NLO evolution kernels: Monte Carlo vs. $\overline{\text{MS}}$

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Introduction

The Large Hadron Collider (LHC) is operating succesfully giving very precise measurements. Input from the theory side is require:

- need of high accuracy QCD calculations not only for inclusive quantities (vs. Tevatron)
- especially in form of Parton Shower Monte Carlo (PS MC)
- huge range of energy scale requiring beyond LO calculations

- Need for NLO Parton Shower MC !

Factorization

Every Parton Shower Monte Carlo is based on the concept of *factorization*. For pp collisions it can be written as:

$$\sigma = \sum_{ij} \int dx_1 dx_2 \, f_i(x_1, \mu) f_j(x_2, \mu) \, \hat{\sigma}_i(p_1, p_2, \alpha_S(\mu), Q^2/\mu^2)$$

We can calculate both

- coefficient functions $\hat{\sigma}$ (hard process)
- parton distribution functions f_i (PDF)

using perturbative QCD

Factorization

- Different factorization schemes $\rightarrow$ different PDFs, different coefficient functions
- Both PDF and coefficient functions depend on this scheme
- Physical observables don't!

The most common scheme is $\overline{MS}$ scheme

- hard process (coefficient function) is process dependent
- parton distribution function (PDF) is universal

Aims

Our interest is in the PDF part of the *pp* collisions

PDFs are ruled by *QCD evolution equations (DGLAP)* → building blocks for the construction of precise PDF are the *evolution kernels*

- ultimate aim - construct stochastic simulation of the QCD evolution in the Next to Leading Order (NLO) of perturbation theory in fully exclusive form! so far exists only improved LO MCPS

To do it we need to know the form and the properties of NLO evolution kernels at both inclusive and exclusive level

- aim of this presentation - investigate the dependence of the 4D NLO evolution kernels on the choice of *evolution time variable used in MC*

Framework

axial gauge $n^2 = 0$

Sudakov parametrisation: $k_i = \alpha_i p + \beta_i n + k_{i\perp}$

Two sets of variables:

- transvers momentum $k_\perp$
- angular scale $\mathbf{a}_i = \mathbf{k}_{i\perp}/\alpha_i$

Two parton phase space for real emission kernels:

$$d\psi \sim \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{da_1}{a_1}\frac{da_2}{a_2}d\phi_{12}\alpha_1^2\alpha_2^2a_1^2a_2^2$$

Transverse phase space:

$$d\psi_\perp = \frac{da_1}{a_1}\frac{da_2}{a_2}d\phi_{12}$$

Structure of the kernel

General structure of the NLO evolution kernel

$$\Gamma \sim \mathrm{PP} \int d\psi \underbrace{W\,\Theta(s(k_1,k_2) \le Q)}_{\textit{exclusive kernel}}$$

- $s(k_1,k_2)$ function (enclosing the phase space) defines the *evolution time variable in MC*
- W originates from γ-trace of Feynman diagrams it depends on $\alpha_i, k_{i\perp}, \phi_{12}$
- PP is Pole Part operator extracting poles in ϵ ($\overline{MS}$ scheme). In 4D "MC scheme" singularities manifest as large logarithms.

Evolution time

The enclosing of the phase space $\Theta(s(k_1, k_2) \leq Q)$ defines the evolution time variable $\rightarrow$ ordering

Typically people use two choices for the evolution time variable: transverse momentum $k_\perp$ and angular scale a (related directly to rapidity: $y = \ln |\mathbf{a}|$)

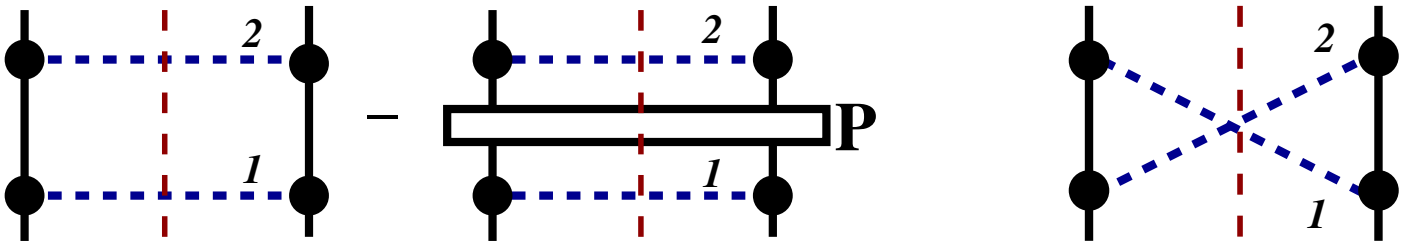
- $k_\perp$-ordering $s(k_1, k_2) = \max\{k_{1\perp}, k_{2\perp}\}$
- angular ordering $s(k_1, k_2) = \max\{a_1, a_2\}$

Aims for this presentation

I discuss the dependence of *4D kernels used in MC* on the choice of the MC **evolution time variable** (upper phase space limit) and in the end I briefly comment on similar properties of $\overline{MS}$ kernels.

Diagrams contributing to the non-singlet kernel

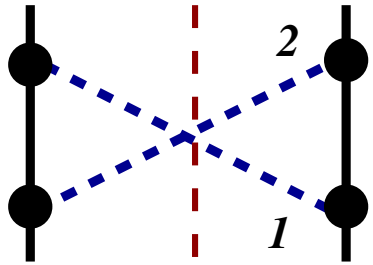
I choose a subset of C_F^2 Feynman diagrams contributing to the non-singlet evolution kernels and discuss their *dependence on the choice of upper phase space limit*.



- All graphs contributing to the NLO kernel are at least single logarithmic (single $\frac{1}{\epsilon}$ pole) divergent

case of single logarithmic graphs

Crossed ladder: example of single logarithmic divergent graph



In this case change of the upper phase space limit

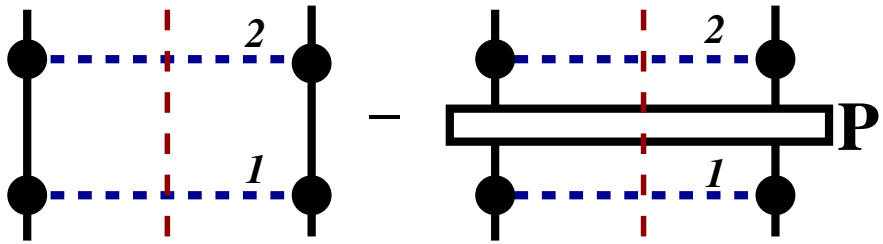
$$\max\{a_1, a_2\} \to \max\{k_{1\perp}, k_{2\perp}\}$$

does not affect kernel contribution

$$\Gamma^{a}_{Bx} = \Gamma^{k_\perp}_{Bx}$$

case of double logarithmic graphs

Bremsstrahlung ladder and its countegraph (subtracting the LO singularity)



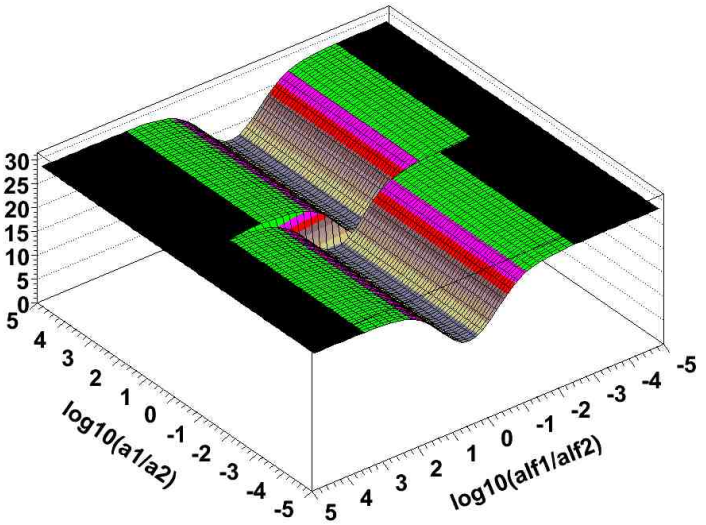
The difference between contributions for maximal $\max\{a_1, a_2\}$ and $\max\{k_{1\perp}, k_{2\perp}\}$ can be encapsulated in form of an integral:

$$\Gamma^a_{Br-ct} - \Gamma^{k_\perp}_{Br-ct} = \left(\frac{\alpha_S}{2\pi}\right)^2 4C_F^2 \ln\left(Q/q_0\right)$$
$$\times \int \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\delta(1-x-\alpha_1-\alpha_2)\, T_2\, \frac{\alpha_1^2}{(1-\alpha_1)^2}\int_1^{\alpha_2/\alpha_1}\frac{dy_1}{y_1} \equiv \Gamma^a_{ct} - \Gamma^{k_\perp}_{ct}$$

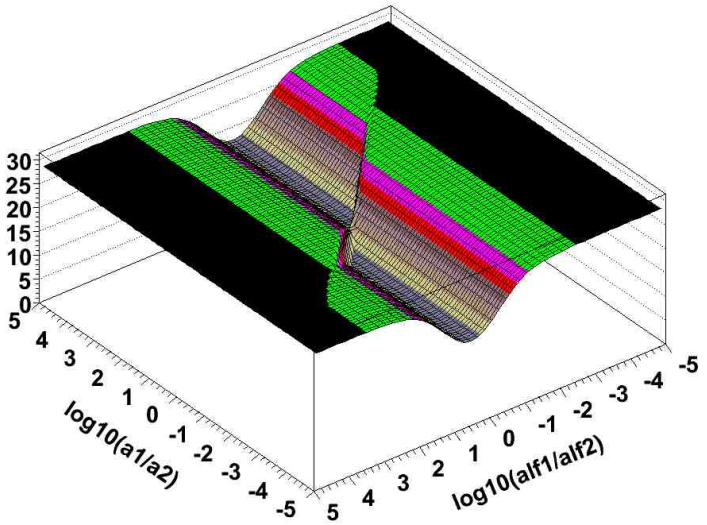
where y_1 is dimensionless variable related with angular scale of the softer gluon.
or in fully inclusive way

$$\Gamma^a_{Br-ct} - \Gamma^{k_\perp}_{Br-ct} = \left(\frac{\alpha_S}{2\pi}\right)^2 4C_F^2 \ln\left(Q/q_0\right)\left[\frac{1}{2}(1+x)\ln^2(x) - (1-x)\ln(x)\right]$$

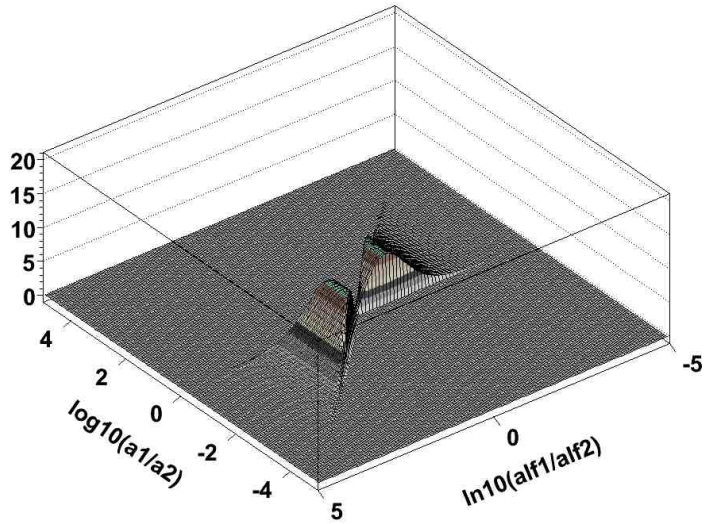
$\max\{a_1, a_2\}$



$\max\{k_{1\perp}, k_{2\perp}\}$

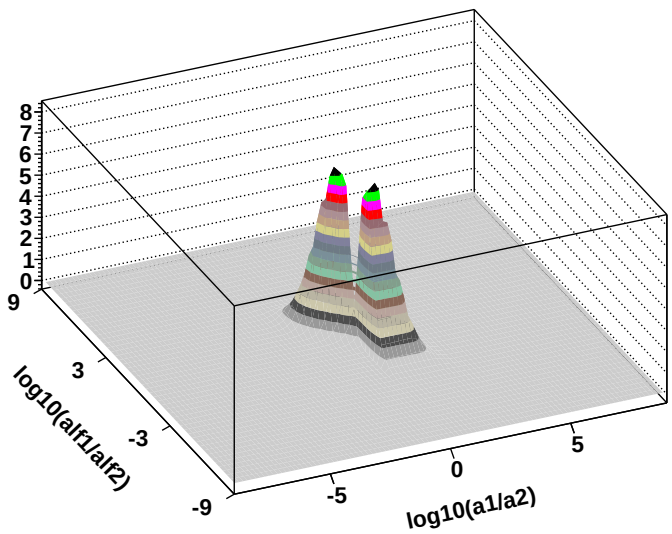


MonteCarlo, ME=BConDif; x in (0.100, 0.120)

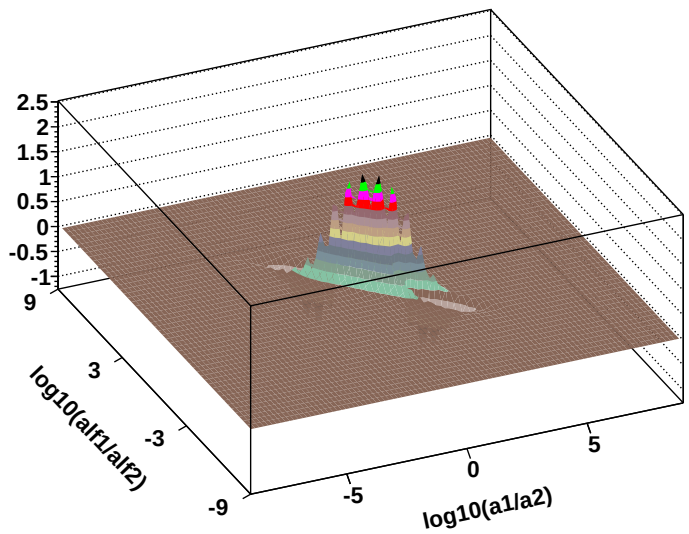


All C_F^2 non-singlet graphs

$\max\{a_1, a_2\}$



$\max\{k_{1\perp}, k_{2\perp}\}$



upper phase space limit 4D vs. $\overline{MS}$

Analysis of example kernel contributions in 4D MC-like scheme shows that both **exclusive** and **inclusive** NLO DGLAP kernels **depend** on choice of upper limit (ordering)!

But we know that in $\overline{MS}$ the inclusive kernel doesn't depend on this choice!!!

WHY???

Sources of differences between $\overline{MS}$ and 4D evolution kernels

Differences originates from the fact that in the $\overline{MS}$ we are using $n = 4 + 2\epsilon$ dimensions.
The mixing of double ϵ poles with terms proportional to ϵ give rise to additional terms which protects the independence of $\overline{MS}$ kernels on the choice of upper limit.

The mixing terms ($\frac{1}{\epsilon^2} \times \epsilon = \frac{1}{\epsilon}$) originates from:

- γ-traces of Feynman graphs
- n-dim. phase space

Differences between $\overline{MS}$ and 4D evolution kernels

We have identified sources of differences and we have **analitical expresions** for them. If we want to include these terms in MC kernels we need to do it in the inclusive manner (like for the virtual corrections) since their phase space is degenerate (transvers degrees of freedom are integrated).

Summary

- I investigated the dependence of NLO DGLAP evolution kernels in 4D on the choice of upper phase space limit which is related to evolution time varaible in MC.
- I discussed the sources of differences between $\overline{MS}$ kernels and 4D kernels applicable in MC.
- Having the analitical formulas for this deifferences allows us to correct the 4D kernels to the $\overline{MS}$ scheme.