# Infra-red nonabelian cancellations in the unintegrated NLO kernel

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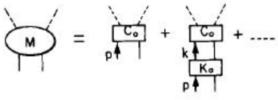
# Construction of NLO kernel

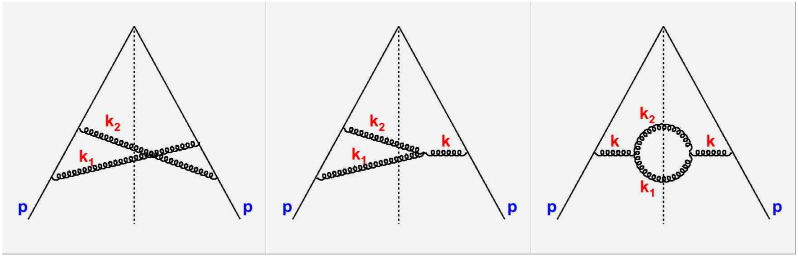
- We follow Curci, Furmanski, Petronzio (CFP) scheme (axial gauge, dimensional regularisation).
- Cross-section for the process factorises:



$$M = C_0(1 + K_0 + K_0^2 + ...) = C_0\Gamma_0$$

- All infra-red singularities in $\Gamma_0$.
- Kernel extracted from a single pole in $\Gamma_0$

# Diagrams contributing to non-abelian part of the kernel (subset)




$$k_1 = \alpha_{\mathbf{1}} p + \alpha_1^- n + \mathbf{k}_{\mathbf{1}\perp}$$

$$k_2 = \alpha_{\mathbf{2}} p + \alpha_2^- n + \mathbf{k}_{\mathbf{2}\perp}$$

$$k = k_1 + k_2$$

$$q = p - k_1 - k_2$$

# Unintegrated NLO kernel

Two-gluons phase space parametrised with Sudakov variables: $\alpha_1$, $\alpha_2$, $k_{1\perp}$, $k_{2\perp}$.

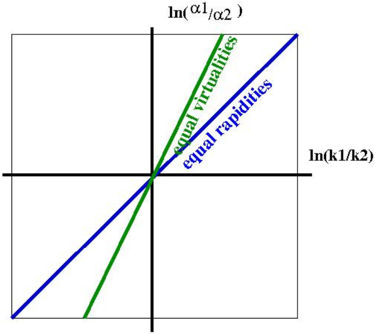
- $\max\{k_{1\perp}, k_{2\perp}\} = Q^2$ fixed.
- $1 - x = \alpha_1 + \alpha_2$ (set to 0.6 on all plots).
- angle between $k_{1\perp}$ and $k_{2\perp}$ integrated over.

In the following we will use a dimensionless

$$y = \begin{cases} \frac{k_{1\perp}^2}{Q^2} & \text{if } k_{1\perp} < k_{2\perp} \\ \frac{k_{2\perp}^2}{Q^2} & \text{if } k_{2\perp} < k_{1\perp} \end{cases}$$

Trace over color $\rightarrow$ abelian and non-abelian parts of the kernel.
Phase space normalised as follows: $d\Psi = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\ \frac{dk_{1\perp}}{k_{1\perp}}\frac{dk_{2\perp}}{k_{2\perp}}$

# Sudakov variables for two real gluons



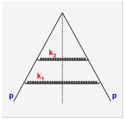

$$q_1 = \frac{k_{1\perp}}{\alpha_1} = \frac{k_{2\perp}}{\alpha_2} = q_2$$

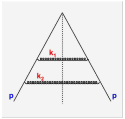
$$v_1 = \frac{k_{1\perp}^2}{\alpha_1} = \frac{k_{2\perp}^2}{\alpha_2} = v_2$$

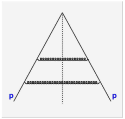
In log variables singularities moved to the domain:

- $l_0$ - **single logarithmic** - infinite lines ("rows") in these variables.
- $l_1$ - **double logarithmic** - infinite in two dimensions ("triangles").

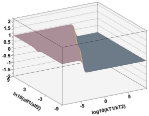
# Short interlude - cancellations in the abelian part




Double gluon emission ladders contribute in different regions of phase-space.

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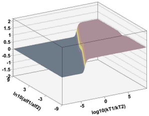
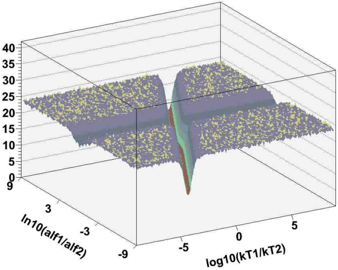
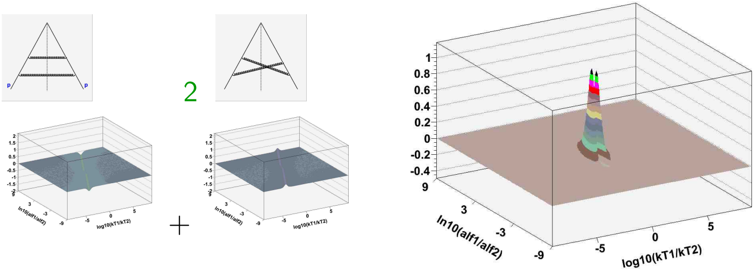



Their sum has a single-log singularity along the line of equal virtualities...

# Cancellations in the abelian part

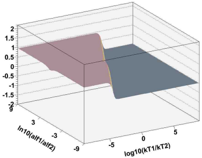
... which, after adding the crossed-ladder diagram:



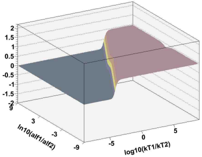

turns out to be regular in the soft sudakov limit!

# Abelian cancellations - analytically

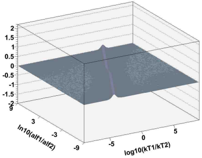
$$\frac{2(1+x^2)}{(1+xy)^2}$$




$$+\frac{2(1+x^2)x^2y^2}{(1+xy)^2}$$

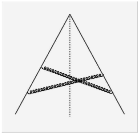



$$+2\,\frac{2(1+x^2)xy}{(1+xy)^2}$$

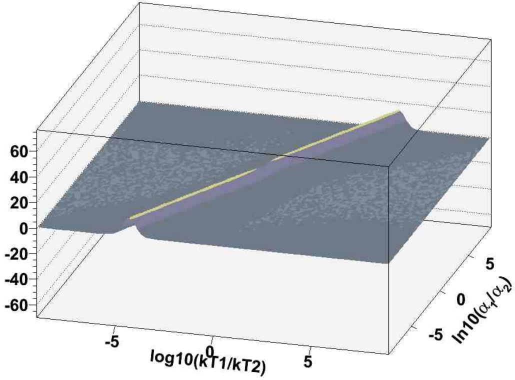


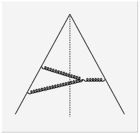

$$= 2(1+x^2)\frac{1+2xy+x^2y^2}{(1+xy)^2} = 2(1+x^2)$$

## Nonabelian diagrams:

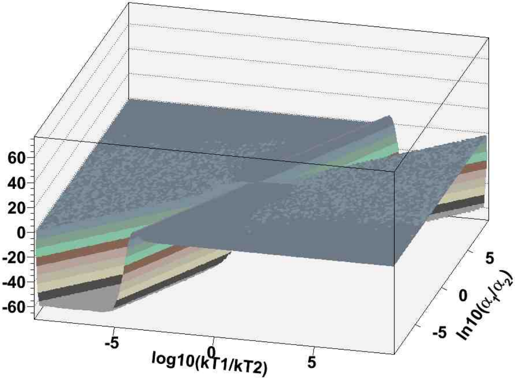


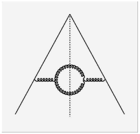
Single log singularity $l_0$ along the line of equal virtualities





Double log singlarity $l_1$ - infinite plateau between the line of equal rapidities and virtualities
Single log singularity $l_0$ along the line of equal virtualities

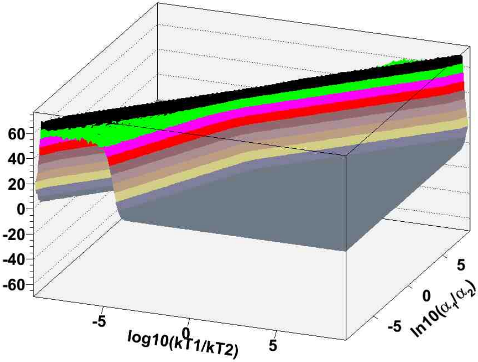




Double log singlarity $l_1$ - infinite plateau between the line of equal rapidities and virtualities

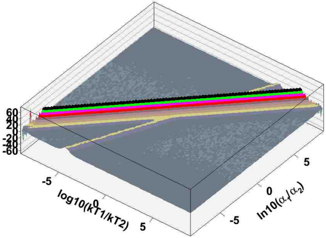
Single log singularity $l_0$ along the line of equal virtualities

Singularity in effective mass $k^2$ of the gluon pair (not discussed here)



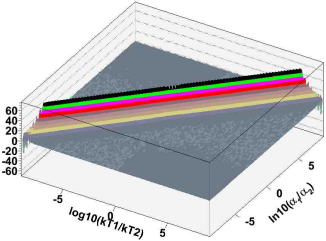
# Cancellations - numerical test

$Yg + Vg =$




Full cancellation of $I_1$!

$Yg + Vg - Bx =$




$I_0$ cancelled!!!

# Conclusions for Monte Carlo

Work still in progress, but regular infra-red structure in both QCD and QED-like part already observed.

- Cancellation of double- and **single**-logarithmic (colour coherence effects) soft sudakovian divergencies!
- in both abelian and non-abelian diagrams!
- Color coherence effects important!
- Regular structure of the unintegrated kernel