On threshold resumation in superparticle production at hadron collid

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Overview

SUSY at LHC

NLO corrections

Soft gluon resummation

Soft anomalous dimension matrices

Phenomenological results

Based on A. Kulesza and LM, PRL **102**:111802,2009; Phys.Rev.D80:095004,2009;
W. Beenakker et al. JHEP 0912:041,2009

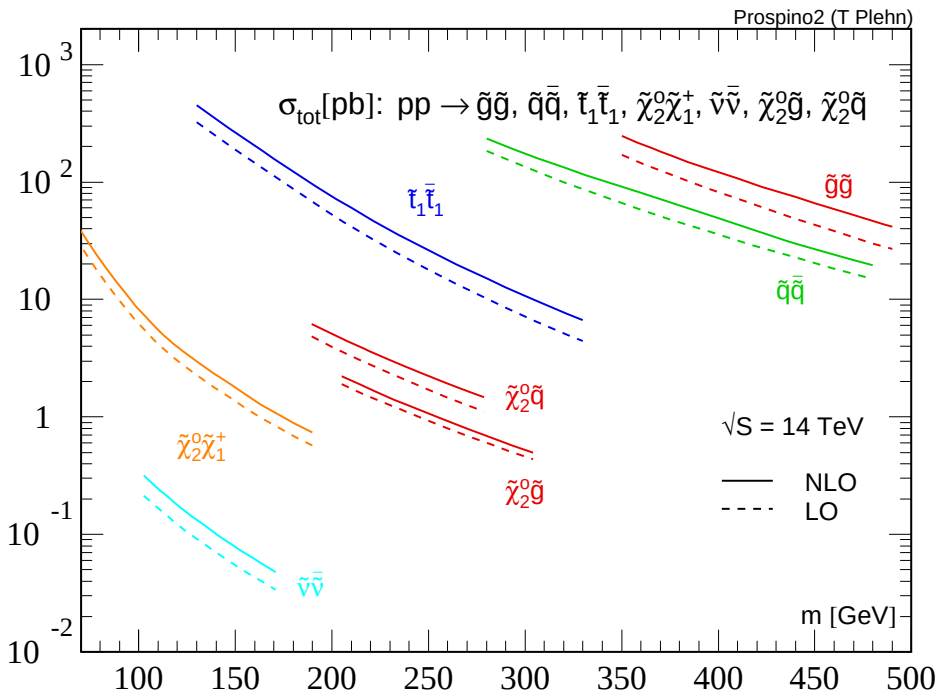
SUSY at LHC

Supersymmetric extension of the SM may be The Theory of physics at Terascale (naturalness, coupling unification, WIMPS, relation to superstrings)

SUSY spectrum contains new heavy particles: e.g. scalar partners of quarks (squarks) and Majorana fermions: gluinos

R-parity: only pair production of SUSY particles possible

Minimal Supersymmetric Standard Model has free parameters, but typically, $\tilde{g}\tilde{g}$ and $\tilde{q}\bar{\tilde{q}}$ production processes are expected to have the largest cross sections at LHC



[T. Plehn, Prospino]

Need for precise predictions

If not Discovery: estimates of total cross section $\longrightarrow$ exclusion limits

If Discovery: Direct determination of masses of SUSY partners may be difficult:

- long decay cascades, leading to multi-particle final states
- some final state particles should escape detection

Ways out:

- end-points of kinematic distributions
- kinematic fits
- precise measurement of total cross-sections

In general: total cross sections may provide precision tests of SUSY parameters.

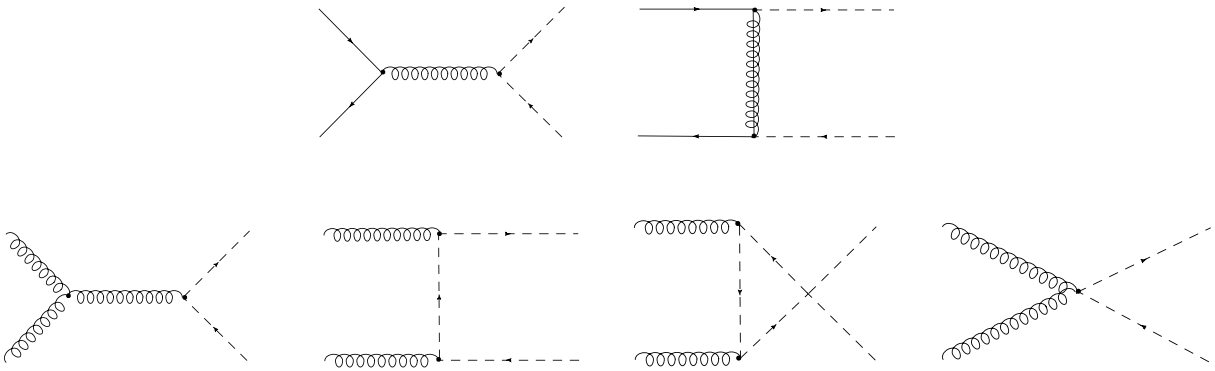
Quality of the tests and parameter determination depends critically on theoretical precision.

LO, NLO results for $\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$ SUSY-QCD corrections are known, but soft gluon corrections are expected to be sizable beyond NLO

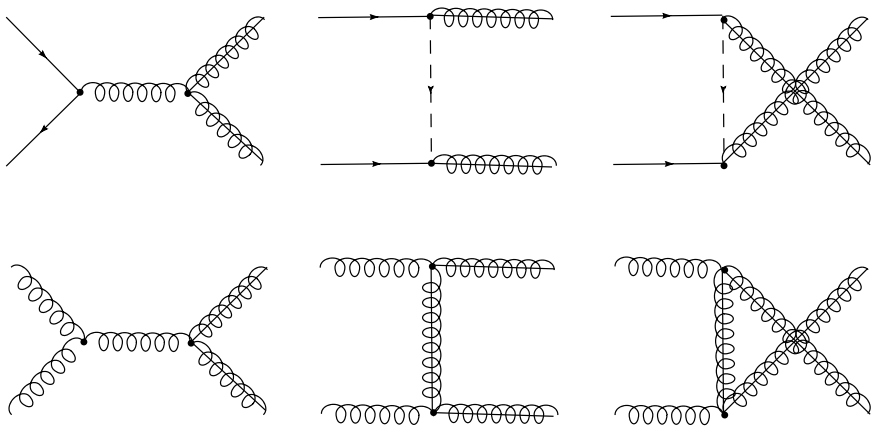
Squark–antisquark and gluino pair production

[Eichten, Dawson, Quigg, 85]

$q_i\bar{q}_j \longrightarrow \tilde{q}\bar{\tilde{q}},\ gg \longrightarrow \tilde{q}\bar{\tilde{q}}$



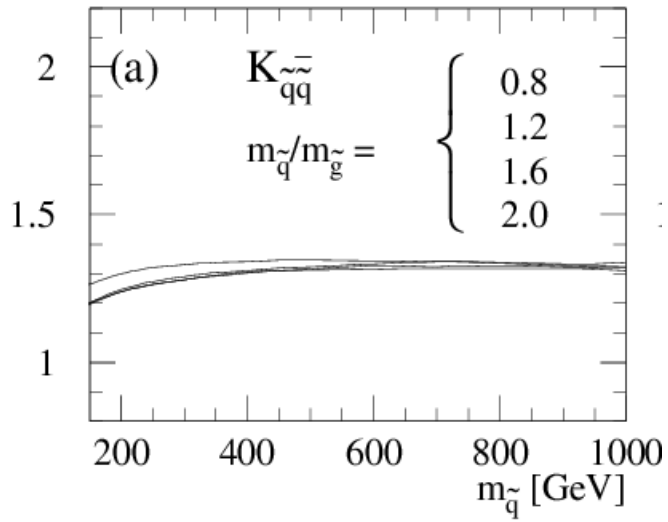
$q\bar{q} \longrightarrow \tilde{g}\tilde{g},\ gg \longrightarrow \tilde{g}\tilde{g}$



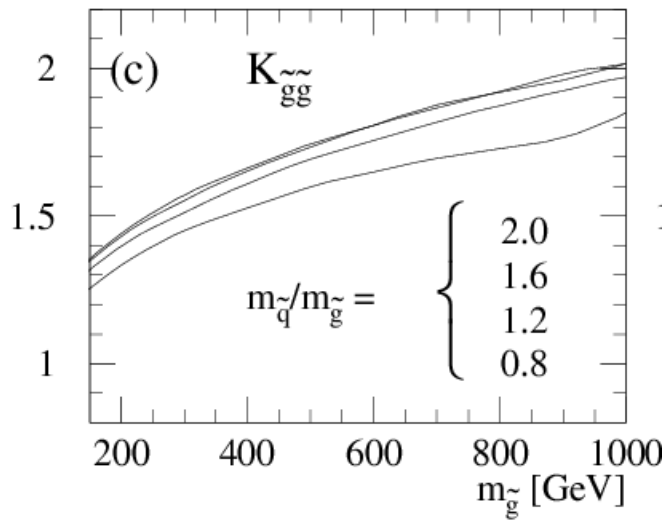
SUSY-QCD one loop corrections

[Beenakker, Hopker, Spira, Zerwas]

Inclusion of one loop corrections at $\mathcal{O}(\alpha_s)$ from quarks, gluons, squarks and gluinos



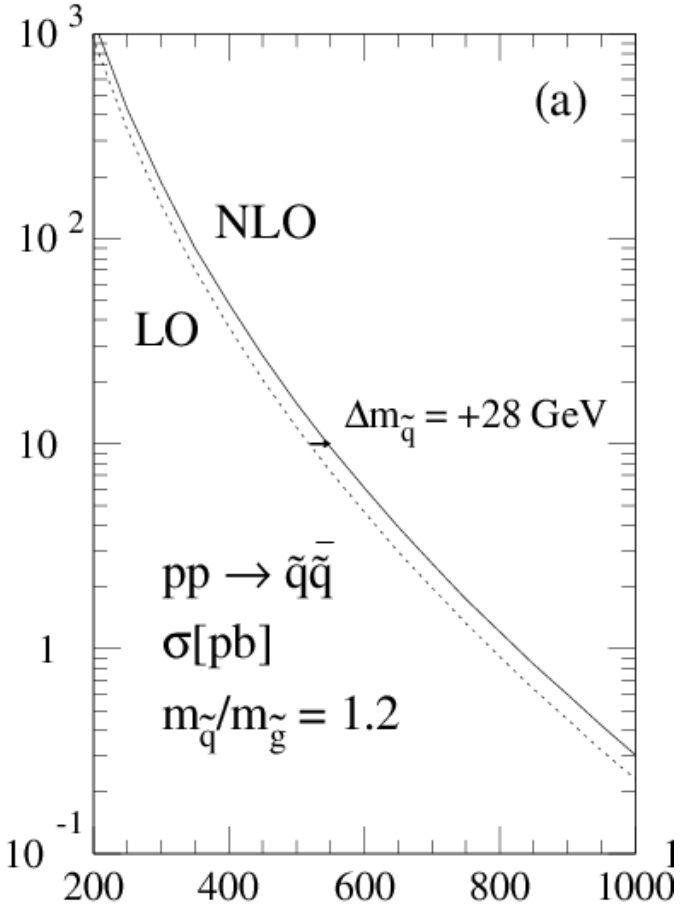
Large NLO K-factors predicted, $K \simeq 1.3$ for $\tilde{q}\bar{\tilde{q}}$ production at the LHC, for $m_{\tilde{q}} = 1$ TeV

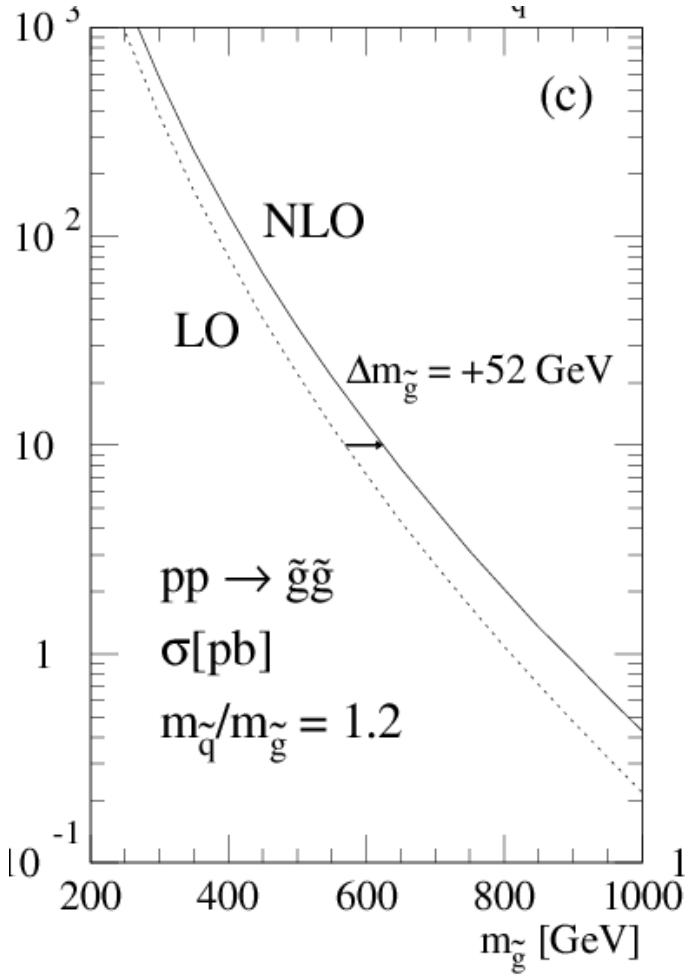


$K \simeq 2$ for $\tilde{g}\tilde{g}$ production at the LHC, for $m_{\tilde{g}} = 1$ TeV

Impact of corrections on sparticle mass determination

[Beenakker, Hopker, Spira, Zerwas]

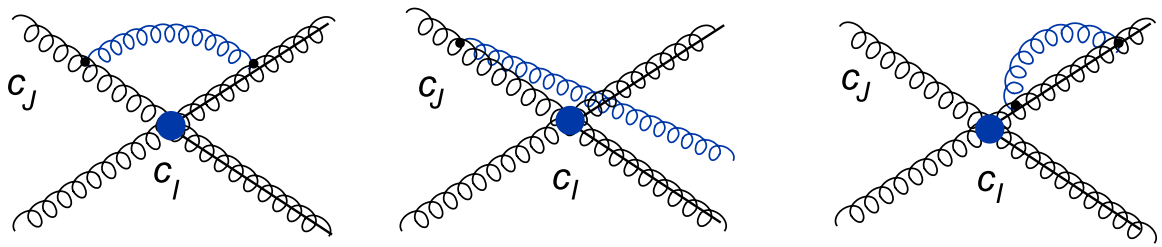




Soft gluon corrections at one loop

Threshold limit is defined by $\hat{s} \longrightarrow (2m)^2$

Velocity of produced particle in the c.m.s. $\beta = \sqrt{1 - \frac{4m^2}{\hat{s}}}$



Energy, ω of emitted real gluon close to threshold is kinematically limited,cut-off $\sim m\beta^2$

$\longrightarrow$ lack of cancellations between real and virtual corrections

Soft gluons do not resolve details of the short-distance interaction

Soft-collinear logarithms for incoming parton, incoherent, depending on parton color charge:
$\sim \alpha_s \log^2 \beta^2 \, \sigma^{(0)}$

Soft non-collinear logarithms for the whole matrix element, coherent, depending on color flow:
$\sim \alpha_s \log \beta^2 \sigma^{(0)}$

Additionally, Coulomb corrections $\sim \alpha_s/\beta \; \sigma^{(0)}$

It is necessary to resum soft logarithms and Coulomb corrections

Threshold resummation

[Sterman, Kidonakis, Oderda], [Catani, Trentadue]

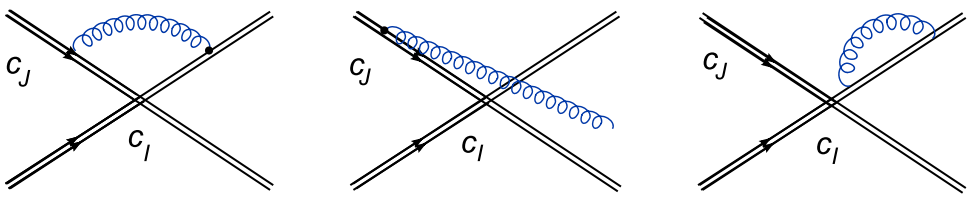
The basis: hard factorization of soft gluon corrections and renolmalization group
Iteration of soft gluon contributions leads to a tower of corrections

$$\delta\sigma \;\sim\; \sigma^{(0)}\alpha_{\rm s}^{n}\log^{2n}\beta$$

$$\delta\sigma \;\sim\; \sigma^{(0)}\alpha_{\rm s}^{n}\log^{2n-1}\beta$$

In part of phase space $\alpha_{\rm s}\log^2\beta > 1$ $\longrightarrow$ resummation is necessary $\longrightarrow$ Mellin moments N

In general, for non-trivial color flow, soft gluon effects lead to mixing between different color amplitudes



$$\sigma^N = \sum_{a,b,I,J} f_{a/h_a}(N)\, f_{b/h_b}(N)\, \underbrace{\Delta_a(N+1)\,\Delta_b(N+1)}_{collinear}\, h_J^*\, \underbrace{\tilde{S}_{JI}(N+1)}_{soft}\, h_I$$

Scale evolution of soft matrix: renormalisation group

[Sterman], [Catani, Trentadue]

$$\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}S_{IJ}=-(\Gamma_S^\dagger)_{IK}S_{KI}-S_{IL}\Gamma_{LJ}$$

Solution in orthogonal color basis:

$$S_{IJ}(N,\mu^2)=S_{IJ}^{(0)}(1)(N,\mu^2)\exp\left[\int_{\mu^2}^{4m^2/N}\frac{dq^2}{q^2}(\lambda_I^*(\alpha_{\rm s}(q^2))+\lambda_J(\alpha_{\rm s}(q^2)))\right]$$

For color basis in which S is diagonal

$$S_{II}(N,\mu^2)=S_{II}^{(0)}(1,\mu^2)\exp\left[\int_{\mu^2}^{4m^2/N}\frac{dq^2}{q^2}2{\rm Re}(\lambda_I(\alpha_{\rm s}(q^2)))\right]$$

At NLL, anomalous dimension matrix Γ_{IJ} obtained from IR poles of one-loop diagrams of effective eikonal theory:

$$\Gamma_{IJ}\;=\;-\frac{g}{2}\frac{\partial}{\partial g}\,{\rm Res}_{\epsilon\to 0}\,Z_{IJ}(g,\epsilon)$$

Resummed cross section at NLL

$$\sigma^{\rm res}_{h_a,h_b\to kl}(N,\mu^2) \;=\; \sum_{a,b,\mathbf{I}} f_{a/h_a,\,N+1}\, f_{b/h_b,\,N+1}\, \hat{\sigma}^{(0)}_{ij\to kl,\mathbf{I},N}\, \Delta^a_{N+1}\, \Delta^b_{N+1}\, \Delta^{(\rm int)}_{ij\to kl,\mathbf{I},N+1}$$

Universal, soft-collinear factors: $\ln \Delta^i_N = \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \int_{\mu_F^2}^{4m^2(1-z)^2} \frac{dq^2}{q^2} A_i(\alpha_{\rm s}(q^2))$

Soft, non-collinear factor: $\ln \Delta^{(\rm int)}_{ij\to kl,I,N} \;=\; \int_0^1 dz \frac{z^{N-1}-1}{1-z}\, D_{ij\to kl,\mathbf{I}}(\alpha_{\rm s}(4m^2(1-z)^2))$

$$A_i \;=\; \left(\frac{\alpha_{\rm s}}{\pi}\right) A^{(1)} + \left(\frac{\alpha_{\rm s}}{\pi}\right)^2 A^{(2)} + \ldots, \qquad A_i^{(1)},\; A_i^{(2)} \;\propto C_i$$

$$D_{ij\to kl,\mathbf{I}} \;=\; \left(\frac{\alpha_{\rm s}}{\pi}\right) D^{(1)} + \ldots$$

Soft anomalous dimension matrix $\longrightarrow$ eigenvalues $\lambda_{ij\to kl,\mathbf{I}}$

$$D^{(1)}_{ij\to kl,\,\mathbf{I}} = 2{\rm Re}(\lambda_{ij\to kl,\mathbf{I}})$$

Anomalous dimension matrices for squark-antisquark production and $q\bar{q} \to \tilde{g}\tilde{g}$

For $\tilde{q}\bar{\tilde{q}}$ results for heavy quarks apply: $(2\mathrm{Re}\,\lambda) = (0, -C_A)$ or $(2\mathrm{Re}\,\lambda) = (0, -C_A, -C_A)$

Color basis for $q\bar{q} \to \tilde{g}\tilde{g}$: $\quad c_1^q = \delta^{\alpha_1\alpha_2}\,\delta^{a_3a_4}, \quad c_2^q = T^b_{\alpha_2\alpha_1} d^{ba_3a_4}, \quad c_3^q = iT^b_{\alpha_2\alpha_1} f^{ba_3a_4},$

$$\Gamma^{q\bar{q}\to\tilde{g}\tilde{g}} = \frac{\alpha_s}{\pi}\left[\begin{pmatrix} 6\bar{S} & 0 & -\Omega \\ 0 & 3\bar{S}+\frac{3}{2}\Lambda & -\frac{3}{2}\Omega \\ -2\Omega & -\frac{5}{6}\Omega & 3\bar{S}+\frac{3}{2}\Lambda \end{pmatrix} - \frac{4}{3}i\pi\,\hat{\mathbf{I}}\right]$$

At threshold, $\Omega \to 0$, $\Gamma \longrightarrow$ diagonal form: diag(λ_i): $(2\mathrm{Re}\,\lambda) \longrightarrow (0, -C_A, -C_A)$

Anomalous dimension matrices $gg \to \tilde{g}\tilde{g}$ production

Color structure: 8 tensors. s-channel basis, $(\mathbf{1}, \mathbf{8_S}, \mathbf{8_A}, \mathbf{10} + \overline{\mathbf{10}}, \mathbf{27}; \mathcal{R})$

$$\Gamma^{gg\to\tilde{g}\tilde{g}} = \frac{\alpha_s}{\pi} \left[\begin{pmatrix} \Gamma_5 & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \Gamma_3 \end{pmatrix} - 3i\pi\, \hat{\mathbf{I}} \right]$$

$$\Gamma_5 = \begin{pmatrix} 6\bar{S} & 0 & 6\Omega & 0 & 0 \\ 0 & 3\bar{S} + \frac{3}{2}\Lambda & \frac{3}{2}\Omega & 3\Omega & 0 \\ \frac{3}{4}\Omega & \frac{3}{2}\Omega & 3\bar{S} + \frac{3}{2}\Lambda & 0 & \frac{9}{4}\Omega \\ 0 & \frac{6}{5}\Omega & 0 & 3\Lambda & \frac{9}{5}\Omega \\ 0 & 0 & \frac{2}{3}\Omega & \frac{4}{3}\Omega & 4\Lambda - 2\bar{S} \end{pmatrix}$$

$$\Gamma_3 = \mathrm{diag}\,(\, 3(\bar{S} + \bar{U})\,,\ 3(\bar{S} + \bar{T})\,,\ 3(\bar{T} + \bar{U})\,)$$

At threshold, $\Gamma \longrightarrow$ diagonal; $(2\,\mathrm{Re}\,\lambda) \longrightarrow (0, -3, -3, -6, -8; -3, -3, -6)$

Squark-gluino and squark-squark channels

Considered in [W. Beenakker, S. Brensing, M. Kramer, A. Kulesza, E. Laenen and I. Niessen, JHEP 0912 (2009) 041]

$qq \to \tilde{q}\tilde{q}$: $3 \otimes 3 = \bar{3} + 6$

$qg \to \tilde{q}\tilde{g}$: $3 \otimes 8 = 3 + \bar{6} + 15$

The results in the threshold limit:

$$D_{qq\to\tilde{q}\tilde{q},I} = \{-4/3, -10/3\}$$

$$D_{qg\to\tilde{q}\tilde{g},I} = \{-4/3, -10/3, -16/3\}$$

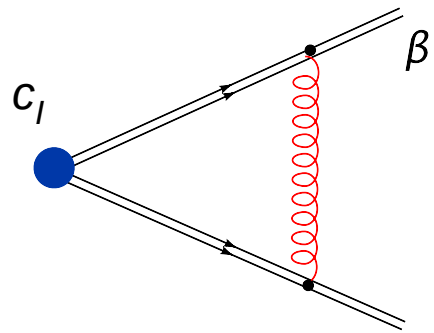
$\longrightarrow D_{\mathbf{I}}^{(1)}$ at threshold $\sim$ quadratic Casimir operator for the outgoing state

Shown in general, at one loop, in [M. Beneke, P. Falgari and C. Schwinn, Nucl. Phys. B 828 (2010) 69.]

Coulomb corrections

[Sommerfeld],[Catani,Mangano,Nason,Trentadue]

Heavy particles produced at threshold are slow, $\beta \ll 1$



Coulomb type exchanges between final state particles at one loop may be large

$$\delta\sigma(\beta) \sim \frac{\alpha_s}{\beta}\sigma^{(0)}(\beta)$$

Resummation of Coulomb corrections proposed by Sommerfeld:

$$\sigma_{\mathbf{I}}^{C} = \frac{\kappa_{\mathbf{I}}\frac{\pi\alpha_s}{\beta}}{1-\exp\left(-\kappa_{\mathbf{I}}\frac{\pi\alpha_s}{\beta}\right)}$$

and $\kappa_{\mathbf{I}}$ is the color factor

Summary of calculational framework

Born level partonic amplitudes: $\mathcal{M}$

$\Downarrow$

Projection in orthogonal color basis: $h_I = \langle c_I | \mathcal{M} \rangle$ and calculation of partonic cross sections

$$\hat{\sigma}_I^{(0)} = \frac{h_I^* h_I}{\langle c_I | c_I \rangle}$$

$\Downarrow$

Mellin transformation of partonic cross sections: $\hat{\sigma}_I^{(0)}(\rho) \longrightarrow \tilde{\hat{\sigma}}_I^{(0)}(N)$

$\Downarrow$

Inclusion of partonic densities and resummation of soft logs in Mellin space

$$\tilde{\sigma}_I^{(R)}(N) \;=\; \tilde{f}_a(N)\, \tilde{f}_b(N)\, \tilde{\hat{\sigma}}_I^{(0)}(N)\, \Delta(N)$$

$\Downarrow$

Numerical inverse Mellin transform and integration over parton densities

$\Downarrow$

Resummed hadronic cross sections in color channels

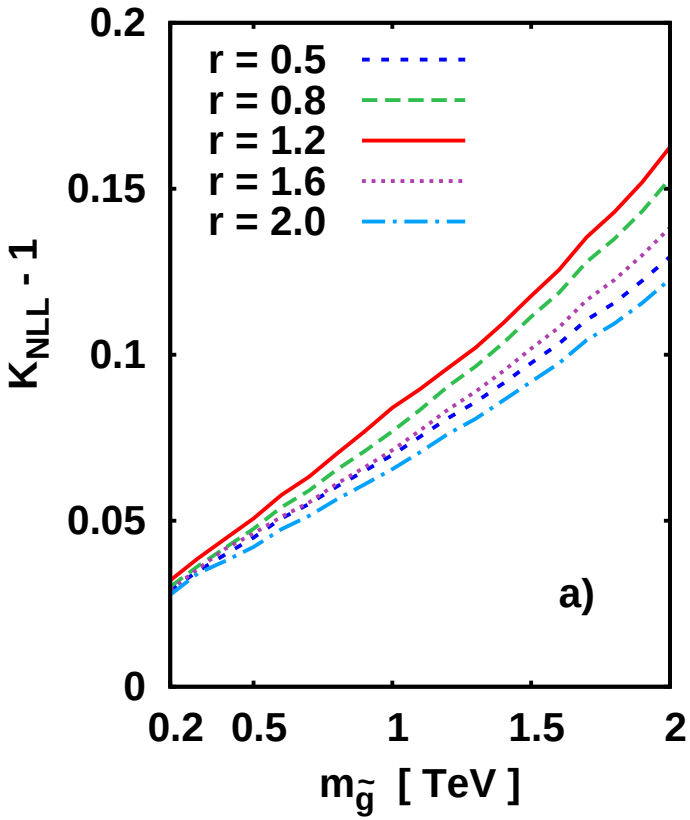
$\Downarrow$

Summation over colors and matching to the NLO result **[PROSPINO]**:

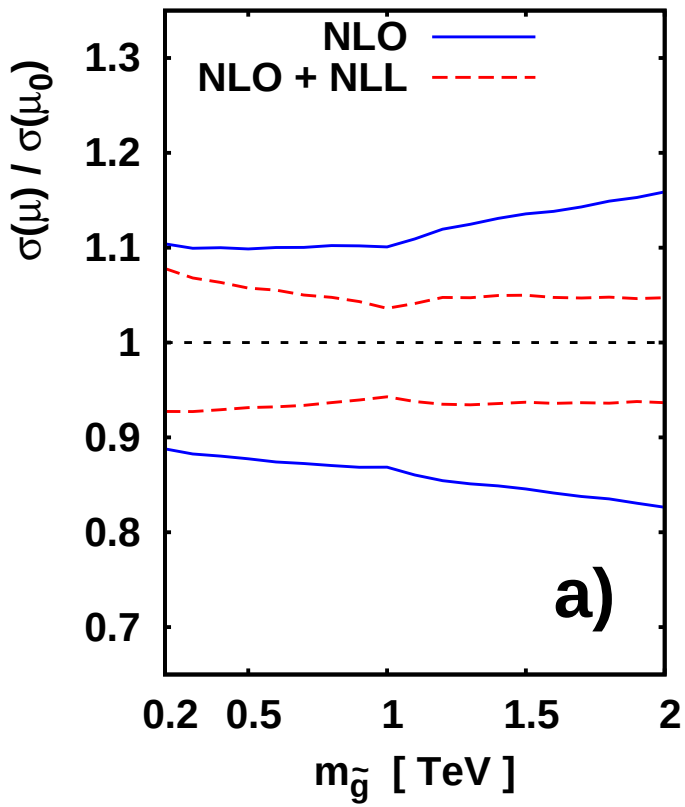
$$\sigma^{\rm match} = \sigma^{\rm NLO} + \left[\sigma^{\rm NLL} - \sigma^{\rm NLL}|_{\rm NLO}\right]$$

NLL results: $\tilde{g}\tilde{g}$

Relative NLL correction $K_{\rm NLL} - 1$



Scale dependence



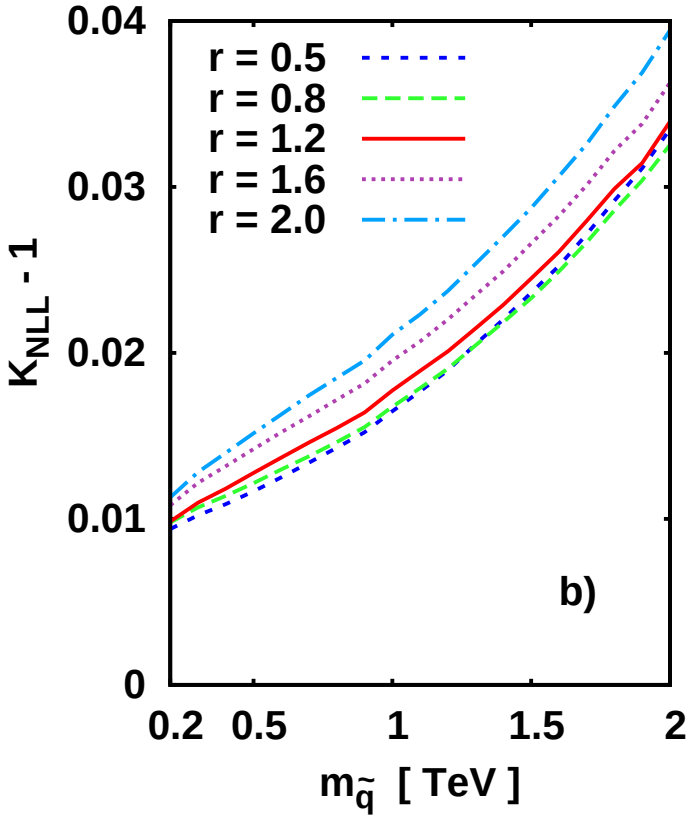
$r = m_{\tilde{g}}/m_{\tilde{q}}$

$m/2 < \mu_R = \mu_F < 2m$

For large gluino mass, sizable effects of soft gluons and reduction of scale dependence by factor of ~ 3

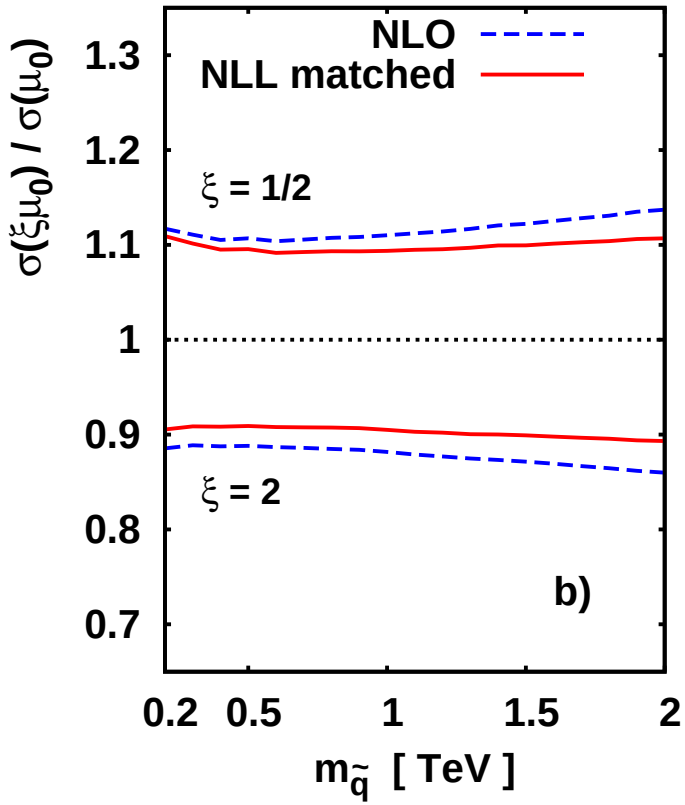
NLL results: $\tilde{q}\bar{\tilde{q}}$

Relative NLL correction $K_{\rm NLL} - 1$



$r = m_{\tilde{g}}/m_{\tilde{q}}$

Scale dependence



$m/2 < \mu_R = \mu_F < 2m$

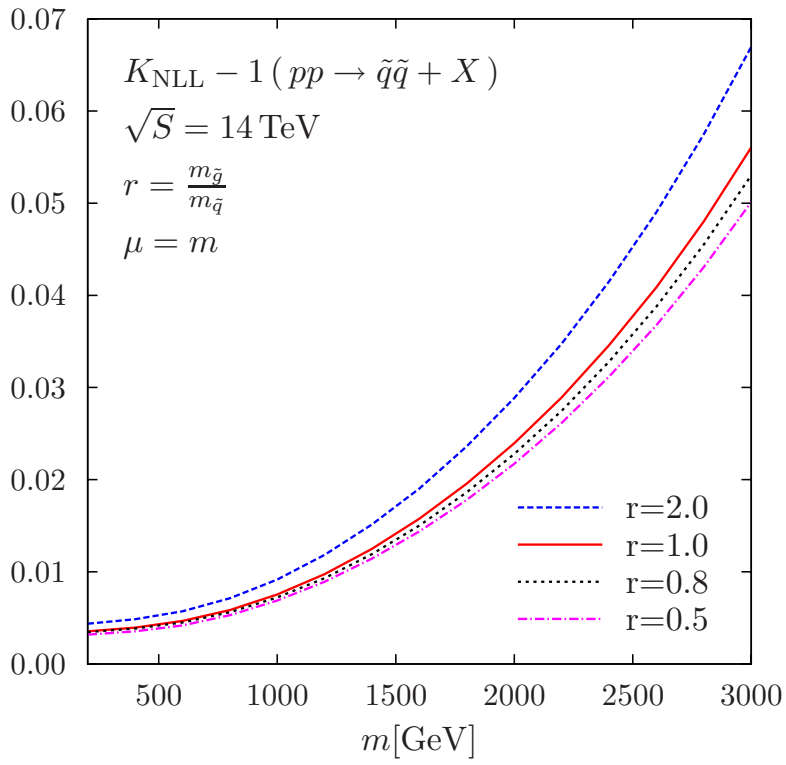
Modest reduction of scale dependence

Also: **[U. Langenfeld and S. O. Moch, 2009]**, **[M. Beneke, P. Falgari and C. Schwinn, 2009]**

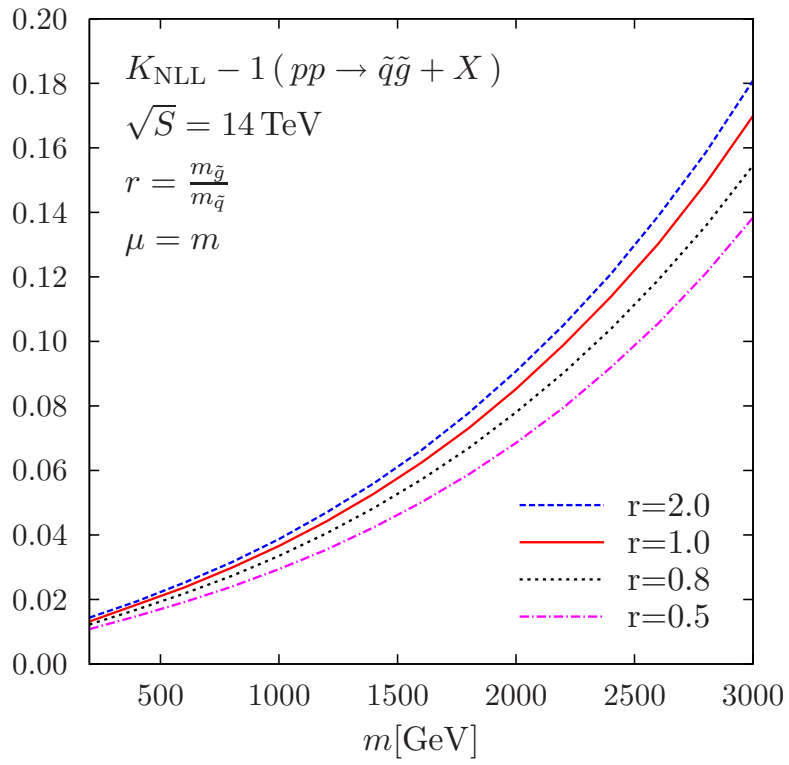
NLL results: $\tilde{q}\tilde{q}$, $\tilde{q}\tilde{g}$ at LHC

[W. Beenakker, S. Brensing, M. Kramer, A. Kulesza, E. Laenen and I. Niessen]

Relative NLL correction $K_{\rm NLL} - 1$



$pp \longrightarrow \tilde{q}\tilde{q}$

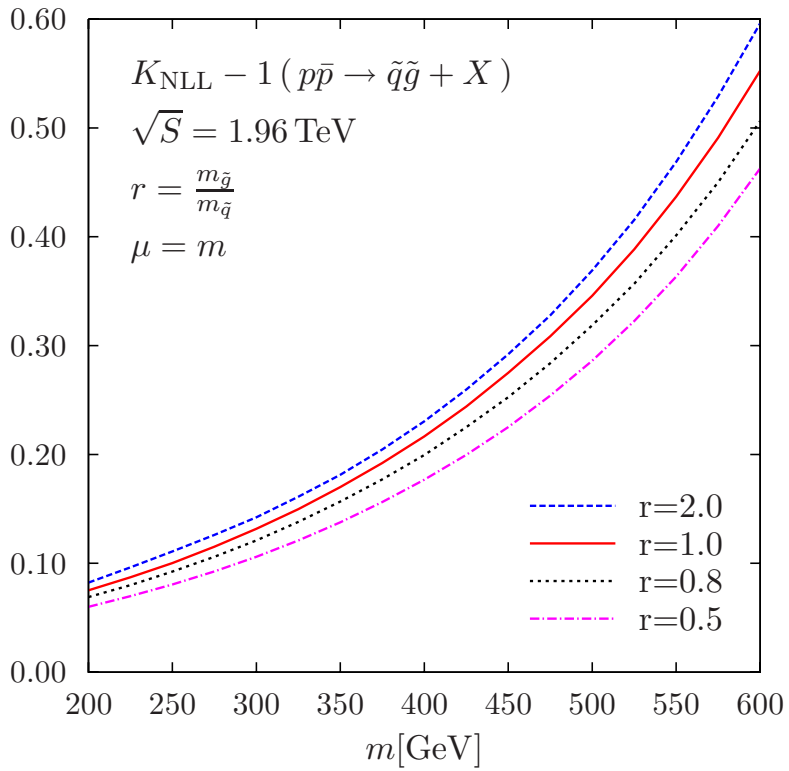


$pp \longrightarrow \tilde{q}\tilde{g}$

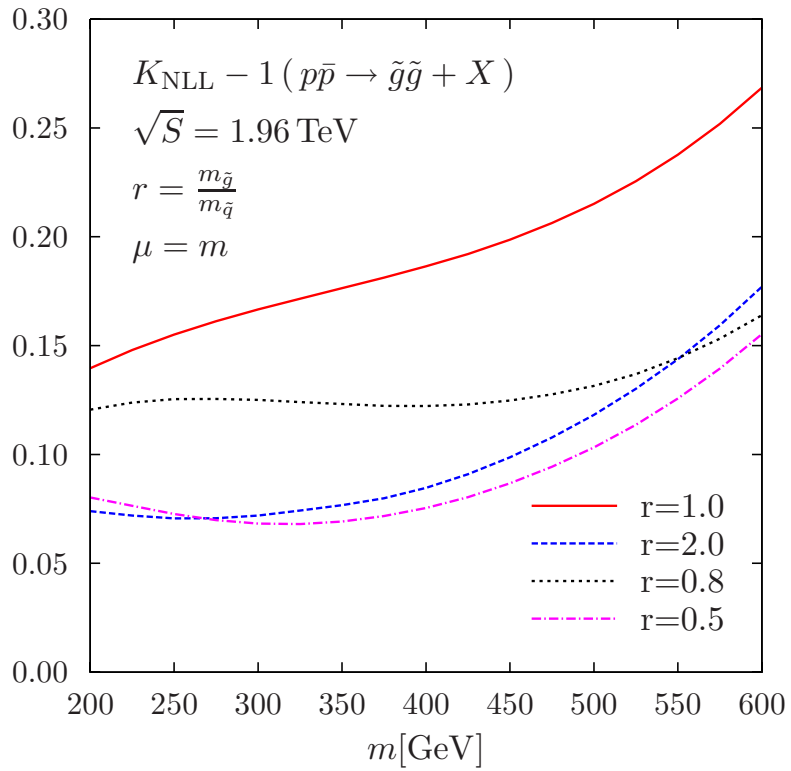
NLL results: $\tilde{q}\tilde{g}$, $\tilde{g}\tilde{g}$ at Tevatron

[W. Beenakker, S. Brensing, M. Kramer, A. Kulesza, E. Laenen and I. Niessen]

Relative NLL correction $K_{\rm NLL} - 1$



$pp \longrightarrow \tilde{q}\tilde{q}$



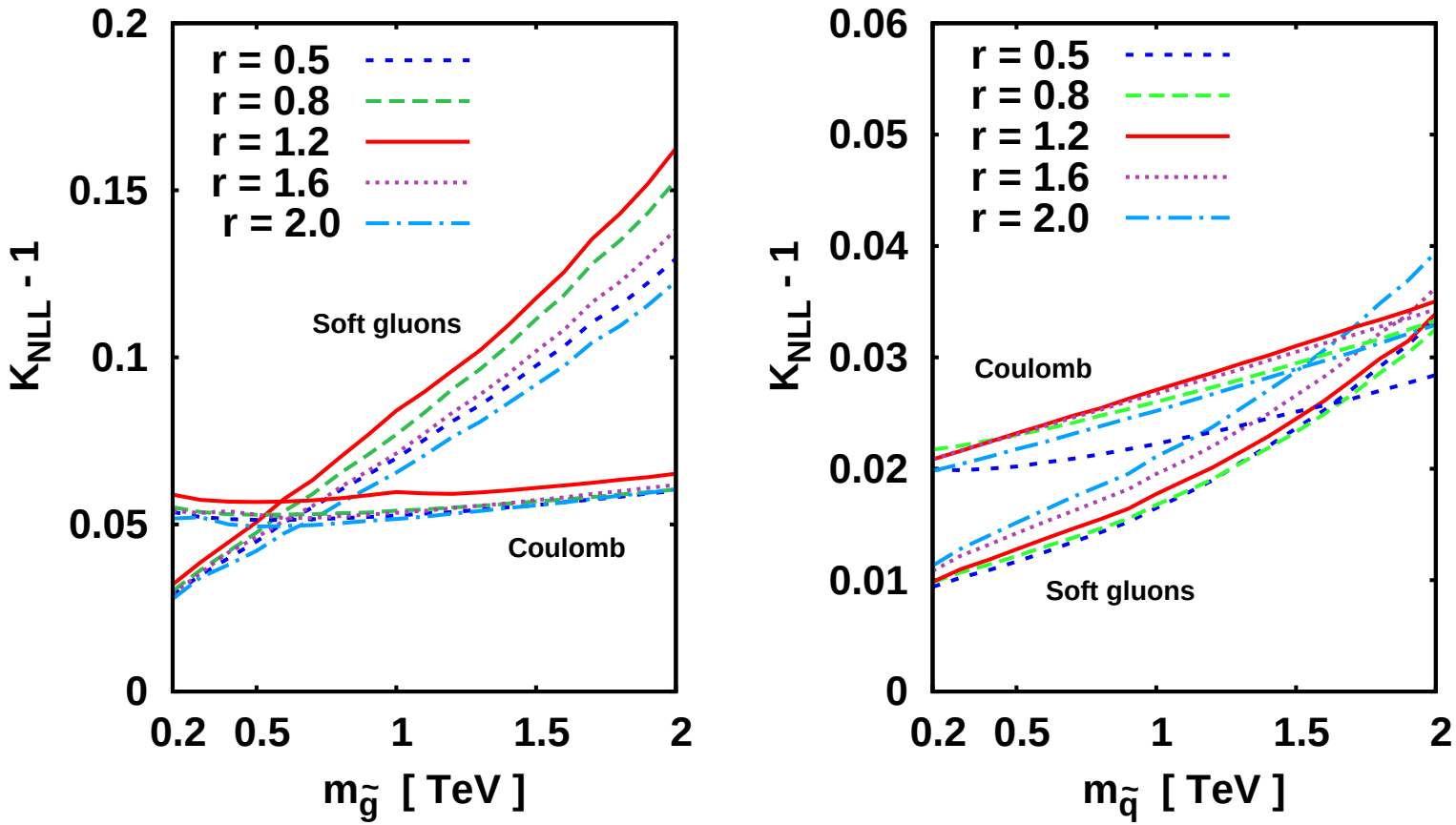
$pp \longrightarrow \tilde{q}\tilde{g}$

$\longrightarrow$ Impact on the exclusion limits

Inclusion of Coulomb corrections

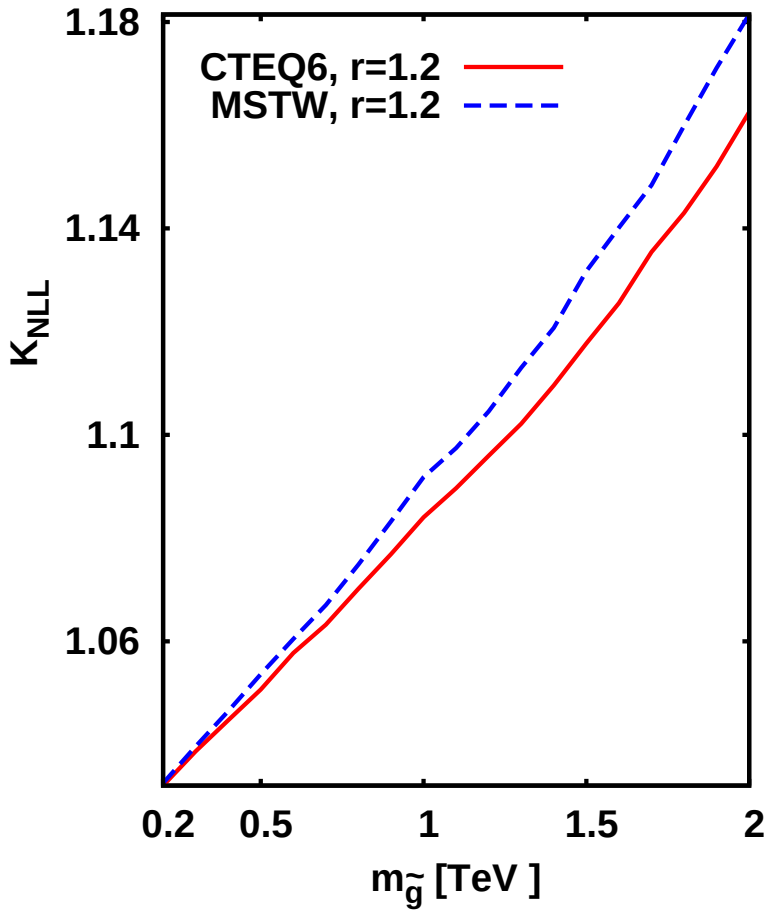
Ratios of resummed soft gluon corrections and Coulomb corrections, beyond NLO to the NLO cross sections

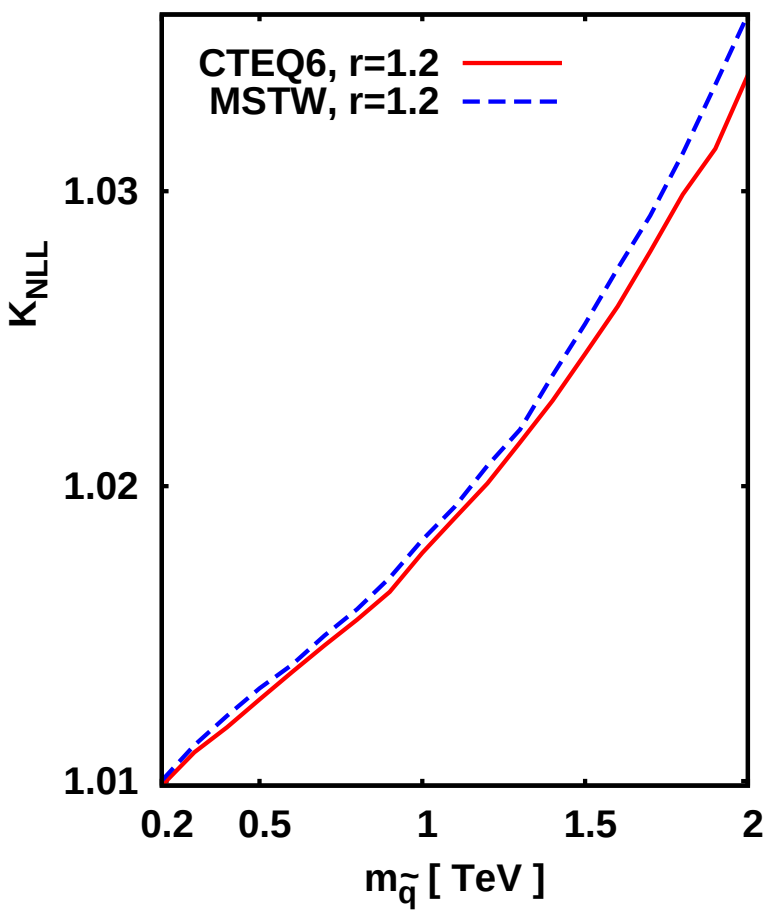
$\tilde{g}\tilde{g}$ $\tilde{q}\bar{\tilde{q}}$



Coulomb corrections are also important

Pdf dependence: K-factors





Recent news

Detailed analysis of combined soft gluon effects and Coulomb effects — with intereference of both these effects — and application to $\tilde{q}\bar{\tilde{q}}$ channel has been performed within Soft Collinear Effective Theory **[M. Beneke, P. Flagari, C. Schwinn, 2010]** (results agree with ours)

Updates of results:

The newest parton distributions CT10 and MSTW are implemented

Results for the LHC at lower energies (to appear soon)

Summary of phenomenological results

- Soft gluon corrections enhance cross sections and grow with color charges of incoming partons and the final state
- Sizable soft gluon corrections found in gg initial states
- Dominance of $gg \longrightarrow \tilde{g}\tilde{g}$ channel $\longrightarrow$ large soft gluon effects in $\tilde{g}\tilde{g}$ production
- Small soft gluon effects in $q\bar{q}$ initiated processes
- Small soft gluon effects in $\tilde{q}\bar{\tilde{q}}$ production
- Soft gluon resummation reduces significantly theory error for $\tilde{g}\tilde{g}$ production related to scale dependence, but it is not the case for $\tilde{q}\bar{\tilde{q}}$ production
- Significant Coulomb corrections
- Weak dependence of the K-factors on choice of PDFs
- Still significant dependence of cross sections on choice of PDFs

Conclusions

1. Soft anomalous dimension matrices have been found for pair production of gluina (or other heavy $SU(3)$ octet particles)

2. At NLL accuracy, the eigenvalues of soft anomalous dimension matrices are proportional to total color charge of the final (eigen)states

3. We computed cross sections for $\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$ production at the LHC, at the NLL accuracy, including matching to known NLO results and Coulomb corrections

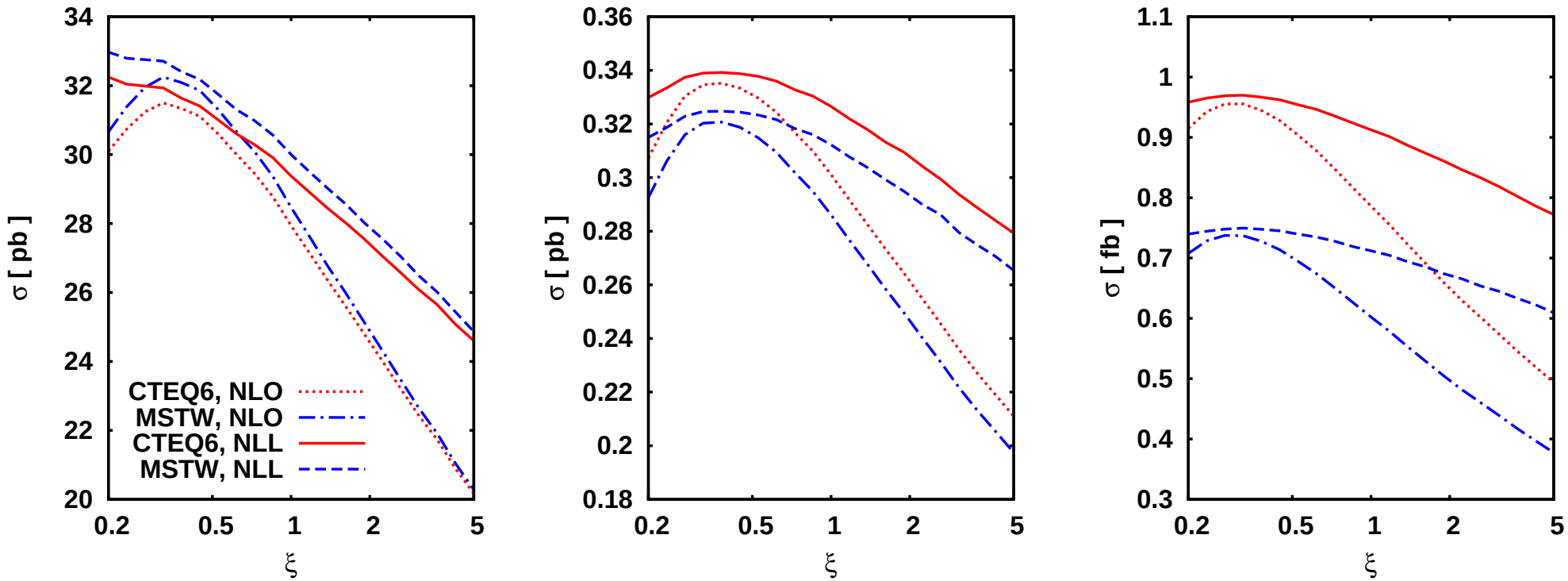
4. Relative soft gluon corrections beyond NLO amount $\mathcal{O}(10\%)$ and Coulomb corrections $\mathcal{O}(5\%)$ for $\tilde{g}\tilde{g}$ production cross section, for $m_{\tilde{g}} = 1$ TeV, smaller effect found for $\tilde{q}\bar{\tilde{q}}$ production

5. Soft gluon resummation reduces significantly theory error for $\tilde{g}\tilde{g}$ production due to scale variations
6. Also available $\tilde{q}\tilde{q}$ and $\tilde{q}\tilde{g}$ results and the full analysis for the case of Tevatron

BACKUP

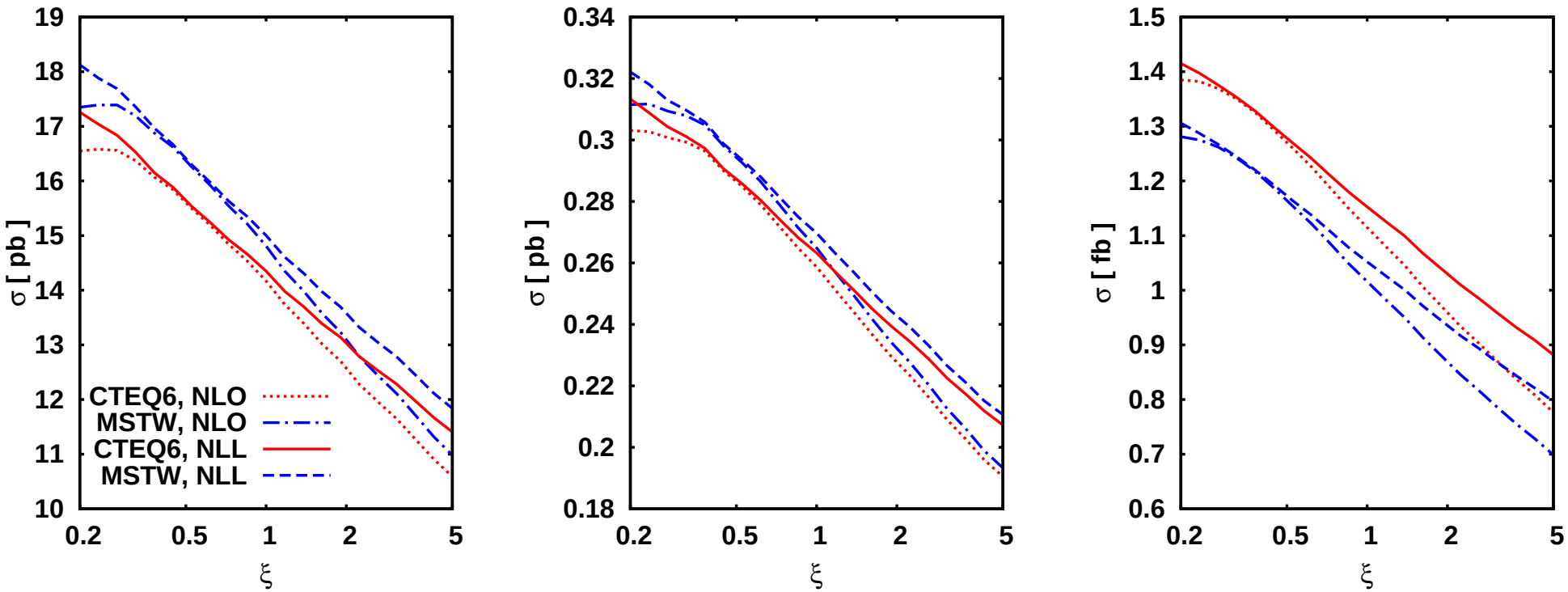
Pdf dependence: cross-sections for gluina

$$\xi = \mu/\mu_0$$



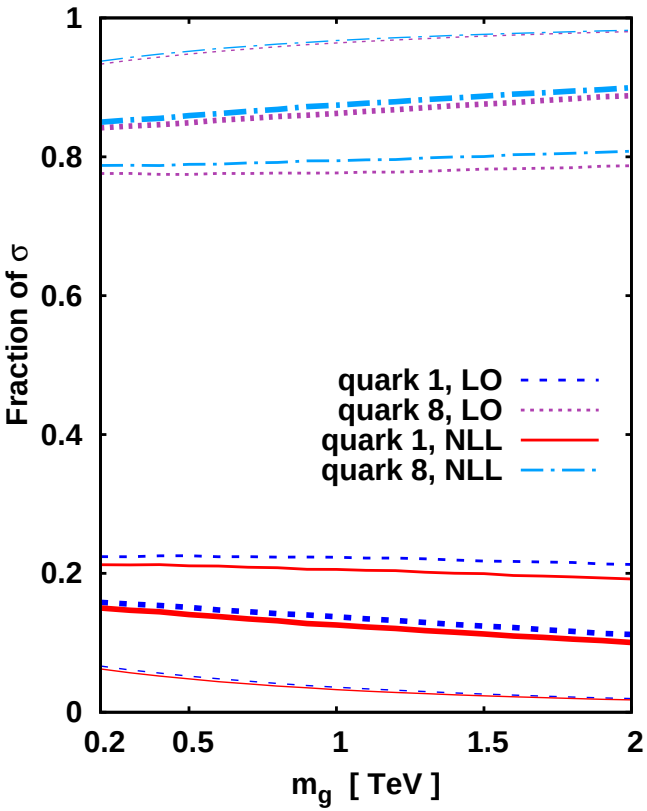
Pdf dependence: cross-sections for squarks

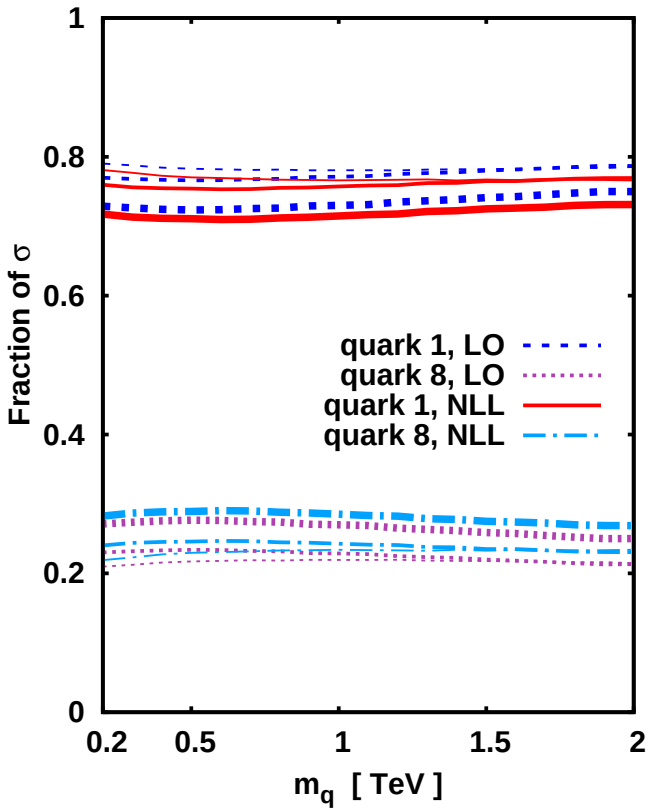
$$\xi = \mu/\mu_0$$



$q\bar{q}$ initiated subprocess: color channels

$q\bar{q} \longrightarrow \tilde{g}\tilde{g}$ and $q\bar{q} \longrightarrow \tilde{q}\bar{\tilde{q}}$





Thick lines $r = 0.5$, Medium lines to $r = 1.2$, thin lines $r = 2.0$