Selected topics in theoretical neutrino physics

Cracow Epiphany Conference,

5-8 January 2010

Marek Zrałek,

University of Silesia

Outline

- Introduction
- Neutrinos in the Standard Model (SM)
- Experimental facts
- Neutrinos in the SM with small ν mass
- Neutrinos beyond the SM
- Conclusions and perspectives

1) INTRODUCTION – What is behind us?

Neurinos always give a new and unexpected informations about elementary interactions

- In 1930, Pauli, remedy for the energy crisis observed in the beta decay,
- In 1934, Fermi, neutrinos were used to construct the first theory of week interaction – Fermi theory for beta decay,
- Majorana in 1937, particles which are the same as their antiparticles can exist – Majorana particles,
- Lee & Yang (1956), Wu (1957) - P and C symmetries are broken in Nature,

- Gargamelle (1973), neutral currents exist – first indication about Z boson,

- (Years 60-70), neutrinos are parts of the leptons doublets – Glashow, Weinberg and Salam - the unified model of electromagnetic and week interaction was created – the Model Standard,

- In 1987 neutrinos from the supernova SN87A were observed – the theory of supernova explosion was confirmed,

- In LEP, 1989, in Z decay - first observation that only three generations of quarks and leptons exist in Nature,

- Between 1998 and 2003 – solar neutrino problem was resolved – first independent proof that Bethe model of energy creation in stars – hydrogen nucleosynthesis - is correct

- Superkamiokande, in 1998, first real confirmation that atmospheric neutrinos oscillate – neutrinos are massive particles – the Standard Model has to be extended

How to extend the Standard Model????

1956	3
7	8
8	2
9	2
1960	9
1	4
2	6
3	20
4	18
5	12
6	9
7	9
8	29
9	70
1970	91
1	81
2	92
3	132
4	196
5	245
6	311
7	298
8	367
9	311
1980	432
1	412
2	318
3	227
4	375
5	344
6	541
7	598
8	498
9	449
1990	481
1	536
2	693
3	540
4	540
5	563
6	591
7	642
8	876
9	1006
2000	1195
1	1155
2	1119
3	1168
4	1001
5	1031
6	1094
7	829
8	613

Neutrinos

in

the Standard Model

1. Neutrinos are massless
2. There are three neutrino flavours
3. Left – handed interactions

In the SM

1) There is no RH neutrinos
2) There is only one Higgs doublet of $SU(2)_L$,
3) Theory is renormalizable,
4) Non-perturbative effects do not introduce m_ν

As a consequence:

Neutrinos are massless

But vanishing of neutrino mass is not guaranteed by any fundamental symmetry

There are three flavour neutrinos:

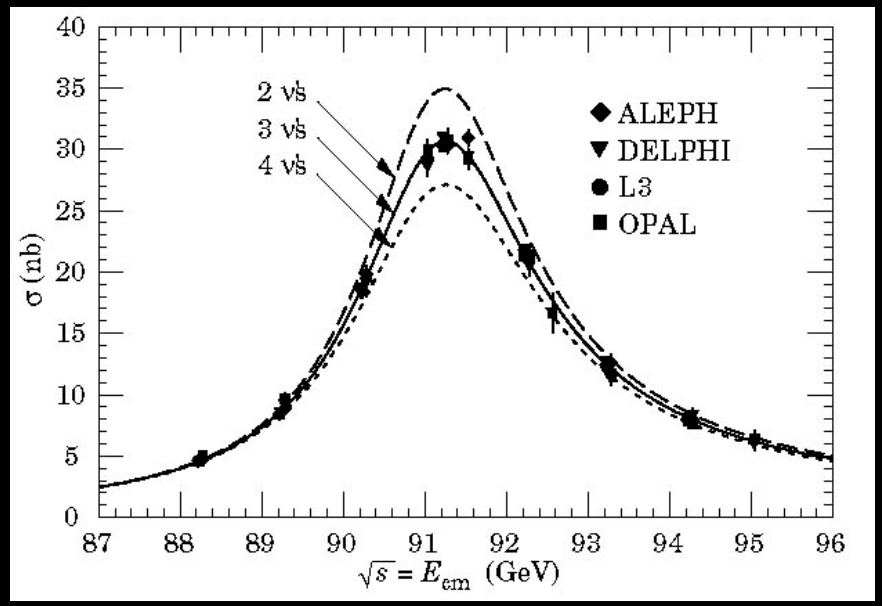
Distinguished by three flavour numbers

L_e, L_μ, L_τ

ν_e,
- F.Reines & C. Cowan (1956) - ν_e

ν_μ,
- M. Schwartz, L. Lederman, J. Steinberger (1962) – ν_μ

ν_τ
- DONAT Collaboration in Fermilab (2000) - ν_μ .



From Z_0 decay- LEP- there are only three flavour neutrinos

Neutrino interaction in the Standard Model

$$L_{CC} = \frac{e}{2\sqrt{2}\sin\theta_W} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha \gamma^\mu (1-\gamma_5) l_\alpha W_\mu^+ + h.c.$$

$$L_{CC} = \frac{e}{4\sin\theta_W \cos\theta_W} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha \gamma^\mu (1-\gamma_5) \nu_\alpha Z_\mu$$

As a consequence:

1. There is no lepton mixing,
2. There is no CP violation in the lepton sector,
3. In electromagnetic structure - only charge radius $\langle r^2 \rangle$,
4. Neutrinos are stable.

What we know from experiments??

1. Neutrino masses,
2. Neutrino mixing,
3. What must to be measured?

C. Amsler et al.
Particle Data Group, 2009

1) We know masses of charged leptons

$$m_e = 0.510998910 \pm 0.000000013\ MeV, \quad \Delta m_e = 0.013\ eV,$$
$$m_\mu = 105.6583668 \pm 0.0000038\ MeV, \quad \Delta m_\mu = 3.8\ eV,$$
$$m_\tau = 1776.84 \pm 0.17\ MeV \quad \Delta m_\tau = 170000.0\ eV$$

2) Neutrino masses we know indirectly

From tritium beta decay

$$\sqrt{|U_{ei}|^2 m_i^2} \le 2\ eV$$

$$m_1 = 0.0$$
$$m_2 = 0.00871^{+0.00008}_{-0.00012}\ eV$$
$$m_3 = 0.0501 \pm 0.0013 eV$$

From oscillation experiments

$$\Delta m_{21}^2 = (7.59^{+0.14}_{-0.21}) \times 10^{-5}\ eV^2$$
$$\Delta m_{32}^2 = (2.43 \pm 0.13) \times 10^{-3}\ eV^2$$

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

From oscillation experiments:

$$\sin^2(2\theta_{12}) = 0.87 \pm 0.03$$

$$\sin^2(2\theta_{23}) > 0.92$$

$$\sin^2(2\theta_{13}) < 0.19$$

$$\delta,\ \phi_1,\ \phi_2 = ???$$

Small neutrino masses and two large mixing angles contrast sharply with the quark masses and their mixing angles

Good motivation in the search for the origin of quarks and lepton flavour – a resurgence of interest in this subject

D. A. Eby, P. H. Frampton and S. Matsuzaki, arXiv:0907.3425 [hep-ph]; F. Bazzocchi and I. de Medeiros Varzielas, arXiv:0902.3250 [hep-ph]; A. Blum and C. Hagedorn, Nucl. Phys. B **821 (2009) 327 [arXiv:0902.4885 [hep-ph]]; G. Altarelli and D. Meloni, J. Phys. G 36 (2009)** 085005 [arXiv:0905.0620 [hep-ph]]; G. Altarelli, F. Feruglio and L. Merlo, JHEP **0905 (2009)** 020 [arXiv:0903.1940 [hep-ph]]; L. Merlo, arXiv:0907.1781 [hep-ph]; S. Goswami, S. Khan and W. Rodejohann, arXiv:0905.2739 [hep-ph];M. Mitra and S. Choubey, Phys. Rev. D **78 (2008) 115014** [arXiv:0806.3254 [hep-ph]]; L. L. Everett and A. J. Stuart, arXiv:0812.1057 [hep-ph]; K. S. Babu and Y. Meng, arXiv:0907.4231 [hep-ph]; P. H. Frampton, S. T. Petcov and W. Rodejohann, Nucl. Phys. B **687 (2004) 31 [arXiv:hep-ph/0401206]; F. Plentinger and W. Rodejohann, Phys. Lett. B 625 (2005)** 264 [arXiv:hep-ph/0507143]; R. N. Mohapatra and W. Rodejohann, Phys. Rev. D **72 (2005) 053001** [arXiv:hep-ph/0507312]; K. A. Hochmuth, S. T. Petcov and W. Rodejohann, arXiv:0706.2975 [hepph]; T. Ohlsson and G. Seidl, Nucl. Phys. B **643 (2002) 247 [arXiv:hep-ph/0206087]; E.Ma and G. Rajasekaran,** Phys. Rev. D **64 (2001) 113012 [arXiv:hep-ph/0106291]; E.Ma, arXiv:0709.0507 [hep-ph];** E. Ma, arXiv:hep-ph/0701016; E. Ma, Mod. Phys. Lett. A **22 (2007) 101 [arXiv:hep-ph/0610342];** E. Ma, Mod. Phys. Lett. A **21 (2006) 2931 [arXiv:hep-ph/0607190]; E. Ma, Mod. Phys. Lett. A 21 (2006) 1917 [arXiv:hep-ph/0607056]; E. Ma, H. Sawanaka and M. Tanimoto, Phys. Lett. B 641** (2006) 301 [arXiv:hep-ph/0606103]; E. Ma, Phys. Rev. D **73 (2006) 057304; B. Adhikary, B. Brahmachari,** A. Ghosal, E. Ma and M. K. Parida, Phys. Lett. B **638 (2006) 345 [arXiv:hep-ph/0603059];** E. Ma, Mod. Phys. Lett. A **20 (2005) 2601 [arXiv:hep-ph/0508099]; E. Ma, Phys. Rev. D 72 (2005)** 037301 [arXiv:hep-ph/0505209]; S. L. Chen, M. Frigerio and E. Ma, Nucl. Phys. B **724 (2005) 423** [arXiv:hep-ph/0504181]; E.Ma, Phys. Rev. D **70 (2004) 031901 [arXiv:hep-ph/0404199]; F. Feruglio,** C. Hagedorn, Y. Lin and L. Merlo, Nucl. Phys. B **775 (2007) 120 [arXiv:hep-ph/0702194]. C. Luhn,** S. Nasri and P. Ramond, Phys. Lett. B **652 (2007) 27 [arXiv:0706.2341 [hep-ph]]; F. Plentinger** and G. Seidl, Phys. Rev. D **78 (2008) 045004 [arXiv:0803.2889 [hep-ph]]; C. Csaki, C. Delaunay,** C. Grojean and Y. Grossman, arXiv:0806.0356 [hep-ph]; M.-C. Chen and K. T. Mahanthappa, Phys. Lett. B **652 (2007) 34 [arXiv:0705.0714 [hep-ph]]; M.-C. Chen and K. T. Mahanthappa,** arXiv:0710.2118 [hep-ph];M.-C. Chen and K. T.Mahanthappa, arXiv:0812.4981 [hep-ph]; R. N.Mohapatra, S. Nasri and H. B. Yu, Phys. Lett. B **639 (2006) 318 [arXiv:hep-ph/0605020]; R. N. Mohapatra** and H. B. Yu, Phys. Lett. B **644 (2007) 346 [arXiv:hep-ph/0610023]; X. G. He, Nucl.** Phys. Proc. Suppl. **168 (2007) 350 [arXiv:hep-ph/0612080]; A. Aranda, arXiv:0707.3661 [hep-ph];** A. H. Chan, H. Fritzsch and Z. z. Xing, arXiv:0704.3153 [hep-ph]; Z. z. Xing, Phys. Lett. B **618** (2005) 141 [arXiv:hep-ph/0503200]; Z. z. Xing, H. Zhang and S. Zhou, Phys. Lett. B **641 (2006)** 189 [arXiv:hep-ph/0607091]; S. K. Kang, Z. z. Xing and S. Zhou, Phys. Rev. D **73 (2006) 013001** [arXiv:hep-ph/0511157]; S. Luo and Z. z. Xing, Phys. Lett. B **632 (2006) 341 [arXiv:hep-ph/0509065];** M. Hirsch, E. Ma, J. C. Romao, J. W. F. Valle and A. Villanova del Moral, Phys. Rev. D **75 (2007)** 053006 [arXiv:hep-ph/0606082]; N. N. Singh, M. Rajkhowa and A. Borah, arXiv:hep-ph/0603189; X. G. He and A. Zee, Phys. Lett. B **645 (2007) 427 [arXiv:hep-ph/0607163]; N. Haba, A. Watanabe** and K. Yoshioka, Phys. Rev. Lett. **97 (2006) 041601 [arXiv:hep-ph/0603116]; Z. z. Xing, Phys. Lett.** B **533 (2002) 85 [arXiv:hep-ph/0204049]; Y. Lin, Nucl. Phys. B 813 (2009) 91 [arXiv:0804.2867**[hep-ph]]; L. Yin, arXiv:0903.0831 [hep-ph].

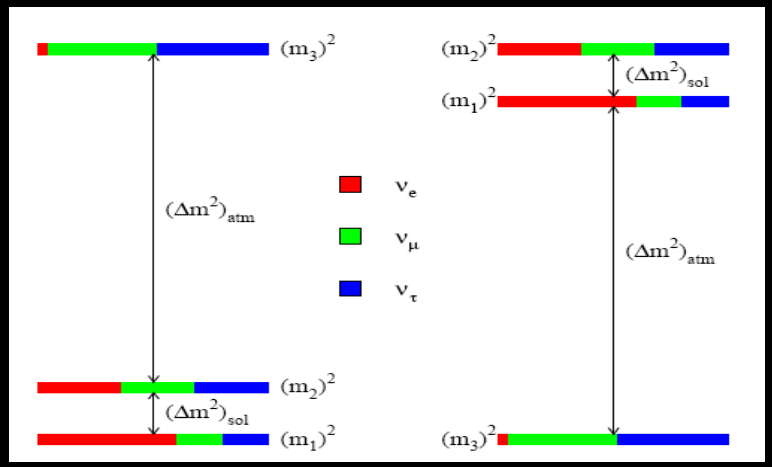
We do not know!!

A)The neutrino mass scheme:

☆ Mass hierarchy

☆ Inverse mass hierarchy

☆ Degenerate



B) Neutrino nature - Dirac or Majorana

C) Are the CP symmetry violated or no, phases

☆ Dirac - δ,

☆ Majorana – α_1, α_2

D) Is mixing angle θ_{13} different from zero

Neutrinos

in the

New Standard Model

(νSM)

- 1 Neutrino mass terms,
- 2 Mössbauer neutrinos,
- 3 GSI anomaly.

Since 1998 we know that neutrinos are massive -- what kind of mass term we have to add to the SM?

Dirac mass? $$m_i^D(\bar{\nu}_{iR}\nu_{iL} + \bar{\nu}_{iL}\nu_{iR})$$

or Majorana mass??

L $$m_{Li}^M(\bar{\nu}_{iR}^c\nu_{iL} + \bar{\nu}_{iL}\nu_{iR}^c)$$

R $$m_{Ri}^M(\bar{\nu}_{iL}^c\nu_{iR} + \bar{\nu}_{iR}\nu_{iL}^c)$$

So in the frame of the new Standard Model (νSM)

$$L_{CC} = \frac{g}{2\sqrt{2}} \overline{N_a} \gamma^\mu (1 - \gamma_5) K_{al} l_l W_\mu^+ + h.c,$$

$$L_{NC} = \frac{g}{4 \cos \theta_W} \overline{N_a} \gamma^\mu (1 - \gamma_5) \Omega_{ab} N_b Z_\mu$$

For three neutrinos:

$$K_{a,l} = U^*_{l,a}$$

$$\Omega_{a,b} = \delta_{a,b}$$

In the present experiments: $m_i \ll E$

Practical Dirac – Majorana Confusion Theorem

L.F. Li and F. Wilczek, Phys. Rev. D25 (1982) 143;
B. Kayser and R.E. Shrock, Phys. Lett. B112 (1982) 137;
B. Kayser, Phys. Rev. D26 (1982) 1662.

Differences in all observables for the Dirac and Majorana neutrinos smoothly vanish for $m_\nu \longrightarrow 0$

Durinng last two years two important properties of „neutrino oscillation" were discused:

- Mössbauer Neutrinos,
- Anomaly observed in the GSI

MÖSSBAUER NEUTRINOS

W.M.Visscher, 1959
W.P. Kells, J.P. Schiffer,1983
R.S. Raghavan,2006

$$A(Z-1) \rightarrow A(Z) + e^{-} + \bar{\nu}_e$$

$$M_{Z-1} = E_\nu + m_\nu + E_R + M_Z + E_e + m_e$$

$$\Delta = M_{Z-1} - M_Z \qquad E_B = -E_e - m_e$$

$$E_{\nu_e} = \Delta + E_B - E_R$$

$$E_B \approx const \qquad E_R \approx 0$$

For Tritium embedded into a cristal

$$^3\mathrm{H} \rightarrow {}^3\mathrm{He} + \mathrm{e}^- + \bar{\nu}_e \qquad \tau = 17.81\ years$$

$$^3\mathrm{He} + \mathrm{e}^- + \bar{\nu}_e \rightarrow {}^3\mathrm{H} \qquad \Gamma = \frac{\hbar}{\tau} = 1.17 \times 10^{-24}\ eV$$

Even if we assume that various brodening effects degrade this value:

$$\Gamma_{\exp} \approx 10^{-11}\ eV$$

W. Potzel, 2006,
R.S. Raghavan, 2006,
but W. Potzel in Ustron 2009 – probably observation of the effect in ^3H - ^3He will be unsuccessful

Energy difference for two relativistic neutrinos with energy E

$$\Delta E_{ik} = \frac{\Delta m_{ik}^2}{2E}$$

Then for:

$$\Delta m^2 \sim 2.5 \times 10^{-3}\, eV$$

$$E = 18.6\ keV$$

Atmospheric neutrinos

We obtain:

$$\Delta E \approx 7 \times 10^{-8}\, eV \gg \Gamma_{\exp}$$

Neutrinos should not oscillate

Do Mössbauer neutrinos oscillate ???

$$m^2 = E^2 - p^2$$

$$\Delta m^2 = \sqrt{(2E\Delta E)^2 + (2p\Delta p)^2}$$

E.K.Akhmedov, J. Kopp, M. Lindner, 2008

If: $\Delta m^2 = 2E\Delta E \approx 4\times 10^{-7} eV^2$ No oscillation

But if: $2p\Delta p \geq 2.5\times 10^{-3} eV^2$ Neutrinos oscillate

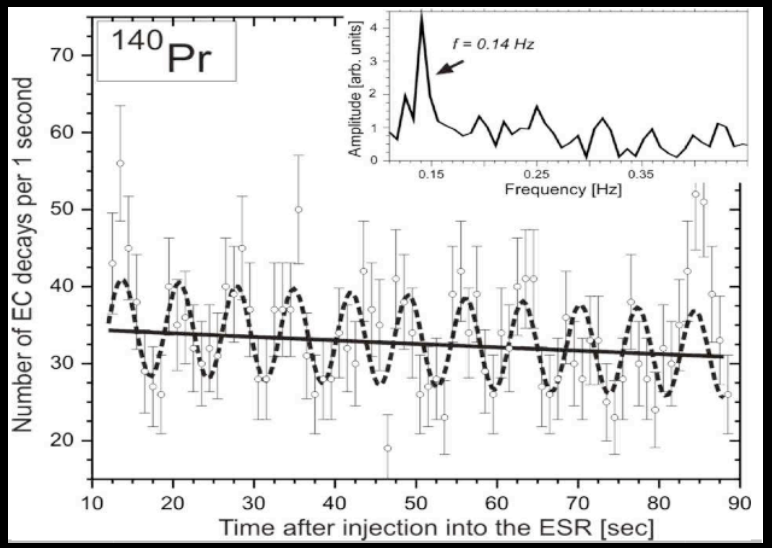
Why is possible to have such big Δp??
For particles on mass shell:

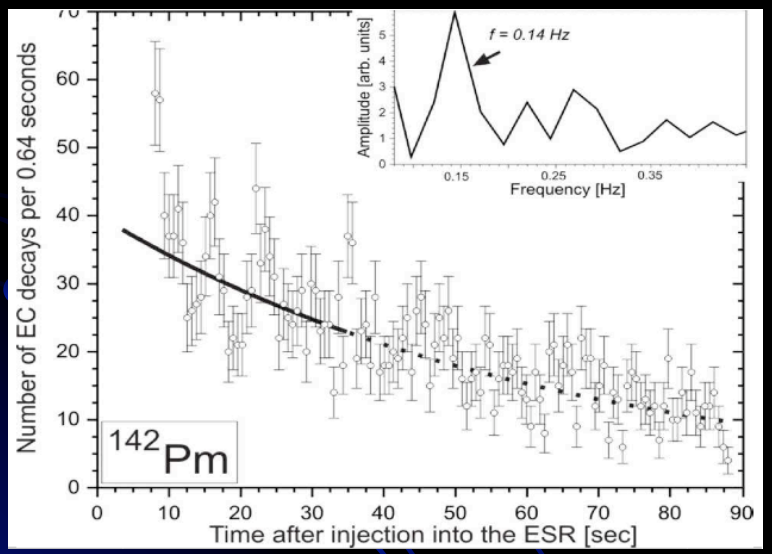
$$p\Delta p = E\Delta E$$

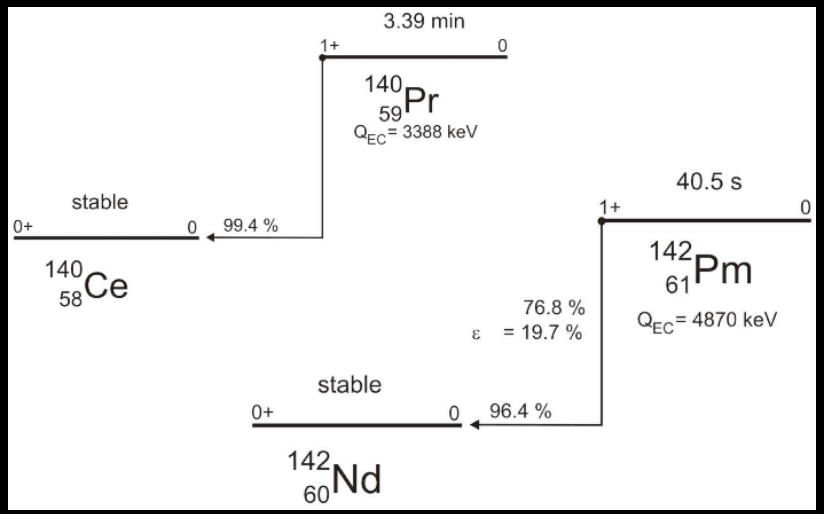
In bound states, energy and momentum are not connected by the "on mass shell" relation *(e.g. eigenstate of harmonic oscillator has definite energy but not momentum)*

GSI, ArXiv:0801.2079

Observation of Non-Exponential Orbital Electron Capture Decays of Hydrogen-Like ^{140}Pr and ^{142}Pm Ions







$$\frac{dN_{EC}(t)}{dt} = N(0) \cdot e^{-\lambda t} \cdot \widetilde{\lambda_{EC}}(t),$$

$$\widetilde{\lambda_{EC}}(t) = \lambda_{EC} \cdot [1 + a \cdot cos(\omega t + \phi)]$$

T = 7.06 (8) s
φ = - 0.3 (3)

T = 7.10 (22) s
φ = - 1.3 (4)

Giunti; Ivanov, Reda and Kienle; Lipkin; Peshkin; Burkardt, Lowe, Stephenson; Ivanov, Kryshen, Pitschmann, Kiele; Giunti; Lipkin….

There are attempts to explain these observations as neutrino oscillation

Litvinov et al. (GSI), Phys. Lett. B664, 163 (2008)

A. N. Ivanov at. al., arXiv:0801.2121,

H. J. Lipkin, arXiv: 0801.1465, arXiv: 0805.0435 (*The GSI method for studying neutrino mass difference – For Pedestrians*),

H. Kleinert, P. Kienle, arXiv:0803.2938.

Neutrino oscillation with the period:

$$T_d = \frac{2\pi}{\omega} = \frac{\gamma M_d}{\Delta m_{21}^2}$$

1) C.Giunti, arXiv:0801.4639,
2) H Kienert at al., arXiv:0808.2389,
3) M.Peshkin, arXiv:0804.4891, arXiv:0811.1765,

No oscillation

Probability that measurement

of any observable **A** gives one of the eigenvalue $a_1, a_2, \ldots. a_N \subset \Delta$:

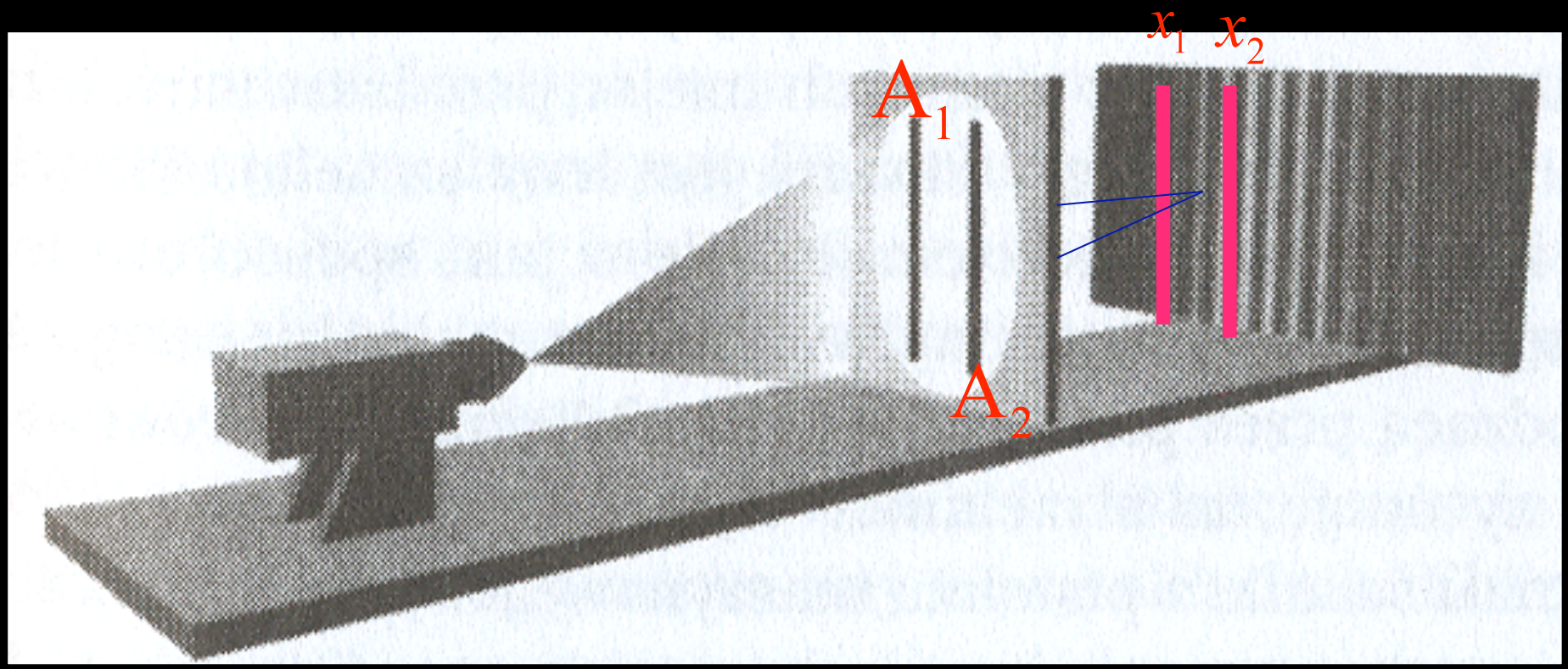
$$\boxed{p_\Delta = Tr\{\hat{P}_\Delta \rho\}}$$

$$\hat{P}_\Delta = \hat{P}_{a_1} + \hat{P}_{a_2} + \ldots. \hat{P}_{a_N}$$

$$p_\Delta = p_{a_1} + p_{a_2} + \ldots + p_{a_N}$$

There is no interference

In the two slits experiments:



$$p(x_1, x_2) = \int_{x_1}^{x_2} dx\, p(x)$$

$$p(x) = \left| A_1(x) + A_2(x) \right|^2$$

Neutrinos beyond the Standard Model

1. MiniBOONE anomaly
2. Problem of neutrino mass and mixing
3. Neutrino oscillation beyond the SM

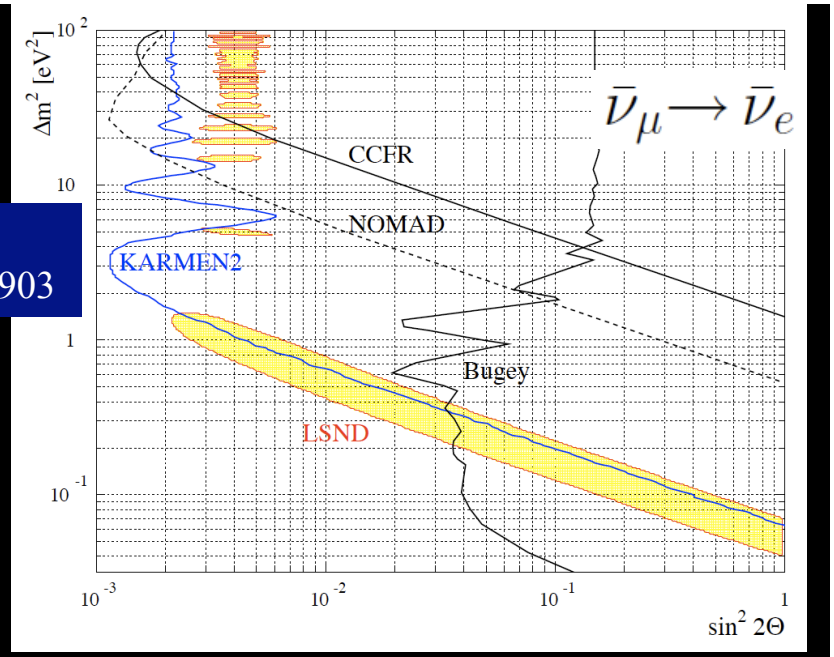
MiniBooNE Anomaly

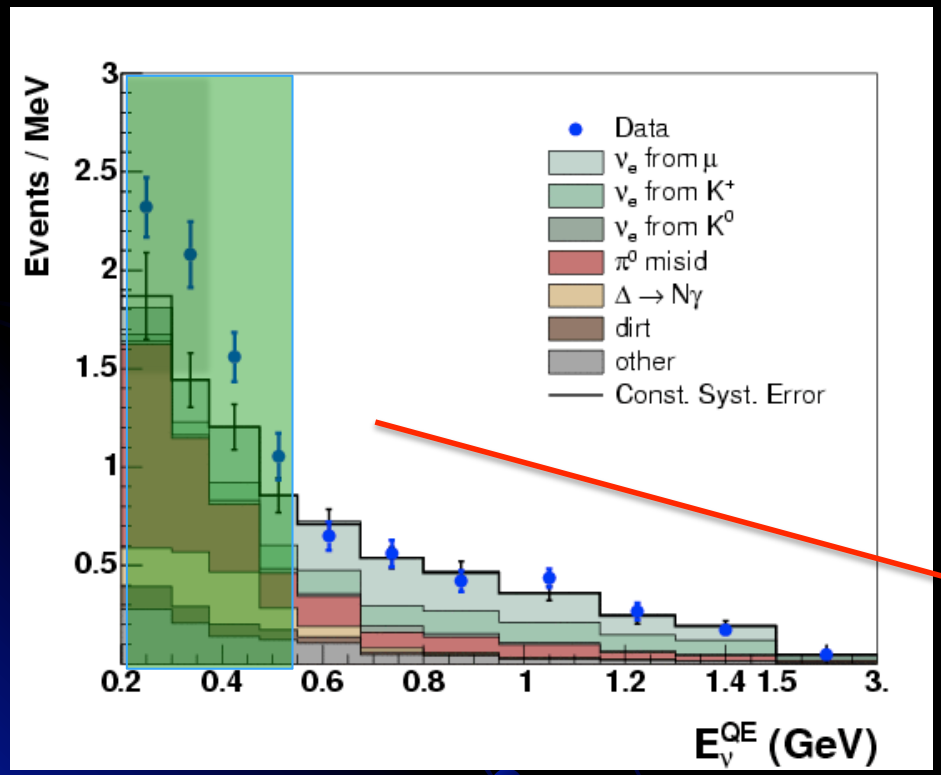
$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \, \sin^2(1.27\Delta m^2 L/E)$$

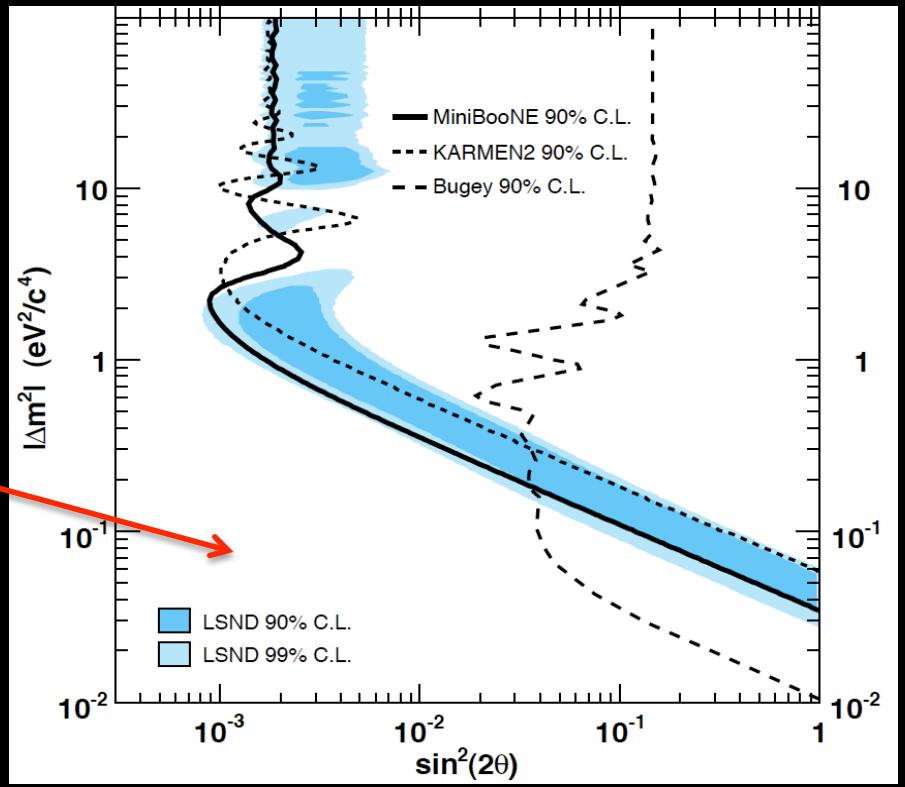
neutrino energy (E):		baseline (L):	
MiniBooNE:	~500 MeV	MiniBooNE:	~500 m
LSND:	~30 MeV	LSND:	~30 m

$$\nu_\mu = 98.1\%, \quad \nu_e = 0.6\%, \quad \bar{\nu}_\mu = 1.2\%, \quad \bar{\nu}_e = 0.03\%$$

K. Eitel, hep-ex/990903

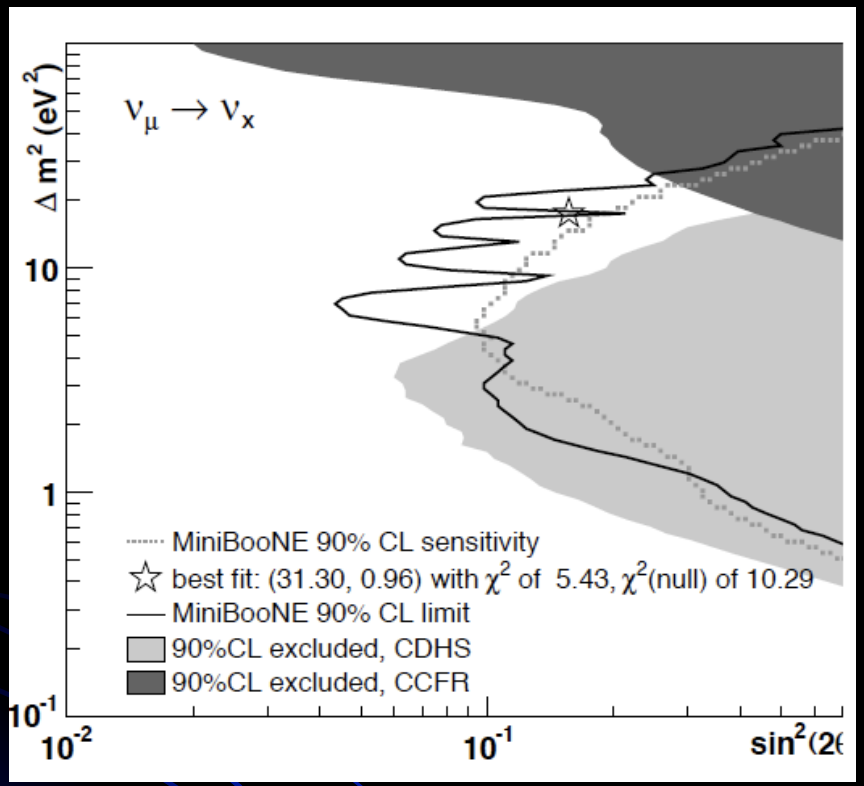


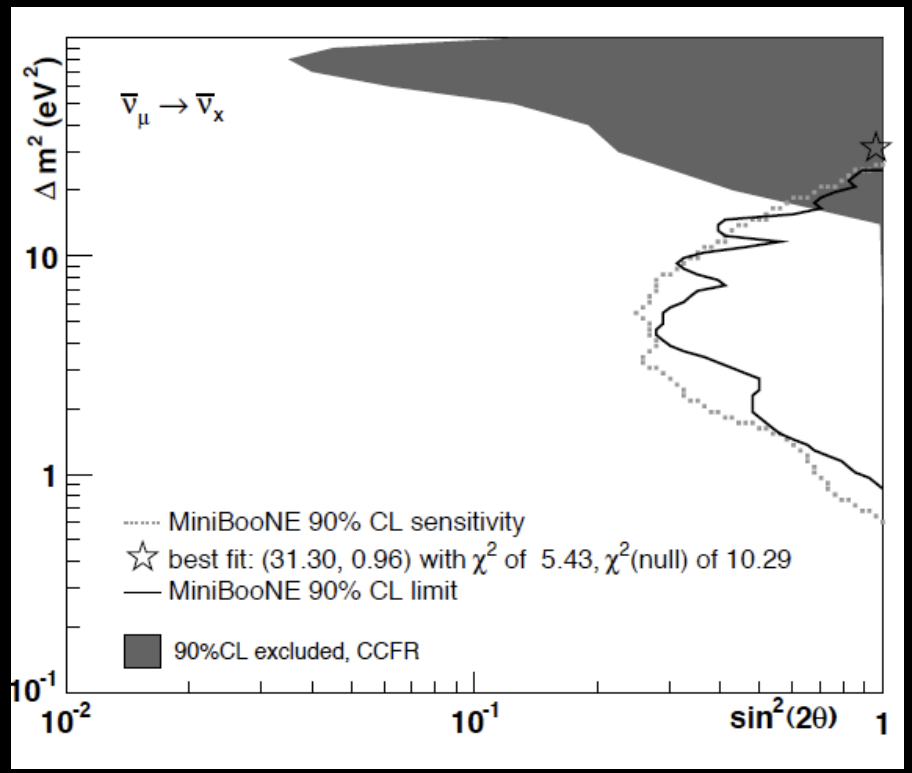




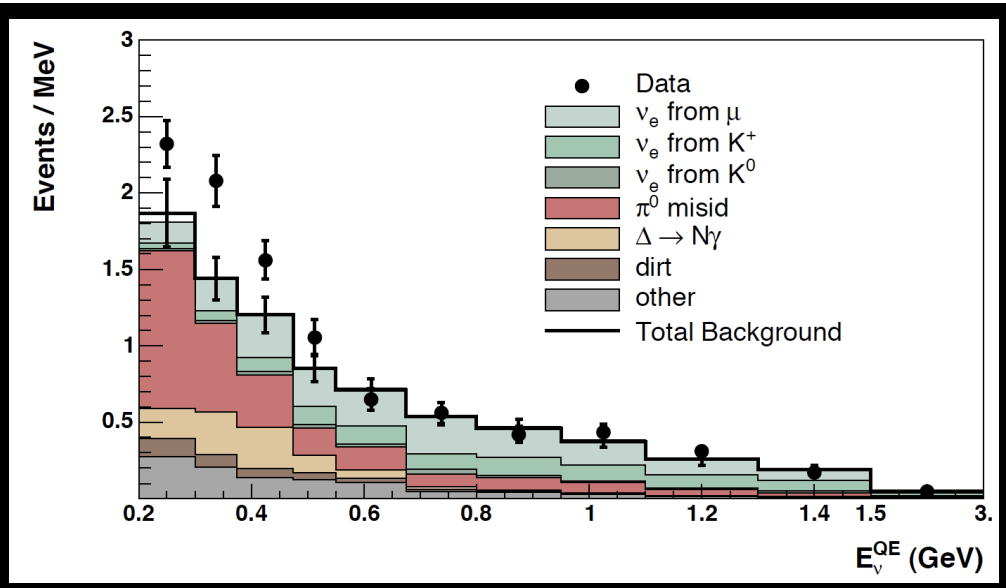
MiniBooNE Neutrino & Antineutrino Disappearance Limits

A.A. Aguilar-Arevalo et al., PRL 103, 061802 (2009)





Improved results soon from MiniBooNE/SciBooNE Joint Analysis!

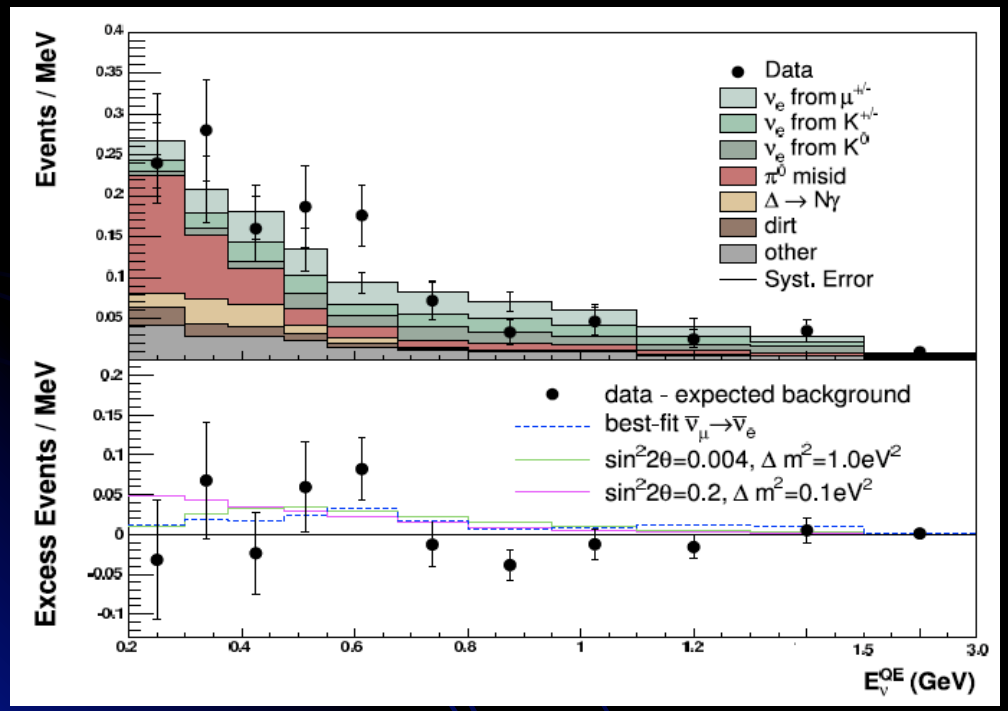


MiniBooNe coll.
arXiv:0812.2243

Excess (128.8 ± 43.4) at low energy (200-475 MeV) for

$$\nu_\mu \rightarrow \nu_e$$

transition, is observed.



MiniBooNE coll.
arXiv:0904.1958

No significant excess of events has been observed for

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

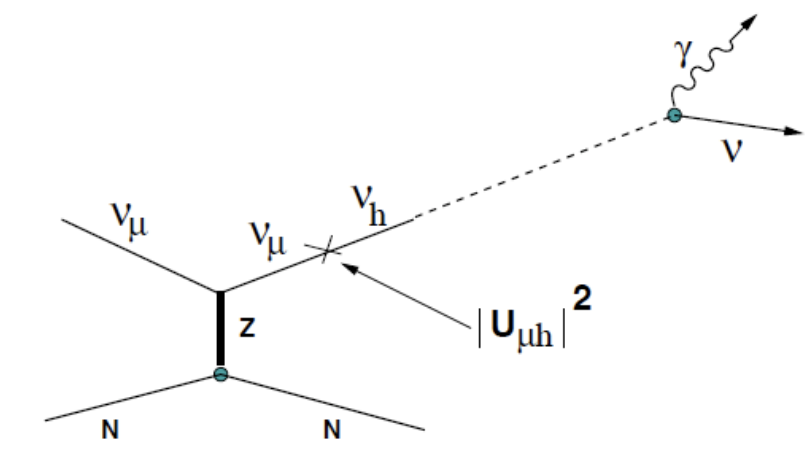
oscillation at law (200-475 MeV) and at high energy (475 -1250 MeV) regions.

In the same channel excess was observed by LSND

There are a lot of papers which try to explain the electron neutrino excess:

Michel Sorel, Janet Conrad, and Michael Shaevitz, Phys. Rev. D **70**, 073004 (2004); G. Karagiorgi *et al.*, Phys. Rev. D **75**, 013011 (2007); Alessandro Melchiorri *et al.*, [arXiv:0810.5133]; Michele Maltoni and Thomas Schwetz, Phys. Rev. D **76**, 093005 (2007); G. Karagiorgi, Z. Djurcic, J. M. Conrad, M. H. Shaevitz and M. Sorel, sumbitted to Phys. Rev. D, arXiv:0906.1997.
Jeffrey A. Harvey, Christopher T. Hill, and Richard J. Hill, Phys. Rev. Lett. **99**, 261601 (2007); Phys. Rev. D **77**, 085017 (2008).
Heinrich Paes, Sandip Pakvasa, and Thomas J. Weiler, Phys. Rev. D **72**, 095017 (2005).
T. Goldman, G. J. Stephenson Jr., and B. H. J. McKellar, Phys. Rev. D **75**, 091301 (2007).
Ann E. Nelson and Jonathan Walsh, Phys. Rev. D **77**, 033001 (2008).
S. Hollenberg, O. Micu, H. Pas, arXiv:0906.5072 [hep-ph].

S.N. Ganienko, arXiv:0902.3802

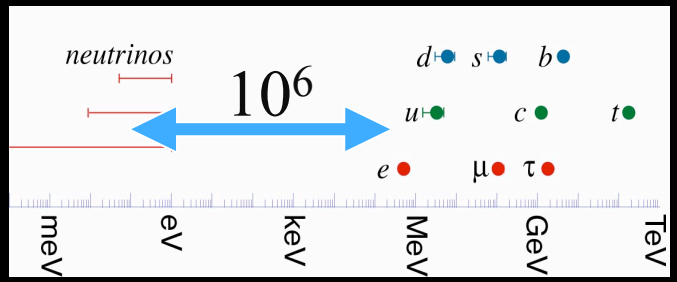


- ⌘ Extra – dimensions,
- ⌘ Sterile neutrinos,
- ⌘ CPT violating interaction,
- ⌘ Neutrino –antineutrino oscillation

Is there anomalous difference between neutrino and antineutrino properties ?? - result are inconclusive

Problem of Neutrino Mass and Mixing

- Why neutrino masses are so small, much smaller then charged leptons and quarks masses



- Why there two large mixing angles for leptons which contrasts sharply with the smallnest of the quark mixing angles.

Problem of particle mass is not solved, why we try to answer those questions about the neutrinos.

$$\frac{m_e}{m_\nu} \leq 10^6$$

but also

$$\frac{m_t}{m_e} \approx 10^5$$

There are several reasons:

$$\frac{m_t}{m_e} \approx 10^5$$

We compare I-st and III-rd generation, in frame of the same family, masses of charged quarks and leptons conform within about an order of magnitude

$$\frac{m_e}{m_\nu} \leq 10^6$$

Smallness of neutrino mass remains a question even within one family

Problem of neutrino mass is probably connected with their nature

Even if the problem of mass is not resolve, the large difference of lepton masses within a single family can shed light on the extension of the SM

Simply solution – add right-handed neutrinos

$$L_Y = -\sum_{\alpha,\beta} f_{\alpha,\beta} \bar{\Psi}_{\alpha L}(-i\sigma_2 \varphi^*)\nu_{\beta R} + h.c. \qquad M_{\alpha,\beta} = \frac{v}{\sqrt{2}} f_{\alpha,\beta}$$

But

Yukawa couplings are completely arbitrary

There is no good reason why $f_{\alpha,\beta}$ must be so small, but

There is no fundamental reason why we can not do that but we do not like this solution.

We can add the right-handed mass term

$$L_{bare} = -\frac{1}{2}\sum_{\alpha,\beta} g_{\alpha,\beta} \bar{\nu}^c_{\alpha L} \nu_{\beta R} + h.c$$

1. B-L is broken, Majorana neutrinos
2. See-saw mechanism,
3. Hierarchy problem

- Quantum gravity can give neutrino masses of order 10^{-5}eV
- Physics beyond the SM must explain observed neutrino masses

Different models of neutrino mass

A. Models with the SM gauge group

A.1. Models with extended fermion sector

- Dirac neutrinos
- Majorana neutrinos

A.2 Models with enlarged Higgs sector

- Higgs triplet (Δ)
- Singly charged singlet (h_-, Zee model)
- Doubly charged singlet (k_{++})

A.3 Models with spontaneous B-L symmetry breaking (Majoron)

- Gauge singlet + right-handed neutrinos
- Extended Higgs sector (Δ, h_-)

B. Models with beyond the SM gauge group

B.1 Models with Left-Right symmetry

- Two doublets Higgs
- Two doublets and triplets

B.2 Models of Grand Unification

- SU(5)
- SO(10)
- E_6

B.3 Supersymmetric models

- Supersymmetric SM –MSSM
- Supersymmetric Left-Right model

B.4 Models with Family Symmetry

Why neutrino masses are very small, answer depends on neutrino nature

(I) Dirac neutrinos ,

$$m_i^D(\bar{\nu}_{iR}\nu_{iL} + \bar{\nu}_{iL}\nu_{iR})$$

or

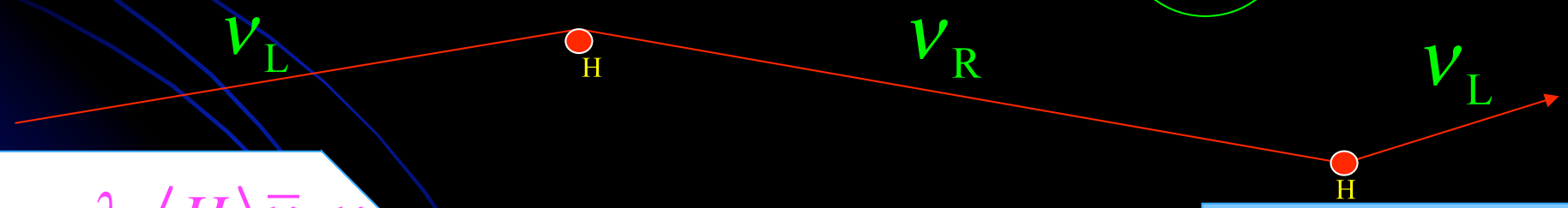
(II) Majorana neutrinos

$$m_{Li}^M(\bar{\nu}_{iR}^c\nu_{iL} + \bar{\nu}_{iL}\nu_{iR}^c)$$

$$m_{Ri}^M(\bar{\nu}_{iL}^c\nu_{iR} + \bar{\nu}_{iR}\nu_{iL}^c)$$

(I) Dirac Neutrinos

Righ handed neutrino fields have to be added ν_R



$$m_\nu \sim \lambda_\nu \langle H \rangle \bar{\nu}_{iR}\nu_{iL}$$

$$\langle H \rangle \approx 175\ GeV$$

$$m_\nu \approx 0.2\ eV \Rightarrow \lambda_\nu \approx 10^{-12}$$

Resolutions:
EXTRA
DIMENSIONS

(II) Neutrina Majorany

See-saw mechanism

Higgs triplet Δ: (ΔLL) $\rho \approx 1 \Rightarrow \langle \Delta \rangle < 8 GeV$

(See-saw II type)

Directly testable at LHC

Two Higgs doublet H: $\frac{1}{M}(HHLL)$ $m_\nu \sim \frac{1}{M} \lambda_\nu \langle H \rangle^2$

R fermion singlet: $\frac{1}{M} HLR$ $M \approx 10^{13} - 10^{16} GeV$

(Type I see-saw)

R_3 fermion triplet: $\frac{1}{M} HLR_3$

(Type III see-saw)

Hierarchy problem scale ≈ 1TeV

GUT scale ≈ 10^{16} GeV

Planck skale ≈ 10^{19} GeV

There is balance between "naturalness" and "testability"

Problem of neutrino mass is connected with their mixing

$$U_{(3\sigma)} = \begin{pmatrix} 0.77-0.86 & 0.50-0.63 & 0.-0.22 \\ 0.22-0.56 & 0.44-0.73 & 0.57-0.8 \\ 0.21-0.55 & 0.40-0.71 & 0.59-0.82 \end{pmatrix}$$

Tri-bimaximal (TB) mixing pattern

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s)$$

$$\sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a)$$

$$\sin\theta_{13} = \frac{r}{\sqrt{2}}$$

If $r,\ |s|,\ |a| \ll 1$ then TB is satisfied and would demand explanation.

If r, s, a are closed to their current 2σ bounds, then TB mixing would only be realized approximately

If TB is realized –
signal of underlying family symmetry

Then from spectral theorem neutrion mass matrix can be decomposed:

$$\mathrm{M}_\nu = \frac{m_1}{6}\Phi_1\Phi_1^+ + \frac{m_2}{3}\Phi_2\Phi_2^+ + \frac{m_3}{2}\Phi_3\Phi_3^+$$

Where m_i are the neutrino masses,

and Φ_i - appropriate eigenvectors, so

$$U_{MNS} = \left(\frac{1}{\sqrt{6}} \Phi_1, \frac{1}{\sqrt{3}} \Phi_2, \frac{1}{\sqrt{2}} \Phi_3 \right)$$

$$\Phi_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \qquad \Phi_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \qquad \Phi_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Large numbers of different flavour symmetry groups

continuous: SO(3),SU(3),….

and discrete: Z, S, A,…

Neutrino oscillation beyond the SM

(A.1) Produced and detected neutrinos are described by the pure QM states:

$$|\nu_\alpha(0)\rangle = \sum_{i=1}^{3} U^*_{\alpha i}|\nu_i\rangle$$

Z. Maki, M. Nakagawa, S. Sakata, Prog.Theor.Phys. 28(1962)870

$$|\nu_\alpha \downarrow\rangle = \sum_i U^*_{\alpha\, i}|\nu_i \downarrow\rangle \qquad |\bar{\nu}_\alpha \uparrow\rangle = \sum_i U_{\alpha\, i}|\bar{\nu}_i \uparrow\rangle$$

(A.2) Number of neutrinos in a detector is described by the factorized formula:

$$\sum_{\beta=e,\mu,\tau} P_{\alpha\to\beta}(E,L) = 1$$

Density of the α initial neutrinos

Detection cross section of β neutrinos

$$N(E,L) = \rho_\alpha(E)\ P_{\alpha\to\beta}(E,L)\ \sigma_\beta(E)\ N_T$$

Number of β neutrinos with energy E, which reach detector in a unit time

Probability of α to β neutrino conversion

Number of active scattering centres in a detector

(A.3) Dirac and Majorana neutrinos oscillate in the same way, so it is impossible to distinguish both types of neutrinos in any oscillation experiments.

- Neutrino masses are too small, they are relativistic particles, and there is no chance to distinguish Dirac from Majorana neutrinos in a production and detection process

- The CP Majorana phases disappear from the oscillation probabilities

New physics can modify neutrino oscillation

- at the source (production process),
- propagation in matter
- at the detector

Beyond the Standard Model

As an example for the production process we take:

μ^- decay

For Dirac neutrinos

$$\mu^-(\lambda_\mu) \to e^-(\lambda_e) + \overline{\nu}_n(\lambda_n) + \nu_m(\lambda_m)$$

In the SM (+1) (-1)

Beyond the SM (+1,-1)

For Majorana neutrinos

$$\mu^-(\lambda_\mu) \to e^-(\lambda_e) + \nu_n(\lambda_n) + \nu_m(\lambda_m)$$

For Dirac nautrinos

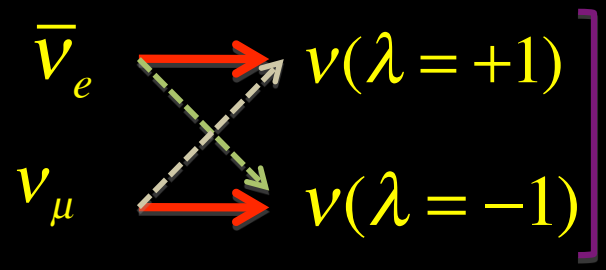
In the MS:

$$\bar{\nu}_e = \nu(\lambda = +1) \qquad |\bar{\nu}_e\rangle = \sum_{i=1}^{3} U_{e\,i} |\bar{\nu}_i\rangle$$

$$\nu_\mu = \nu(\lambda = -1) \qquad |\nu_\mu\rangle = \sum_{i=1}^{3} U^*_{\mu i} |\nu_i\rangle$$

Pure state

Beyond the SM:

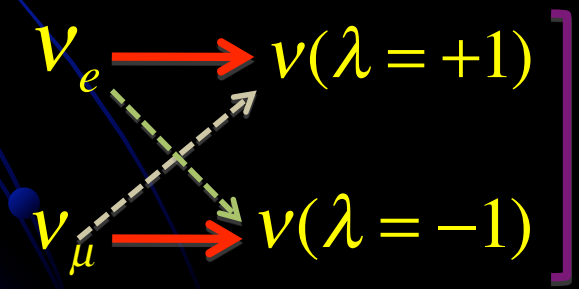


Mixed state

Density Matrix

For Majorana neutrinos

In the SM or beyond the SM:



Mixed states

(B.1) Neutrino density matrix is calculated in the standard way

State of the neutrinos produced in the process

$$A \rightarrow B + \bar{l}_\alpha + \nu_i$$

M.Ochman,R. Szafron,MZ, J.Phys.G35:065003,2008

is described by the density matrix (if initial particle **(A)** is not polarized and polarizations of the final particles **(B,l)** are not measured):

$$\rho^{\alpha}_{\lambda,\, i;\mu,\, k} = \frac{1}{N_\alpha} \sum \int f_i^{\alpha}(\lambda_A;\lambda_B,\lambda_l,\lambda) f_k^{\alpha *}(\lambda_A;\lambda_B,\lambda_l,\mu)$$

$$Tr(\rho^{\alpha}) = 1$$

where $f_i^{\alpha}(\lambda_A;\lambda_B,\lambda_l,\lambda)$ are the amplitudes for the production process $A \rightarrow B + \bar{l}_\alpha + \nu_i$.

But even if only left – handed neutrinos are produced

The necessary and sufficient condition for pure initial state of produced neutrinos with negative helicities is the factorization for spin and mass of the production amplitudes

$$f_i^\alpha(\lambda_A;\lambda_B,\lambda_l,\lambda=-1)=g^\alpha(\lambda_A;\lambda_B,\lambda_l,\lambda=-1)*h_i^\alpha\equiv g_\mu^\alpha h_i^\alpha$$

If we introduce the shortcut notation:

$$\mu=(\lambda_A;\lambda_B,\lambda_l)$$

Then:

$$f_i^\alpha(\mu)=g^\alpha(\mu,\lambda=-1)*h_i^\alpha\equiv g_\mu^\alpha h_i^\alpha$$

Then the density matrix is given by:

$$\rho^{\alpha}(i,\lambda=-1;k,\mu=-1)=\chi_i^{\alpha}\chi_k^{\alpha *}$$

where

$$\chi_i^{\alpha}=\frac{h_i^{\alpha}}{\sqrt{\sum_i \left|h_i^{\alpha}\right|^2}}$$

which is equivalent to the pure QM state:

$$\left|\nu_{\alpha}\right\rangle=\sum_i \chi_i^{\alpha}\left|\nu_i\right\rangle \qquad \left|\nu_{\alpha}\right\rangle=\sum_i U_{\alpha\, i}^{*}\left|\nu_i\right\rangle$$

$$\left\langle\nu_{\alpha}\middle|\nu_{\beta}\right\rangle=\sum_i \chi_i^{\alpha}\chi_i^{\beta *}\neq\delta_{\alpha\beta}(=1\, for\ \alpha=\beta)$$

νSM

$$\chi_i^{\alpha}=U_{\alpha\, i}^{*}$$

(B.2) Factorization for the final oscillation rate

$$\sigma_{\alpha\to\beta}(L,E)=$$

$$\frac{1}{32\pi s}\frac{p_f}{p_i}\frac{1}{2s_f+1}\sum_{i,k,\lambda,\lambda_C,\lambda_l,\lambda_D}\int d\Omega\, A_i^{\beta}(\lambda,\lambda_C,\lambda_l,\lambda_D;\Omega)\rho^{\alpha}(i,\lambda;k,\lambda;L,E)A_k^{\beta *}(\lambda,\lambda_C,\lambda_l,\lambda_D;\Omega)$$

The density matrix after oscillation

$$\rho^{\alpha}(i,\lambda=-1;k,\mu=-1;L,E)=\rho_{i,k}^{\alpha}(L,E)e^{-\frac{\delta m_{i,k}^2}{2E}L}$$

If the detection amplitudes factorize

$$A_i^{\beta}(\lambda=-1,\lambda_C,\lambda_l,\lambda_D;\Omega)=e_{\eta}^{\beta}(\theta,\varphi)*k_i^{\beta}$$

Then the final cross section factorize

$$\sigma_{\alpha\to\beta}(L,E)=$$

$$\frac{1}{32\pi s}\frac{p_f}{p_i}\frac{1}{2s_f+1}\sum_{i,k,\eta}\int d\Omega\,(e_{\eta}^{\beta}*k_i^{\beta})(\rho_{i,k}^{\alpha}e^{-\frac{\delta m_{i,k}^2}{2E}L})(e_{\eta}^{\beta *}*k_k^{\beta *})=$$

$$=P_{\alpha\to\beta}(L,E)\,\sigma_{\beta}(E)$$

The oscillation probability is given by:

$$P_{\alpha\to\beta}(L,E)=\sum_{i,k}\rho^{\alpha}_{i,k}(L,E)\;k_i^{\beta}k_k^{\beta*}e^{-\frac{\delta m^2_{i,k}}{2E}L}$$

And the final detection cross section

$$\sigma_{\beta}(E)=\frac{1}{32\pi s}\frac{p_f}{p_i}\frac{1}{2s_f+1}\sum_{\eta}\int d\Omega\;\left|e^{\beta}_{\eta}(\theta,\varphi)\right|^2$$

The sum over all final flavours

$$\sum_{\beta=e,\mu,\tau}P_{\alpha\to\beta}(L,E)=\sum_{i,k}\sum_{\beta=e,\mu,\tau}(k_i^{\beta}k_k^{\beta*})\rho^{\alpha}_{i,k}e^{-\frac{\delta m^2_{i,k}}{2E}L}=\sum_{i}\rho^{\alpha}_{i,i}=1$$

if

$$\sum_{\beta=e,\mu,\tau}(k_i^{\beta}k_k^{\beta*})=\delta_{i,k}$$

The probability is conserved if the final states are orthogonal

(B3) Dirac and Majorana neutrinos propagate in matter in a different way, so in principle both types of neutrinos can be distinguished in future oscillation experiments.

- Production and detection states are the same if one neutrino is produced or detected
- If two neutrinos are produced (like in the muon decay) or detected (like in the inverse muon decay), even in the production and detection processes Dirac and Majorana neutrinos are distinguished
- Propagation in a matter distinguishes both types of neutrinos (neutral currents are crucial).

Conclusions and Perspectives

The road ahead:

EXPERIMENTS:

- Are neutrino Dirac or Majorana particles?
- What are absolute neutrino massea
- Is the 23 angle maximal?
- Does the 13 angle vanish?
- Is there CP volation in the lepton sector?
- Do the sterile neutrino exist?

THEORY

MASS PROBLEM

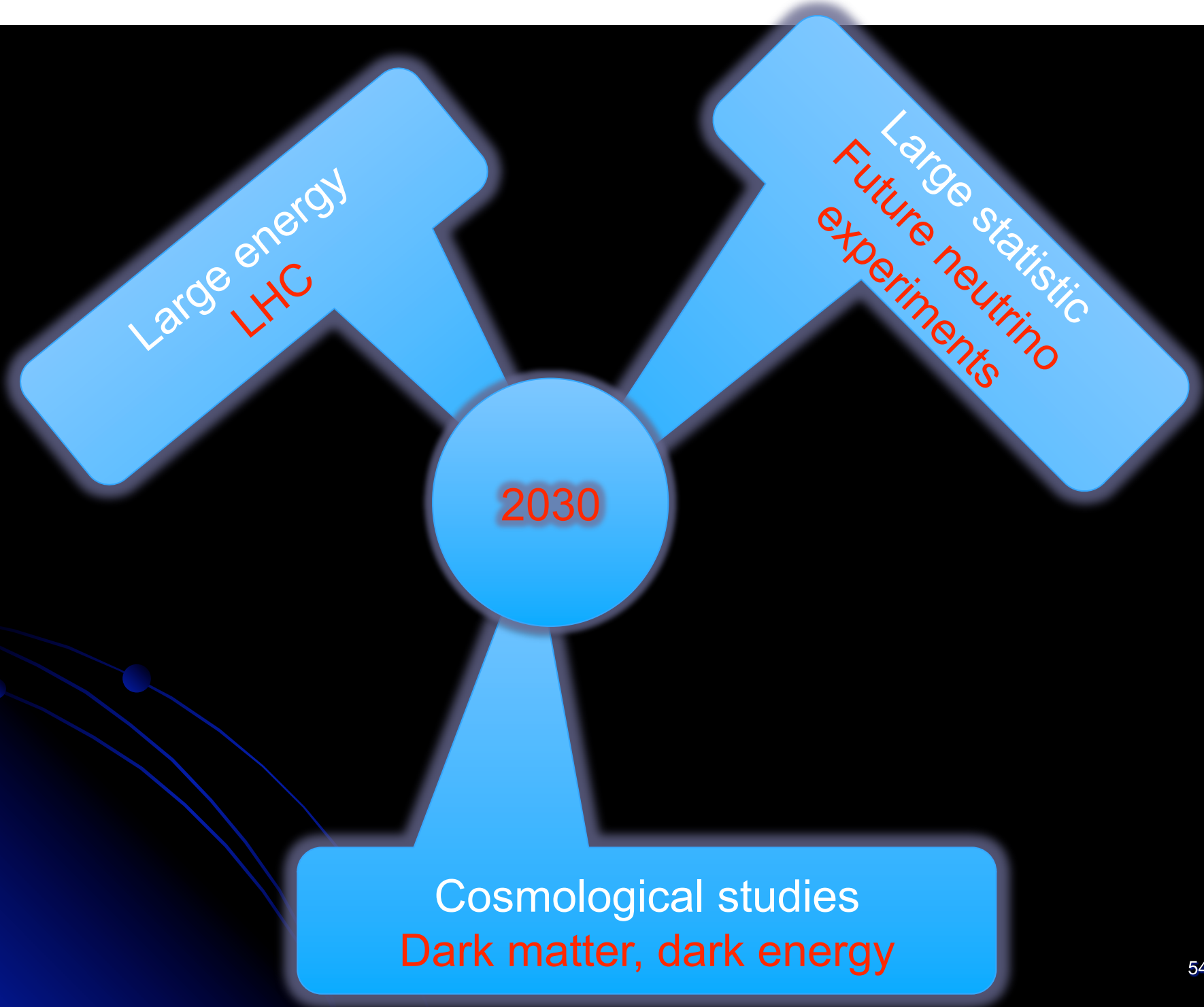
- To understand why neutrino mass is so tiny

FLAVOUR PROBLEM

- Why the leptons and quarks mixing angles are so different?
- To understand CP symmetry breaking in the lepton sector

PROBLEM OF INTERACTION

- Are there non-standard neurtino interaction in the TeV scale?


Large energy
LHC
Large statistic
Future neutrino
experiments
2030
Cosmological studies
Dark matter, dark energy