Traveling waves and impact-parameter correlations in high-energy QCD

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Palaiseau, France

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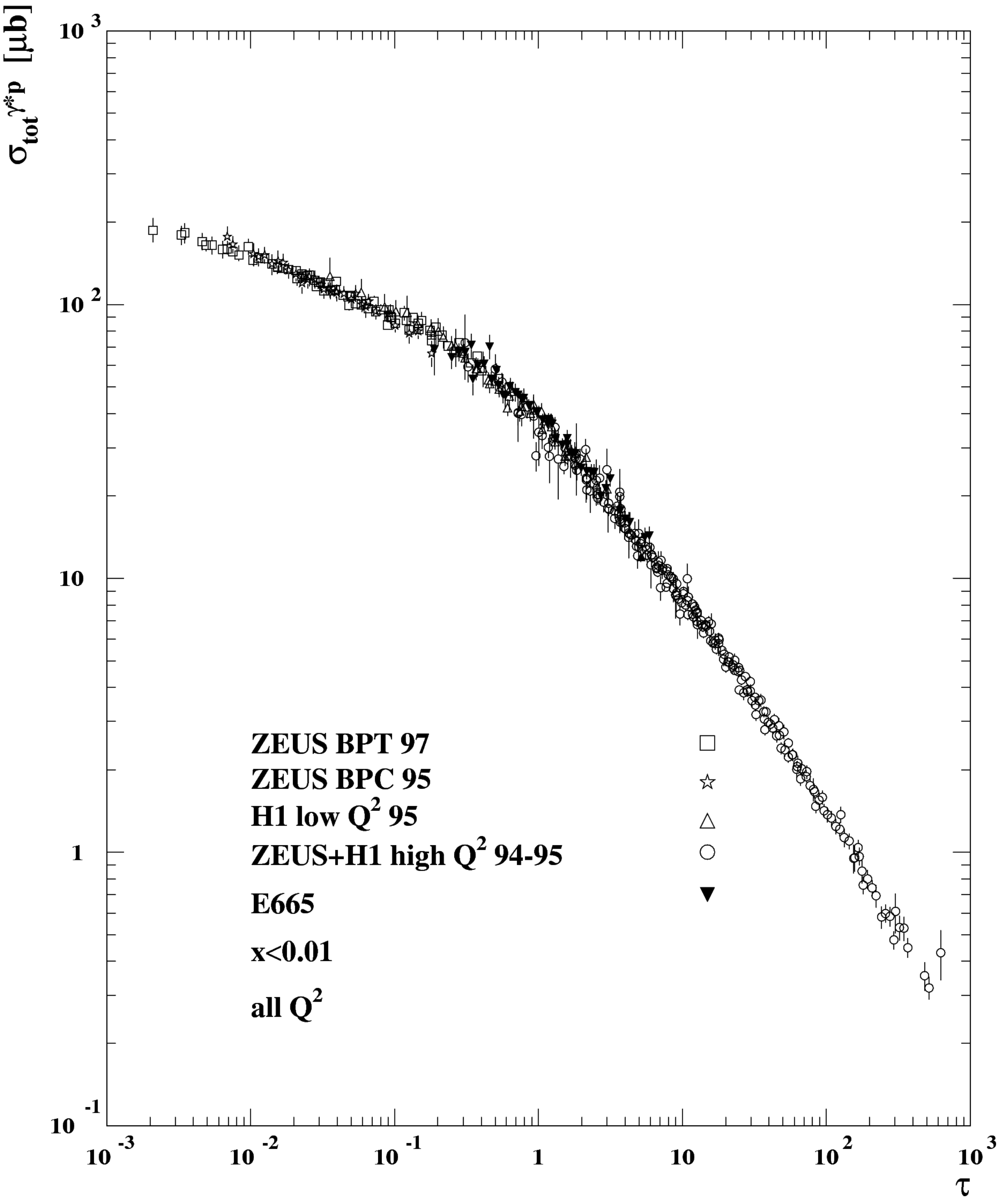
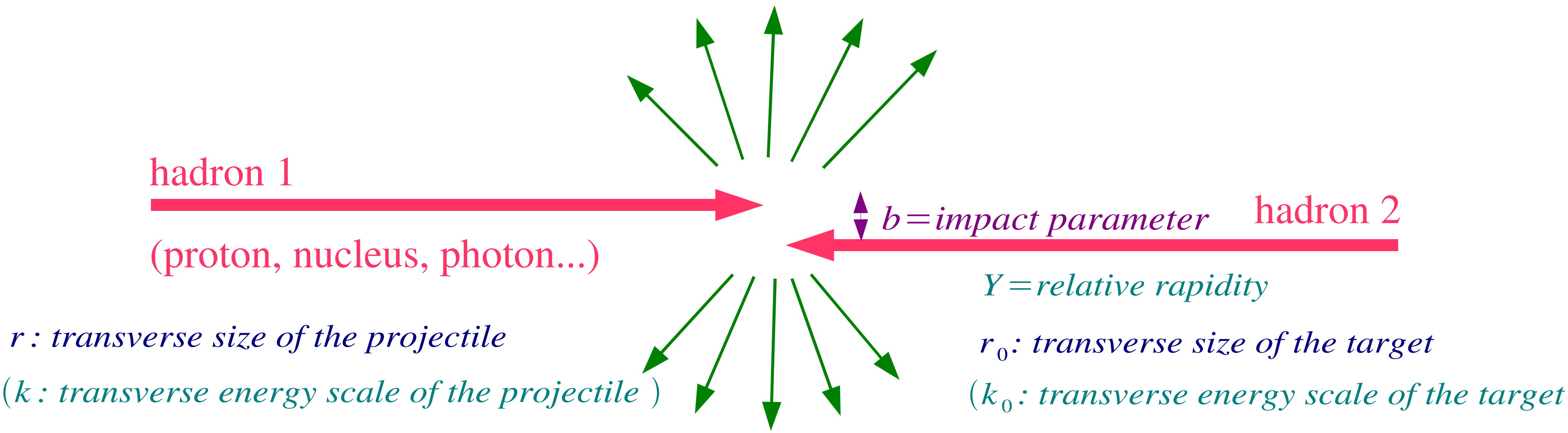
Epiphany conference dedicated to the memory of Jan Kwieciński

Geometric scaling



Staśto,
Golec-Biernat,
Kwieciński,
PRL (2001)

High energy QCD



$$A(Y, \overset{k}{r}) = \int d^2 b \, A(Y, b, \overset{k}{r}) = \textit{elastic amplitude}$$

$$A(Y, b, \overset{k}{r}) = \textit{fixed impact parameter amplitude} \quad \boxed{\leq 1}$$

(High) energy dependence of QCD amplitudes?

QCD and statistical physics

Iancu, Mueller, Munier (2004)

***We identified the <u>universality class</u> of high-energy scattering at fixed impact parameter as that of one-dimensional** (« space » variable: $\log k^2$)*
***reaction-diffusion processes** evolving in « time » Y*

- simple picture of high energy scattering, based on the parton model
- connects the QCD problem to more general physics and mathematics
- new results for QCD amplitudes!

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***We identified the <u>universality class</u> of high-energy scattering at fixed impact parameter as that of one-dimensional** (« space » variable: $\log k^2$)*
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- simple picture of high energy scattering, based on the parton model
- connects the QCD problem to more general physics and mathematics
- new results for QCD amplitudes!

But: this identification is still a conjecture!
In particular, it is not completely clear that QCD may be reduced
to a one-dimensional problem

$$A(Y, \not{b}, r)$$

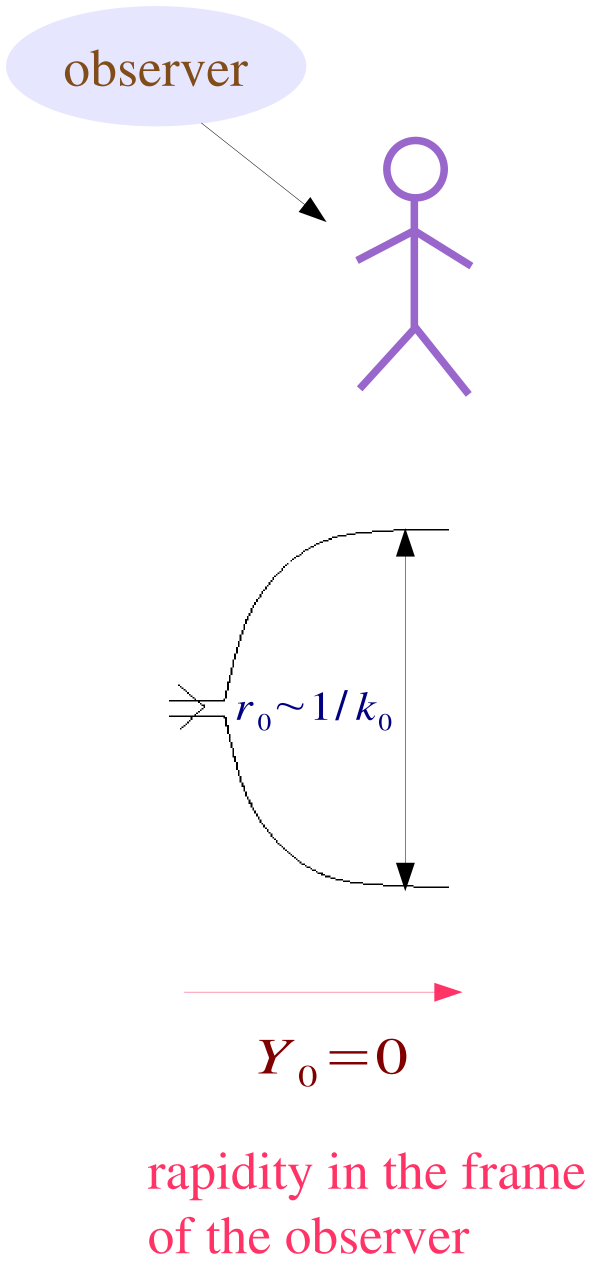
Condition: each impact parameter evolves independently.
This talk: a numerical check of this statement

Munier, Salam, Soyez (2008)

Outline

- High energy QCD and one-dimensional stochastic processes
- Independence of different impact parameters in a toy model

How a high rapidity hadron looks



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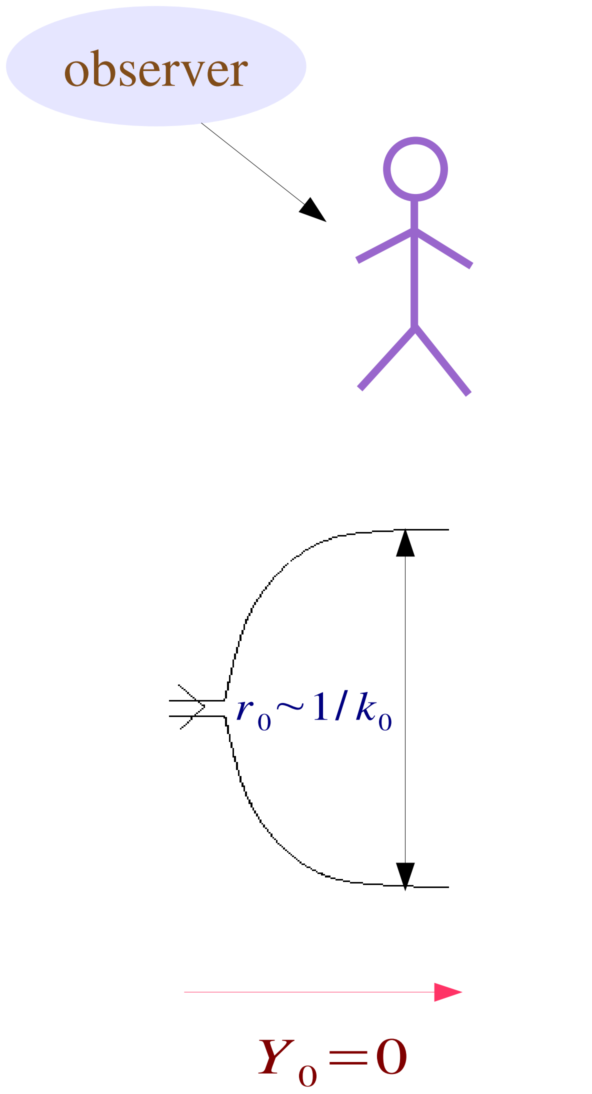
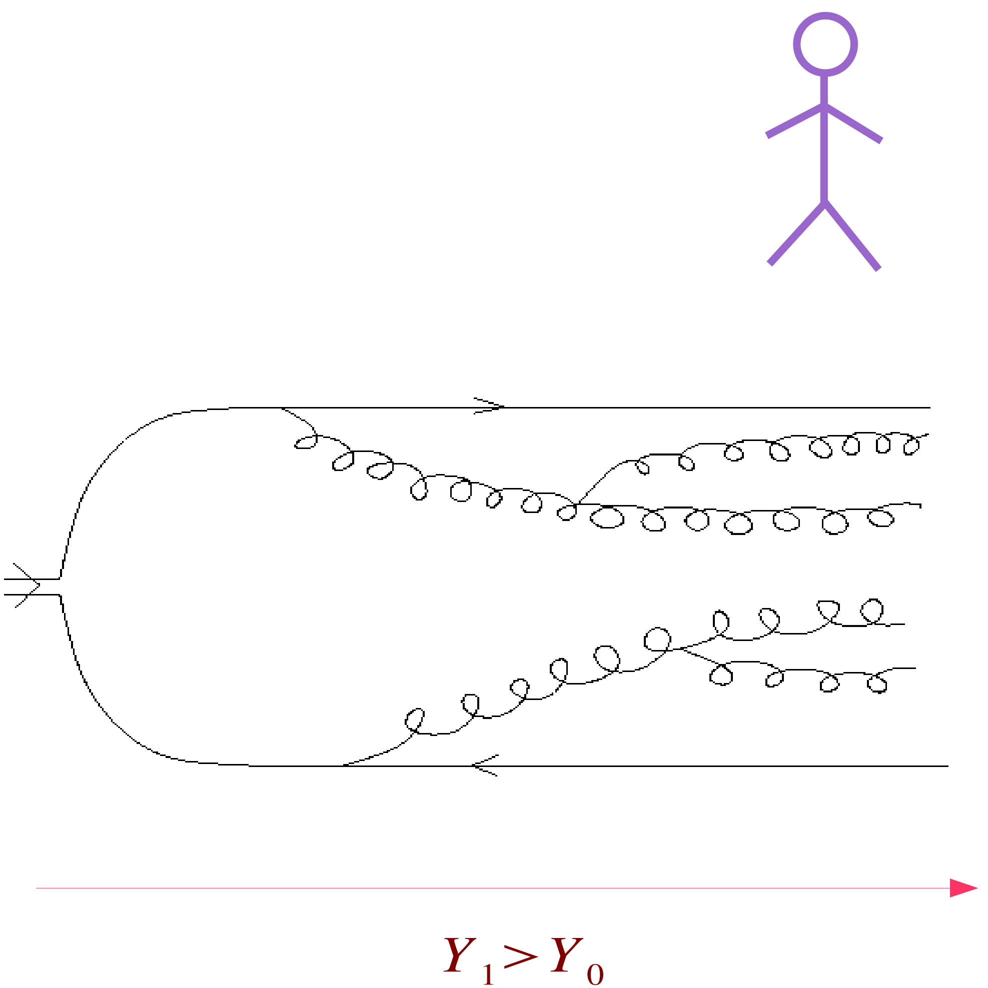


rapidity in the frame of the observer



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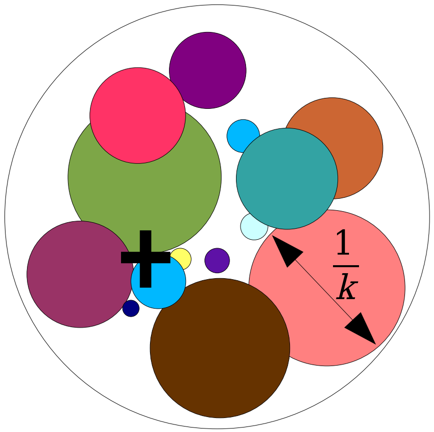

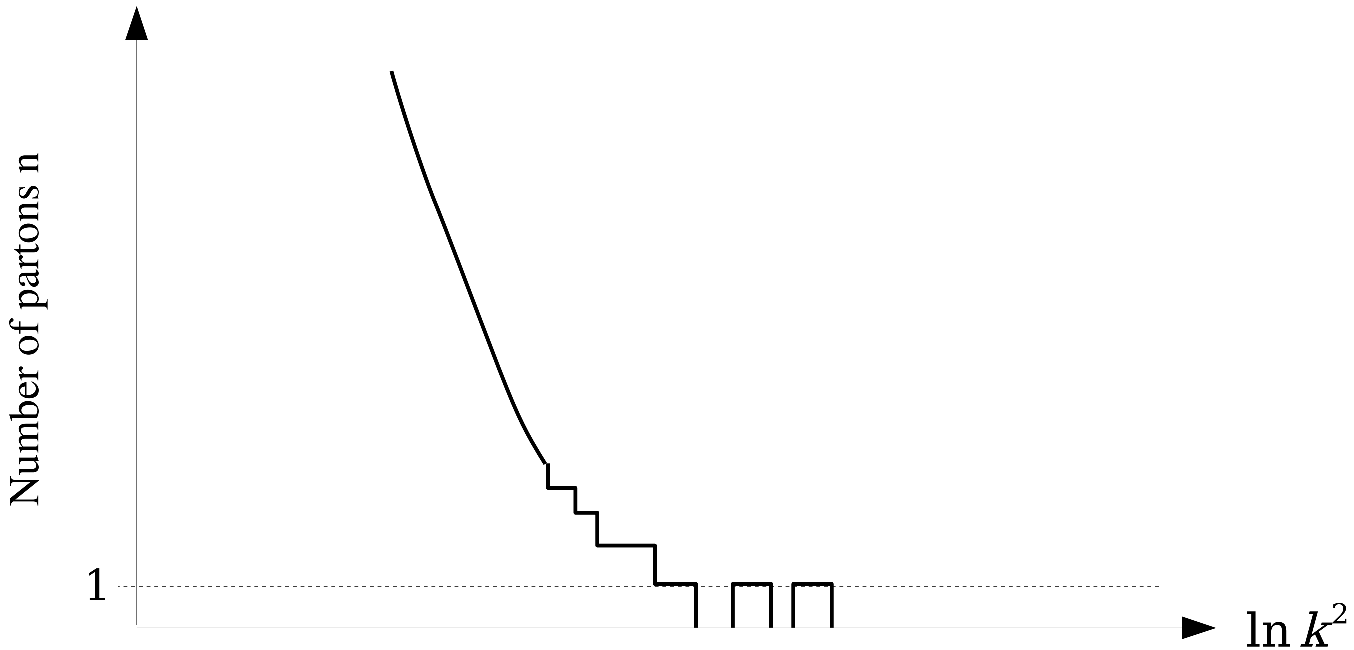

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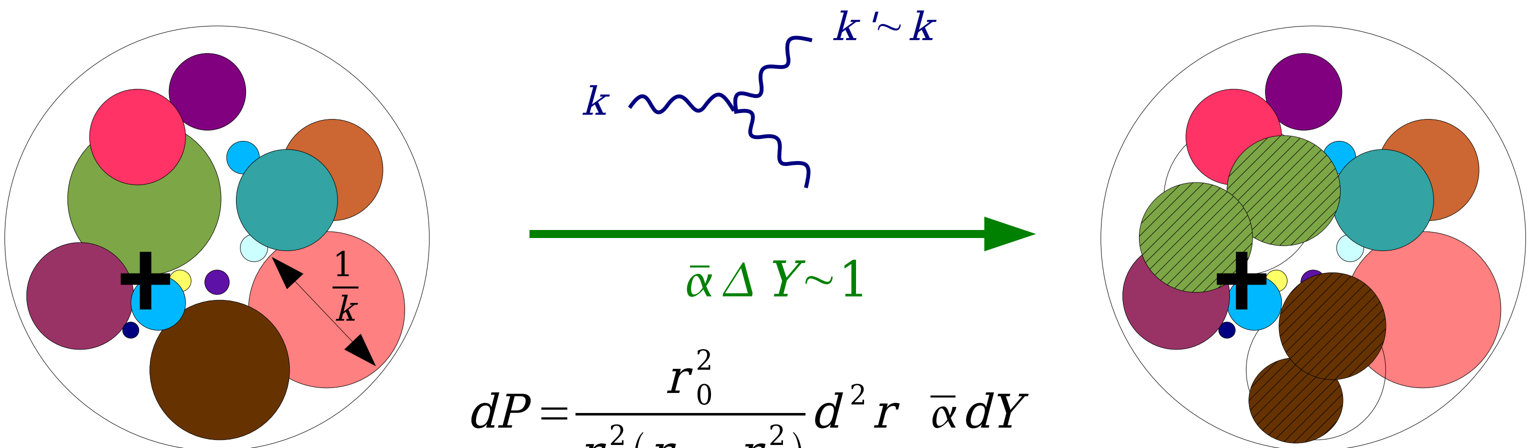
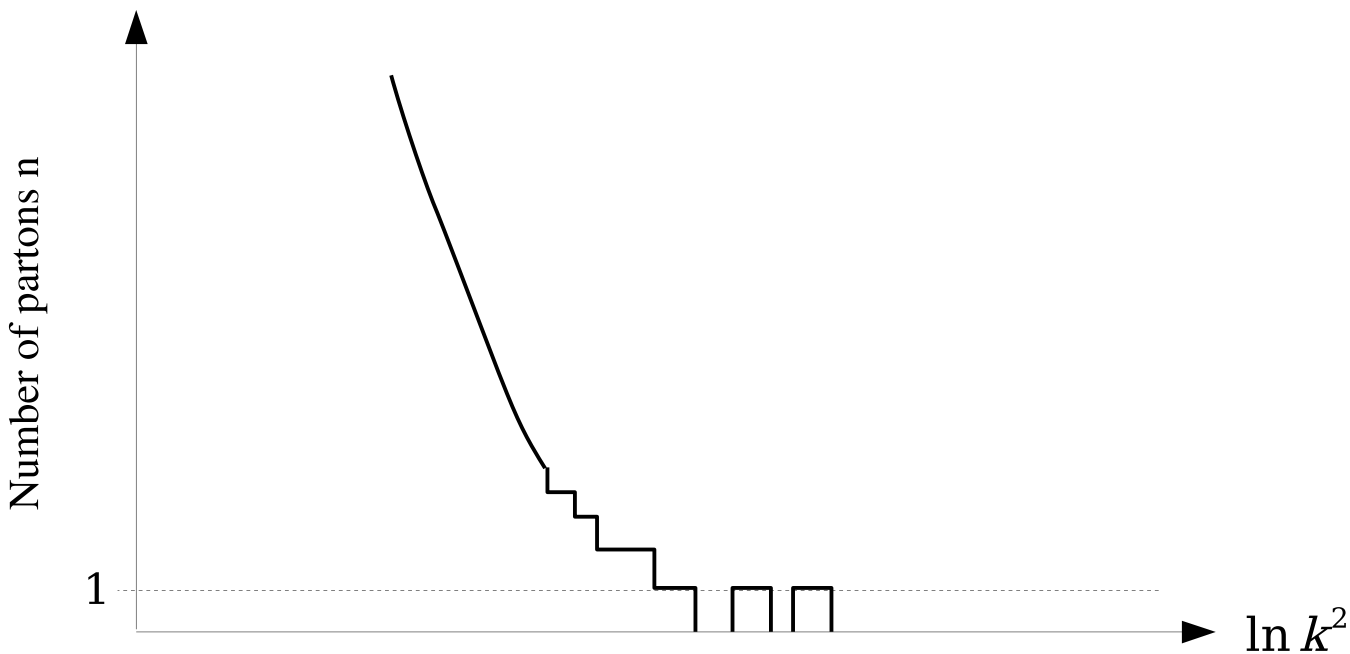
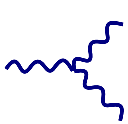


$$dP = \frac{r_0^2}{r^2 (r_0 - r^2)} d^2 r \;\; \bar{\alpha}\, dY$$



$$\partial_{\bar{\alpha} Y}\, n = \chi(-\partial_{\ln k^2})\, n \qquad + \sqrt{n}\;\; \nu$$



$BFKL \sim \partial^2_{\ln k^2} n + n$

Noise term due to discreteness

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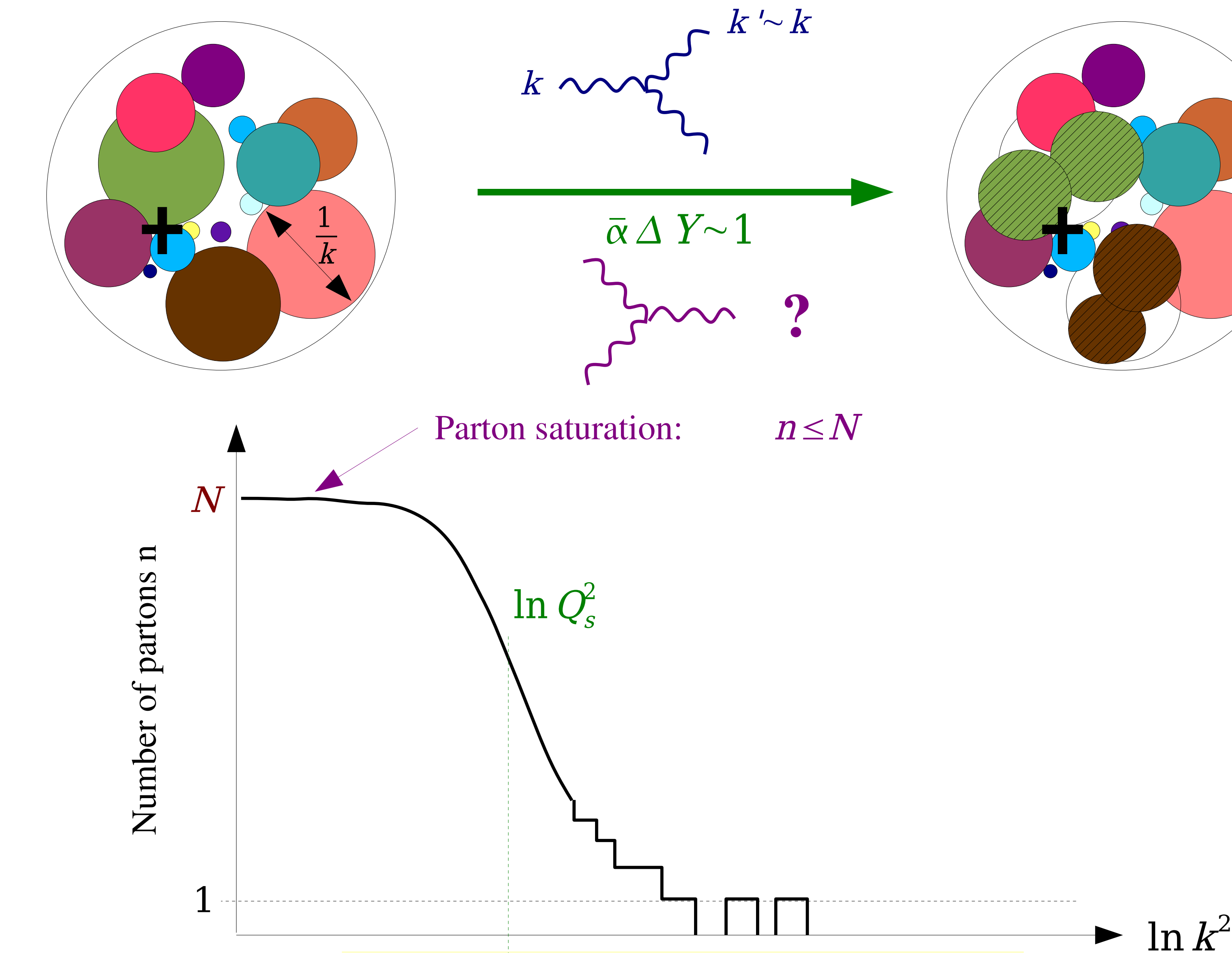

$$\partial_{\bar{\alpha} Y}\, n = \chi\left(-\partial_{\ln k^2}\right) n - \frac{n^2}{N} + \sqrt{n}\;\; \nu$$

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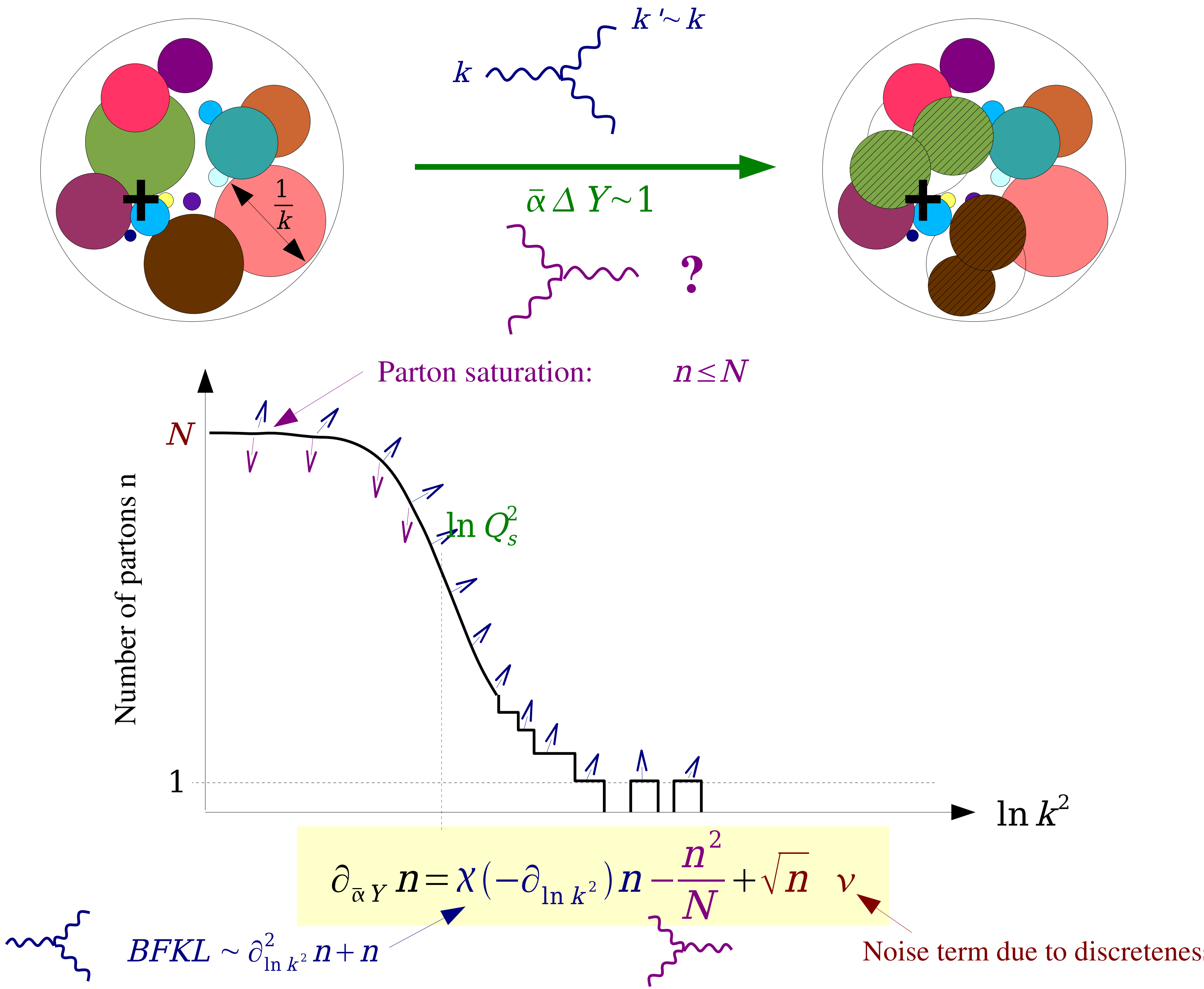

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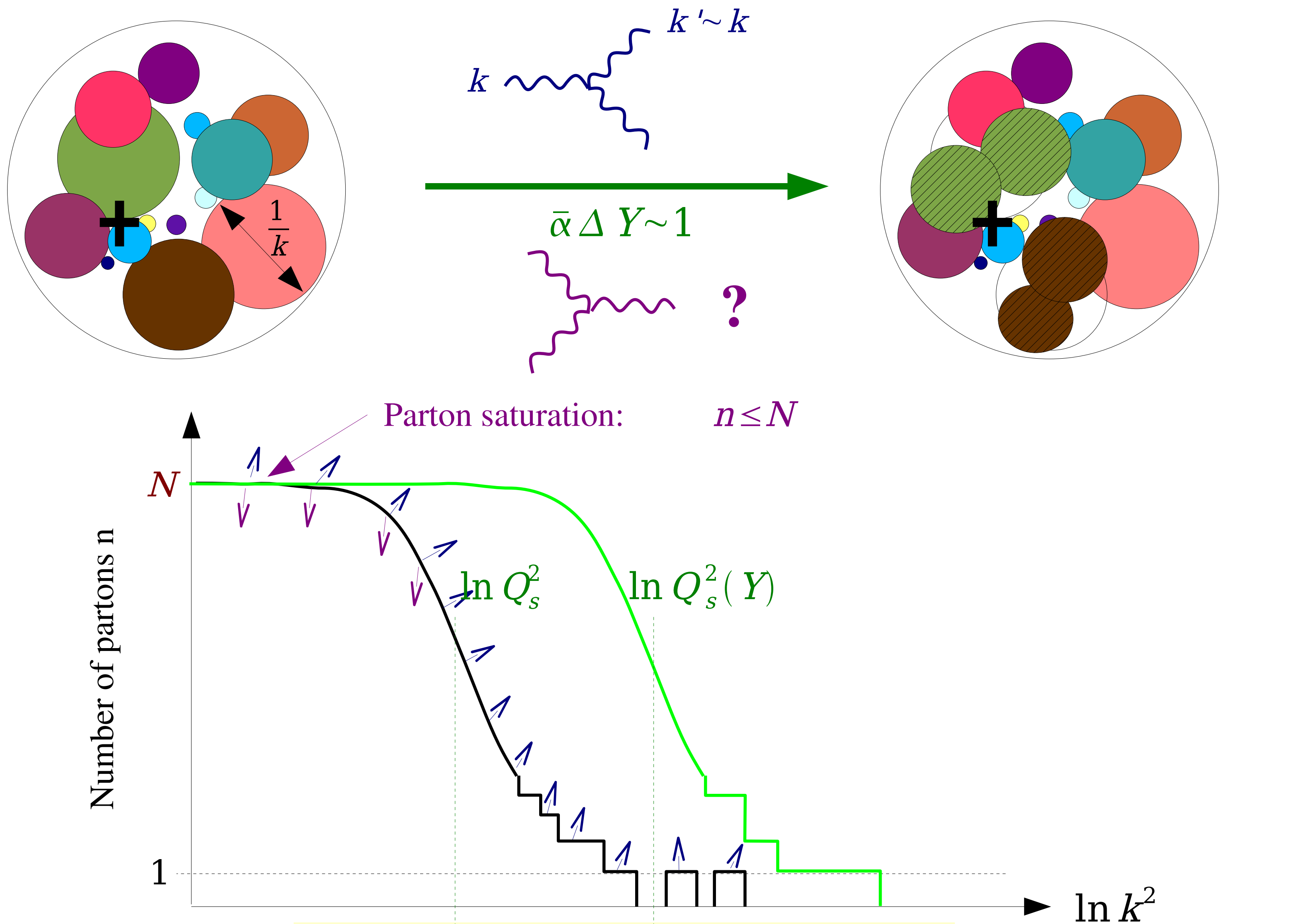

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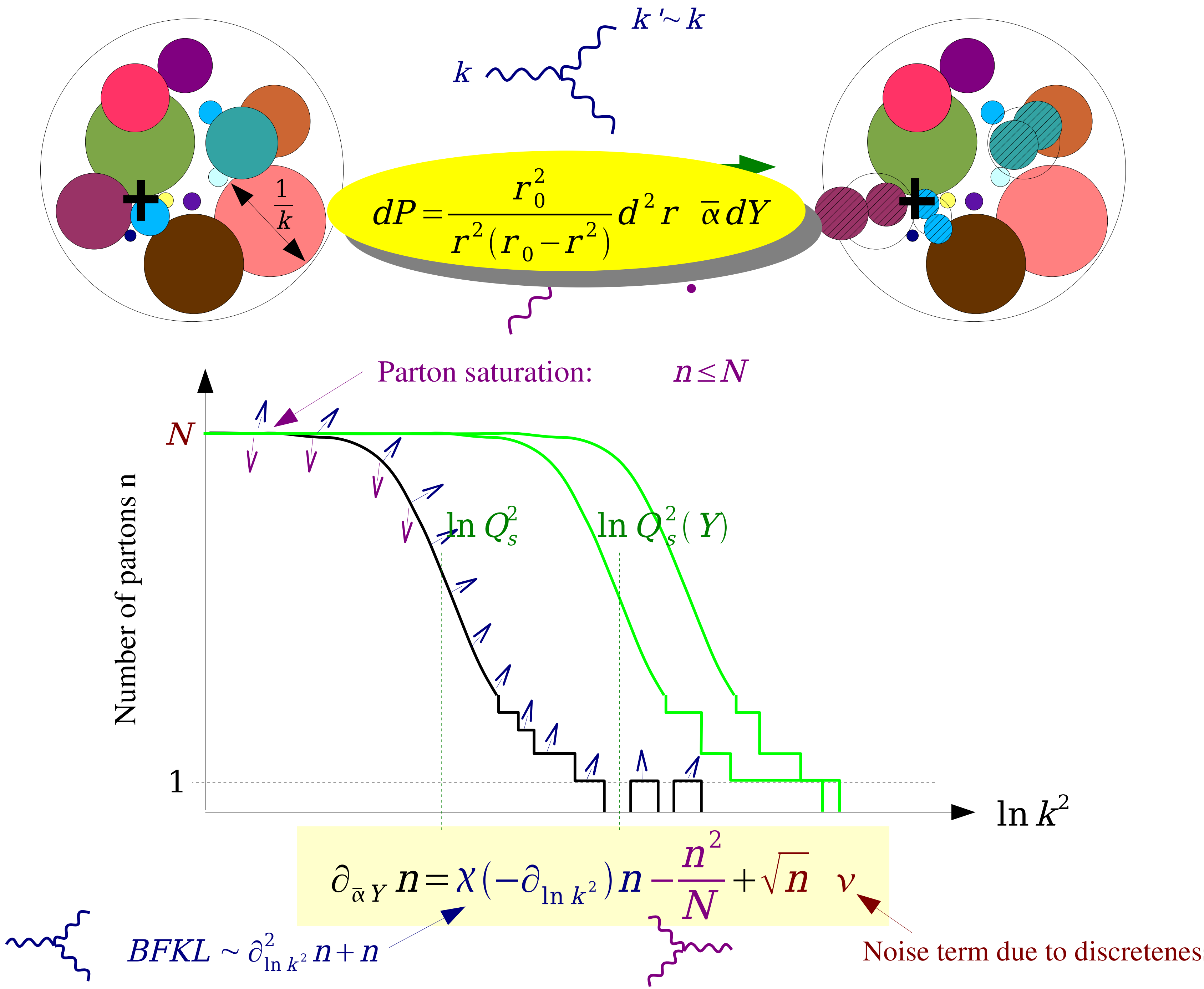

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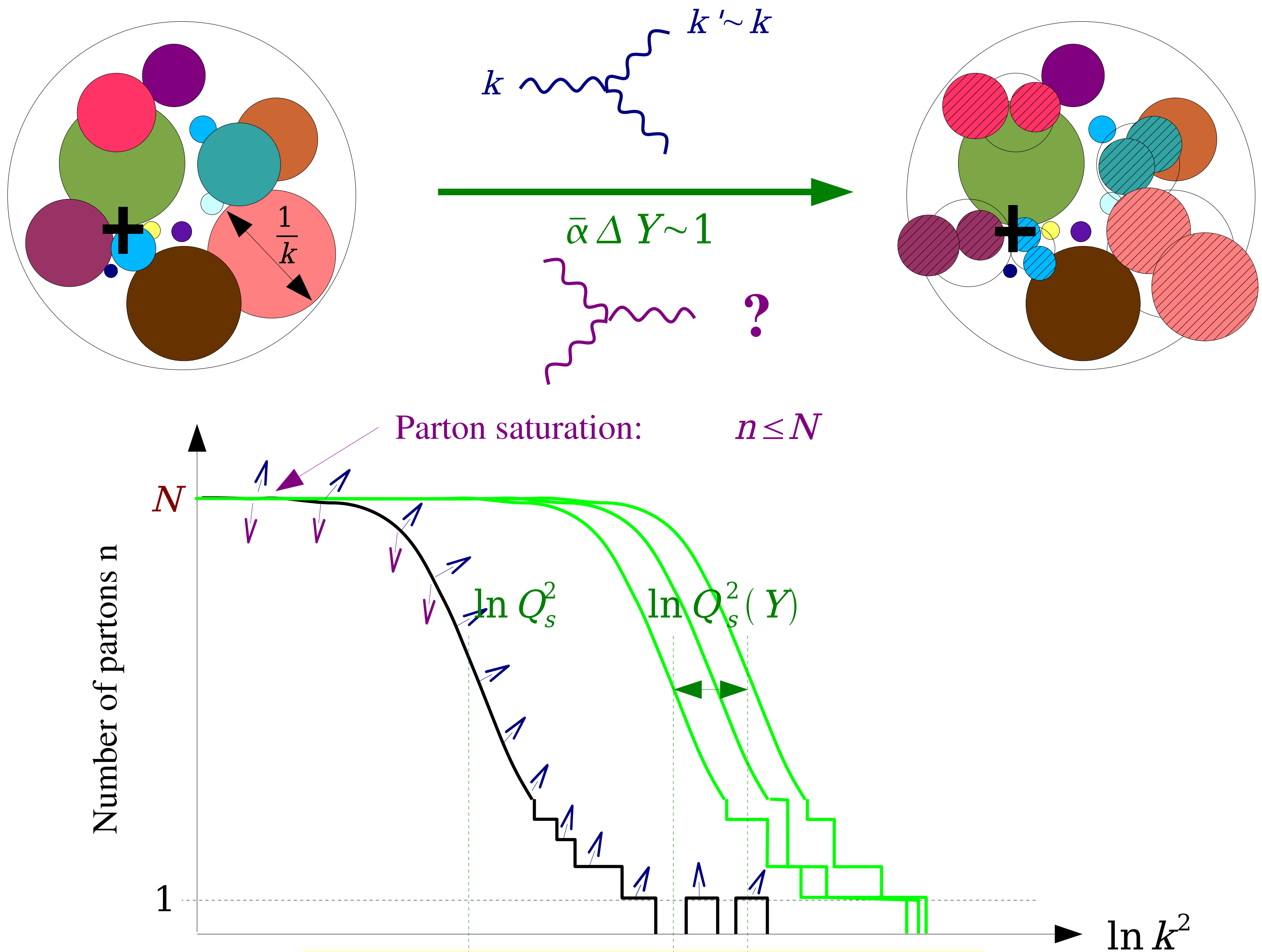

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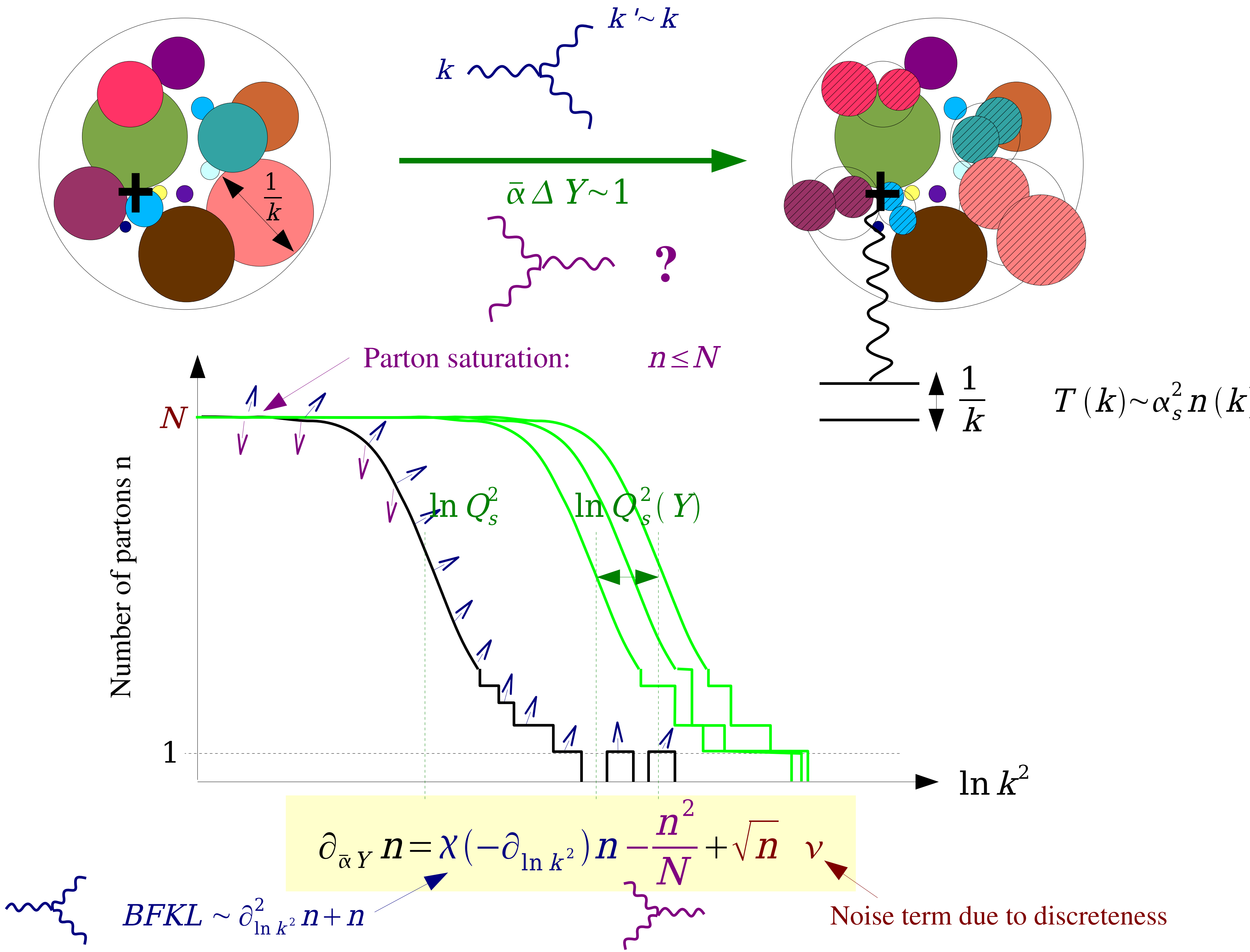

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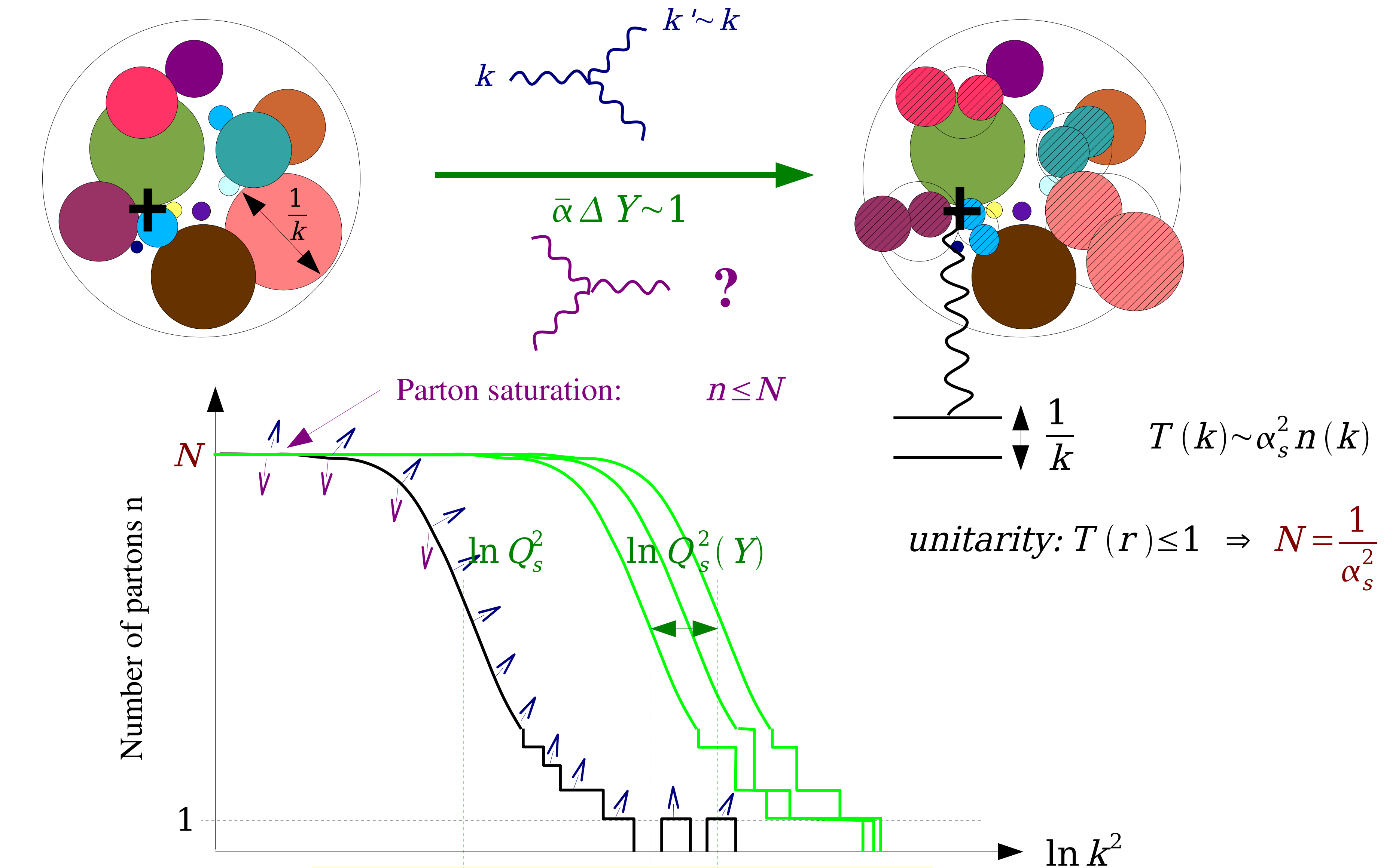

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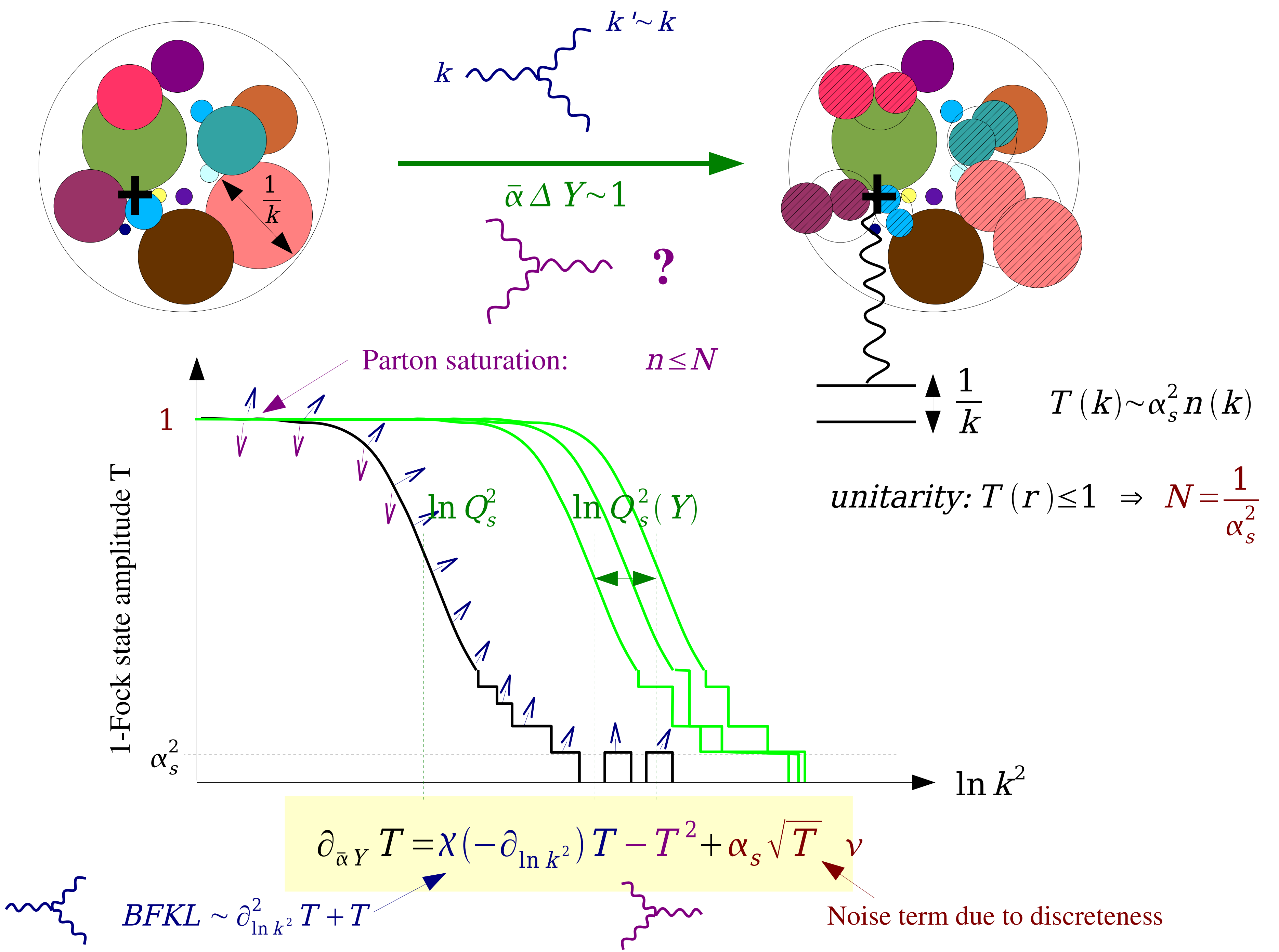

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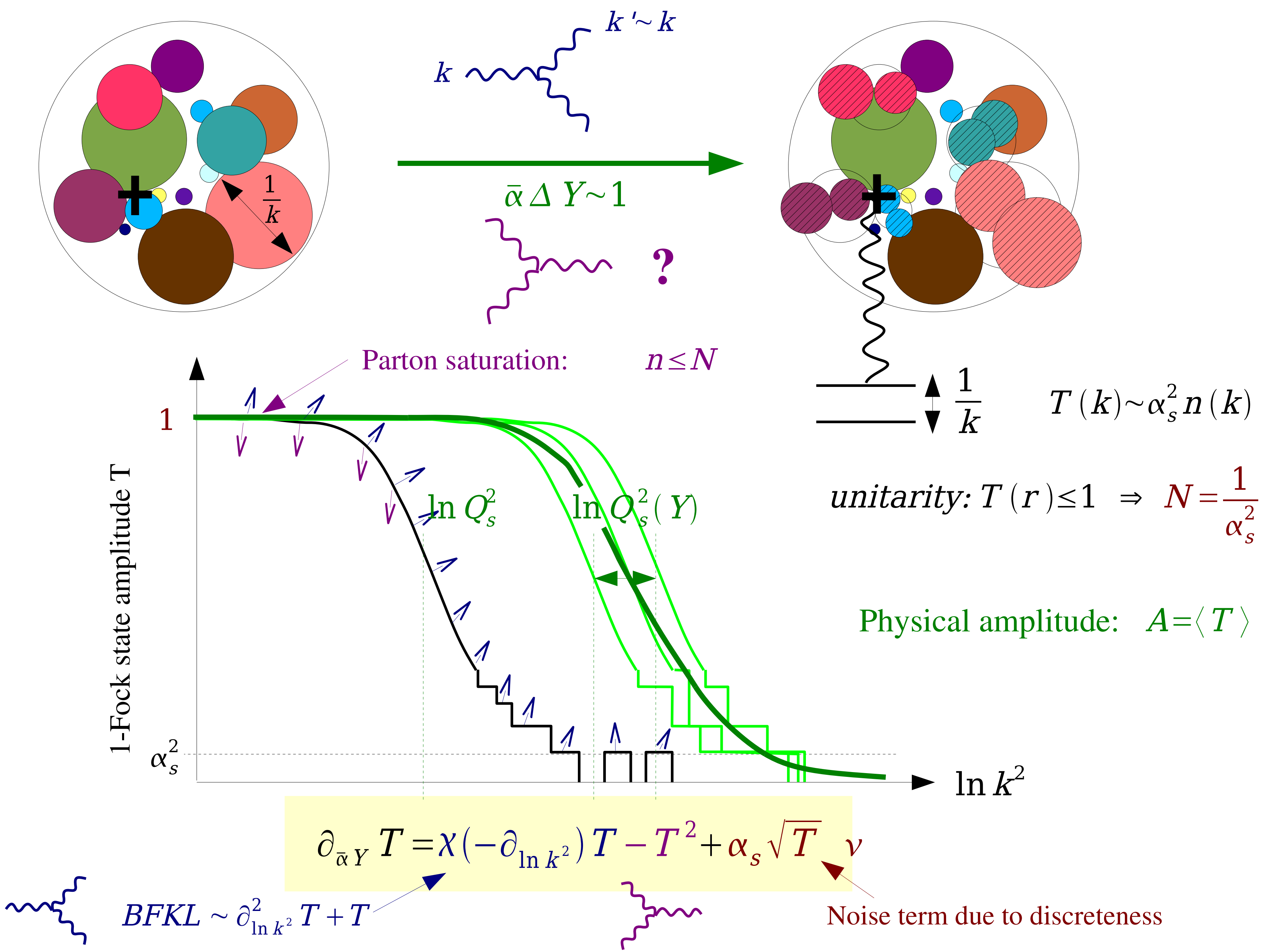

QCD and reaction-diffusion

$$\partial_{\bar{\alpha} Y} T = \chi(-\partial_{\ln k^2})\, T - T^2 + \alpha_s \sqrt{T}\ \nu$$

Similar to the sFKPP equation $\partial_t T = \partial_x^2 T + T - T^2 + \sqrt{\frac{2}{N} T}\ \nu$

Fisher; Kolmogorov, Petrovsky, Piscunov (1937)

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Predictions for QCD amplitudes

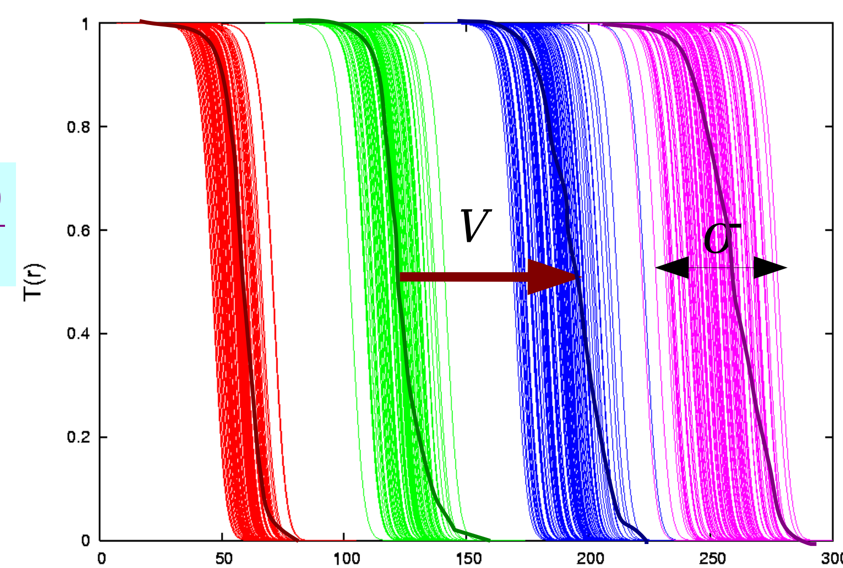
Shape of the *partonic* amplitude: $T \sim (r Q_s(Y))^{2\gamma_0}$

Saturation scale:

$$V = \frac{d}{d(\bar{\alpha} Y)} \langle \ln Q_s^2 \rangle = \frac{\chi(\gamma_0)}{\gamma_0} - \frac{\pi^2 \gamma_0 \chi''(\gamma_0)}{2 \ln^2(1/\alpha_s^2)} + \pi^2 \gamma_0^2 \chi''(\gamma_0) \frac{3 \ln\ln(1/\alpha_s^2)}{\gamma_0 \ln^3(1/\alpha_s^2)}$$

$$\langle \ln^n Q_s^2 \rangle_{cumulant} = \pi^2 \gamma_0^2 \chi''(\gamma_0) \frac{n!\,\zeta(n)}{\gamma_0^n} \left[\frac{\bar{\alpha} Y}{\ln^3(1/\alpha_s^2)} \right]$$

$$\Rightarrow A \sim A\left(\frac{r^2 Q_s^2(Y)}{\sqrt{\frac{\bar{\alpha} Y}{\ln^3(1/\alpha_s^2)}}} \right)$$

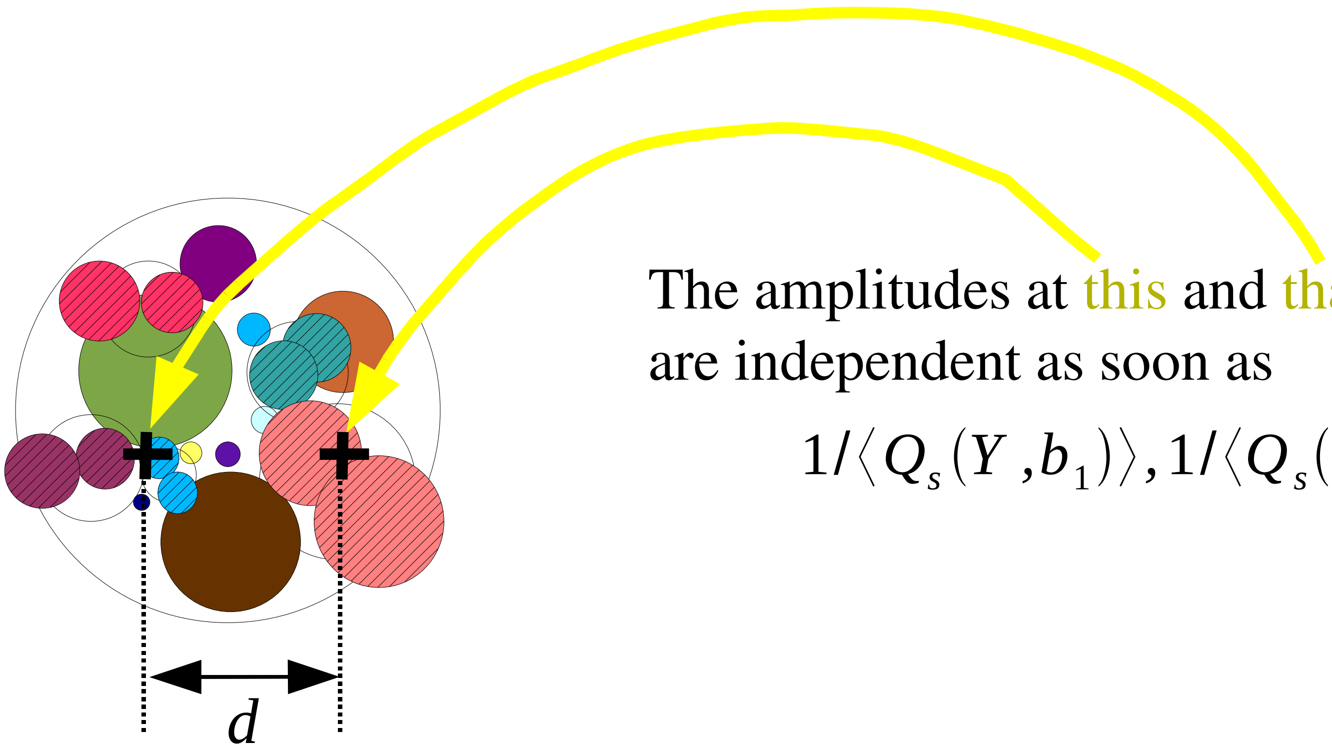


Traveling waves

Brunet, Derrida, Mueller, Munier (2004)

These formulas are independent of the precise form of the stochasticity and of the nonlinearity.

Independence of the different impact parameters?



The amplitudes at this and that impact parameters are independent as soon as

$$1/\langle Q_s(Y,b_1)\rangle, 1/\langle Q_s(Y,b_2)\rangle < d$$

As soon as the distance between the probed impact parameters is larger than the relevant distance scale of the evolution (=the inverse saturation scale), the amplitudes measured at the two impact parameters should be independent.

Supported by (too) simple analytical estimates (fluctuations neglected...)

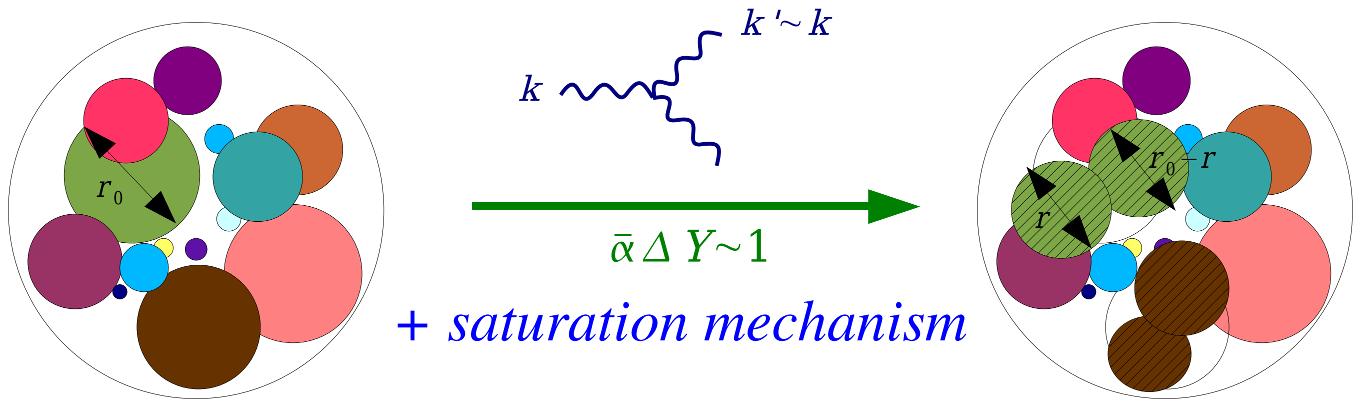
Outline

- High energy QCD and one-dimensional stochastic processes
- Independence of different impact parameters in a toy model

Toy model with impact-parameter dependence

QCD:

$$dP = \frac{r_0^2}{r^2 (r_0 - r^2)} d^2 r \quad \bar{\alpha}\, dY$$



Too complicated!

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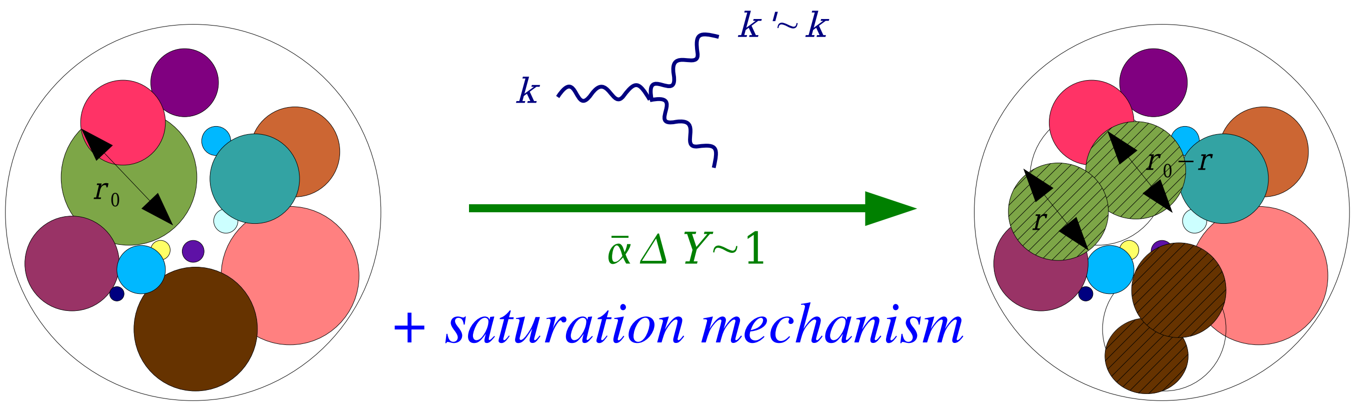


Too complicated!

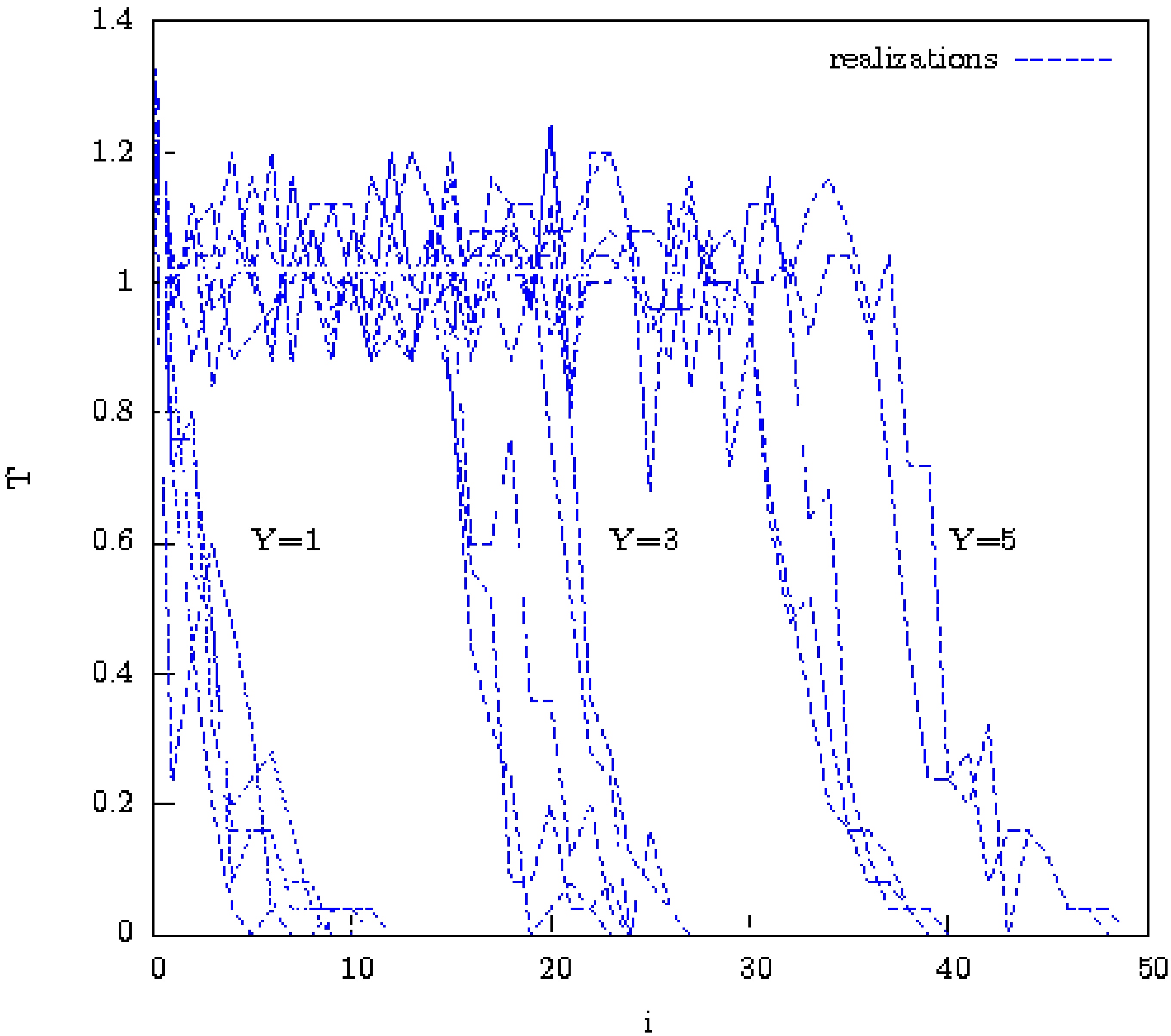
Toy model:

$$dP = \frac{|r_0|}{|r||r_0 - r|} d r \;\; dY$$

+ discretized sizes + saturation mechanism

We have 2 variables (r,b), and we keep the singularity structure of QCD

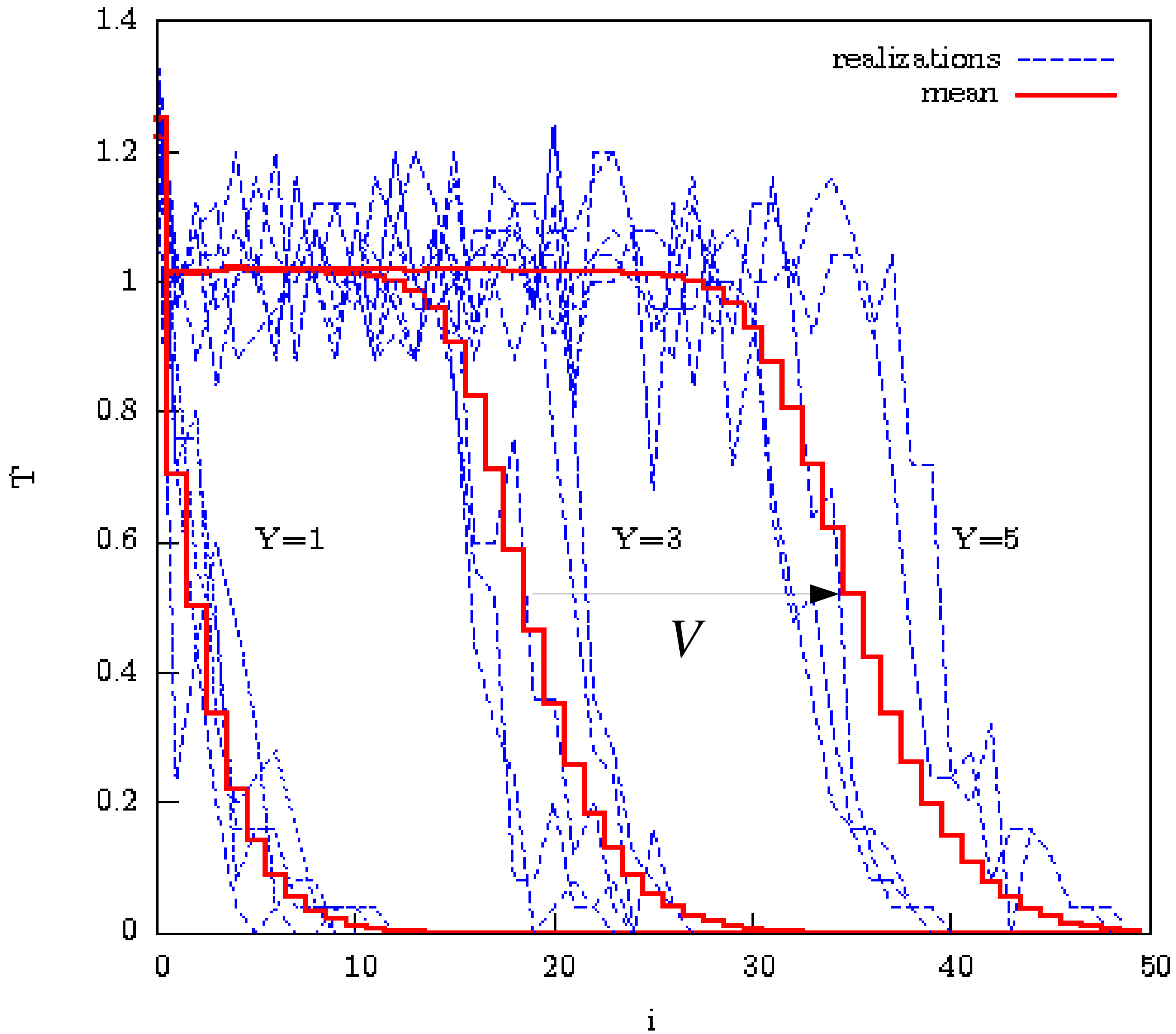
Traveling waves



N=25

One given impact parameter

Traveling waves



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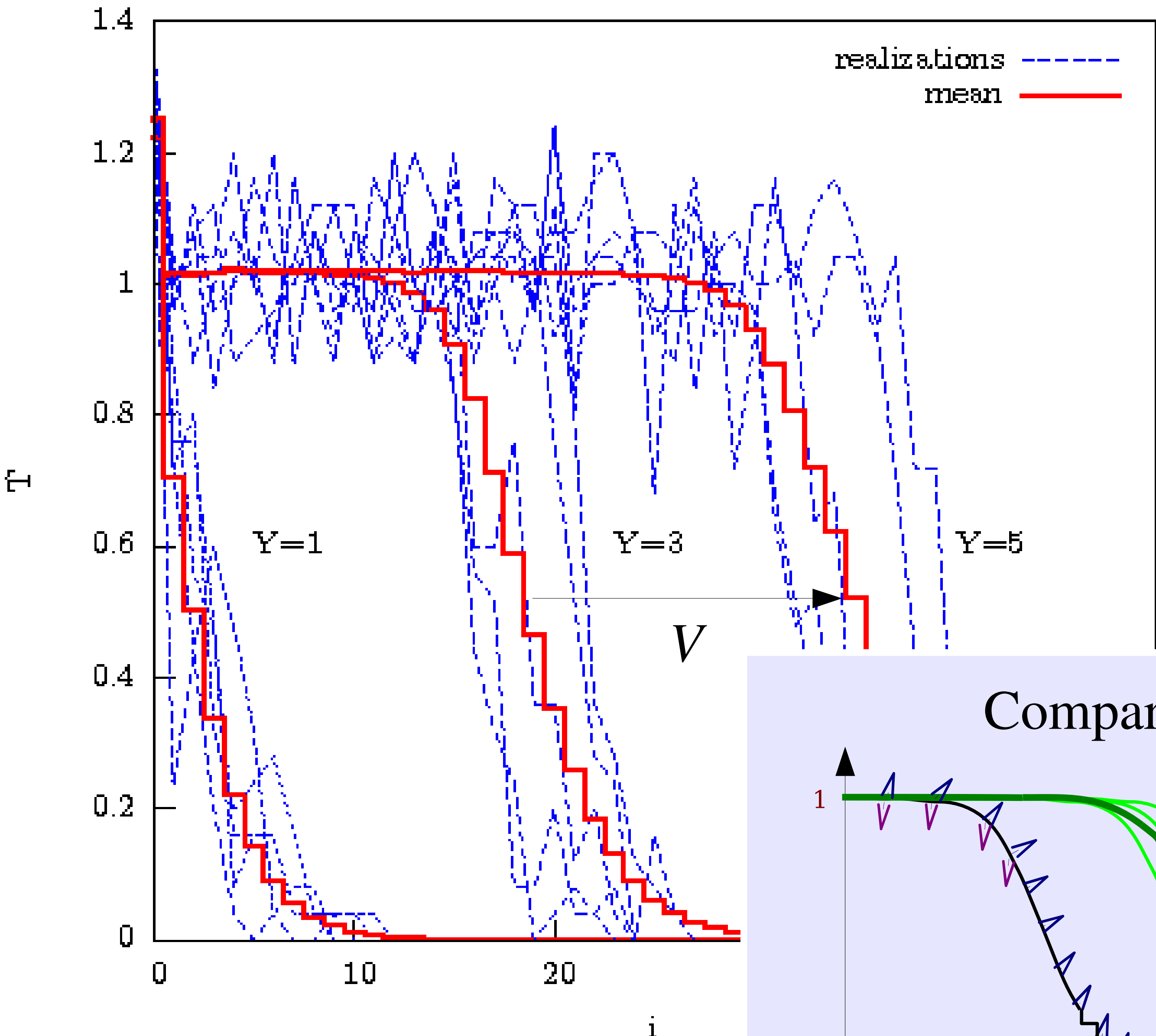

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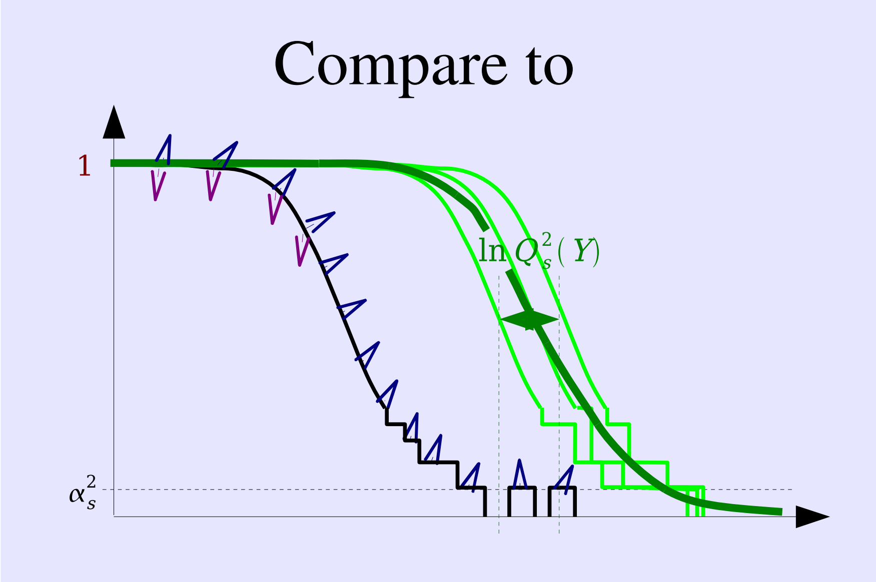
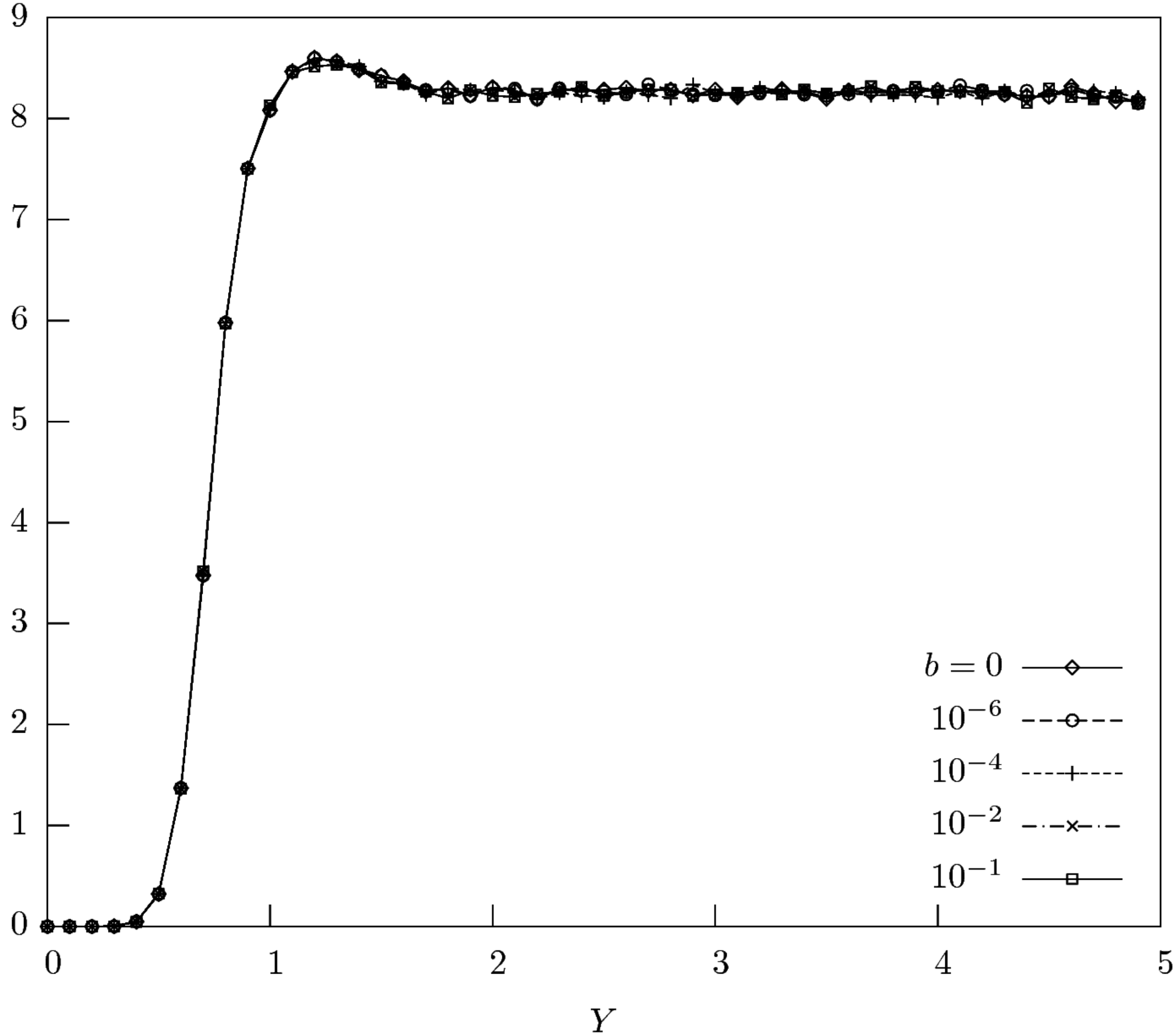


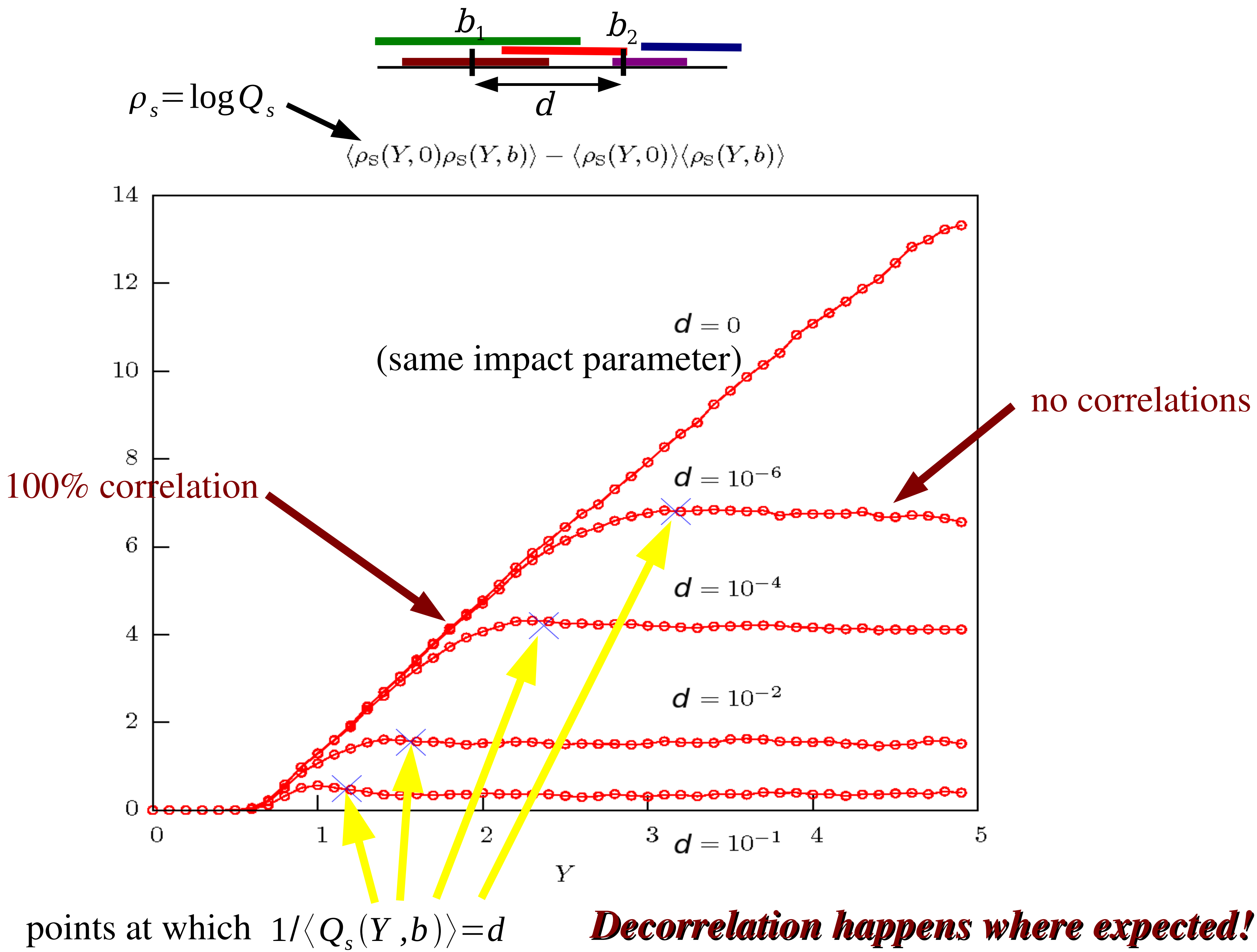
One given impact pa

Traveling wave velocity

$V = \langle \rho_S(Y + dY, b) - \rho_S(Y, b) \rangle / dY$



Correlation of the saturation scales



Decorrelation happens where expected!

A more refined look

Comparison with a fixed impact-parameter version of the model

The toy model is defined by its interval splitting rate

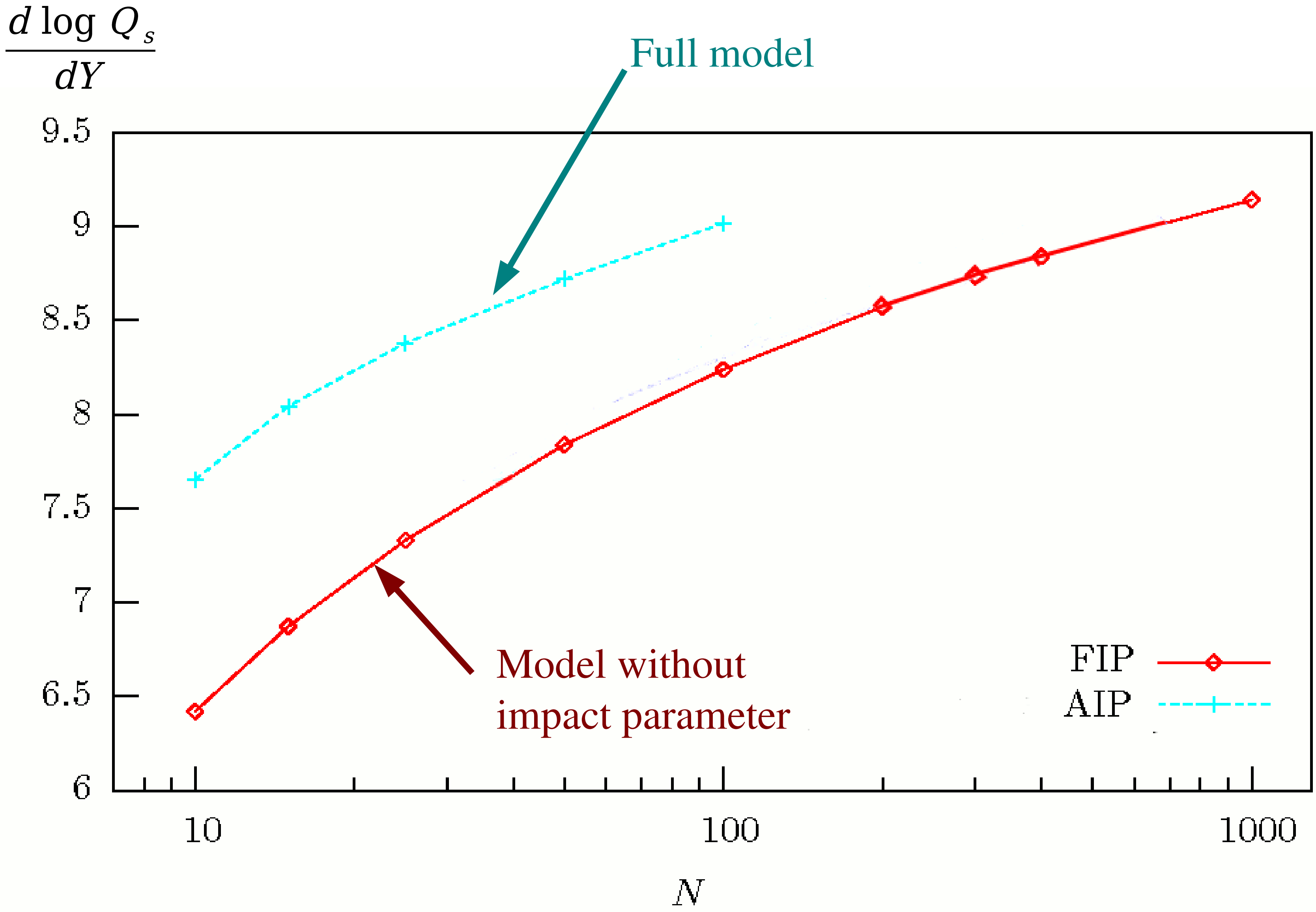
$$dP = \frac{|r_0|}{|r||r_0 - r|} d r \quad dY$$ (+ saturation condition)

which generates a distribution of sizes and impact parameters of intervals.

One may *discard the impact parameter dependence* (this implies a rescaling of the splitting rate) and get a true ***one-dimensional model*** for which only the size matters.

A more refined look

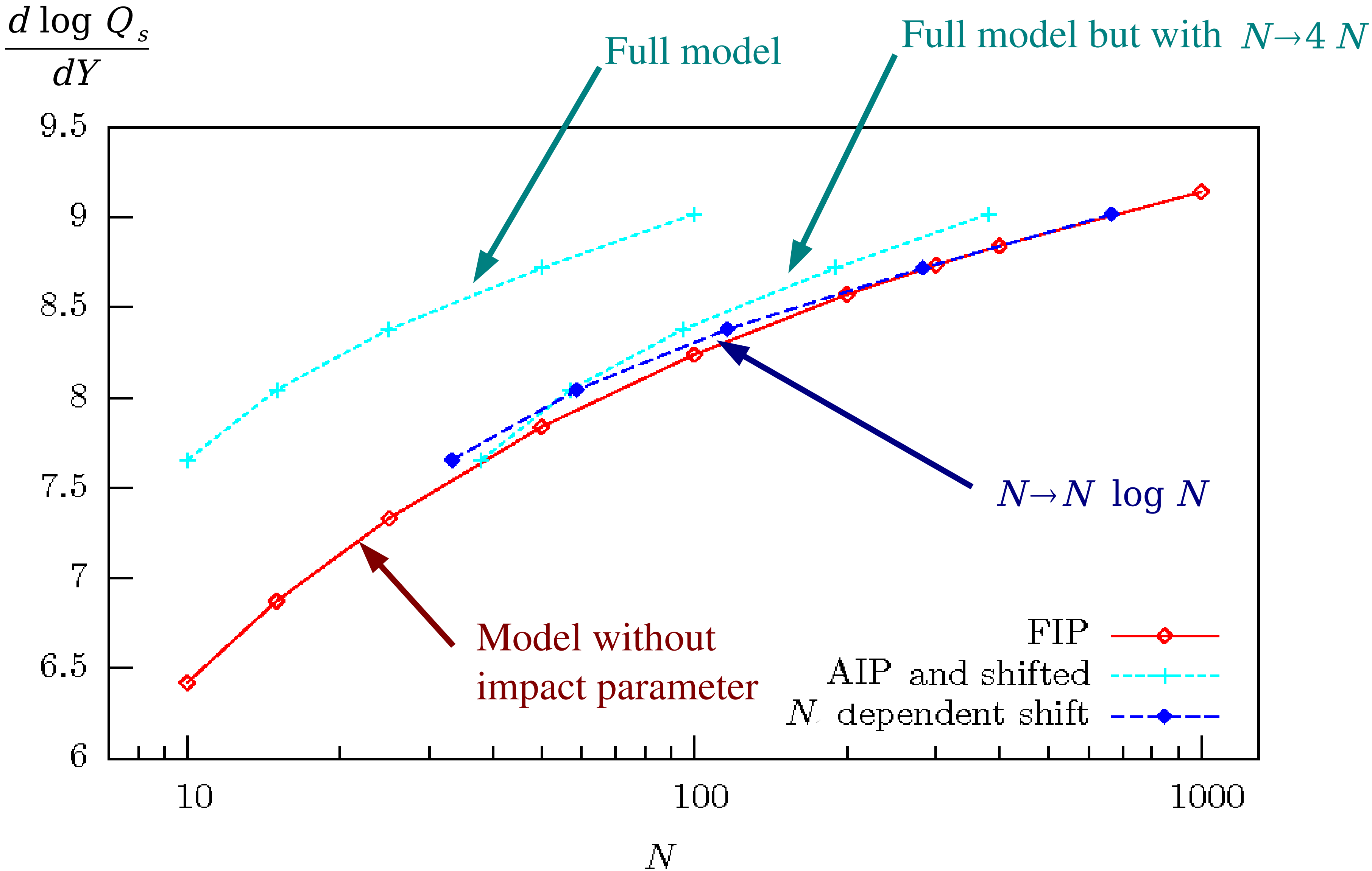
Comparison with a fixed impact-parameter version of the model



Significant disagreement!

A more refined look

Comparison with a fixed impact-parameter version of the model



The disagreement seems to amount to a mere rescaling of N!
(=rescaling of the QCD coupling)

Summary

We have identified, from the physics, the universality class of high energy QCD as the one of *one-dimensional* reaction-diffusion processes, whose dynamics are governed by an equation of the form

$$\partial_{\bar{\alpha} Y} T = \chi(-\partial_{\ln k^2})\, T - T^2 + \alpha_s \sqrt{T}\ \nu$$

We went back to the assumption that the QCD evolutions at different impact parameters decouple.

In a toy model, we have found that this is true.

However, a detailed comparison with a fixed-impact parameter version of the model shows some discreapancy, indicating that **the fixed-impact parameter model has more fluctuations**.