Exact kinematics in the gluon cascade

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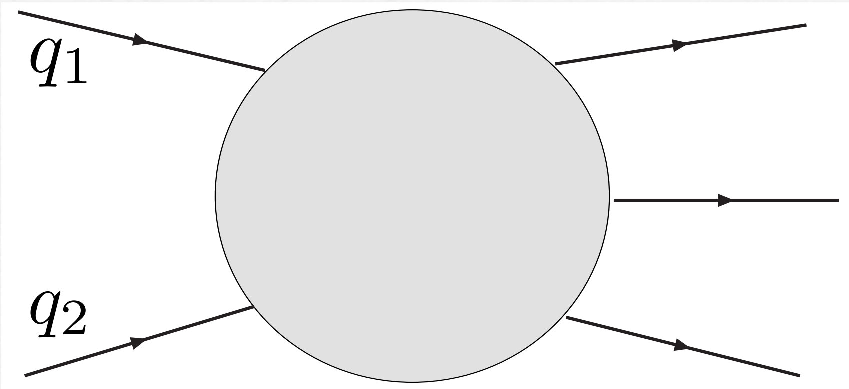
Epiphany Conference, Kraków, January 6th, 2009

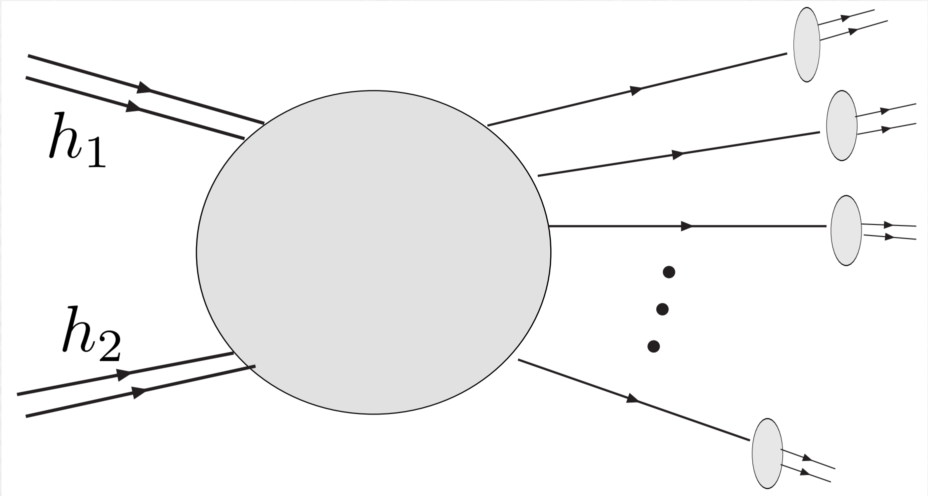
Outline

- Motivation and introduction. High energy dipole evolution.
- Modified kernel for dipole evolution.
- Light cone wave functions with exact kinematics.
- Gluon fragmentation amplitudes.
- Relation with the maximally helicity violating (MHV) amplitudes.

Work done in collaboration with Leszek Motyka

Motivation

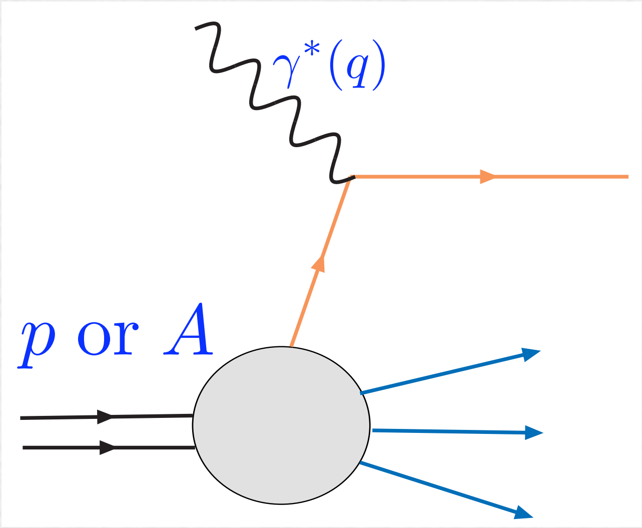




Exact kinematics straightforward when considering small number of particles

In hadronic collisions at high energy: initial state is a hadron not a parton. Many partons can be produced which further hadronize. Efficient description in terms of parton distributions and fragmentation functions.

DIS process



Parton distributions

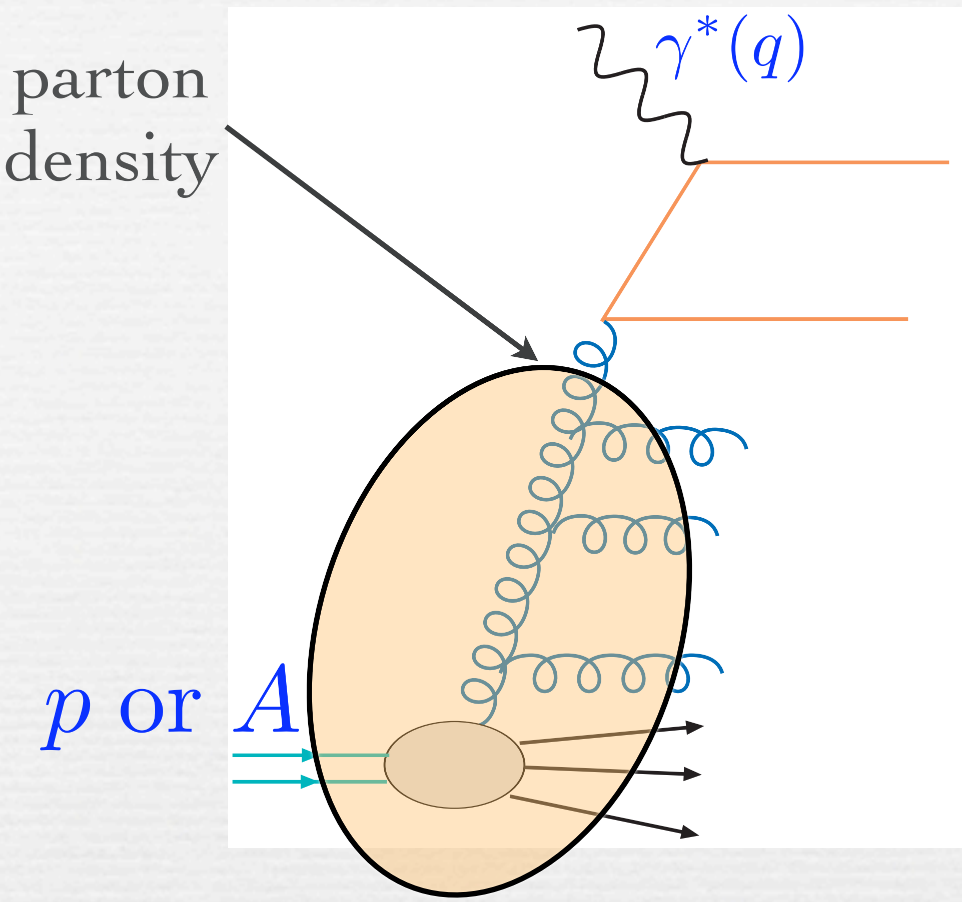
fraction of longitudinal momentum

$f(x, Q)$ or $f(x, k_T)$

Hard scale

Transverse momentum

QCD description of the parton density



Evolution of parton density

$\updownarrow$

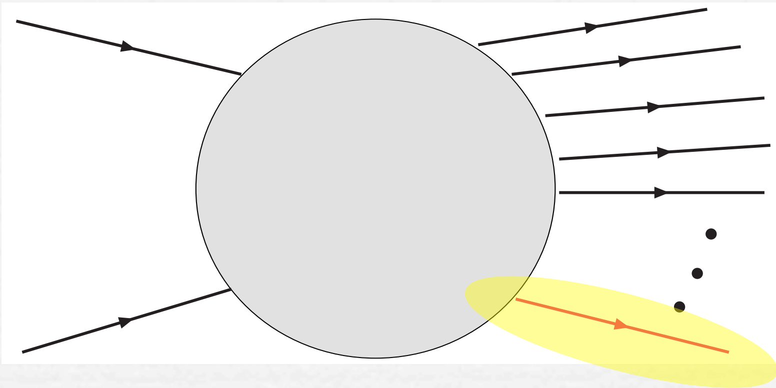
Multiple parton emissions
+virtual processes

Evolution of parton density from RG-type closed equation

$$\frac{\partial}{\partial \log \mu} f = K \otimes f \qquad \mu = Q, s$$

Branching kernel has perturbative expansion

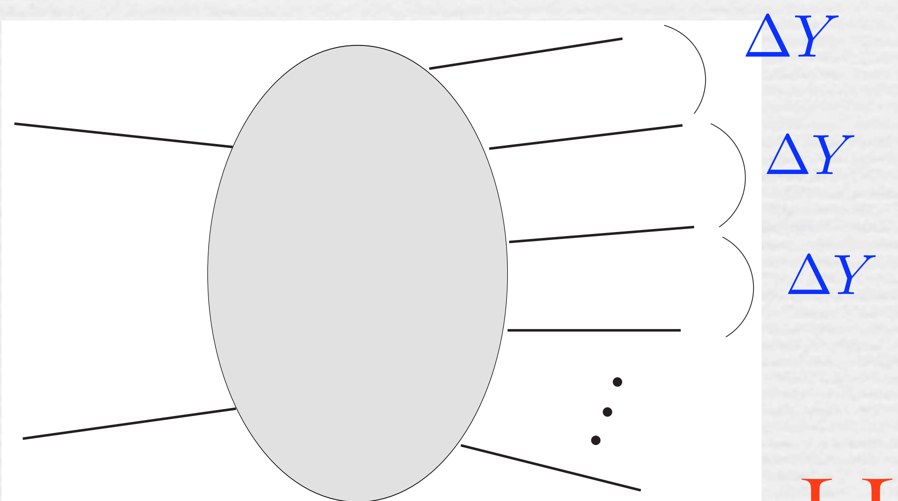
$$K = K^{(0)} \alpha_s + K^{(1)} \alpha_s^2 + K^{(2)} \alpha_s^3 + \ldots$$



In this framework: kinematic approximations on the emissions of the partons

Branching kernel in higher orders does contain not only higher loop diagrams, but also topologically equivalent diagrams but with external partons in a different kinematic region.

Example: high energy limit $s \to \infty$



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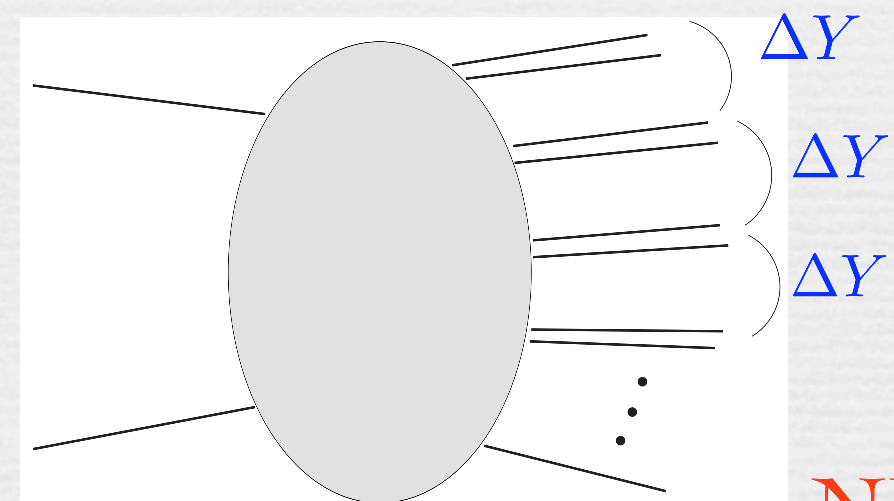


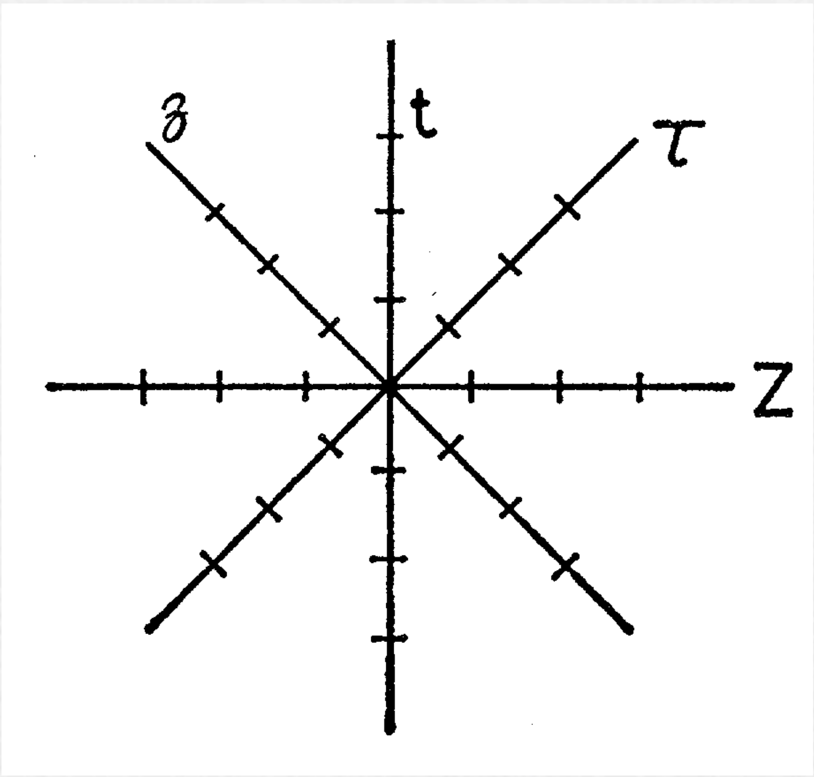
NLL

Clusters of particles in rapidity.

When $s \to \infty$, α_s is **not** a small parameter, hence expansion is slowly convergent. Perturbative methods not very efficient in correcting the kinematic approximations done on phase space.

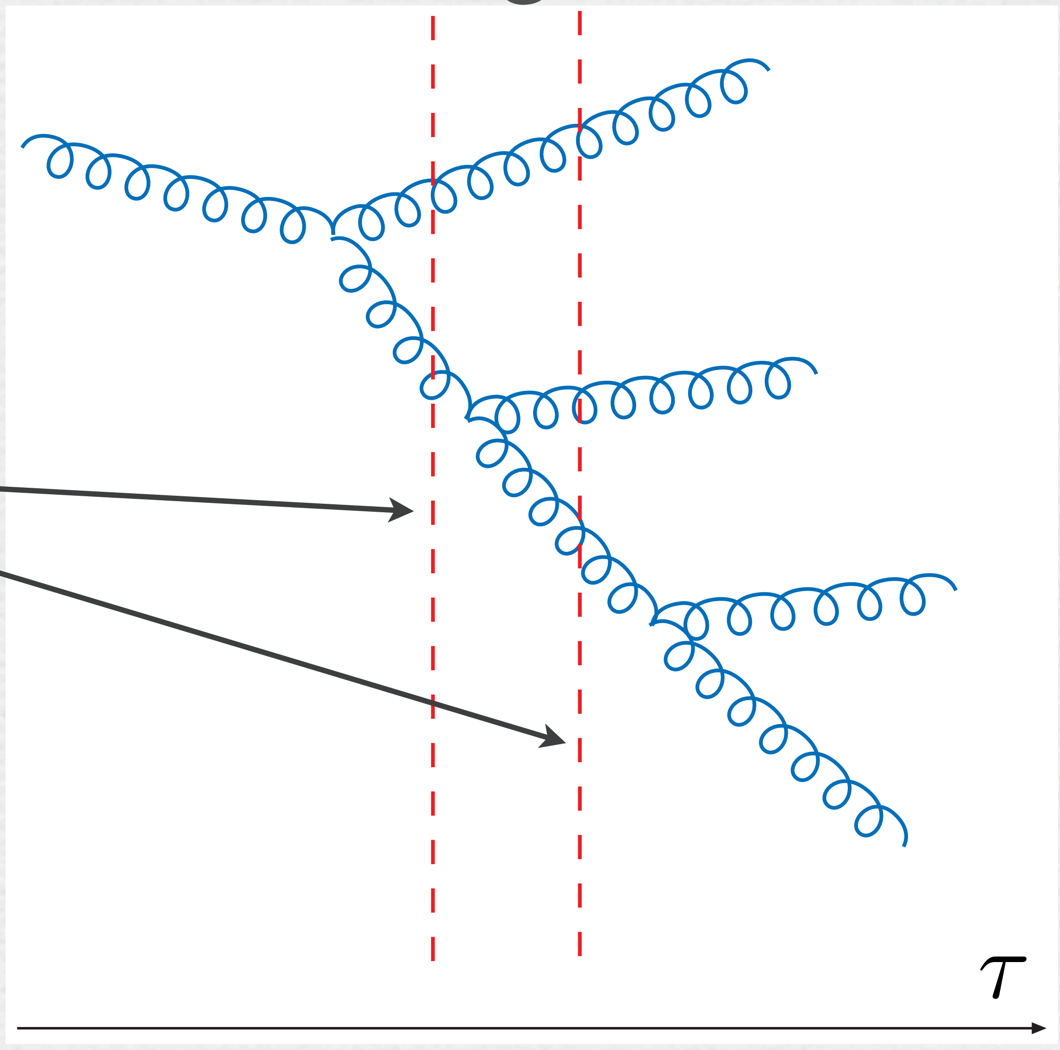
Multiple gluon emissions in the light-cone formalism

Light-cone formalism



Infinite momentum frame: a limit of a Lorentz frame moving in the -z direction with a (nearly) the speed of light.

Time ordered diagram



Energy denominators

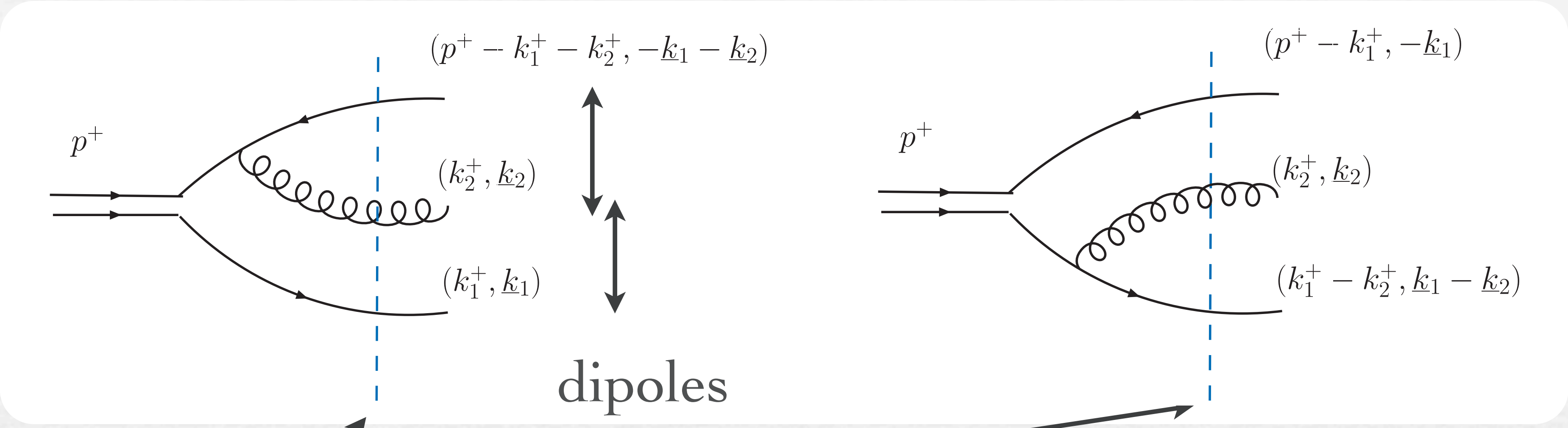
Difference of light - cone energies:

$$D_n = P^- - \sum_i k_i^-$$

Dipole evolution at high energy

A.Mueller

Quark-antiquark pair emitting longitudinally soft gluon. Dipole evolution.



Energy denominators in light cone perturbation theory

High energy limit: strong ordering in longitudinal momenta

$$\bar{D}_1 = \frac{1}{P^- - [(P-k_1-k_2)^- + k_1^- + k_2^-]} \simeq \frac{1}{k_2^-}$$

$$k_2^+ \ll k_1^+, P^+ \qquad k_2^- \equiv \frac{\underline{k}_2^2}{2k_2^+}$$

$$\Psi^{(1)}(\underline{k}_1, \underline{k}_2, z_1, z_2) = 2gt_a \frac{\epsilon_2 \cdot \underline{k}_2}{\underline{k}_2^2} [\Psi^{(0)}(\underline{k}_1, z_1) - \Psi^{(0)}(\underline{k}_1 + \underline{k}_2, z_1)]$$

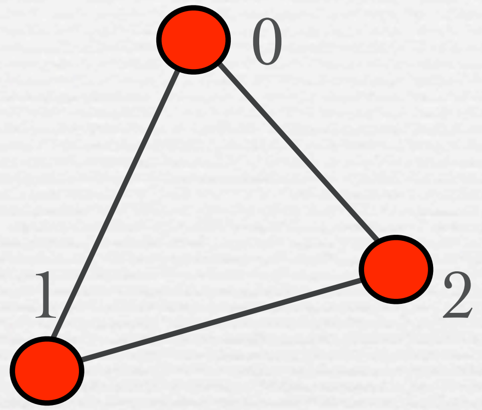
Wave function with 1 gluon

Wave function without gluons

In transverse coordinate space

$$\Phi^{(1)}(\underline{x}_{01},\underline{x}_{02};z_1,z_2) = -\frac{igt_a}{\pi}\left(\frac{\underline{x}_{20}}{x_{20}^2}-\frac{\underline{x}_{21}}{x_{21}^2}\right)\cdot\underline{\epsilon}_2\,\Psi^{(0)}(\underline{x}_{01};z_1)$$

Soft gluons factorize in the transverse spae



Dipole kernel in the limit of high energy:

$$\frac{d^2\underline{x}_{02}\,\underline{x}_{01}^2}{\underline{x}_{02}^2\,\underline{x}_{12}^2}$$

Dipole evolution in rapidity:

$$\frac{\partial N_{01}}{\partial Y} = \bar{\alpha}_s \int \frac{d^2x_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2x_{12}^2}\left[N_{02}+N_{12}-N_{01}\right]$$

Y rapidity

N_{01} dipole scattering amplitude (related to the gluon density)

No restrictions on the transverse coordinates (or momenta).

In the high energy limit:

$$\bar{D}_1 = \frac{1}{P^- - [(P - k_1 - k_2)^- + k_1^- + k_2^-]} \simeq \frac{1}{k_2^-}$$

$$k_2^+ \ll k_1^+, P^+ \qquad k_2^- \equiv \frac{\underline{k}_2^2}{2k_2^+}$$

For the consistency of the calculation we should take:

$$\frac{\underline{k}_2^2}{k_2^+} > \frac{\underline{k}_1^2}{k_1^+}$$

For more emissions

$$\ldots \frac{\underline{k}_4^2}{k_4^+} > \frac{\underline{k}_3^2}{k_3^+} > \frac{\underline{k}_2^2}{k_2^+} > \frac{\underline{k}_1^2}{k_1^+}$$

Ordering in the fluctuation time: Dokshitzer, Marchesini, Salam

$$\tau \sim \frac{k^+}{\underline{k}^2}$$

Longitudinal and transverse momenta tied together.

Modified dipole kernel

Quasi-local approximation

$$D_1 \simeq \frac{1}{k_1^- + k_2^-}$$

Keep the energy of the parent emitter

$$\Psi^{(1)}(\underline{k}_1, \underline{k}_2, z_1, z_2) = 2gt_a \frac{\epsilon_2 \cdot \underline{k}_2}{\underline{k}_2^2 + \underline{k}_1^2 \frac{k_2^+}{k_1^+}} [\Psi^{(0)}(\underline{k}_1, z_1) - \Psi^{(0)}(\underline{k}_1 + \underline{k}_2, z_1)]$$

Approximate Fourier transform

$$\Phi^{(1)}(\underline{x}_{02}, \underline{x}_{12}; z) \sim gt^a \left(\bar{Q}_{01} K_1(\bar{Q}_{01} x_{02}) \frac{\epsilon_2 \cdot \underline{x}_{02}}{x_{02}} - \bar{Q}_{01} K_1(\bar{Q}_{01} x_{12}) \frac{\epsilon_2 \cdot \underline{x}_{12}}{x_{12}} \right) \Phi^{(0)}(\underline{x}_{01}; z)$$

Modified Bessel functions of the second kind.

$$\overline{Q}_{01} \simeq \frac{1}{x_{01}} \sqrt{\frac{k_2^+}{k_1^+}} = \frac{1}{x_{01}} \sqrt{z}$$

z longitudinal momentum fraction

$\ln 1/z \sim y$ rapidity

Modified dipole kernel

$$d^2\underline{x}_2\left(\overline{Q}_{01}K_1(\overline{Q}_{01}x_{02})\frac{\epsilon_2\cdot\underline{x}_{02}}{x_{02}}-\overline{Q}_{01}K_1(\overline{Q}_{01}x_{12})\frac{\epsilon_2\cdot\underline{x}_{12}}{x_{12}}\right)^2 \xrightarrow[\frac{x_{02}}{x_{01}}\sqrt{z}\to 0]{} \frac{d^2\underline{x}_{02}\,\underline{x}_{01}^2}{\underline{x}_{02}^2\,\underline{x}_{12}^2}$$

$$\overline{Q}_{01}\simeq\frac{1}{x_{01}}\sqrt{\frac{k_2^+}{k_1^+}}=\frac{1}{x_{01}}\sqrt{z}$$

Dipole kernel with Bessel-Macdonald functions:

- Energy dependent cutoff in impact parameter: exponential tails, range depends on the energy.
- Violation of conformal invariance in 2-dimensions.
- Recovering original dipole kernel in the high energy limit.

Impact parameter and NLL correction

Cutoff on configuration of large dipoles

$$x_{02} \gg x_{01} \ \& \ x_{12} \sim x_{02}$$

exponential behavior

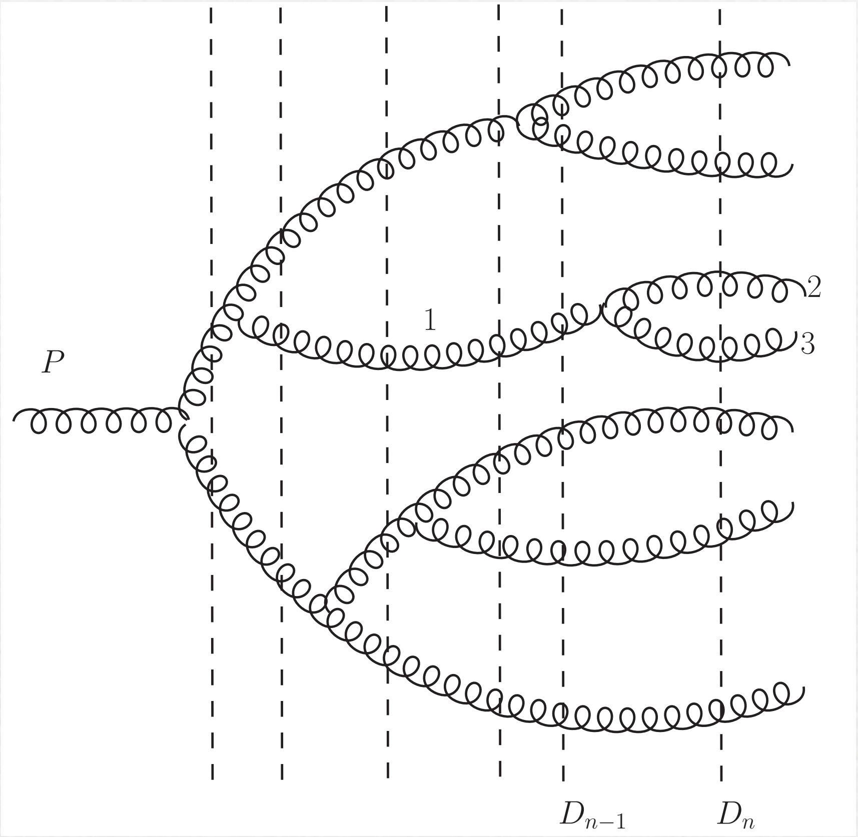
$$\mathcal{K} \sim \exp\left(-\frac{x_{02}}{x_{01}}\sqrt{z}\right)$$

Recovering part of NLL contribution from explicit calculation by Balitsky and Chirilli (non-conformal part).

$$\mathcal{K}^{\rm NLO}_{\rm non-conf.} \otimes N_Y \quad \to \quad -\frac{\bar{\alpha}_s^2}{\pi}\int \frac{d^2\underline{x}_2\, x_{01}^2}{x_{02}^4} \log^2\left(\frac{x_{02}}{x_{01}}\right)[\ldots]$$

Light cone wave functions

∗ Previously: modified kernel, only some corrections in the energy denominators. Still eikonal vertices.
∗ Keep kinematics exact through the complete evolution: both vertices and energy denominators kept exact.



Gluon in the initial state. Dynamics similar to the dipole model.

Helicity conserved through the whole cascade.

Recurrence relations between wave functions

$$\Psi_{n+1}(k_0, k_1, \ldots, k_n) = \frac{g}{\sqrt{\xi_{01}}} \frac{\underline{\epsilon}^{(-)} \underline{v}_{01}}{D_n + \xi_{01}\, \underline{v}_{01}^2} \Psi_n(k_{01}, k_2, \ldots, k_n)$$

$$\xi_{01} = \frac{z_0 z_1}{z_0 + z_1} \qquad \underline{v}_{01} = \frac{\underline{k}_0}{z_0} - \frac{\underline{k}_1}{z_1}$$

Light cone wave function

Case of the on-shell incoming gluon.

Can resum the wave function completely.

$$- D_{n+1}\,\Psi_{n+1}(1,2,\ldots,n+1) \;=\; g\sum_{i=1}^{n} \frac{v^*_{(i,i+1)}}{\sqrt{\xi_{(i,i+1)}}}\Psi_n(1,2,\ldots,(i\,i+1),\ldots,n+1) \qquad n \to n+1$$

$$-D_n\Psi_n(1,2,\ldots,n) \;=\; g\sum_{k=1}^{n-1} \frac{v^*_{(k,k+1)}}{\sqrt{\xi_{(k,k+1)}}}\Psi_{n-1}(1,2,\ldots,(k\,k+1),\ldots,n) \qquad n-1 \to n$$

...

Tree-level gluon wave function with exact kinematics

$$\Psi_n(1,2,\ldots,n) \;=\; (-1)^{n-1}g^{n-1}\,\Delta^{(n)}\,\frac{1}{\sqrt{z_1 z_2\ldots z_n}}\,\frac{1}{\xi_{(12\ldots n-1)n}\,\xi_{(12\ldots n-2)(n-1\,n)}\,\cdots\,\xi_{1(2\ldots n)}}$$

$$\times\,\frac{1}{v_{(12\ldots n-1)n}\,v_{(12\ldots n-2)(n-1\,n)}\,\cdots\,v_{1(2\ldots n)}}\,.$$

$$v_{(i_1 i_2\ldots i_p)(j_1 j_2\ldots j_q)} = \frac{k_{i_1}+k_{i_2}+\ldots+k_{i_p}}{z_{i_1}+z_{i_2}+\ldots+z_{i_p}} - \frac{k_{j_1}+k_{j_2}+\ldots+k_{j_q}}{z_{j_1}+z_{j_2}+\ldots+z_{j_q}},$$

$$\xi_{(i_1 i_2\ldots i_p)(j_1 j_2\ldots j_q)} = \frac{(z_{i_1}+z_{i_2}+\ldots+z_{i_p})(z_{j_1}+z_{j_2}+\ldots+z_{j_q})}{z_{i_1}+z_{i_2}+\ldots+z_{i_p}+z_{j_1}+z_{j_2}+\ldots+z_{j_q}},$$

Relation to Parke-Taylor amplitudes

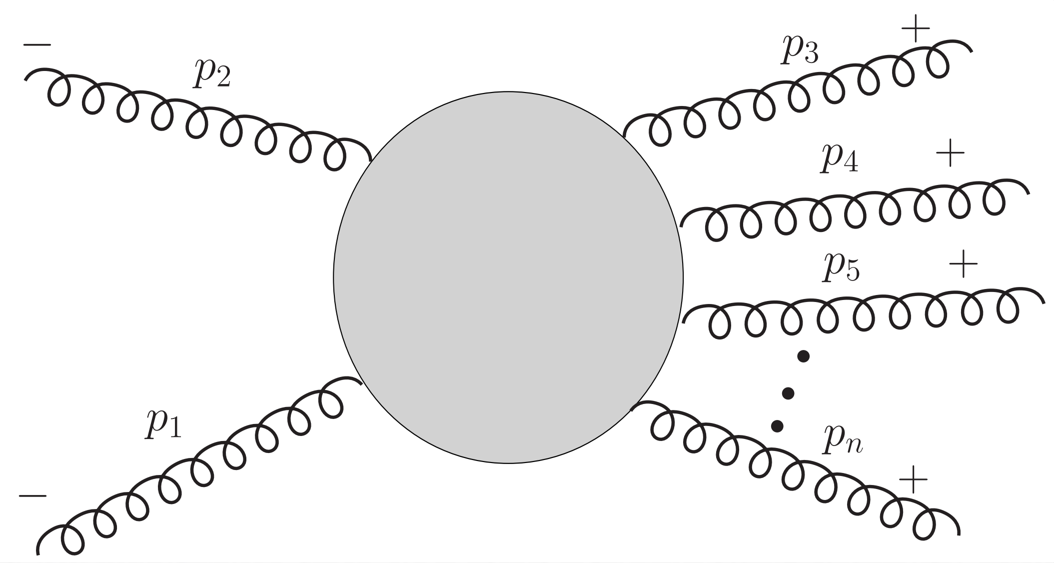
$$\mathcal{M}_n = \sum_{\{1,\ldots,n\}} \mathrm{tr}(t^{a_1} t^{a_2} \ldots t^{a_n})\, m(p_1, \epsilon_1; p_2, \epsilon_2; \ldots ; p_n, \epsilon_n) \,,$$

Color part

Kinematical part

Maximally Helicity Violating amplitude for gluons: 2 to n

Here: all gluons are outgoing



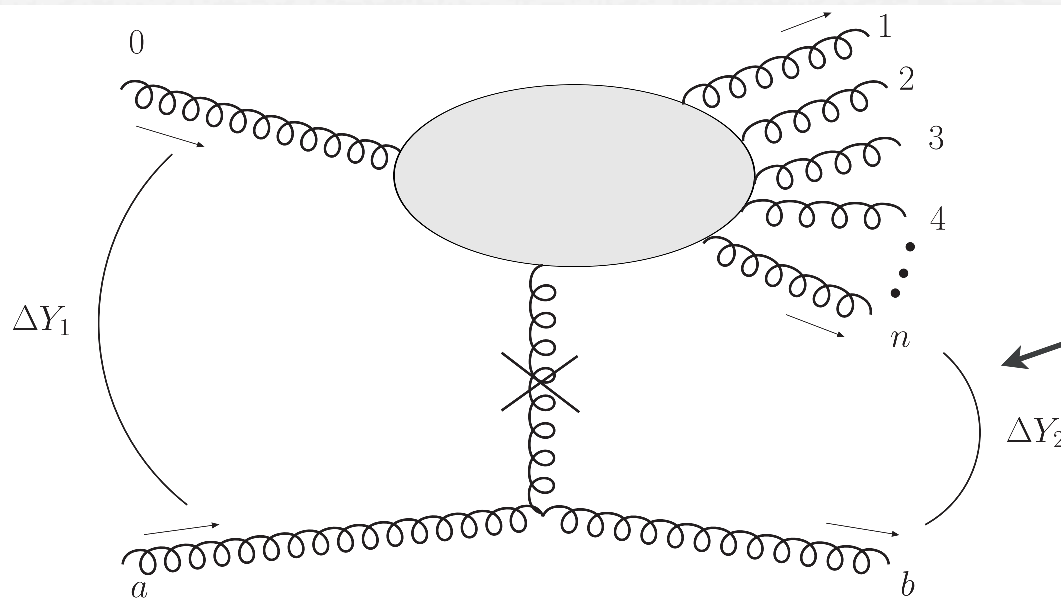
$$\langle ij \rangle = \sqrt{z_i z_j}\, \underline{\epsilon}^{(+)} \cdot \left(\frac{\underline{k}_i}{z_i} - \frac{\underline{k}_j}{z_j} \right)$$

$$\langle ij \rangle = \sqrt{z_i z_j}\, \underline{\epsilon}^{(+)} \cdot \underline{v}_{ij},$$

Tree level, Parke-Taylor formula

$$m(1^-, 2^-, 3^+, \ldots, n^+) = i g^{n-2} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n{-}2\; n{-}1 \rangle \langle n{-}1\; n \rangle \langle n1 \rangle} \,,$$

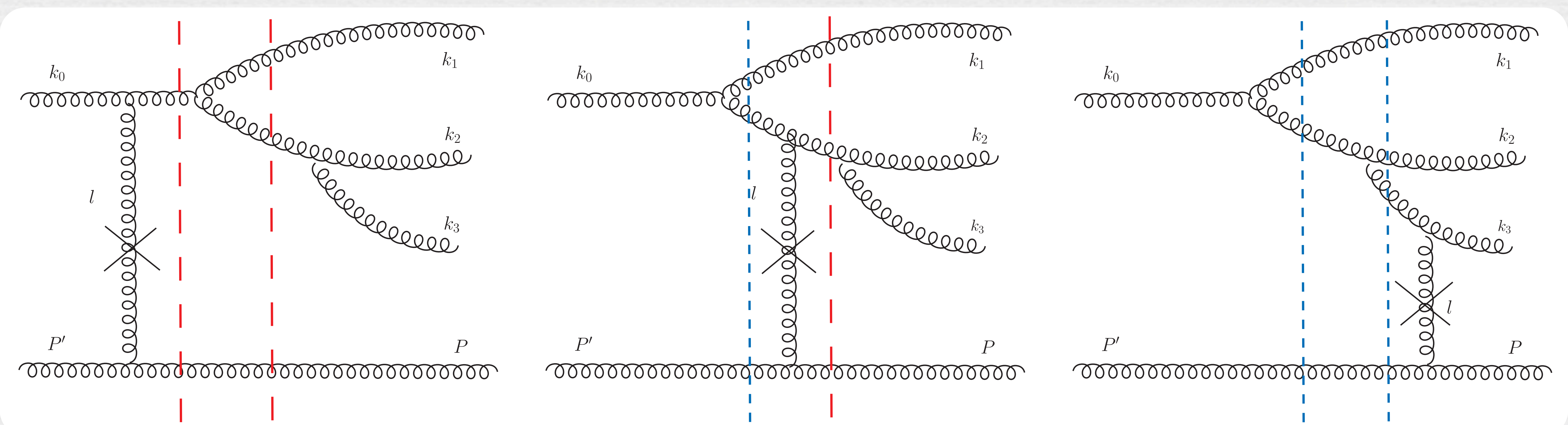
Scattering from light -cone wave functions



Rapidity gap

Helicity conserving.
High energy approximation: instantaneous gluon in the light-cone gauge

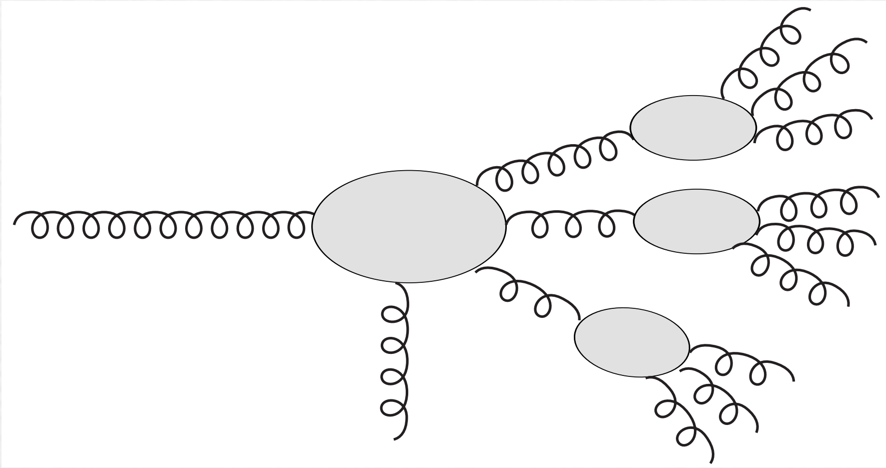
Sum over initial and final state emissions



Final state emissions: gluon fragmentation

Fragmentation in the final state

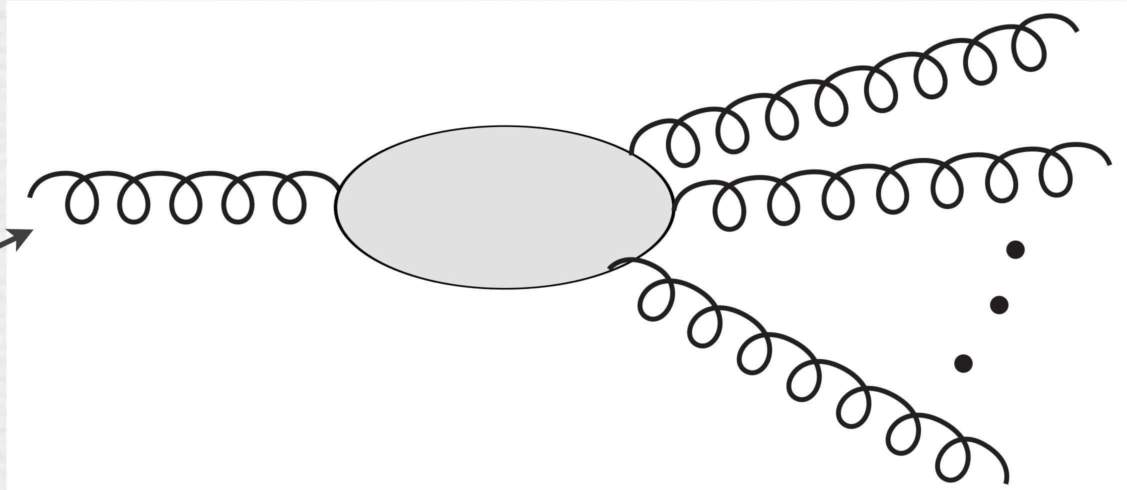
initial state



scattering

n (on-shell) gluons in the final state

initial off-shell gluon



Amplitude for fragmentation

$$T[(12\ldots n) \to 1, 2, \ldots, n] \;=\; g^{n-1} \left(\frac{z_{(12\ldots n)}}{z_1 z_2 \ldots z_n} \right)^{3/2} \quad \frac{1}{v_{12} v_{23} \ldots v_{n-1\,n}}$$

Duality: wave function vs fragmentation

Wave function
initial state

↔

Fragmentation
final state

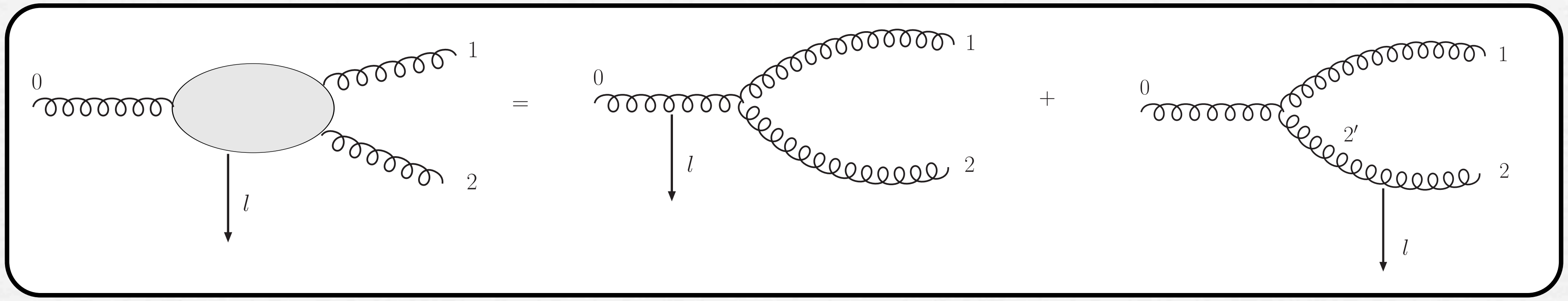
$$\Psi_n \sim \frac{1}{v_{(12\ldots n-1)n}\, v_{(12\ldots n-2)(n-1\,n)} \;\cdots\; v_{1(2\ldots n)}}$$

$$T_n \sim \frac{1}{v_{12} v_{23} \ldots v_{n-1\,n}}$$

Nearly identical expressions (the same topology of graphs): different combinations of momenta

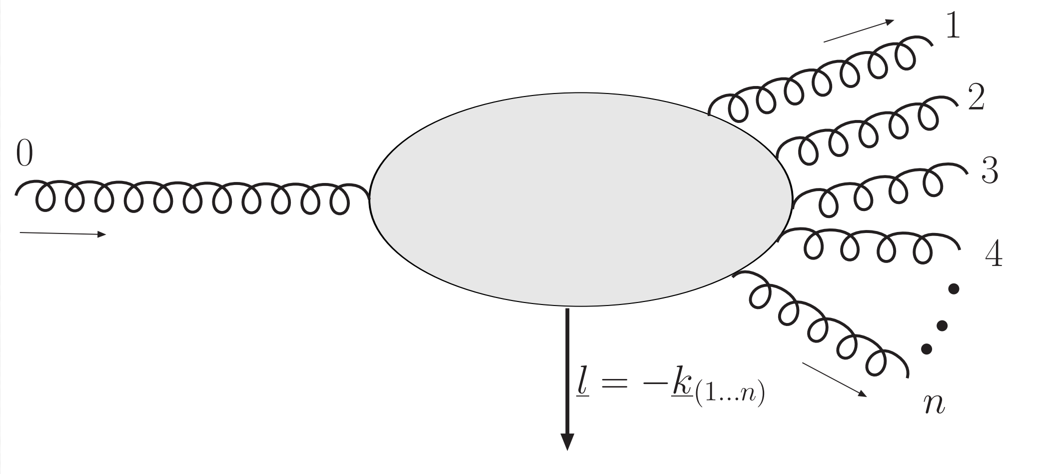
$$v_{(i_1 i_2 \ldots i_p)(j_1 j_2 \ldots j_q)} = \frac{k_{i_1} + k_{i_2} + \ldots + k_{i_p}}{z_{i_1} + z_{i_2} + \ldots + z_{i_p}} - \frac{k_{j_1} + k_{j_2} + \ldots + k_{j_q}}{z_{j_1} + z_{j_2} + \ldots + z_{j_q}},$$

Relation with MHV



$$\tilde{\Psi}_2(1,2) \;=\; T[(12)\to 1,2] + \Psi_2(1,2')\,,$$

Master formula for arbitrary number of gluons



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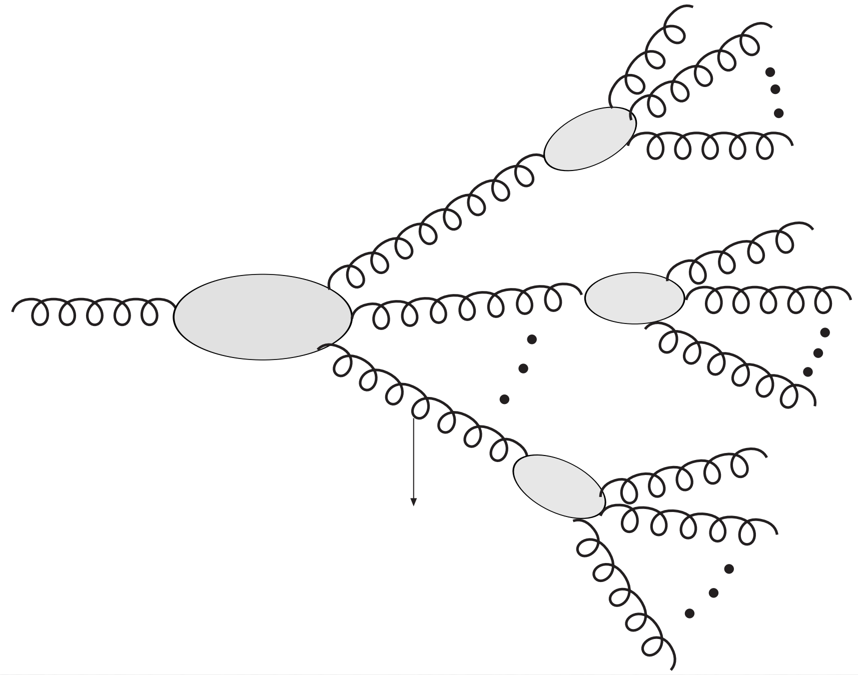
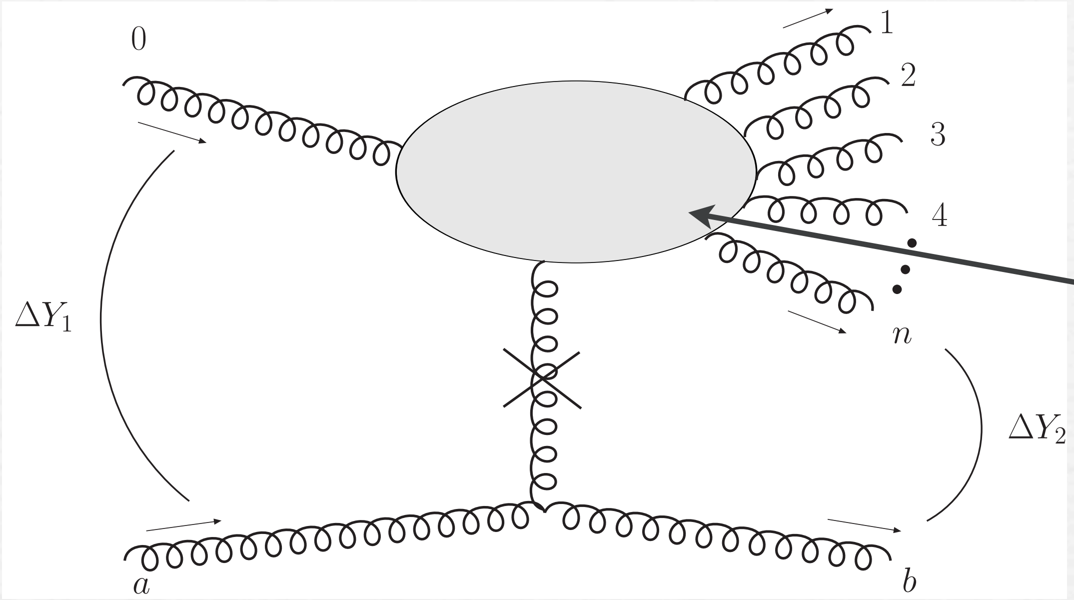


$$\tilde{\Psi}_n(1,2,\ldots,n) \;=\; \sum_{m=1}^{n} \sum_{(1\le n_1<n_2<\ldots<n_{m-1}\le n)} \Psi_m((1\ldots n_1)(n_1+1\ldots n_2)\ldots(n_{m-1}+1\ldots n))$$
$$\times\; T[(1\ldots n_1)\to 1,\ldots,n_1]\, T[(n_1+1\ldots n_2)\to n_1+1,\ldots,n_2]\;\ldots\; T[(n_{m-1}+1\ldots n)\to n_{m-1}+1,\ldots,n]\,.$$



Upper part: 1 to n with momentum transfer. Obtained by summing all possible attachments.

$$M(0; a \to 1, \ldots, n; b) \simeq \frac{s}{t} \times \tilde{\Psi}_n$$

2 to 2 amplitude

Spinor products:

$$\langle i i+1 \rangle = \sqrt{z_i z_{i+1}} v_{i i+1}$$

Recover MHV amplitude in the light cone formalism

$$M(0; a \to 1, \ldots, n; b) \simeq g^{n+1} \frac{\langle a0 \rangle^4}{\langle a0 \rangle \langle 01 \rangle \langle 12 \rangle \langle n-1\, n \rangle \langle nb \rangle \langle ba \rangle},$$

Summary

- Kinematical effects in the gluon cascades are of major importance.
- Reformulation of the dipole kernel to include part of these effects.
- Impact parameter dependence significantly modified: exponential tails with the energy-dependent cutoff.
- Lack of the 2-dimensional conformal invariance: consistent with the exact NLL calculation.
- Resummation of the light cone wave function with exact kinematics.
- Resummation of the fragmentation amplitudes. Duality between the fragmentation and the wave functions.
- Derivation of the scattering amplitudes in the light cone formalism.
- Consistency check with (new derivation of) the MHV amplitudes.