Soft gluon effects in SUSY particle production at the LHC

Dedicated to the memory of Prof. Jan Kwieciński

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Overview

SUSY at LHC

NLO corrections

Soft gluon resummation

Soft anomalous dimension matrices

Phenomenological results

Based on A. Kulesza and LM, arXiv:0807.2405[hep-ph] accepted for publication in Phys. Rev. Lett.

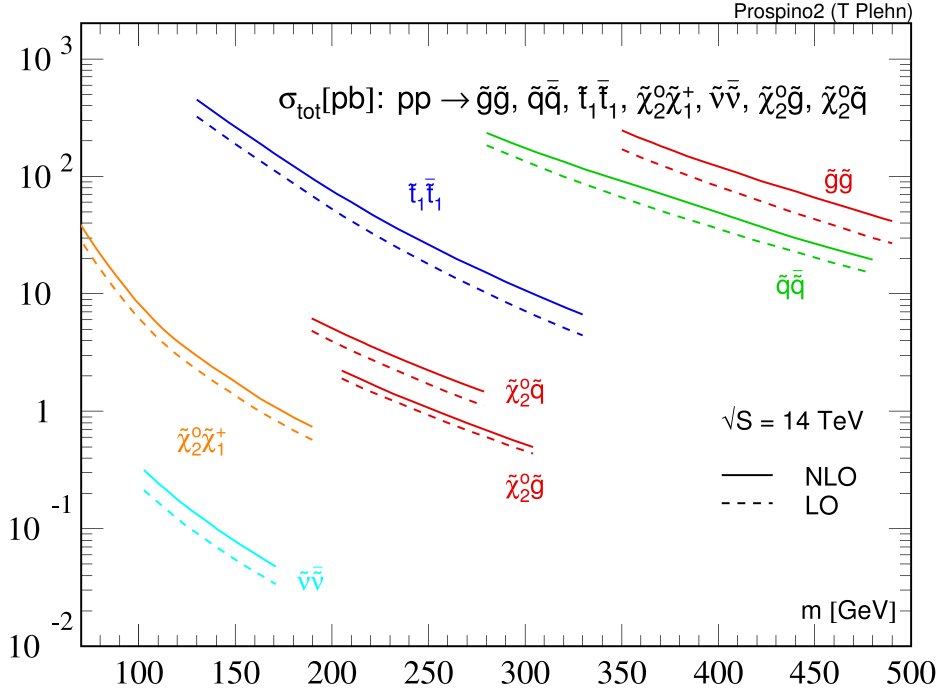
SUSY at LHC

Supersymmetric extension of the SM may be The Theory of physics at Terascale (naturalness, coupling unification, WIMPS, relation to superstrings)

SUSY spectrum contains new heavy particles: e.g. scalar partners of quarks (squarks) and Majorana fermions: gluinos

R-parity: only pair production of SUSY particles possible

Minimal Supersymmetric Standard Model has free parameters, but typically, $\tilde{g}\tilde{g}$ and $\tilde{q}\bar{\tilde{q}}$ production processes are expected to have the largest cross sections at LHC



[T. Plehn, Prospino]

Need for precise predictions

Direct determination of masses of SUSY partners may be difficult:

- long decay cascades, leading to multi-particle final states
- some final state particles should escape detection

Ways out:

- end-points of kinematic distributions
- kinematic fits
- precise measurement of total cross-sections

In general: total cross sections may provide precision tests of SUSY parameters.

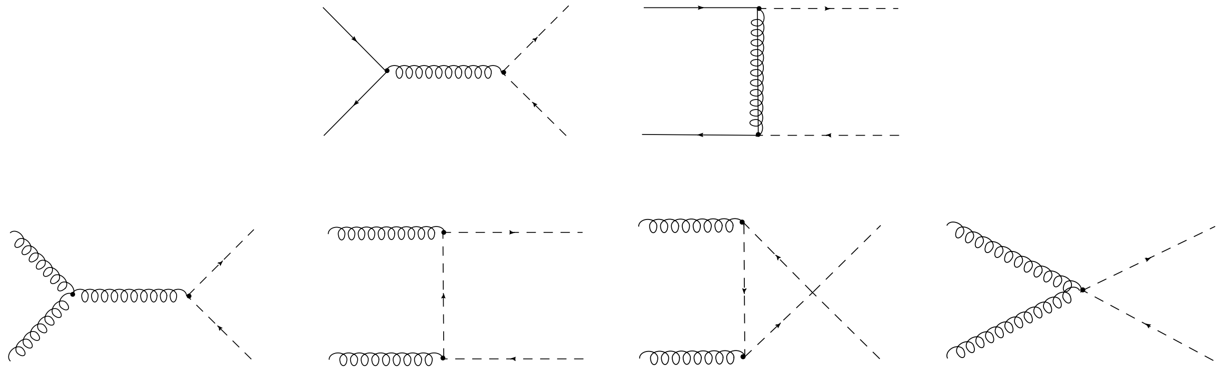
Quality of the tests and parameter determination depends critically on theoretical precision.

LO, NLO results for $\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$ SUSY-QCD corrections are known, but soft gluon corrections are expected to be sizable beyond NLO

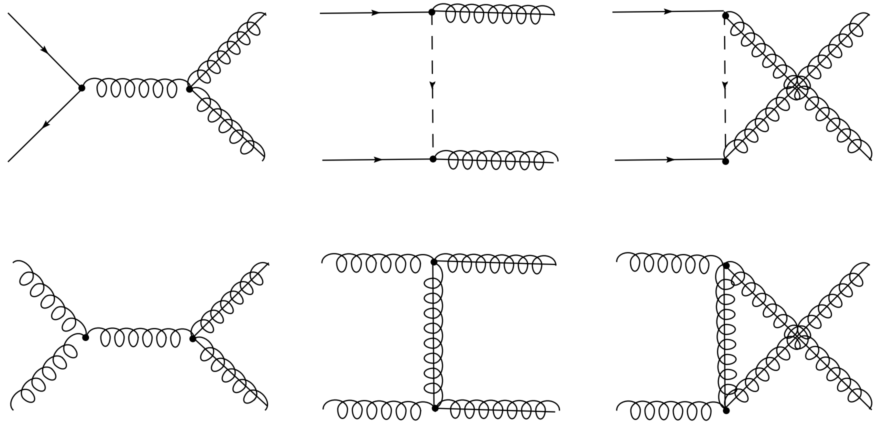
Squark–antisquark and gluino pair production

[Eichten, Dawson, Quigg, 85]

$$q_i\bar{q}_j \longrightarrow \tilde{q}\bar{\tilde{q}},\; gg \longrightarrow \tilde{q}\bar{\tilde{q}}$$



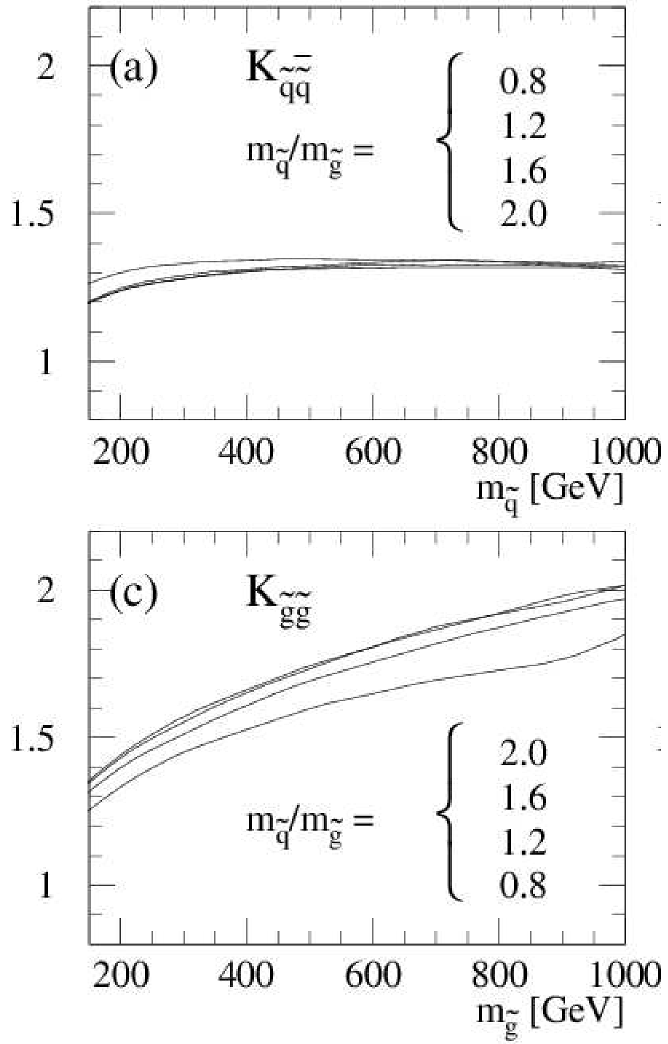
$$q\bar{q} \longrightarrow \tilde{g}\tilde{g},\; gg \longrightarrow \tilde{g}\tilde{g}$$



SUSY-QCD one loop corrections

[Beenakker, Hopker, Spira, Zerwas]

Inclusion of one loop corrections at $\mathcal{O}(\alpha_s)$ from quarks, gluons, squarks and gluinos

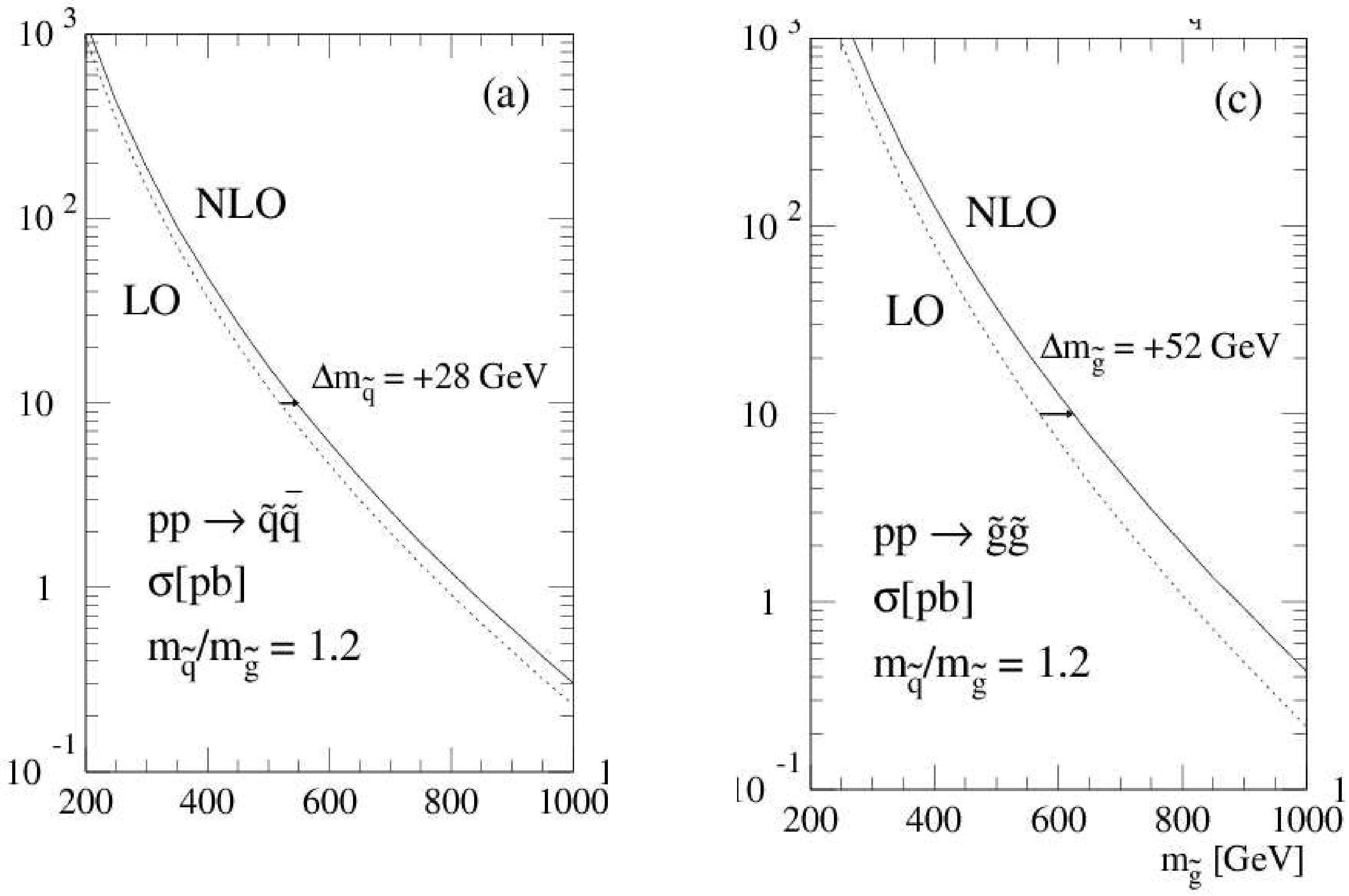


Large NLO K-factors predicted, $K \simeq 1.3$ for $\tilde{q}\bar{\tilde{q}}$ production at the LHC, for $m_{\tilde{q}} = 1$ TeV

$K \simeq 2$ for $\tilde{g}\tilde{g}$ production at the LHC, for $m_{\tilde{g}} = 1$ TeV

Impact of corrections on sparticle mass determination

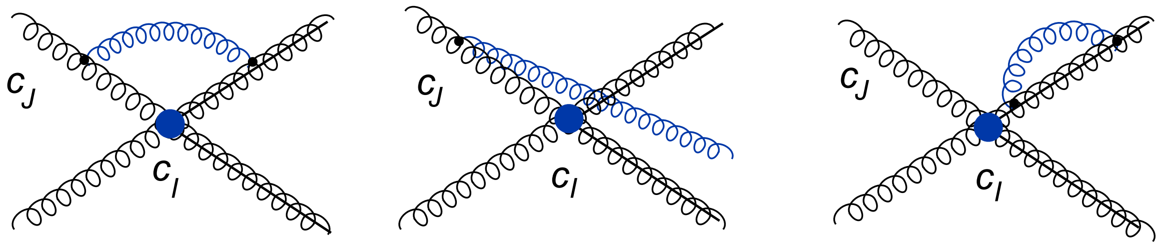
[Beenakker, Hopker, Spira, Zerwas]



Soft gluon corrections at one loop

Threshold limit is defined by $\hat{s} \longrightarrow (2m)^2$

Velocity of produced particle in the c.m.s. $\beta = \sqrt{1 - \frac{4m^2}{\hat{s}}}$



Energy, ω of emitted real gluon close to threshold is kinematically limited,cut-off $\sim m\beta^2$

$\longrightarrow$ lack of cancellations between real and virtual corrections

Soft gluons do not resolve details of the short-distance interaction

Soft-collinear logarithms for incoming parton, incoherent, depending on parton color charge:

$\sim \alpha_s \log^2 \beta^2 \, \sigma^{(0)}$

Soft non-collinear logarithms for the whole matrix element, coherent, depending on color flow: $\sim \alpha_s \log \beta^2 \sigma^{(0)}$

Additionally, Coulomb corrections $\sim \alpha_s / \beta \; \sigma^{(0)}$

It is necessary to resum soft logarithms and Coulomb corrections

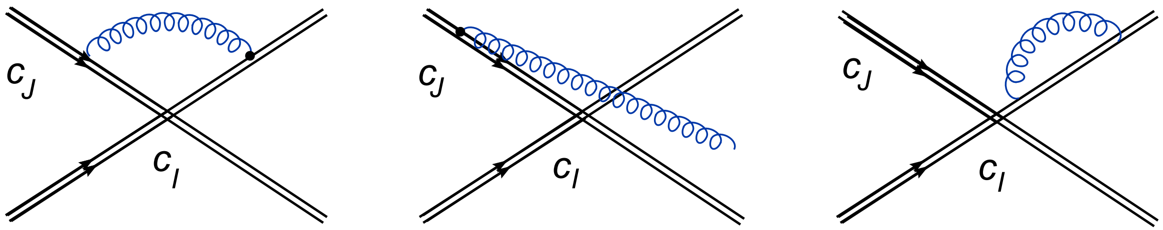
Factorisation of soft gluon effects and eikonal approximation

[Sterman], [Catani, Trentadue]

Soft gluons effects can be factored out from nonperturbative effects and from the hard matrix element

$\longrightarrow$ Factorisation of infra-red singularities

Leading soft gluons effects emerge in the eikonal approximation, $\omega \ll m$



For trivial color flow:

$$\sigma \;\sim\; \underbrace{[f_{i/h_a}\, f_{j/h_b}]}_{\text{parton distributions}} \; \underbrace{\left[\frac{J_i}{\mathcal{J}_i}\,\frac{J_j}{\mathcal{J}_j}\right]}_{\text{matching jet factors}} \otimes \underbrace{\hat{\sigma}_{ij}^{(H)}}_{\text{hard part}} \otimes \underbrace{\mathcal{S}}_{\text{soft part}}$$

In Mellin space: convolutions $\longrightarrow$ products

$$\sigma^N = \sum_{a,b} f_{a/h_a}(N)\, f_{b/h_b}(N)\, \Delta(N+1)\, \sigma^{(H)}(N)$$

Threshold resummation

[Sterman, Kidonakis, Oderda], [Catani, Trentadue]

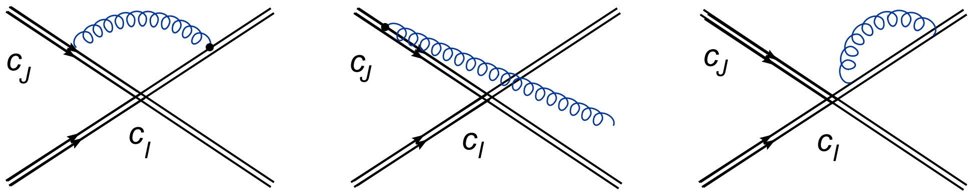
Iteration of soft gluon contributions leads to a tower of corrections

$$\delta\sigma \;\sim\; \sigma^{(0)}\alpha_s^n \log^{2n}\beta$$

$$\delta\sigma \;\sim\; \sigma^{(0)}\alpha_s^n \log^{2n-1}\beta$$

In part of phase space $\alpha_s \log^2\beta > 1$ $\longrightarrow$ resummation is necessary

In general, for non-trivial color flow, soft gluon effects lead to mixing between different color amplitudes



$$\sigma^N = \sum_{a,b,I,J} f_{a/h_a}(N)\, f_{b/h_b}(N)\, \underbrace{\Delta_a(N+1)\,\Delta_b(N+1)}_{collinear}\, h_J^*\, \underbrace{\tilde{S}_{JI}(N+1)}_{soft}\, h_I$$

Scale evolution of soft matrix: renormalisation group

[Sterman], [Catani, Trentadue]

$$\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} S_{IJ} = -(\Gamma_S^\dagger)_{IK} S_{KI} - S_{IL} \Gamma_{LJ}$$

Solution in orthogonal color basis:

$$S_{IJ}(N, \mu^2) = S_{IJ}^{(0)}(1)(N, \mu^2) \exp\left[\int_{\mu^2}^{4m^2/N} \frac{dq^2}{q^2} (\lambda_I^*(\alpha_s(q^2)) + \lambda_J(\alpha_s(q^2)))\right]$$

For color basis in which S is diagonal

$$S_{II}(N, \mu^2) = S_{II}^{(0)}(1, \mu^2) \exp\left[\int_{\mu^2}^{4m^2/N} \frac{dq^2}{q^2} 2\mathrm{Re}(\lambda_I(\alpha_s(q^2)))\right]$$

At NLL, anomalous dimension matrix Γ_{IJ} obtained from IR poles of one-loop diagrams of effective eikonal theory:

$$\Gamma_{IJ} = -\frac{g}{2} \frac{\partial}{\partial g} \mathrm{Res}_{\epsilon \to 0}\, Z_{IJ}(g, \epsilon)$$

Resummed cross section at NLL

$$\sigma^{\rm res}_{h_a,h_b\to kl}(N,\mu^2) \;=\; \sum_{a,b,\mathbf{I}} f_{a/h_a,\,N+1}\, f_{b/h_b,\,N+1}\, \hat{\sigma}^{(0)}_{ij\to kl,\mathbf{I},N}\, \Delta^a_{N+1}\, \Delta^b_{N+1}\, \Delta^{(\rm int)}_{ij\to kl,\mathbf{I},N+1}$$

Universal, soft-collinear factors: $\ln \Delta^i_N = \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \int_{\mu_F^2}^{4m^2(1-z)^2} \frac{dq^2}{q^2} A_i(\alpha_{\rm s}(q^2))$

Soft, non-collinear factor: $\ln \Delta^{(\rm int)}_{ij\to kl,I,N} \;=\; \int_0^1 dz \frac{z^{N-1}-1}{1-z}\, D_{ij\to kl,\mathbf{I}}(\alpha_{\rm s}(4m^2(1-z)^2))$

$$A_i \;=\; \left(\frac{\alpha_{\rm s}}{\pi}\right) A^{(1)} + \left(\frac{\alpha_{\rm s}}{\pi}\right)^2 A^{(2)} + \ldots, \qquad A_i^{(1)},\; A_i^{(2)} \;\propto C_i$$

$$D_{ij\to kl,\mathbf{I}} \;=\; \left(\frac{\alpha_{\rm s}}{\pi}\right) D^{(1)} \;+\ldots$$

Soft anomalous dimension matrix $\longrightarrow$ eigenvalues $\lambda_{ij\to kl,\mathbf{I}}$

$$D^{(1)}_{ij\to kl,\,\mathbf{I}} = 2{\rm Re}(\lambda_{ij\to kl,\mathbf{I}})$$

Anomalous dimension matrices for squark-antisquark production and $q\bar{q} \to \tilde{g}\tilde{g}$

For $\tilde{q}\bar{\tilde{q}}$ results for heavy quarks apply: $(2\mathrm{Re}\,\lambda) = (0, -C_A)$ or $(2\mathrm{Re}\,\lambda) = (0, -C_A, -C_A)$

Color basis for $q\bar{q} \to \tilde{g}\tilde{g}$: $c_1^q = \delta^{\alpha_1\alpha_2}\,\delta^{a_3a_4}$, $c_2^q = T^b_{\alpha_2\alpha_1} d^{ba_3a_4}$, $c_3^q = iT^b_{\alpha_2\alpha_1} f^{ba_3a_4}$,

$$\Gamma^{q\bar{q}\to\tilde{g}\tilde{g}} = \frac{\alpha_s}{\pi}\left[\begin{pmatrix} 6\bar{S} & 0 & -\Omega \\ 0 & 3\bar{S}+\frac{3}{2}\Lambda & -\frac{3}{2}\Omega \\ -2\Omega & -\frac{5}{6}\Omega & 3\bar{S}+\frac{3}{2}\Lambda \end{pmatrix} - \frac{4}{3} i\pi\,\hat{\mathbf{I}}\right]$$

$$\bar{T} \equiv \ln\left(\frac{m^2-\hat{t}}{\sqrt{m^2\hat{s}}}\right) - \frac{1-i\pi}{2}, \quad \bar{U} \equiv \ln\left(\frac{m^2-\hat{u}}{\sqrt{m^2\hat{s}}}\right) - \frac{1-i\pi}{2},$$

$$\bar{S} \equiv -\frac{L_\beta+1}{2}, \quad \Lambda \equiv \bar{T}+\bar{U}, \quad \Omega \equiv \bar{T}-\bar{U}$$

$$\hat{s} = (p_1+p_2)^2, \quad \hat{t} = (p_1-p_3)^2, \quad \hat{u} = (p_1-p_4)^2, \quad L_\beta = \frac{1}{\beta}(1-2m^2/\hat{s})\left(\ln\frac{1-\beta}{1+\beta}+i\pi\right)$$

At threshold, $\Omega \to 0$, $\Gamma \longrightarrow$ diagonal form: diag(λ_i): $(2\mathrm{Re}\,\lambda) \longrightarrow (0, -C_A, -C_A)$

Anomalous dimension matrices $gg \to \tilde{g}\tilde{g}$ production

Color structure: 8 tensors. s-channel basis, $(\mathbf{1}, \mathbf{8_S}, \mathbf{8_A}, \mathbf{10} + \overline{\mathbf{10}}, \mathbf{27}; \mathcal{R})$

$$\Gamma^{gg\to\tilde{g}\tilde{g}} = \frac{\alpha_s}{\pi} \left[\begin{pmatrix} \Gamma_5 & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \Gamma_3 \end{pmatrix} - 3i\pi\,\hat{\mathbf{I}} \right]$$

$$\Gamma_5 = \begin{pmatrix} 6\bar{S} & 0 & 6\Omega & 0 & 0 \\ 0 & 3\bar{S} + \frac{3}{2}\Lambda & \frac{3}{2}\Omega & 3\Omega & 0 \\ \frac{3}{4}\Omega & \frac{3}{2}\Omega & 3\bar{S} + \frac{3}{2}\Lambda & 0 & \frac{9}{4}\Omega \\ 0 & \frac{6}{5}\Omega & 0 & 3\Lambda & \frac{9}{5}\Omega \\ 0 & 0 & \frac{2}{3}\Omega & \frac{4}{3}\Omega & 4\Lambda - 2\bar{S} \end{pmatrix}$$

$$\Gamma_3 = \mathrm{diag}\,(\,3(\bar{S} + \bar{U})\,,\; 3(\bar{S} + \bar{T})\,,\; 3(\bar{T} + \bar{U})\,)$$

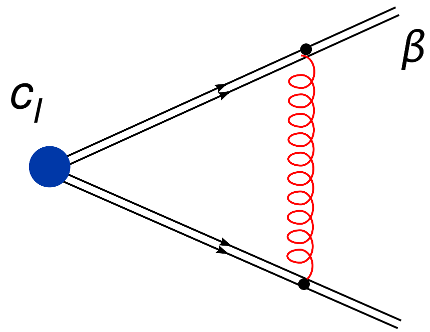
At threshold, $\Gamma \longrightarrow$ diagonal; $(2\,\mathrm{Re}\,\lambda) \longrightarrow (0, -3, -3, -6, -8; -3, -3, -6)$

$\longrightarrow D^{(1)}_{\mathbf{I}}$ at threshold $\sim$ quadratic Casimir operator for the outgoing state

Coulomb corrections

[Sommerfeld],[Catani,Mangano,Nason,Trentadue]

Heavy particles produced at threshold are slow, $\beta \ll 1$



Coulomb type exchanges between final state particles at one loop may be large

$$\delta\sigma(\beta) \;\sim\; \frac{\alpha_s}{\beta}\sigma^{(0)}(\beta)$$

Resummation of Coulomb corrections proposed by Sommerfeld:

$$\sigma_{\mathbf{I}}^{C} \;=\; \frac{\kappa_{\mathbf{I}}\frac{\pi\alpha_s}{\beta}}{1 - \exp\left(-\kappa_{\mathbf{I}}\frac{\pi\alpha_s}{\beta}\right)}$$

and $\kappa_{\mathbf{I}}$ is the color factor

Summary of calculational framework

Born level partonic amplitudes: $\mathcal{M}$

$\Downarrow$

Projection in orthogonal color basis: $h_I = \langle c_I | \mathcal{M} \rangle$ and calculation of partonic cross sections

$$\hat{\sigma}_I^{(0)} = \frac{h_I^* h_I}{\langle c_I | c_I \rangle}$$

$\Downarrow$

Mellin transformation of partonic cross sections: $\hat{\sigma}_I^{(0)}(\rho) \longrightarrow \tilde{\hat{\sigma}}_I^{(0)}(N)$

$\Downarrow$

Inclusion of partonic densities and resummation of soft logs in Mellin space

$$\tilde{\sigma}_I^{(R)}(N) \; = \; \tilde{f}_a(N)\, \tilde{f}_b(N)\, \tilde{\hat{\sigma}}_I^{(0)}(N)\, \Delta(N)$$

$\Downarrow$

Numerical inverse Mellin transform and integration over parton densities

$\Downarrow$

Resummed hadronic cross sections in color channels

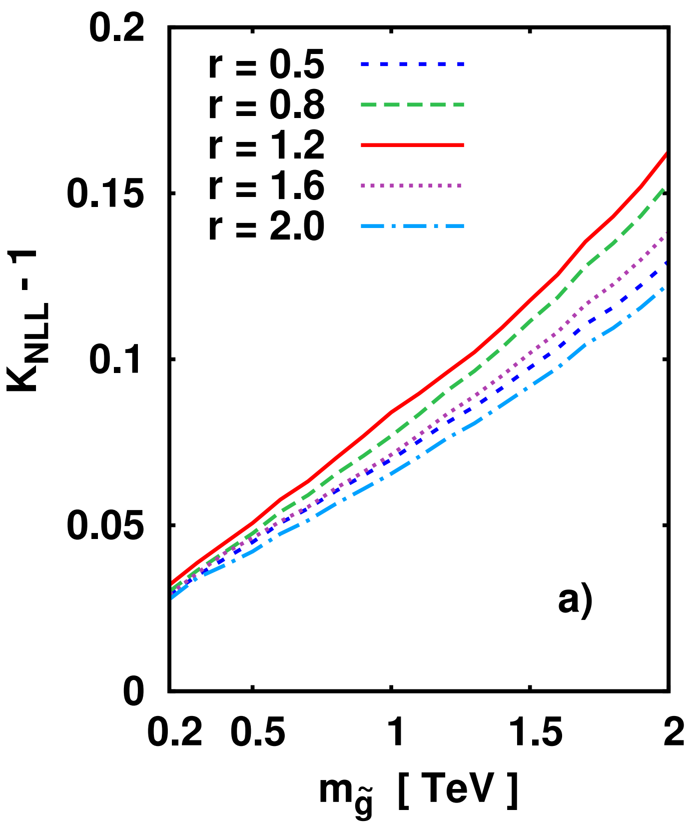
$\Downarrow$

Summation over colors and matching to the NLO result **[PROSPINO]**:

$$\sigma^{\mathrm{match}} = \sigma^{\mathrm{NLO}} + \left[\sigma^{\mathrm{NLL}} - \sigma^{\mathrm{NLL}}|_{\mathrm{NLO}}\right]$$

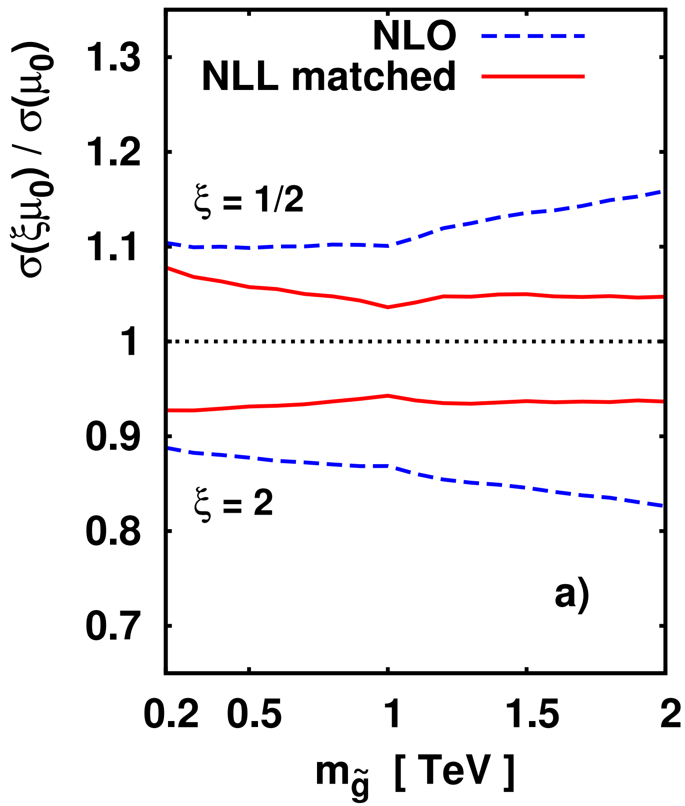
NLL results: $\tilde{g}\tilde{g}$

Relative NLL correction $K_{\rm NLL} - 1$



$r = m_{\tilde{g}}/m_{\tilde{q}}$

Scale dependence

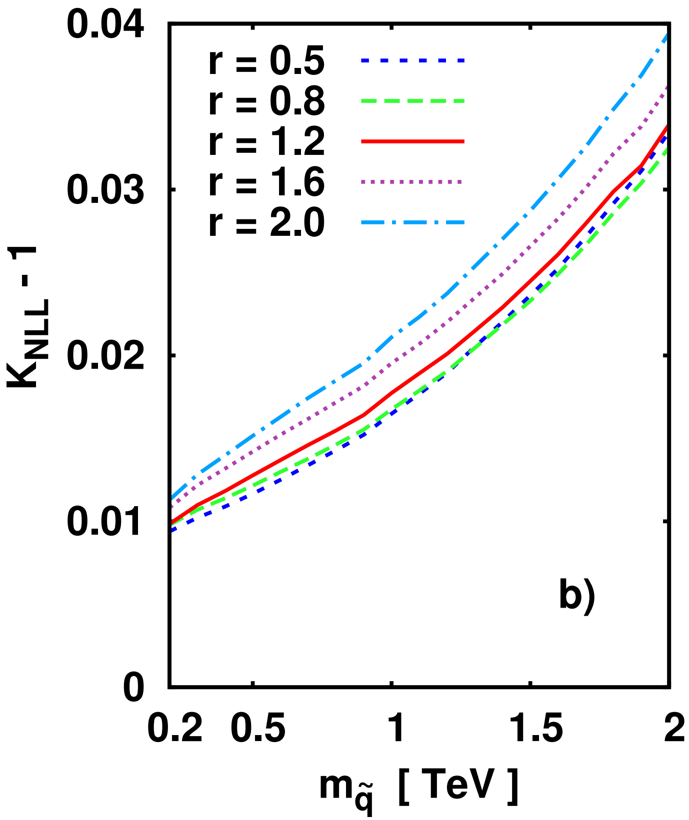


$m/2 < \mu_R = \mu_F < 2m$

For large gluino mass, sizable effects of soft gluons and reduction of scale dependence by factor of ~ 3

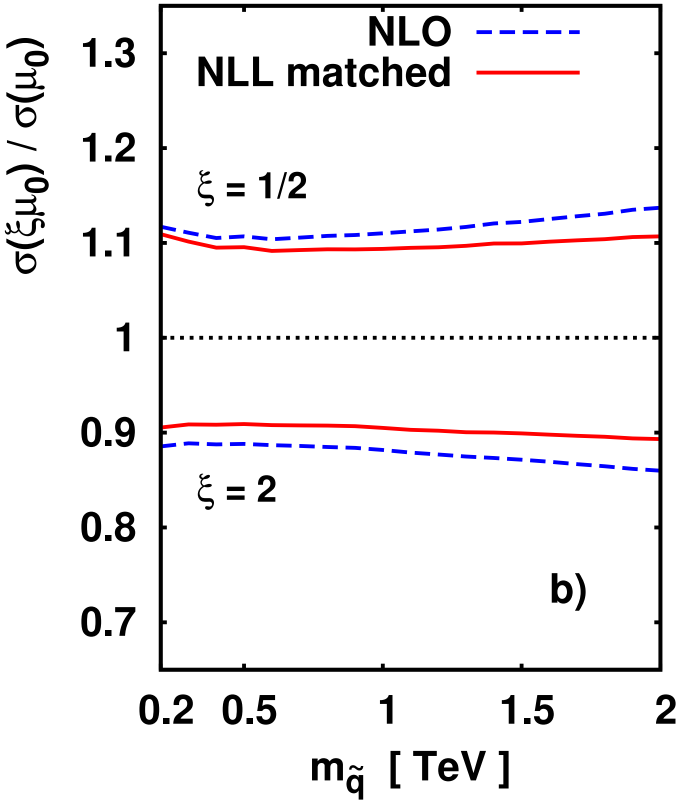
NLL results: $\tilde{q}\bar{\tilde{q}}$

Relative NLL correction $K_{\rm NLL} - 1$



$r = m_{\tilde{g}}/m_{\tilde{q}}$

Scale dependence



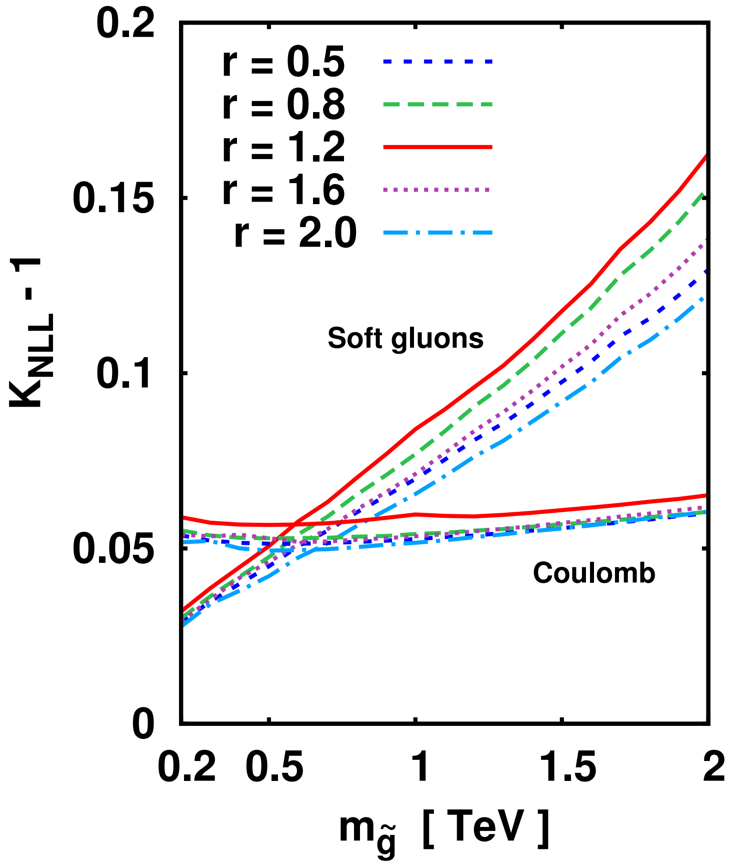
$m/2 < \mu_R = \mu_F < 2m$

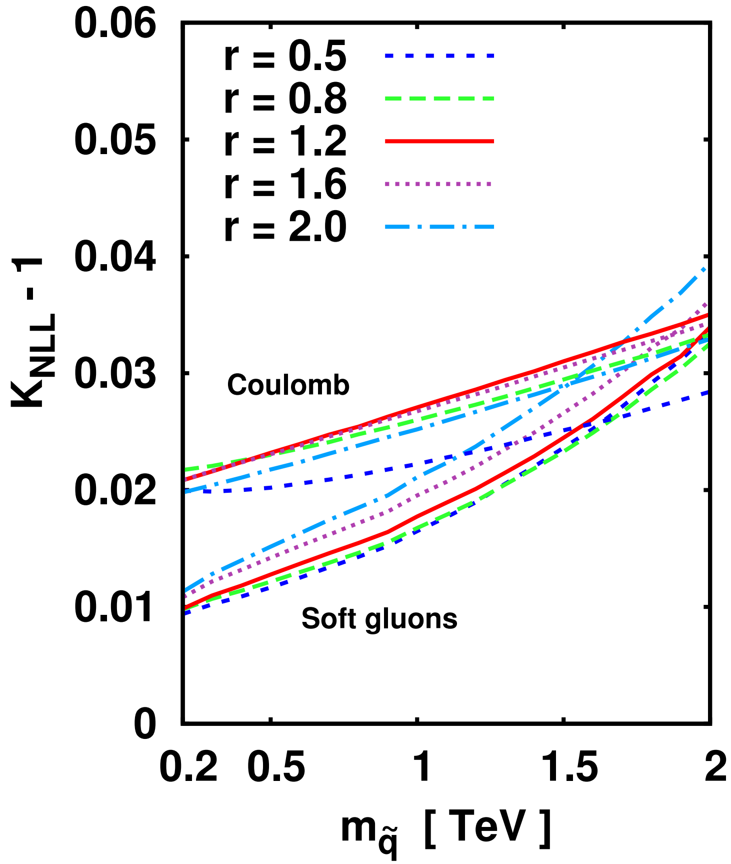
Modest reduction of scale dependence

Inclusion of Coulomb corrections

Ratios of resummed soft gluon corrections and Coulomb corrections, beyond NLO
to the NLO cross sections

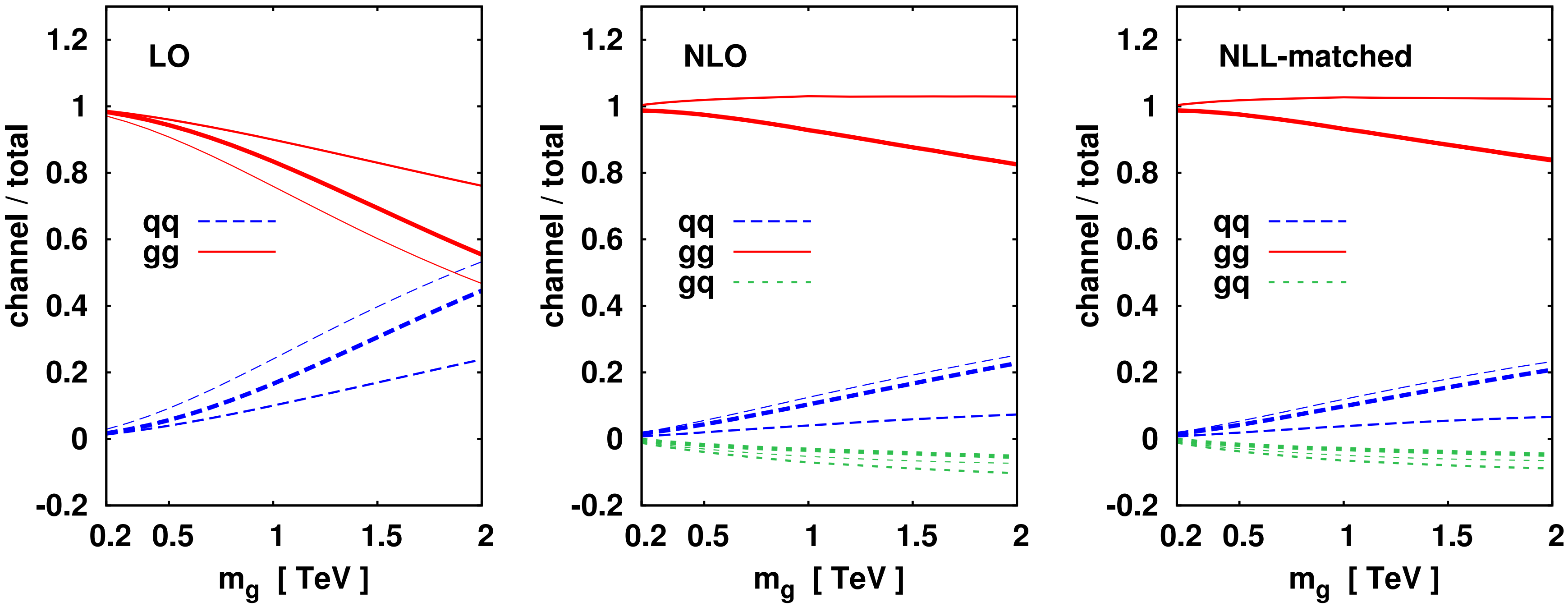
$\tilde{g}\tilde{g}$ $\tilde{q}\bar{\tilde{q}}$





Coulomb corrections are also important

Gluina: subprocess decomposition

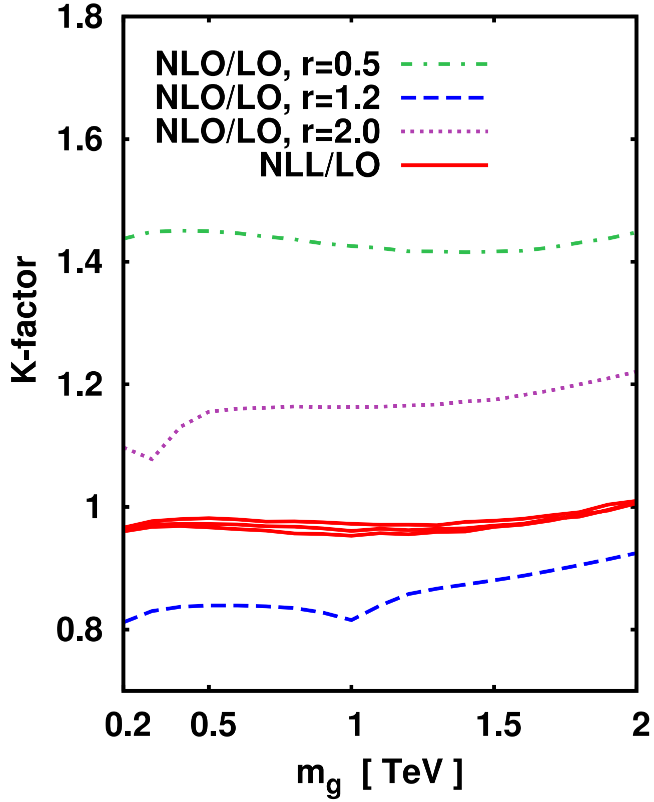


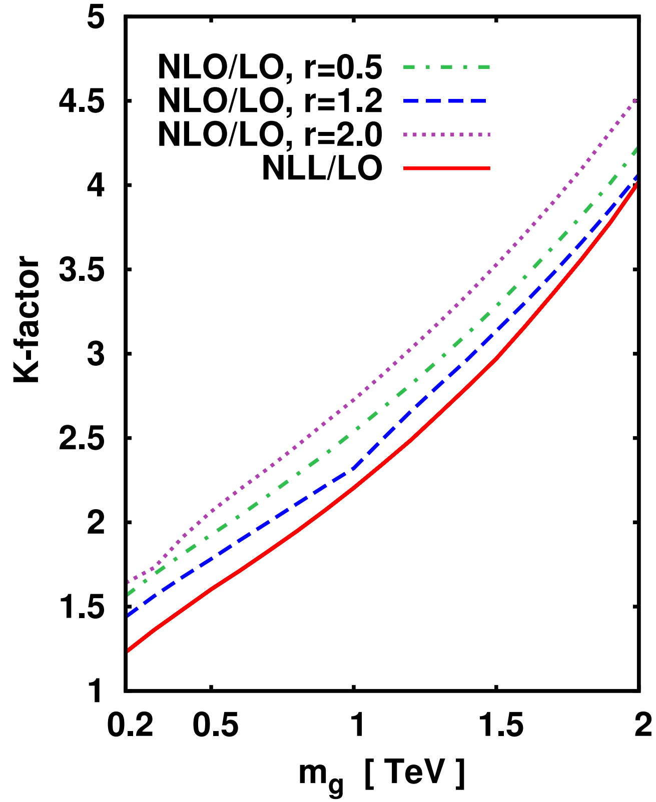
Gluon dominance, enhanced by higher order corrections

Gluina: K-factors for subprocesses

K-factors: $K = \sigma_{\rm NLO}/\sigma_{\rm LO}$ and $K = \sigma_{\rm NLL}/\sigma_{\rm LO}$ for partonic channels

$q\bar{q} \to \tilde{g}\tilde{g}$ $\qquad\qquad$ $gg \to \tilde{g}\tilde{g}$

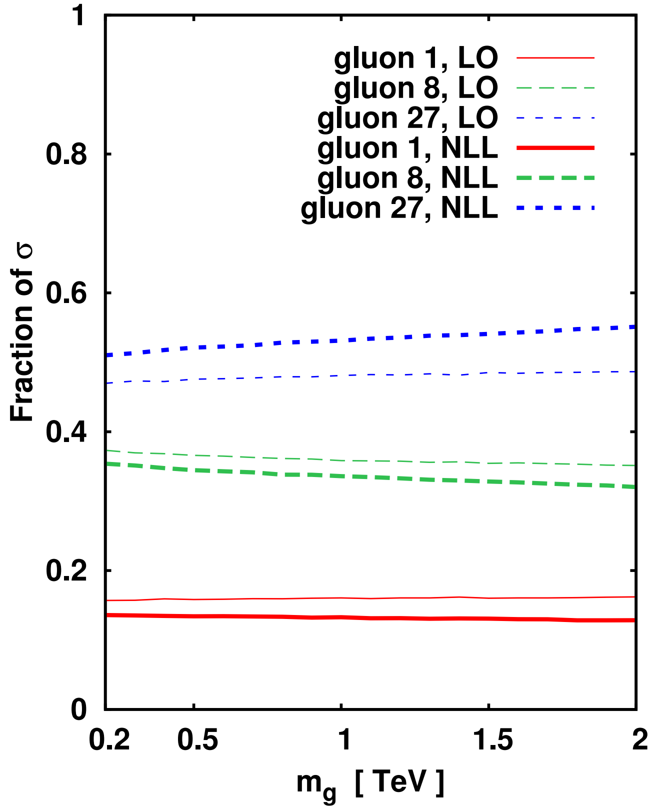


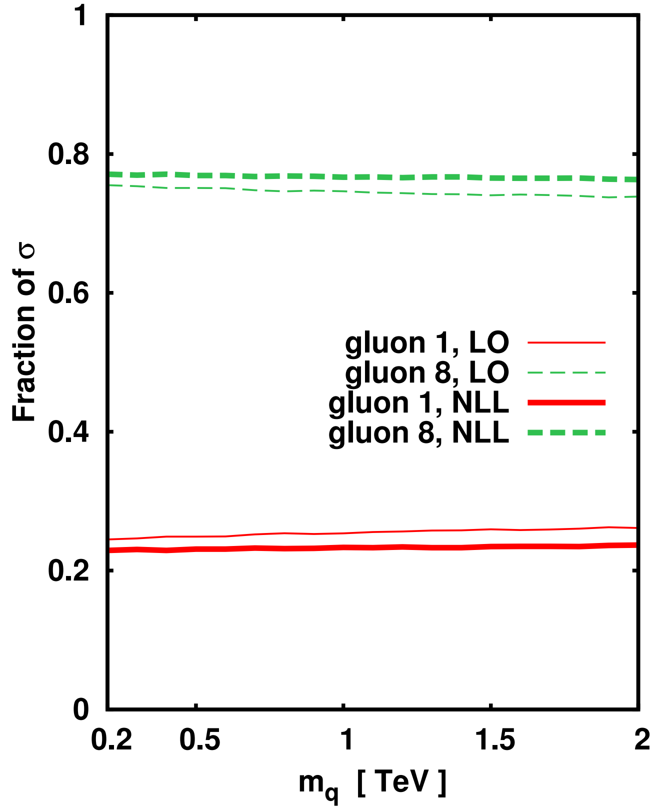


Soft gluons effects are important in gg initiated processes

Gluon initiated subprocess: color channels

$gg \longrightarrow \tilde{g}\tilde{g}$ and $gg \longrightarrow \tilde{q}\bar{\tilde{q}}$ (NLL: thick lines, LO: thin lines)

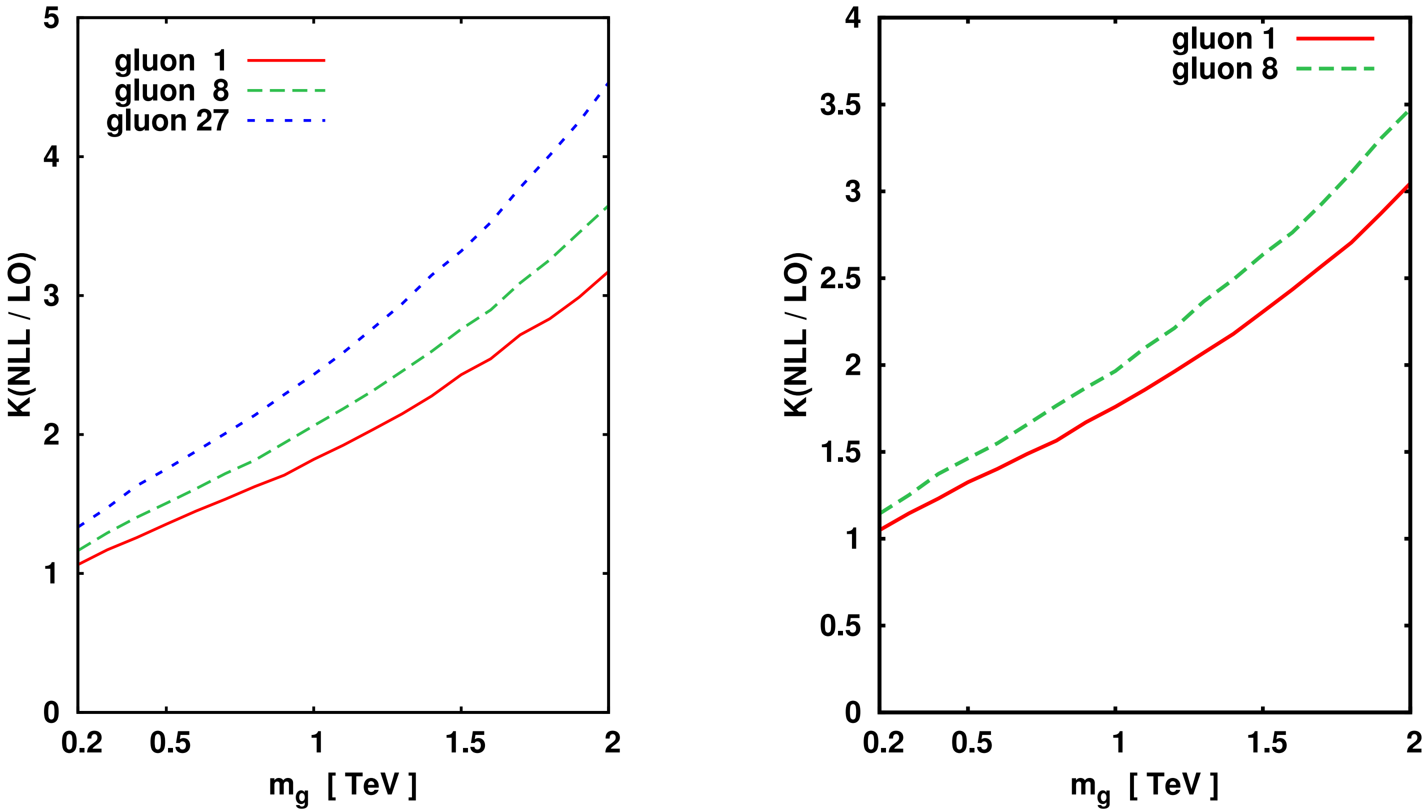




Dominance of higher color charges, additionally enhanced by soft gluon effects

K-factors in gg initiated subprocess: color channels

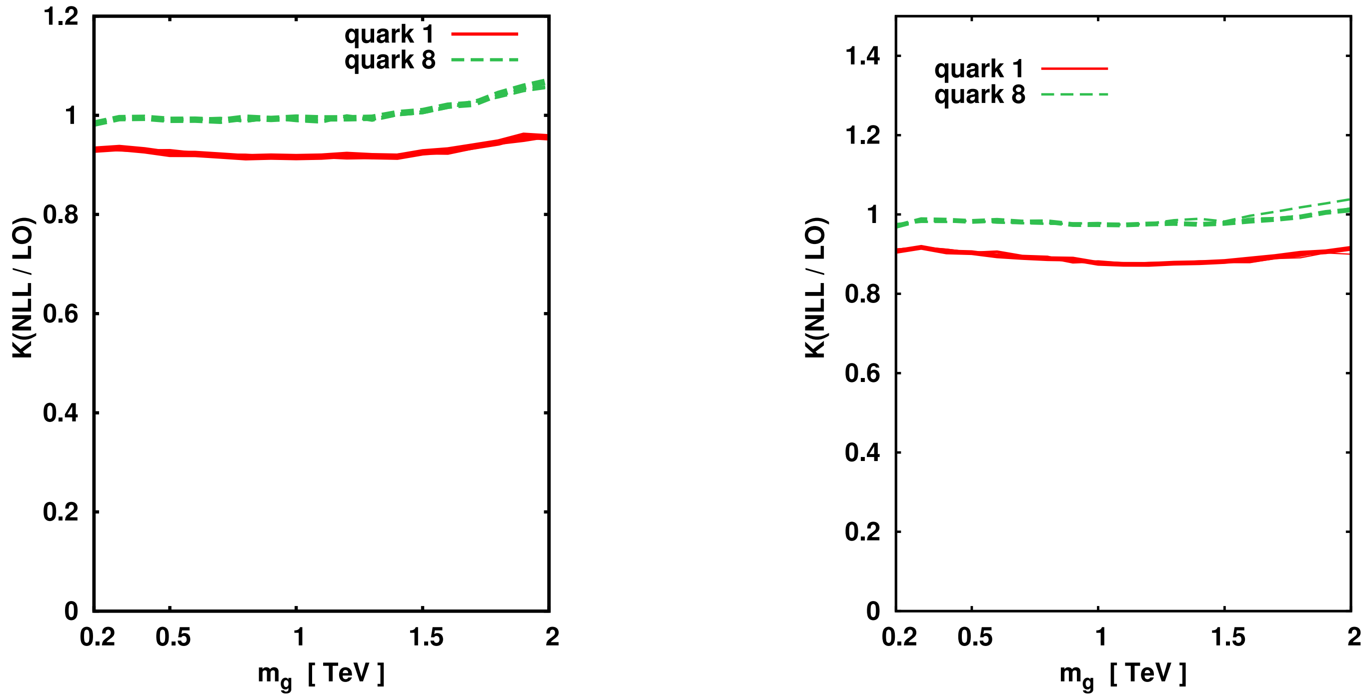
$$gg \longrightarrow \tilde{g}\tilde{g} \text{ and } gg \longrightarrow \tilde{q}\bar{\tilde{q}}$$



Strongest effects for higher color charges

K-factors in $q\bar{q}$ initiated subprocess: color channels

$$q\bar{q} \longrightarrow \tilde{q}\bar{\tilde{q}} \text{ and } q\bar{q} \longrightarrow \tilde{g}\tilde{g}$$



Weak soft gluon effects in $q\bar{q}$ initiated processes

Summary of phenomenological results

- Soft gluon corrections enhance cross sections and grow with color charges of incoming partons and the final state
- Sizable soft gluon corrections found in gg initial states
- Dominance of $gg \to \tilde{g}\tilde{g}$ channel $\longrightarrow$ large soft gluon effects in $\tilde{g}\tilde{g}$ production
- Small soft gluon effects in $q\bar{q}$ initiated processes
- Small soft gluon effects in $\tilde{q}\bar{\tilde{q}}$ production
- Soft gluon resummation reduces significantly theory error for $\tilde{g}\tilde{g}$ production related to scale dependence, but it is not the case for production
- Significant Coulomb corrections

Conclusions

1. Soft anomalous dimension matrices have been found for pair production of gluina (or other heavy $SU(3)$ octet particles)

2. At NLL accuracy, the eigenvalues of soft anomalous dimension matrices are proportional to total color charge of the final (eigen)states

3. We computed cross sections for $\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$ production at the LHC, at the NLL accuracy, including matching to known NLO results and Coulomb corrections

4. Relative soft gluon corrections beyond NLO amount $\mathcal{O}(10\%)$ and Coulomb corrections $\mathcal{O}(5\%)$ for $\tilde{g}\tilde{g}$ production cross section, for $m_{\tilde{g}} = 1$ TeV, smaller effect found for $\tilde{q}\bar{\tilde{q}}$ production

5. Soft gluon resummation reduces significantly theory error for $\tilde{g}\tilde{g}$ production due to scale variations

BACKUP

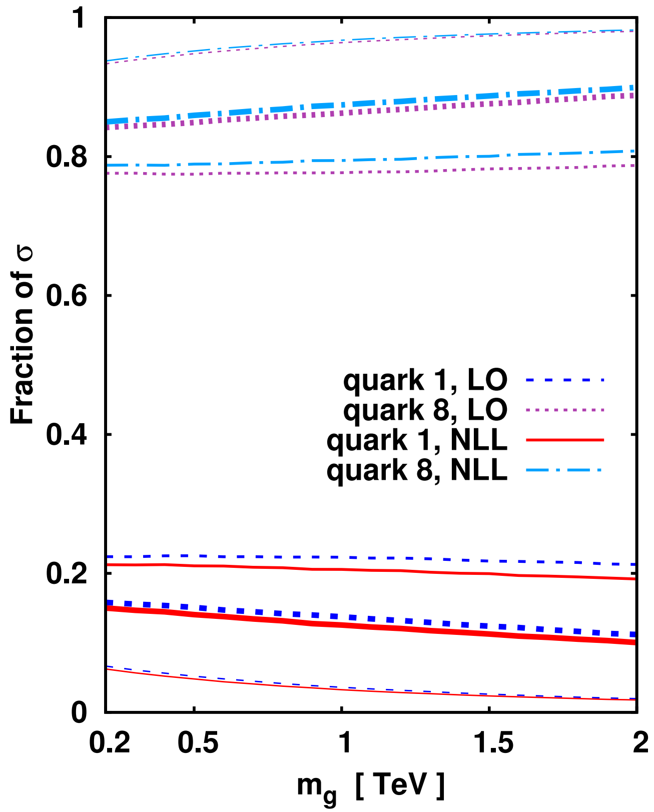
$gg \to \tilde{g}\tilde{g}$: color structures

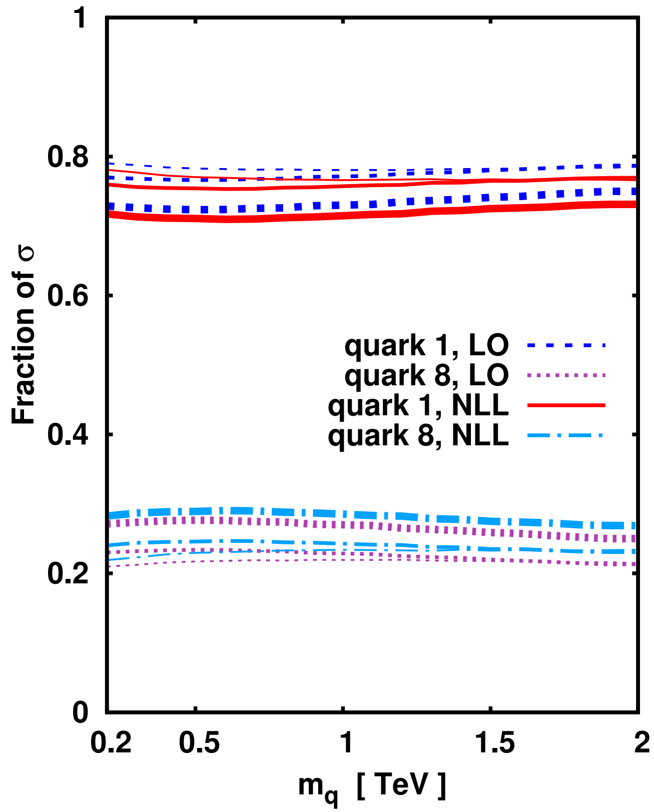
8 tensors, s-channel basis:

$$
\begin{aligned}
c_1^g &= \frac{1}{8}\delta^{a_1a_2}\delta^{a_3a_4}, \\
c_2^g &= \frac{3}{5}d^{a_1a_2b}d^{ba_3a_4}, \\
c_3^g &= \frac{1}{3}f^{a_1a_2b}f^{ba_3a_4}, \\
c_4^g &= \frac{1}{2}\left(\delta^{a_1a_3}\delta^{a_2a_4} - \delta^{a_1a_4}\delta^{a_2a_3}\right) - \frac{1}{3}f^{a_1a_2b}f^{ba_3a_4}, \\
c_5^g &= \frac{1}{2}\left(\delta^{a_1a_3}\delta^{a_2a_4} + \delta^{a_1a_4}\delta^{a_2a_3}\right) - \frac{1}{8}\delta^{a_1a_2}\delta^{a_3a_4} - \frac{3}{5}d^{a_1a_2b}d^{ba_3a_4}, \\
c_6^g &= \frac{i}{4}\left(f^{a_1a_2b}d^{ba_3a_4} + d^{a_1a_2b}f^{ba_3a_4}\right), \\
c_7^g &= \frac{i}{4}\left(f^{a_1a_2b}d^{ba_3a_4} - d^{a_1a_2b}f^{ba_3a_4}\right), \\
c_8^g &= \frac{i}{4}\left(d^{a_1a_3b}f^{ba_2a_4} + f^{a_1a_3b}d^{ba_2a_4}\right).
\end{aligned}
$$

$q\bar{q}$ initiated subprocess: color channels

$q\bar{q} \longrightarrow \tilde{g}\tilde{g}$ and $q\bar{q} \longrightarrow \tilde{q}\bar{\tilde{q}}$





Thick lines $r = 0.5$, Medium lines to $r = 1.2$, thin lines $r = 2.0$