Properties of inclusive vs. exclusive QCD evolution kernels

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Outline

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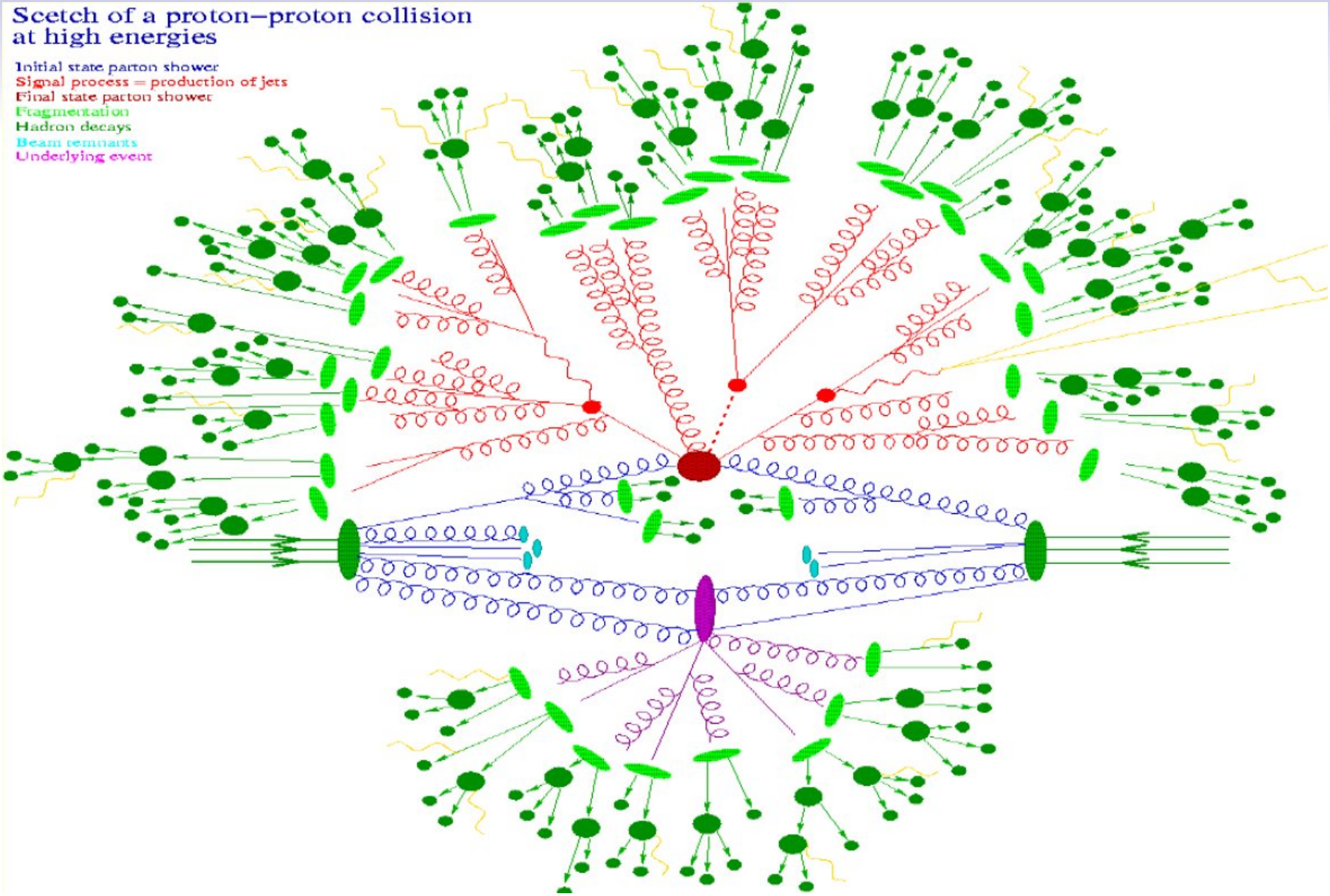
- Factorization and QCD evolution
- Details on calculations of evolution kernels
- Summary

Introduction

The Large Hadron Collider (LHC) just started operating

- very precise measurments
- need of high accuracy QCD calculations (vs. Tevatron)
- especially in form of Parton Shower Monte Carlo (PS MC)


Scetch of a proton–proton collision
at high energies
Initial state parton shower
Signal process = production of jets
Final state parton shower
Fragmentation
Hadron decays
Beam remnants
Underlying event

Factorization

What enables us the theoretical description of this very complicated process is the *factorization theorems*

$$\sigma = \sum_{ij} \int dx_1 dx_2 \; f_i(x_1, \mu) f_j(x_2, \mu) \, \hat{\sigma}_i(p_1, p_2, \alpha_S(\mu), Q^2/\mu^2)$$

We can calculate both

- coefficient functions (hard process)
- parton distribution functions (PDF)

using perturbative QCD

Factorization

- Different factorization schemes $\rightarrow$ different PDFs, different coefficient functions
- Both PDF and coefficient functions depend on this scheme
- Physical observables do not depend on factorization scale μ

The most common scheme is $\overline{MS}$ scheme

- hard process (coefficient function) is process dependent
- parton distribution function (PDF) process independent

Aims

Our interest is in the PDF part of the *pp* collisions

PDFs are ruled by *QCD evolution equations (DGLAP)* $\rightarrow$ building blocks for the construction of precise PDF are the *evolution kernels*

- ultimate aim - construct stochastic simulation of the QCD evolution in the Next to Leading Order (NLO) of perturbation theory in fully exclusive form! so far exists only improved LO MCPS

To do it we need to know the form and the properties of exclusive NLO evolution kernels

- aim of this presentation - show that within $\overline{MS}$ factorization scheme *inclusive evolution kernels* do not depend on the choice of *evolution time variable used in MC*

Framework

Two gluon phase space

$$d\psi = \frac{d^n k_1}{(2\pi)^n} 2\pi\delta^+(k_1^2) \frac{d^n k_2}{(2\pi)^n} 2\pi\delta^+(k_2^2),$$

Sudakov parametrisation:

$$k_i = \alpha_i p + \beta_i n + k_{i\perp}$$

Two set of variables (evolution time)

- transverse momentum $\mathbf{k}_{i\perp}$
- angular scale $\mathbf{a}_i = \frac{\mathbf{k}_{i\perp}}{\alpha_i}$ - rapidity related variable $y = \ln|\mathbf{a}|$

General structure of two-gluon real emission diagram

$$\Gamma \sim \mathrm{PP} \int d\psi \; V \; \Theta(s(k_1, k_2) \leq Q)$$

- the exect form of $s(k_1, k_2)$ function (enclosing the phase space) defines the *evolution time variable in MC*
- V is originating from trace its *dimension* is Q^{-4} it depends on $\alpha_i, k_{i\perp}, \theta$
- PP is Pole Part operator extracting poles in ϵ

Evolution time

The enclosing of the phase space $\Theta(s(k_1, k_2) \leq Q)$ defines the evolution variable

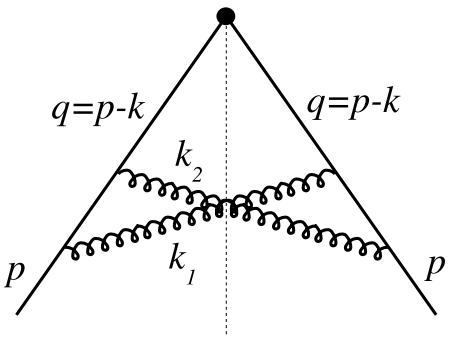
We typically use two choices for the evolution time: transverse momentum $k_\perp$ and angular scale a (related directly to rapidity: $y = \ln|\mathbf{a}|$)

- transverse momentum $s(k_1, k_2) = \max\{k_{1\perp}, k_{2\perp}\}$
- angular scale $s(k_1, k_2) = \max\{a_1, a_2\}$

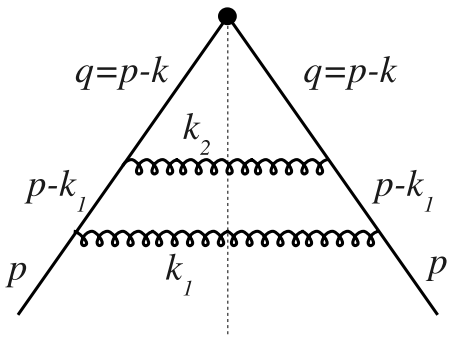
We want to show that within the $\overline{MS}$ renormalization scheme both chioces give the same inclusive evolution kernels but different exclusive kernels

Evolution time independence

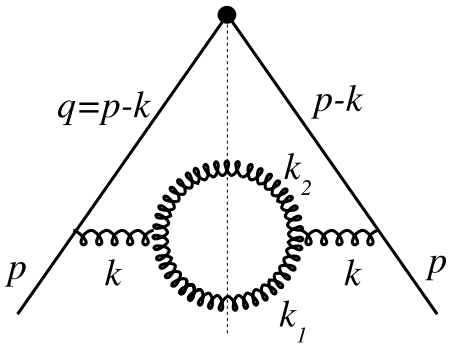
- There are *THREE mechanisms* allowing the *independence* from the choice of evolution time variable



case 1



case 2



case 3

- We show it on a subset of diagrams contributing to NLO DGLAP kernels

Structure of kernel contributions

After integrating out the transverse degrees of freedom dependency

$$\Gamma_{k_\perp} \sim \mathrm{PP} \int \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{1}{\epsilon} \left[\frac{A}{\epsilon} + B + ... \right]$$

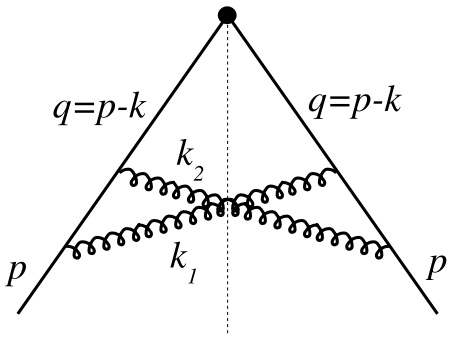
$$\Gamma_a \sim \mathrm{PP} \int \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} (\alpha_1 \alpha_2)^{2\epsilon} \frac{1}{\epsilon} \left[\frac{A}{\epsilon} + B + ... \right]$$

The only difference is the additional phase space factor $(\alpha_1 \alpha_2)^{2\epsilon}$ in case of angular scale (α_i = light cone variable of emited gluon)

Two types of contributions

- case 1 - without additional ϵ pole, $A = 0$
- case 2,3 - with additional ϵ pole $A \neq 0$

case 1 - $A = 0$ contributions



$$(\alpha_1\alpha_2)^{2\epsilon} = 1 + 2\epsilon \ln(\alpha_1\alpha_2)$$

$$\Gamma_{k_\perp} \sim \mathrm{PP}\frac{1}{\epsilon}\int \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\delta(1-x-\alpha_1-\alpha_2)B$$

$$\Gamma_a \sim \mathrm{PP}\frac{1}{\epsilon}\int \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}(1+2\epsilon\ln(\alpha_1\alpha_2))\delta(1-x-\alpha_1-\alpha_2)B$$

Pole Part operator PP kills terms $\mathcal{O}(\epsilon) \rightarrow$ for $A = 0$ case both schemes

- $s(k_1, k_2) = \max\{k_{1\perp}, k_{2\perp}\}$
- $s(k_1, k_2) = \max\{a_1, a_2\}$ leads to the same result!

case 2,3 - $A \neq 0$ contributions

$$\Gamma_{k_\perp} \sim \mathrm{PP}\frac{1}{\epsilon}\int \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\left[\frac{A}{\epsilon}+B\right]$$

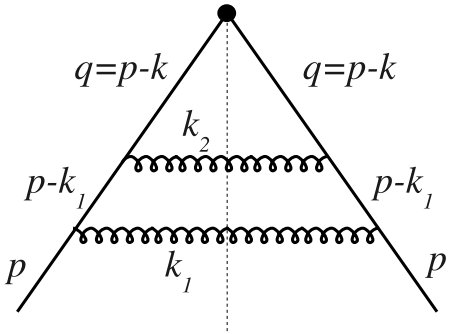
$$\Gamma_a \sim \mathrm{PP}\frac{1}{\epsilon}\int \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\left[\frac{A}{\epsilon}+B+2A\ln(\alpha_1\alpha_2)\right]$$

- additional mixing term $2A\ln(\alpha_1\alpha_2)$
- this is only partial result incomplit!
- take into account virtual diagrams and soft counterterms

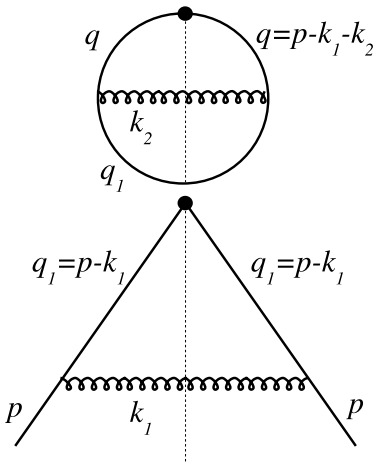
We divide the $A \neq 0$ contributions into two groups:

- case 2 - cancelation by soft counterterm
- case 3 - cancelation by virtual diagram

case 2 - Bremsstrahlung, $A \neq 0$



minus



$$\Gamma_{Br} \sim \mathrm{PP} \int d\psi \frac{1}{q^4} \left[T_0 + T_1 \frac{\mathbf{k}_{1\perp}\mathbf{k}_{2\perp}}{\mathbf{k}_{1\perp}^2} + T_2 \frac{\mathbf{k}_{2\perp}^2}{\mathbf{k}_{1\perp}^2} \right] \Theta(\max\{k_{1\perp}, k_{2\perp}\} \leq Q)$$

- one ϵ pole originating from integration over scale (the $1/q^4$ term)
- second pole from $1/k_{1\perp}^2$ integration

case 2 - Bremsstrahlung

The difference between the two schemes is coming from the phase space factor

$$\Gamma^a_{Br} - \Gamma^{k_\perp}_{Br} \sim \int d\alpha_1 d\alpha_2 \delta(1 - x - \alpha_1 - \alpha_2)[(\alpha_1\alpha_2)^{2\epsilon} - 1]\frac{\alpha_2}{\alpha_1}\frac{T_2}{(1-\alpha_1)^2}$$

The soft counterterm is simply the square of LO kernels

$$\Gamma^a_{ct} - \Gamma^{k_\perp}_{ct} \sim \int d\alpha_1 d\alpha_2 \delta(1 - x - \alpha_1 - \alpha_2)$$
$$\times \ln(\alpha_1\alpha_2)\frac{2 - 2\alpha_1 + \alpha_1^2}{(1-\alpha_1)^2\alpha_1\alpha_2}[2 - 4\alpha_1 - 2\alpha_2 + 2\alpha_1\alpha_2 + 2\alpha_1^2 + \alpha_2^2]$$

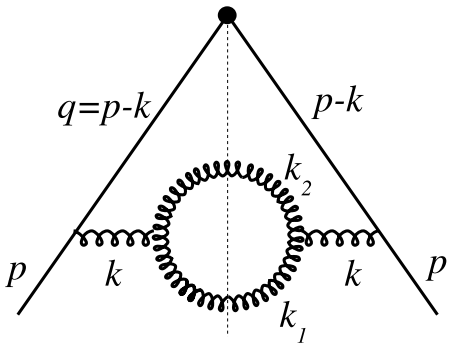
case 2 - Bremsstrahlung

Difference between two schemes exactly compensates!

$$\Gamma^{a}_{Br} - \Gamma^{k_\perp}_{Br} = \Gamma^{a}_{ct} - \Gamma^{k_\perp}_{ct}$$

Counterterm is a square of LO contribution $\rightarrow$ NLO contributions resembels the structure of LO!

case 3 - Cancellation by virtual diagram



$A \neq 0$ + virtual diagram

$$\Gamma^a_{Vg} \sim \mathrm{PP} \int d\psi_a \left[\tilde{T}_0 + \underbrace{\frac{\tilde{T}_1 \mathbf{a}_1^2 + \tilde{T}_2 \mathbf{a}_2^2}{\mathbf{a}^2} + \tilde{T}_3 \frac{(\mathbf{a}_1^2 - \mathbf{a}_2^2)^2}{\mathbf{a}^4}}_{\sim 1/\epsilon} \right] \Theta(\max\{a_1, a_2\} \leq Q)$$

- second pole from $1/\mathbf{a}^2$ integration (collinear singularuty)

case 3 - Cancellation by virtual diagram

$$\Gamma^{a}_{virt} - \Gamma^{k_\perp}_{virt} \sim \frac{1}{\epsilon}\left(-8I_0 - 8\ln(1-x) + \frac{22}{3}\right) \underbrace{2\ln(1-x)}_{2\ln(\alpha)\leftarrow(\alpha^{2\epsilon})}$$

The difference between two schemes is now compensated by virtual diagram

$$\Gamma^{a}_{Vg} - \Gamma^{k_\perp}_{Vg} = \Gamma^{a}_{virt} - \Gamma^{k_\perp}_{virt}$$

Conclusions

- $\overline{MS}$ *inclusive evolution kernels* do not depend on the **evolution time variable in MC** ($k_\perp$, a)
- *unintegrated (exclusive) kernels* do depend on the **evolution time variable in MC** $\rightarrow$ understanding of this differences is critical for the construction of *exclusive MC*
- we investigated also the case of $s(k1, k2) = \max\{v_1, v_2\}$ (eikonal minus variables $v_i = k_{i\perp}/\sqrt{\alpha_i}$ and full virtuality Q^2)
- The presented independence is very likely to be valid also for much more general choice of $s(k1, k2)$ function/evolution time
- This analysis is essential in reconciling of $\overline{MS}$ scheme and exclusive MC implementation of PDF

Phase space parametrization

Spherical coordinates in $k_\perp$ space

$$d\psi_{k_\perp} \sim \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2} dk_{1\perp} dk_{2\perp} k_{1\perp}^{1+2\epsilon} k_{2\perp}^{1+2\epsilon}$$

rapidity related variables $\mathbf{a}_i = \frac{\mathbf{k}_{i\perp}}{\alpha_i}$

$$d\psi_a \sim \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2} \alpha_1^{2+2\epsilon}\alpha_2^{2+2\epsilon} da_1 da_2 a_1^{1+2\epsilon} a_2^{1+2\epsilon}$$

Structure of kernel contributions

$$\Gamma \sim \text{PP} \int \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \int dk_{1\perp} dk_{2\perp} k_{1\perp}^{1+2\epsilon} k_{2\perp}^{1+2\epsilon} \; V \; \Theta(\max\{k_{1\perp}, k_{2\perp}\} \leq Q)$$

Each kernel contribution has at least single ϵ pole
This ϵ pole can be extracted

- additional integration variable $\tilde{Q}$
- dimensionless variables $k_{i\perp} = \tilde{Q} y_i$

$$\Gamma \sim \text{PP} \int \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \underbrace{\int_0^Q d\tilde{Q} \tilde{Q}^{4\epsilon-1}}_{=Q^{4\epsilon}/(4\epsilon)} \int dy_1 dy_2 y_1^{1+2\epsilon} y_2^{1+2\epsilon} \; W \; \delta(\max\{y_1, y_2\} - 1)$$

Structure of kernel contributions for different evolution times

$\mathbf{k}_\perp$ space

$$\Gamma_{k_\perp} \sim \mathrm{PP}\frac{1}{\epsilon}\int \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\int dy_1 dy_2 y_1^{1+2\epsilon} y_2^{1+2\epsilon}\; W\; \delta(\max\{y_1, y_2\} - 1)$$

$\mathbf{a}$ space

$$\Gamma_a \sim \mathrm{PP}\frac{1}{\epsilon}\int \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}(\alpha_1\alpha_2)^{2\epsilon}\int dy_1 dy_2 y_1^{1+2\epsilon} y_2^{1+2\epsilon}\; \tilde{W}\; \delta(\max\{y_1, y_2\} - 1)$$

Structure of kernel contributions

Both in $\mathbf{k}_\perp$ and $\mathbf{a}$ space integrals over *transverse degrees of freedom* are equal!

$$\frac{1}{\epsilon}\int dy_1 dy_2 y_1^{1+2\epsilon} y_2^{1+2\epsilon}\; W\; \delta(\max\{y_1, y_2\} - 1) = \frac{1}{\epsilon}\left(\frac{A}{\epsilon} + B + \ldots\right)$$

$$\frac{1}{\epsilon}\int dy_1 dy_2 y_1^{1+2\epsilon} y_2^{1+2\epsilon} \tilde{W}\; \delta(\max\{y_1, y_2\} - 1) = \frac{1}{\epsilon}\left(\frac{A}{\epsilon} + B + \ldots\right)$$

where A, B depend on α_1, α_2