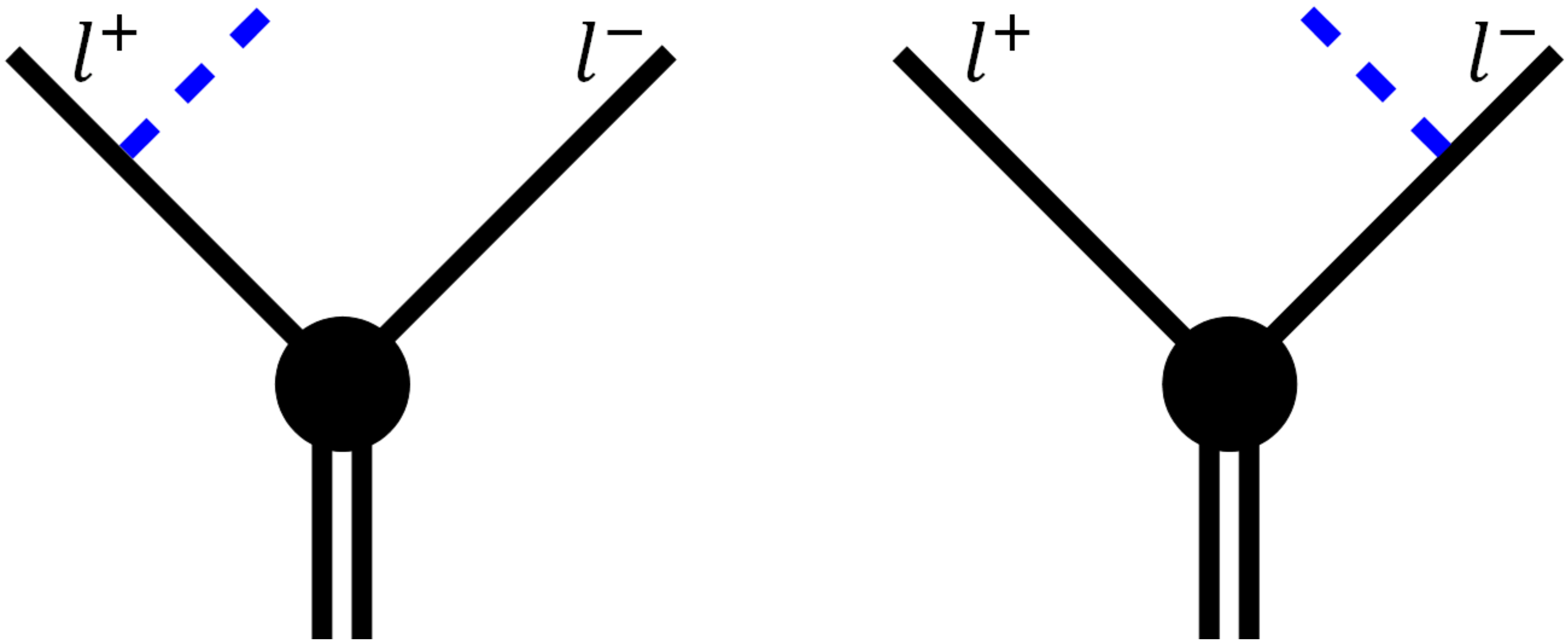
$\Theta_\gamma = \angle(\gamma, \mu^-)$ are defined in (μ^+, μ^-)-pair rest frame

First order spin amplitudes



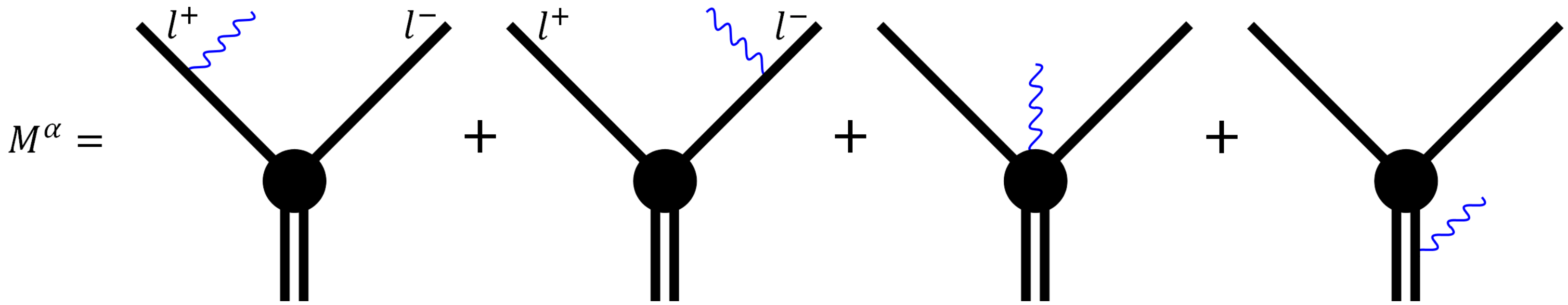
- *The formula which we had on previous slide could be constructed because the Born level matrix elemet (and resulting Born level distribution) relates with the one of first order in α_{QED} through convolution of positively defined function (I will use it as emission kernel) (Berends Kleiss Jadach 1982).*
- *Does such convolution hold for other processes, even if we are concerned with the first order only?*
- *Paper by R. Kleiss from 1992 tells us that it will not hold at level of $(\frac{\alpha}{\pi})^2 \simeq 10^{-5}$.*
- *Comment, these properties are important for all variants of NLO factorizations.*
- *All these issues can be solved with studies of matrix elements only.*

First order spin amplitudes



- *Structure of singularities for the first order corrections to decay of Z/γ^* which we will use as an example.*
- *Two kinematical branches need to be taken into account.*
- *Fortunately kinematical parametrizations for the two branches have identical phase space Jacobians. It simplifies tasks for multiphoton configurations.*

First order spin amplitudes



- *Feynman diagrams for FSR in* Z/γ^* *decays*
- *Out of the first two diagrams distribution for* Z/γ *decay was obtained.*
- *Other two diagrams appear e.g. in scalar QED, and/or in decays of W's or B mesons.*
- *Let us look into sub-structure of these amplitudes.*

$\mathcal{M}$atrix $\mathcal{E}$lement Z/γ^* decay, (formalism $\sim$ Kleiss-Stirling methods):

- $$I = I^A \qquad\qquad + I^B \qquad\qquad + I^C$$
- $$I = \not{J}\left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\right] - \left[\frac{1}{2}\frac{\not{e}_1 \not{k}_1}{p\cdot k_1}\right]\not{J} + \not{J}\left[\frac{1}{2}\frac{\not{e}_1 \not{k}_1}{q\cdot k_1}\right]$$
- Decomposes into 3 parts. Each is independently gauge invariant, valid for "any" $\not{J}$.
- Only $|I^A|^2$ contributes to infrared singularities.
- Terms I^B and I^C contribute to collinear big logarithms.
- We could expect another term I^D which would not contribute neither to collinear nor soft divergent/large logarithms (once integration is performed)

structure of singularities apparent already at amplitude level

First order spin amplitudes

What happens for other decays

1. $W \to l\nu_l\gamma$: I^A, I^B and I^D dependent on electroweak calculation scheme.
2. $B^0 \to \pi^+ K^- \gamma$: I^A only
3. $B^+ \to \pi^0 K^+ \gamma$: I^A only
4. $\gamma^* \to \pi^+ \pi^- \gamma$: I^A, and I^D
5. $\tau^+ \to \pi^+ \nu_\tau \gamma$: I^A and I^D
6. ...

It is important that in all cases, and not only for processes of QED, amplitudes can be constructed from the same building blocks.

These properties of amplitudes translate into properties of distributions and that is why exact PHOTOS algorithm for single photon emission can be constructed.

If non dominat terms can be neglected algorithm simlifies and process dependent weights can be replaced by the ones dependinch on charges and spins of outgoing particles.

Single emission

1. **Solution for single emission works perfect.**
2. **Technical precision controlled to precision better than statistical error of 100 Mevts.**
3. **An example where interference between emission from two charged lines is hidden in exact process dependent kernel, but must be added if basically identical one is used.**
4. **Web page with multitude of automated tests (RECOMENDATION: to be repeated after installation in collaboration software): http://mc-tester.web.cern.ch/MC-TESTER/**
5. **Let us go to iteration, used in solution for double and muliple photon emission modes.**

Elementary test of principle

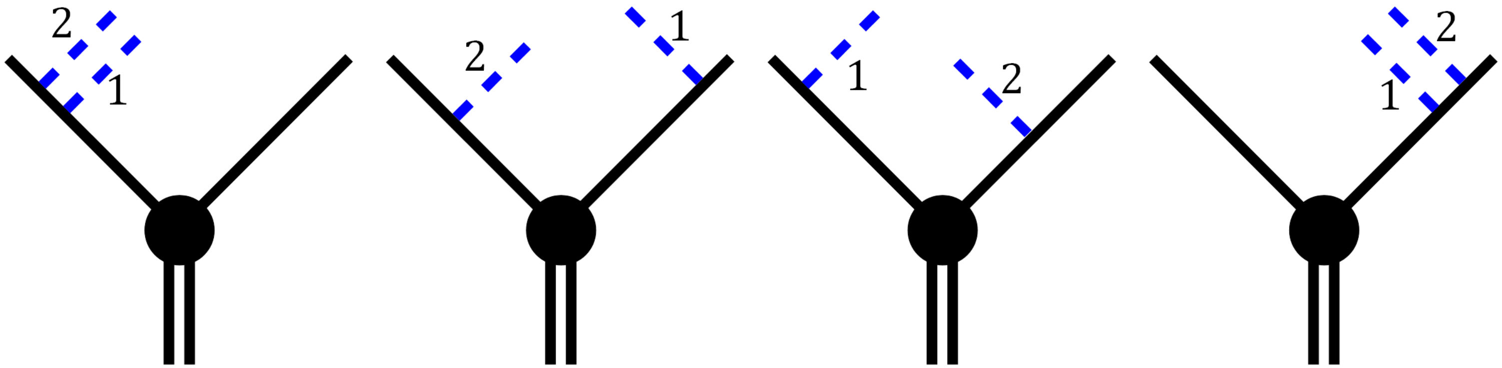
- Do PHOTOS generate the LL contribution to lepton spectra?
- Formal solution of QED evolution equation can be written as:

$$D(x,\beta_{ch}) = \delta(1-x)+\beta_{ch}P(x)+\frac{1}{2!}\beta_{ch}^2\{P\times P\}(x)+\frac{1}{3!}\beta_{ch}^3\{P\times P\times P\}(x)+\ldots \quad (8)$$

where $P(x)=\delta(1-x)(\ln\varepsilon+3/4)+\Theta(1-x-\varepsilon)\frac{1}{x}(1+x^2)/(1-x)$
and $\{P\times P\}(x)=\int_0^1 dx_1\int_0^1 dx_2\delta(x-x_1x_2)P(x_1)P(x_2)$.

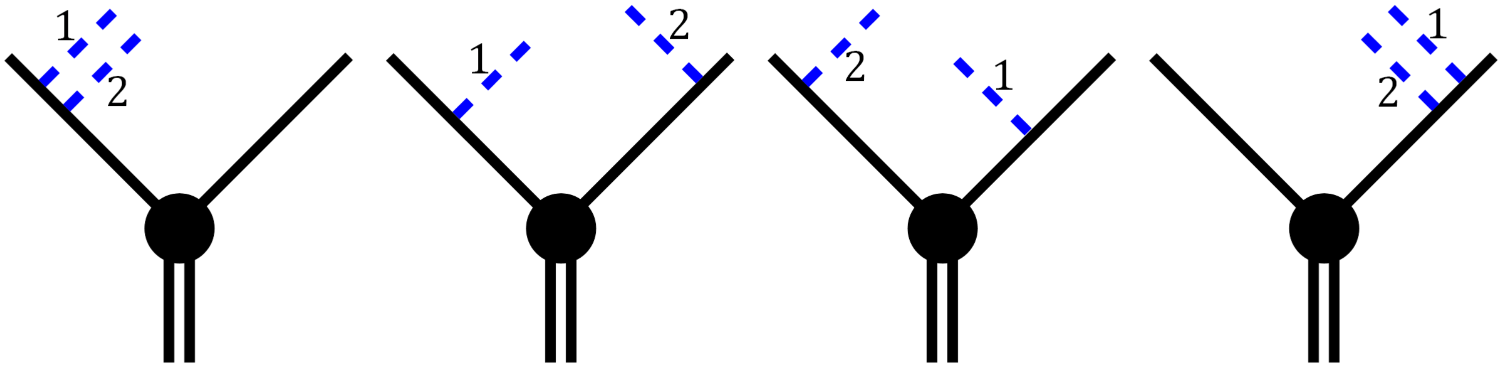
- In LL contributing regions, phase space Jacobian's of PHOTOS trivialize (CPC 1994). and the expression given above is obtained in a straightforward manner. In fact for each of the outcoming charged lines simultaneously.
- But it is only a limit! **PHOTOS treat phase space corners exactly.** We had to understand at spin amplitude, and exact distribution, levels why formula (8) work, keeping in mind what happens with amplitudes non leading parts.

Multiple photon emission



- *To generate consecutive photons, PHOTOS simply iterates its single photon algorithm.*
- *Previously generated photons are treated a any other decay products.*
- *We generate photon 1 (each leg one after another)*
- *We include interference or matrix element weight*
- *And in the same way photon 2.*
- *previously generated photon(s) we remove, for matrix element calculation, from kinematical configuration, using reduction procedure.*
- *Iterative nature is very similar to solution for* $D(x, \beta_{ch}) \times D(y, \beta_{ch})$*, but except collinear limit, here,* $x_1 x_2$ *extends to 3 dimensions and as a consequence order of generating emissions matter* $\hat{x}_1 \hat{x}_2 \neq \hat{x}_2 \hat{x}_1$*. Also generating of* x*-es and* y*-es are affecting each other.*

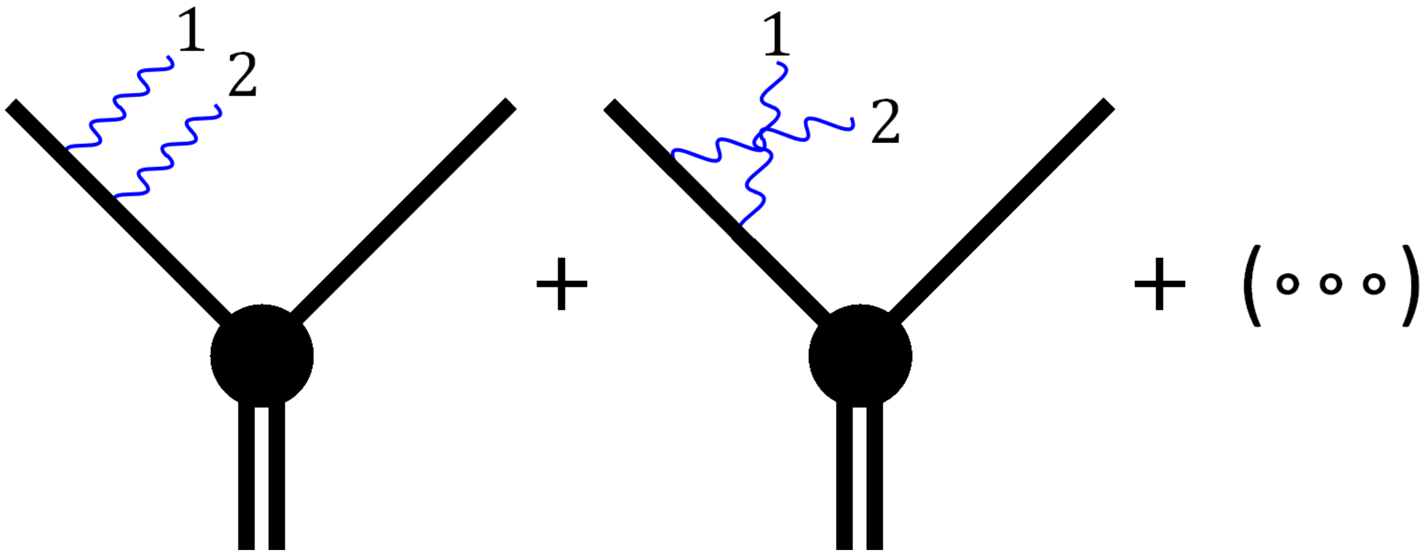
Multiple photon emission



- *We can produce such point in phase space starting with generation of photon 2 and continuing with 1.*
- *Each of the two generation chains cover all phase space. There is no phase space ordering in use. Instead we have statistical factor* $\frac{1}{l!}$ *from*
- *Such solution must be confronted with distributions obtained from matrix elements.*
- *Comparisons with distributions obtained from double and triple photon amplitudes were performed in 1994.*
- *Now let us look at properties of spin amplitudes.*

Multiple photon emission

$M^{\alpha^2} =$



- *We have to check if description given in two previous slides justifies with properties of spin amplitudes.*
- *Iterative algorithm? What with interferences of consecutive emissions?*
- *It is important to check if such properties are process dependent or generalize.*
- *My decade long work under leadership of S. Jadach on e^+e^- generators provided help.*
- *Is double photon emission amplitude build from terms we know from first order?*
- *From calculation it is clear that the structure of $Z/\gamma^* \to l^+l^-\gamma\gamma$ generalizes to other processes.*

Exact Matrix Element: $Z \to \mu^+\mu^-\gamma\gamma$ written explicitly

- We use conventions from paper A. van Hameren, Z.W., EPJC 61 (2009) 33. Expressions are valid for any current J, (also for QCD part proportional to $\{T^A T^B\}$, T^A is for first T^B for second gluon.

- To get complete amplitude sum the gauge invariant parts, add spinors, eg. $\bar{u}(p)$ and $v(q)$; k_1/k_2 e_1/e_2 denotes momenta/polarizations for 1-st/2-nd photon/gluon. Factors of parts coincide with those of first order.

$$I_1^{\{1,2\}} = \frac{1}{2}\not{J}\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right) \qquad eikonal$$

$$I_{2l}^{\{1,2\}} = -\frac{1}{4}\left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\frac{\not{e}_2\not{k}_2}{p\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right)\frac{\not{e}_1\not{k}_1}{p\cdot k_1}\right]\not{J} \qquad \beta_1$$

Multiple photon emission

$$I_{2r}^{\{1,2\}} = \frac{1}{4}\not{J}\left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\frac{\not{k}_2\not{e}_2}{q\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right)\frac{\not{k}_1\not{e}_1}{q\cdot k_1}\right] \qquad \beta_1$$

$$I_{3}^{\{1,2\}} = -\frac{1}{8}\left(\frac{\not{e}_1\not{k}_1}{p\cdot k_1}\not{J}\frac{\not{k}_2\not{e}_2}{q\cdot k_2} + \frac{\not{e}_2\not{k}_2}{p\cdot k_2}\not{J}\frac{\not{k}_1\not{e}_1}{q\cdot k_1}\right) \qquad start for \beta_2 ...$$

$$I_{4p}^{\{1,2\}} = \frac{1}{8}\frac{1}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left(\frac{\not{e}_1\not{k}_1\not{e}_2\not{k}_2}{p\cdot k_1} + \frac{\not{e}_2\not{k}_2\not{e}_1\not{k}_1}{p\cdot k_2}\right)\not{J}$$

$$I_{4q}^{\{1,2\}} = \frac{1}{8}\not{J}\frac{1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left(\frac{\not{k}_2\not{e}_2\not{k}_1\not{e}_1}{q\cdot k_1} + \frac{\not{k}_1\not{e}_1\not{k}_2\not{e}_2}{q\cdot k_2}\right)$$

$$I_{5pA}^{\{1,2\}} = \frac{1}{2}\not{J}\frac{k_1\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1}\right)\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2}\right)$$

$$I_{5pB}^{\{1,2\}} = -\frac{1}{2}\not{J}\frac{1}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left(\frac{k_1\cdot e_2 k_2\cdot e_1}{k_1\cdot k_2} - e_1\cdot e_2\right)$$

$$I_{5qA}^{\{1,2\}} = \frac{1}{2}\not{J}\frac{k_1\cdot k_2}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1}\right)\left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2}\right)$$

Multiple photon emission

$$I_{5qB}^{\{1,2\}} = -\frac{1}{2} \not{J} \frac{1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2} \left(\frac{k_1\cdot e_2 k_2\cdot e_1}{k_1\cdot k_2} - e_1\cdot e_2 \right)$$

$$I_{6B}^{\{1,2\}} = -\frac{1}{4} \frac{k_1\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2} \left[+ \left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_1\cdot k_2} \right) \frac{\not{e}_2 \not{k}_2}{p\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} \right) \frac{\not{e}_1 \not{k}_1}{p\cdot k_1} \right] \not{J}$$

$$I_{7B}^{\{1,2\}} = -\frac{1}{4} \not{J} \frac{k_1\cdot k_2}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2} \left[+ \left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_1\cdot k_2} \right) \frac{\not{k}_2 \not{e}_2}{q\cdot k_2} + \left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} \right) \frac{\not{k}_1 \not{e}_1}{q\cdot k_1} \right]$$

- for **exponentiation** one use **separation** into 3 parts only.
- $I_3^{\{1,2\}}$, $I_{4p}^{\{1,2\}}$, $I_{4q}^{\{1,2\}}$ were studied to improve options for PHOTOS kernel iteration. Things are less transparent, concept of effective fermionic momenta is used in interpretation, eg. $u((p-k_1)_{long})\bar{u}((p-k_1)_{long}) \simeq \not{p} - \not{k}_1$, this makes sense only in some limits, but separation is all over phase space. We got what is necessary! Parts for each kinematical branch. In fact sub-structures for amplitudes for processes of other theories appear as well.
- Separation of β_2 into parts: of no use. No match with singularities of QED.

Multiple photon emission

1. PHOTOS Monte Carlo is for simulation of multiphoton FSR bremsstrahlung.
2. Generates correlated samples: events with and without FSR bremsstrahlung.
3. For processes mediated by Z/γ' and W's high precision is investigated.
4. Important for program construction were studies of spin amplitudes. Their gauge invariant parts are used in definition of photon emission kernel.
5. Remaining parts of amplitudes are needed for discussion of systematic errors, for optimalization or for correcting weights.
6. For some processes eg. where matrix element is obtained from scalar QED introduction of data constrained form factors may be necessary.
7. Program version using C++ `HepMC` event record of is available for tests.
8. For us LL means collinear leading logs. PHOTOS NLL equivalent to NNNLL in double log classification.