QCD factorization beyond leading twist in exclusive processes: ρ_T-meson production

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- Since a decade, there have been much developments in hard exclusive processes.
  - form factors, Distribution Amplitudes $\longrightarrow$ Generalized Distribution Amplitudes
  - DVCS $\longrightarrow$ Generalized Parton Distributions, Transition Distribution Amplitudes
- The key tool is the collinear factorization

Introduction: phenomenology of exclusive processes within collinear factorization

- Experimental tests are possible in fixed target experiments
  - $e^{\pm}p$: HERA (HERMES), JLab, COMPASS...

  as well as in colliders, mainly for medium s
  - $e^{\pm}p$ colliders: HERA (H1, ZEUS)
  - $e^{+}e^{-}$ colliders: LEP, Belle, BaBar, BEPC
- Collinear factorization has been proven only for specific cases:
  e.g.: ρ_T production cannot directly be factorized (appearance of end point singularities)
  $\Rightarrow$ improvement needed for a consistent approach of exclusive processes

Our studies attempt to describe exclusive processes involving the production of ρ-mesons in diffraction-type experiment.

We choose $t = t_{min}$ for simplicity.

- $\gamma^*(q) + \gamma^*(q') \to \rho_T(p_1) + \rho(p_2)$ process in $e^+ e^- \to e^+ e^- \rho_T(p_1) + \rho(p_2)$ with double tagged lepton at ILC
- $\gamma^*(q) + P \to \rho_T(p_1) + P$ at HERA

Polarization effects in $\gamma^* P \to \rho P$ at HERA

- one can experimentally measure all spin density matrix elements

- at $t = t_{min}$ one can experimentally distinguish

$$\begin{cases} \gamma^*_L \to \rho_L : & \text{dominates} & \text{(twist 2 dominance)} \\ \gamma^*_T \to \rho_T : & \text{sizable} & \text{(twist 3)} \end{cases}$$

- S-channel helicity conservation:

$$\begin{cases} \gamma^*_L \to \rho_L & (\equiv T_{00}) \\ \gamma^*_T \to \rho_T , & \end{cases}$$

Dominate with respect to all other transitions.
Experimentally, $\gamma^*_T \to \rho_T$ is dominated by $\gamma^*_{T(-)} \to \rho_{T(-)}$ and $\gamma^*_{T(+)} \to \rho_{T(+)}$ $(\equiv T_{11})$

The processes with vector particle such as rho-meson probes deeper into the fine features of QCD.
It deserves theoretical development to describe
HERA data in its special kinematical range:

- large $s_{\gamma^* P}$ $\Rightarrow$ small-x effects expected, within k_t-factorization
- large Q^2 $\Rightarrow$ hard scale $\Rightarrow$ perturbative approach and collinear factorization
  $\Rightarrow$ the ρ can be described through its chiral even Distribution Amplitudes

$$\begin{cases} \rho_L & \text{twist 2} \\ \rho_T & \text{twist 3} \end{cases}$$

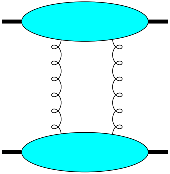
The main ingredient is the $\gamma^* \to \rho$ impact factor SIMPLEST OBJECT !!

- For ρ_T, special care is needed: a pure 2-body description would violate gauge invariance.
- We show that:
  - Including in a consistent way all twist 3 contributions, i.e. 2-body and 3-body correlators, gives a gauge invariant impact factor
  - Our treatment is free of end-point singularities and does not violates the QCD factorization

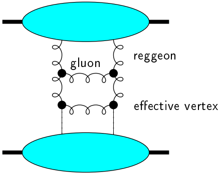
QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in t channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity.

Born order:

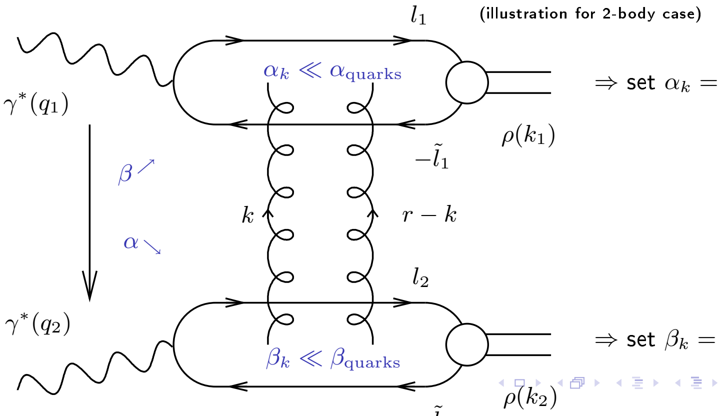


BFKL ladder:



$\gamma^* \, \gamma^* \to \rho \, \rho$ as an example

- Use Sudakov decomposition $k = \alpha p_1 + \beta p_2 + k_\perp$ $(p_1^2 = p_2^2 = 0, \; 2p_1 \cdot p_2 = s)$
- write $d^4k = \frac{s}{2} \, d\alpha \, d\beta \, d^2k_\perp$
- $t-$channel gluons with non-sense polarizations $(\epsilon_{NS}^{up} = \frac{2}{s} \, p_2, \; \epsilon_{NS}^{down} = \frac{2}{s} \, p_1)$ dominates at large s



$\Rightarrow$ set $\alpha_k = 0$ and $\int d\beta$

$\Rightarrow$ set $\beta_k = 0$ and $\int d\alpha$

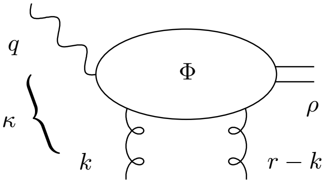
impact representation $\underline{k}$ = Eucl. $\leftrightarrow$ $k_\perp$ = Mink.

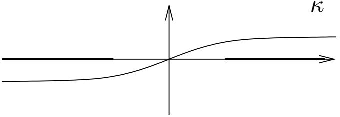
$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2 \underline{k}^2\,(\underline{r}-\underline{k})^2} \Phi^{\gamma^*(q_1)\to\rho(p_1^\rho)}(\underline{k},\underline{r}-\underline{k})\;\Phi^{\gamma^*(q_2)\to\rho(p_2^\rho)}(-\underline{k},-\underline{r}+\underline{k})$$

The $\gamma^*_{L,T}(q)g(k_1) \to \rho_{L,T}\,g(k_2)$ impact factor is normalized as

$$\Phi^{\gamma^*\to\rho}(\underline{k}^2) = e^{\gamma^*\mu}\,\frac{1}{2\,s}\int \frac{d\kappa}{2\pi}\,\mathrm{Disc}_\kappa\,\mathcal{S}_\mu^{\gamma^*\,g\to\rho\,g}(\underline{k}^2),$$

with $\kappa = (q+k)^2 = \beta\,s - Q^2 - \underline{k}^2$

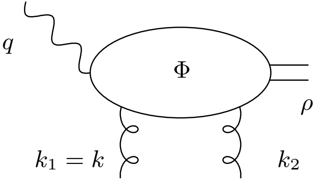




Gauge invariance

- QCD gauge invariance (probes are colorless)
  $\Rightarrow$ impact factor should vanish when $\underline{k} \to 0$ or $\underline{r} - \underline{k} \to 0$

- In the following we will restrict ourselve to the case $t = t_{min}$, i.e. to $\underline{r} = 0$



$$k_1 = \frac{\kappa + Q^2 + \underline{k}^2}{s} p_2 + k_\perp$$

$$k_2 = \frac{\kappa + \underline{k}^2}{s} p_2 + k_\perp ,$$

$$k_1^2 = k_2^2 = -\underline{k}^2$$

This kinematics takes into account skewedness effects along p_2

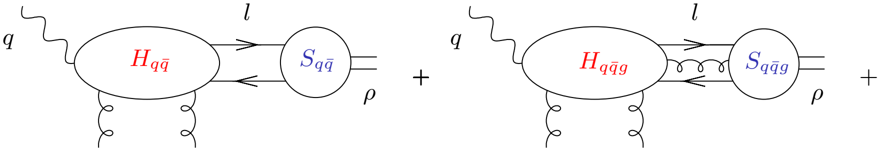
$\Rightarrow$ restriction to the transitions
$$\left\{ \begin{array}{cccl} 0 & \to & 0 & \text{(twist 2)} \\ (+ \text{ or } -) & \to & (+ \text{ or } -) & \text{(twist 3)} \end{array} \right.$$

- At twist 3 level (for $\gamma_T^* \to \rho_T$ transition), gauge invariance is a non trivial statement which requires 2 *and* 3 body correlators

- The impact factor can be written as

$$\Phi = \int d^4l \cdots \mathrm{tr}[H(l\cdots) \quad S(l\cdots)]$$

hard part soft part



- At the 2-body level:

$$S_{q\bar{q}}(l) = \int d^4z\, e^{-il\cdot z} \langle \rho(p)|\psi(0)\,\bar{\psi}(z)|0\rangle,$$

- H and S are related by $\int d^4l$ and by the summation over spinor indices

1 - Momentum factorization (1)

- Use Sudakov decomposition in the form $(p = p_1,\ n = 2\,p_2/s)$

$$\begin{array}{cccccccc} l_\mu & = & x\,p_\mu & + & l^\perp_\mu & + & (l \cdot p)\,n_\mu, & x = l \cdot n \\ \text{scaling:} & & 1 & & 1/Q & & 1/Q^2 & \end{array}$$

- decompose $H(k)$ around the p direction:

$$H(l) = H(xp) \quad + \quad \left.\frac{\partial H(l)}{\partial l_\alpha}\right|_{l=xp} (l - x\,p)_\alpha + \dots \quad \text{with} \quad (l - x\,p)_\alpha \approx l^\perp_\alpha$$

twist 2 — kinematical twist 3 *and* genuine twist 3

- In Fourier space, the twist 3 term $l^\perp_\alpha$ turns into a derivative of the soft term
$\Rightarrow$ one will deal with $\int d^4z\; e^{-il\cdot z} \langle \rho(p)|\psi(0)\, i\, \overleftrightarrow{\partial}_{\alpha\perp} \bar{\psi}(z)|0\rangle$

1 - Momentum factorization (2)

- write

$$d^4 l \longrightarrow d^4 l \; \delta(x - l \cdot n) \; dx$$

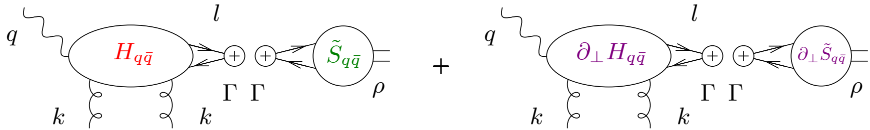
- $\int d^4 l \, \delta(x - l \cdot n)$ is then absorbed in the soft term:

$$\begin{aligned}
(\tilde{S}_{q\bar{q}}, \partial_\perp \tilde{S}_{q\bar{q}}) &\equiv \int d^4 l \, \delta(x - l \cdot n) \int d^4 z \, e^{-il\cdot z} \langle \rho(p) | \psi(0) \, (1, \, i \, \overleftrightarrow{\partial_\perp}) \bar{\psi}(z) | 0 \rangle \\
&= \int \frac{d\lambda}{2\pi} \, e^{-i\lambda x} \int d^4 z \, \delta^{(4)}(z - \lambda n) \, \langle \rho(p) | \psi(0) \, (1, \, i \, \overleftrightarrow{\partial_\perp}) \bar{\psi}(z) | 0 \rangle \\
&= \int \frac{d\lambda}{2\pi} \, e^{-i\lambda x} \langle \rho(p) | \psi(0) \, (1, \, i \, \overleftrightarrow{\partial_\perp}) \bar{\psi}(\lambda n) | 0 \rangle
\end{aligned}$$

- $\int dx$ performs the longitudinal momentum factorization

2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices $\psi(0)\,\bar{\psi}(z)$ and $\psi(0)\, i\, \overleftrightarrow{\partial_\perp}\, \bar{\psi}(z)$:



- Φ has now the simple factorized form:

$$\Phi = \int dx \left\{ \mathrm{tr}\left[H_{q\bar{q}}(x\,p)\,\Gamma\right]\, S^{\Gamma}_{q\bar{q}}(x) + \mathrm{tr}\left[\partial_\perp H_{q\bar{q}}(x\,p)\,\Gamma\right]\, \partial_\perp S^{\Gamma}_{q\bar{q}}(x) \right\}$$

$\Gamma = \gamma^\mu$ and $\gamma^\mu\,\gamma^5$ matrices

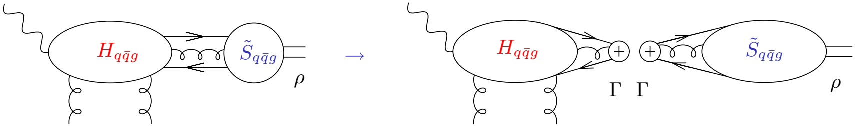
$$S^{\Gamma}_{q\bar{q}}(x) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda x} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\,\psi(0)|0\rangle$$

$$\partial_\perp S^{\Gamma}_{q\bar{q}}(x) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda x} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\, i\, \overleftrightarrow{\partial_\perp}\, \psi(0)|0\rangle$$

- choose axial gauge condition for gluons, *i.e.* $n \cdot A = 0 \Rightarrow$ no Wilson line

Factorization of 3-body contributions

- 3-body contributions start at genuine twist 3
  $\Rightarrow$ no need for Taylor expansion
- Momentum factorization goes in the same way as for 2-body case
- Spinorial (and color) factorization is similar:



2-body non-local correlators

ρ_L	twist 2
ρ_T	kinematical twist 3 (WW) genuine twist 3 genuine + kinematical twist 3

- vector correlator

$$\langle\rho(p)|\bar{\psi}(z)\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \left[\varphi_1(x)\,(e^*\cdot n)p_\mu + \varphi_3(x)\, e^{*T}_\mu\right]$$

- axial correlator

$$\langle\rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\,\varphi_A(x)\,\varepsilon_{\mu\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta$$

- vector correlator with transverse derivative

$$\langle\rho(p)|\bar{\psi}(z)\gamma_\mu\, i\, \overleftrightarrow{\partial^\perp_\alpha}\,\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \varphi^T_1(x)\, p_\mu e^{*T}_\alpha$$

- axial correlator with transverse derivative

$$\langle\rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\, i\, \overleftrightarrow{\partial^\perp_\alpha}\,\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\,\varphi^T_A(x)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta,$$

where x ($\bar{x} = 1 - x$) = momentum fraction along $p \equiv p_1$ of the quark (antiquark) and $\stackrel{\mathcal{F}}{=} \int_0^1 dx \exp\left[ix\, p\cdot z\right]$, with $z = \lambda n$

3-body non-local correlators   **genuine twist 3**

- vector correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^V\, B(x_1,x_2)\, p_\mu\, e_\alpha^{*T},$$

- axial correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_5\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^A\, i\, D(x_1,x_2)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e_\lambda^{*T}\, p_\beta\, n_\delta,$$

where x_1, $\bar{x}_2$, $x_2 - x_1$ = quark, antiquark, gluon momentum fraction

and $\stackrel{\mathcal{F}_2}{=} \int_0^1 dx_1 \int_0^1 dx_2 \; \exp\left[i\, x_1\, p\cdot z_1 + i(x_2 - x_1)\, p\cdot z_2\right],$ with $z_{1,2} = \lambda n$

From C-conjugation on the previous correlators, one gets:

- 2-body correlators:

$$
\begin{aligned}
\varphi_1(y) &= \varphi_1(1-y) \\
\varphi_3(y) &= \varphi_3(1-y) \\
\varphi_A(y) &= -\varphi_A(1-y) \\
\varphi_1^T(y) &= -\varphi_1^T(1-y) \\
\varphi_A^T(y) &= \varphi_A^T(1-y)
\end{aligned}
$$

- 3-body correlators:

$$
\begin{aligned}
B(x_1, x_2) &= -B(1-x_2, 1-x_1) \\
D(x_1, x_2) &= D(1-x_2, 1-x_1)
\end{aligned}
$$

Equations of motion

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- Dirac equation leads to

$$\langle i(\overrightarrow{\not{D}}(0)\psi(0))_\alpha \bar{\psi}_\beta(z)\rangle = 0 \qquad (i\overrightarrow{D}_\mu = i\overrightarrow{\partial}_\mu + A_\mu)$$

- Apply the Fierz decomposition to the above 2 and 3-body correlators

$$-\langle\psi(x)\,\bar{\psi}(z)\rangle = \frac{1}{4}\langle\bar{\psi}(z)\gamma_\mu\psi(x)\rangle\gamma_\mu + \frac{1}{4}\langle\bar{\psi}(z)\gamma_5\gamma_\mu\psi(x)\rangle\gamma_\mu\gamma_5.$$

- $\Rightarrow$ Equation of motion: $\quad \zeta^{V,A}_{3,\,\rho} = f^{V,A}_{3\,\rho}/f_\rho$

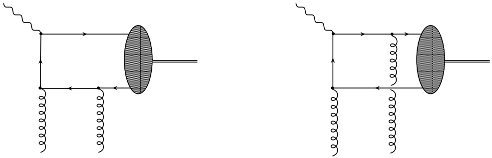
$$\bar{y}_1\,\varphi_3(y_1) + \bar{y}_1\,\varphi_A(y_1) + \varphi_1^T(y_1) + \varphi_A^T(y_1) = \int dy_2\left[\zeta_3^V\,B(y_1,\,y_2) + \zeta_3^A\,D(y_1,\,y_2)\right]$$

$$y_1\,\varphi_3(y_1) - y_1\,\varphi_A(y_1) - \varphi_1^T(y_1) + \varphi_A^T(y_1) = \int dy_2\left[-\zeta_3^V\,B(y_2,\,y_1) + \zeta_3^A\,D(y_2,\,y_1)\right]$$

- In WW approximation: genuine twist 3 = 0

$$\begin{cases} \varphi_A^T(x) = \frac{1}{2}[(x-\bar{x})\,\varphi_A^{WW}(x) - \varphi_3^{WW}(x)] \\ \varphi_1^T(x) = \frac{1}{2}[(x-\bar{x})\,\varphi_3^{WW}(x) - \varphi_A^{WW}(x)] \end{cases}$$

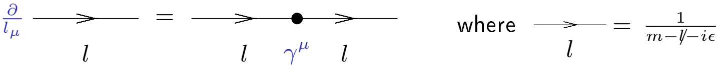
- without derivative

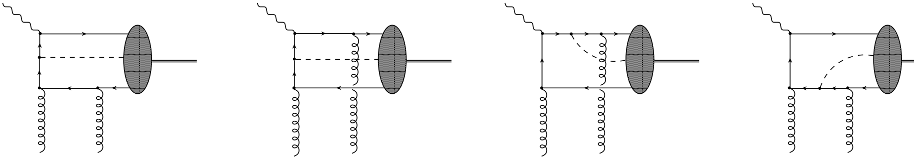


twist 2 $(\gamma_L^* \to \rho_L)$

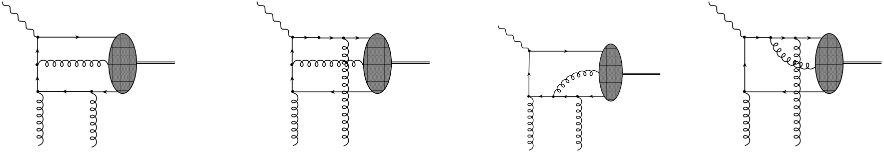
twist 3 $(\gamma_T^* \to \rho_T)$

- practical trick for computing $\partial_\perp H$: use the Ward identity

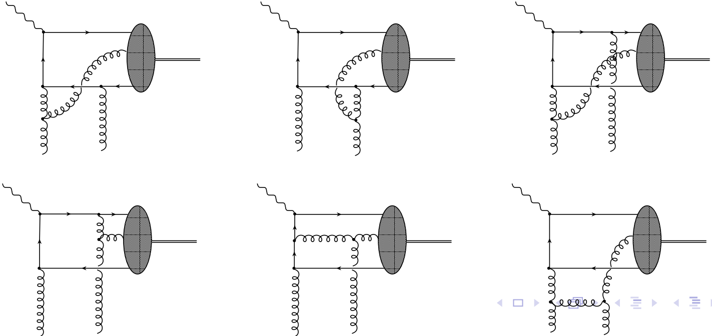




- "abelian" type



- "non-abelian" type



$n-$independence

$$e_\mu^{*T} = e_\mu^* - p_\mu \, e^* \cdot n \quad \text{keeping} \quad n \cdot p = 1$$

$$\frac{d\mathcal{A}}{dn^\mu} = 0 \,, \qquad \text{where} \quad \frac{d}{dn^\mu} = \frac{\partial}{\partial n^\mu} + e_\mu^* \frac{\partial}{\partial (e^* \cdot n)}$$

$$\mathrm{tr}\,[\mathrm{H}_{3\rho}(\mathrm{y}_1, \mathrm{y}_2)\, \mathrm{p}^\rho \, \not{p}]\; \mathrm{B}(\mathrm{y}_1,\, \mathrm{y}_2) \;=$$
$$\frac{1}{y_1 - y_2}\, (\mathrm{tr}\,[\mathrm{H}_2(\mathrm{y}_1)\, \not{p}] - \mathrm{tr}\,[\mathrm{H}_2(\mathrm{y}_2)\, \not{p}]) B(y_1,\, y_2)\,,$$

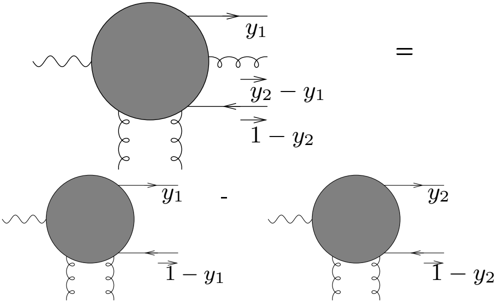


Figure: Reduction of 3-body correlators to 2-body correlators through Ward identity

- vector correlators

$$\frac{d}{dy_1}\varphi_1^T(y_1) = -\varphi_1(y_1) + \varphi_3(y_1)$$
$$-\zeta_3^V \int_0^1 \frac{dy_2}{y_2 - y_1} \times \left(\theta(y_2 - y_1)B(y_1, y_2) + \theta(y_1 - y_2)B(y_2, y_1)\right)$$

- axial correlators

$$\frac{d}{dy_1}\varphi_A^T(y_1) = \varphi_A(y_1) - \zeta_3^A \int_0^1 \frac{dy_2}{y_2 - y_1} \left(\theta(y_2 - y_1)D(y_1, y_2) + \theta(y_1 - y_2)D(y_2, y_1)\right)$$

Solutions: WW approximation

- $B = 0 = D$

$$\varphi_{3/A}^{WW}(y) = \frac{1}{2}\left[\int_0^y \frac{dv}{\bar{v}}\varphi_1(v) \pm \int_y^1 \frac{dv}{v}\varphi_1(v)\right]$$

$$\varphi_{1/A}^{T\,WW}(y) = \frac{1}{2}\left[-\bar{y}\int_0^y \frac{dv}{\bar{v}}\varphi_1(v) \pm y\int_y^1 \frac{dv}{v}\varphi_1(v)\right]$$

Solutions: genuine twist-3

- $\varphi_3(y) = \varphi_3^{WW}(y) + \varphi_3^{gen}(y)$

$$\varphi_3^{gen}(y) = -\frac{1}{2}\int_y^1 \frac{du}{u}\left[\int_0^u dy_2 \frac{d}{du}(\zeta_3^V B - \zeta_3^A D)(y_2, u) - \int_u^1 \frac{dy_2}{y_2 - u}(\zeta_3^V B - \zeta_3^A D)(u, y_2)\right.$$
$$\left. - \int_0^u \frac{dy_2}{y_2 - u}(\zeta_3^V B - \zeta_3^A D)(y_2, u)\right]$$
$$-\frac{1}{2}\int_0^{y_1} \frac{du}{\bar{u}}\left[\int_u^1 dy_2 \frac{d}{du}(\zeta_3^V B + \zeta_3^A D)(u, y_2) - \int_u^1 \frac{dy_2}{y_2 - u}(\zeta_3^V B + \zeta_3^A D)(u, y_2)\right.$$
$$\left. - \int_0^u \frac{dy_2}{y_2 - u}(\zeta_3^V B + \zeta_3^A D)(y_2, u)\right]$$

- $\varphi_1^T(y) = \varphi_1^{T\,WW}(y) + \varphi_1^{T\,gen}(y)$

$$\varphi_1^{T\,gen}(y) = \int_0^y du\, \varphi_3^{gen}(u) - \zeta_3^V \int_0^y dy_1 \int_y^1 dy_2 \frac{B(y_1, y_2)}{y_2 - y_1}$$

- the corresponding expressions for $\varphi_A^{gen}(y)$ and $\varphi_A^{T\,gen}(y)$:

$$\varphi_A(y) = \varphi_A^{WW}(y) + \varphi_A^{gen}(y)$$
$$\varphi_A^T(y) = \varphi_A^{T\,WW}(y) + \varphi_A^{T\,gen}(y)$$

are obtained by the substitutions:

$$\varphi_A^{gen}(y) \quad \Longleftrightarrow_{\zeta_3^V B \leftrightarrow \zeta_3^A D} \quad \varphi_3^{gen}(y)$$
$$\varphi_A^{T\,gen}(y) \quad \Longleftrightarrow_{\zeta_3^V B \leftrightarrow \zeta_3^A D} \quad \varphi_1^{T\,gen}(y)$$

$\gamma_L^* \to \rho_L$ impact factor

$$\Phi^{\gamma_L^* \to \rho_L}(\underline{k}^2) = -i \frac{4 C_F \, e_q \, f_\rho}{Q} \int dx \, \varphi_1(x) \frac{\underline{k}^2}{x \, \bar{x} \, Q^2 + \underline{k}^2}$$

pure twist 2 scaling

$\gamma_T^* \to \rho_T$ impact factor:

Spin Non-Flip/Flip separation appears

$$\Phi^{\gamma_T^* \to \rho_T}(\underline{k}^2) = \Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)\, T_{n.f.} + \Phi_{f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)\, T_{f.}$$

where

$$T_{n.f.} = -(e_\gamma \cdot e^*) \quad \text{and} \quad T_{f.} = \frac{(e_\gamma \cdot k)(e^* k)}{\underline{k}^2} + \frac{(e_\gamma \cdot e^*)}{2}$$

non-flip transitions $\left\{ \begin{array}{l} + \to + \\ - \to - \end{array} \right.$ flip transitions $\left\{ \begin{array}{l} + \to - \\ - \to + \end{array} \right.$

pure twist 3 scaling

$$\Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)$$

$$= -\frac{m_\rho f_\rho}{2\sqrt{2}\,Q^2}\left\{-2\,C_F\int dx_1 \frac{\left(\underline{k}^2+2\,Q^2\,x_1\,(1-x_1)\right)\underline{k}^2}{x_1\,(1-x_1)\,(\underline{k}^2+Q^2\,x_1\,(1-x_1))^2}\left[(2x_1-1)\,\varphi_1^T(x_1)+\varphi_A^T(x_1)\right]\right.$$

$$+2\,\zeta\int dx_1\,dx_2\,[B\,(x_1,x_2)-D\,(x_1,x_2)]\,\frac{x_1\,(1-x_1)\,\underline{k}^2}{\underline{k}^2+Q^2\,x_1\,(1-x_1)}\left[\frac{(2\,C_F-N_c)Q^2}{\underline{k}^2\,(x_1-x_2+1)+Q^2\,x_1\,(1-x_2)}\right.$$

$$\left.-\frac{N_c\,Q^2}{x_2\underline{k}^2+Q^2\,x_1\,(x_2-x_1)}\right]-2\,\zeta\int dx_1\,dx_2\,[B\,(x_1,x_2)+D\,(x_1,x_2)]\left[\frac{2\,C_F+N_c}{1-x_1}\right.$$

$$+\frac{x_1\,Q^2}{\underline{k}^2+Q^2x_1\,(1-x_1)}\left(\frac{(2\,C_F-N_c)\,x_1\,\underline{k}^2}{\underline{k}^2\,(x_1-x_2+1)+Q^2x_1\,(1-x_2)}-2C_F\right)$$

$$\left.\left.+N_c\frac{(x_1-x_2)\,(1-x_2)}{1-x_1}\,\frac{Q^2}{\underline{k}^2\,(1-x_1)+Q^2\,(x_2-x_1)\,(1-x_2)}\right]\right\}$$

and

$$\Phi_{f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2) = -\frac{m_\rho f_\rho}{2\sqrt{2}\,Q^2}\left\{4\,C_F\int dx_1\,\frac{\underline{k}^2\,Q^2}{(\underline{k}^2+Q^2\,x_1\,(1-x_1))^2}\left[\varphi_A^T(x_1)-(2x_1-1)\,\varphi_1^T(x_1)\right]\right.$$

$$-4\,\zeta\int dx_1\,dx_2\,\frac{x_1\,\underline{k}^2}{\underline{k}^2+Q^2\,x_1\,(1-x_1)}\,[D\,(x_1,x_2)\,(-x_1+x_2-1)+B\,(x_1,x_2)\,(x_1+x_2-1)]$$

$$\left.\times\left[\frac{(2\,C_F-N_c)Q^2}{\underline{k}^2\,(x_1-x_2+1)+Q^2\,x_1\,(1-x_2)}-\frac{N_c\,Q^2}{x_2\,\underline{k}^2+Q^2x_1\,(x_2-x_1)}\right]\right\}$$

WW limit

- In the WW limit, only the twist 2 and kinematical twist 3 terms are kept.
- The only remaining contributions come from the two-body correlators
- non-flip transition

$$\Phi^{\gamma^*_T\to\rho_T}_{n.f.}(\underline{k}^2) = \frac{-\,e\,m_\rho f_\rho}{2\sqrt{2}\,Q^2}\frac{\delta^{ab}}{2\,N_c}\int_0^1 \left\{ \frac{(2\,x-1)\,\varphi_1^T(x) + 2\,x\,(1-x)\,\varphi^{WW}{}_3(x) + \varphi_A^T(x)}{x\,(1-x)} - \frac{2\,\underline{k}^2\left(\underline{k}^2 + 2\,Q^2\,x\,(1-x)\right)\left((2\,x-1)\,\phi_1^T(x) + \phi_A^T(x)\right)}{x\,(1-x)\,(\underline{k}^2 + Q^2\,x\,(1-x))^2} \right\}$$

which simplifies, using equation of motion:

$$\int dx_1 [2\,x\,\bar{x}\,\varphi_3^{WW}(x) + (x-\bar{x})\,\varphi_1^T(x) + \varphi_A^T(x)] = 0$$

$$\Phi^{\gamma^*_T\to\rho_T}_{n.f.}(\underline{k}^2) = \frac{e\,m_\rho f_\rho}{\sqrt{2}\,Q^2}\frac{\delta^{ab}}{2\,N_c}\int_0^1 \frac{2\,\underline{k}^2\left(\underline{k}^2 + 2\,Q^2\,x\,(1-x)\right)}{x\,(1-x)\,(\underline{k}^2 + Q^2\,x\,(1-x))^2}\left[(2\,x-1)\,\varphi_1^T(x) + \varphi_A^T(x)\right].$$

- flip transition:

$$\Phi^{\gamma^*_T\to\rho_T}_{n.f.}(\underline{k}^2) = -\frac{e\,m_\rho f_\rho}{\sqrt{2}\,Q^2}\frac{\delta^{ab}}{2\,N_c}\int_0^1 \frac{2\,\underline{k}^2\,Q^2}{(\underline{k}^2 + Q^2\,(1-x)\,x)^2}\left[(1-2\,x_1)\,\varphi_1^T(x) + \varphi_A^T(x)\right].$$

- The obtained results are gauge invariant:

$$\Phi^{\gamma_T^* \to \rho_T} \to 0 \quad \text{when} \quad \underline{k} \to 0$$

  - this is straightforward in the WW limit
  - at the full twist 3 order:
    - the C_F part of the abelian 3-body contribution cancels the 2-body contribution after using the equation of motion
    - the N_c part of the abelian 3-body contribution cancels the 3-body non-abelian contribution
    - thus $\gamma_T^* \to \rho_T$ impact factor is gauge-invariant only provided the 3-body contributions have been taken into account

- Our results are free of end-point singularities, in both WW approximation and full twist-3 order calculation:
  - the flip contribution obviously does not have any end-point singularity because of the $\underline{k}^2$ which regulates them
  - the potential end-point singularity for the non-flip contribution is spurious since $\varphi_A^T(x_1)$, $\varphi_1^T(x_1)$ vanishes at $x_1 = 0, 1$ as well as $B(x_1, x_2)$ and $D(x_1, x_2)$.

Conclusions

- We have performed a full up to twist 3 computation of the $\gamma^* \to \rho$ impact factor, in the $t = t_{min}$ limit.
- Our result respects gauge invariance. This is achieved only after including 2 and 3 body correlators.
- It is free of end-point singularities
  (this should be contrasted with standard collinear treatment, at moderate s, where the k_T-factorization is NOT applicable: see Mankiewicz-Piller).
- In this talk we relied on the Light-Cone Collinear approach (Ellis + Furmanski + Petronzio; Efremov + Teryaev; Anikin + Teryaev), which is non-covariant, but very efficient for practical computations.

Conclusions cntd

- Comparison with a fully covariant approach by Ball+Braun et al
  The dictionnary between the two approaches within a full twist 3 treatment is established

$$B(y_1,\,y_2) = \frac{V(y_1,\,1-y_2,\,y_2-y_1)}{y_2-y_1}\,, \qquad D(y_1,\,y_2) = \frac{A(y_1,\,1-y_2,\,y_2-y_1)}{y_2-y_1}$$

$$\begin{aligned} \varphi_1(y) &= f_\rho m_\rho \phi_{\|}(y), \quad \varphi_3(y) = f_\rho m_\rho g^{(v)}(y)\,, \\ \varphi_A(y) &= -\frac{1}{4} f_\rho m_\rho \frac{\partial g^{(a)}(y)}{\partial y} \end{aligned}$$

- We also performed calculations of the same impact factor within the covariant approach by Ball+Braun et al: calculations proceed in quite different way : eg. no ϕ^T-DAs but Wilson line effects are important !!
- Phenomenological applications will be done in the near future.