Initial conditions of heavy ion collisions and high energy factorization

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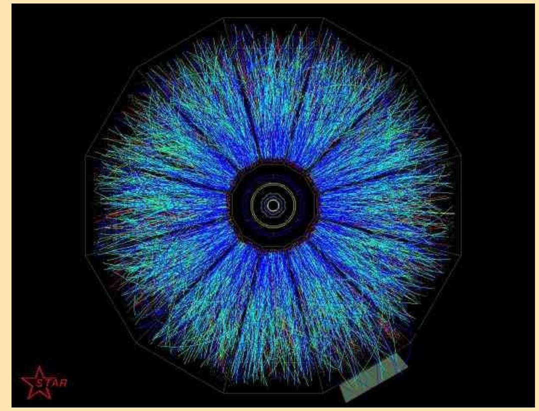
Outline

- Color glass and glasma
- NLO corrections to classical field: JIMWLK factorization
  - Technicalities
  - Single gluon production
  - Rapidity correlations

Talk based on

- F. Gelis, T. L. and R. Venugopalan, "High energy factorization in nucleus-nucleus collisions," Phys. Rev. **D78** 054019 (2008), arXiv:0804.2630 [hep-ph].
- F. Gelis, T. L. and R. Venugopalan, "High energy factorization in nucleus-nucleus collisions II — Multigluon correlations," Phys. Rev. **D78** 054020 (2008), arXiv:0807.1306 [hep-ph].
- F. Gelis, T. L. and R. Venugopalan, "High energy factorization and long range rapidity correlations in the Glasma," arXiv:0810.4829 [hep-ph].

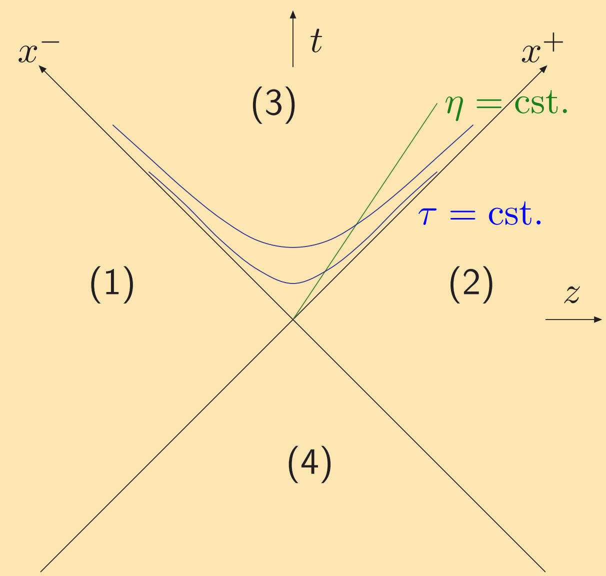
Little bang at RHIC



Collide two heavy nuclei at
$\sqrt{s} = 200 A\mathrm{GeV}$ (RHIC) or $5500 A\mathrm{GeV}$ (LHC).

At early times ($\tau \ll R_A$) expansion is 1-dimensional, to a first approximation boost invariant $\equiv \partial_\eta = 0$.

- Boost invariant *field configurations*
  - ▶ 1 dim. Hubble expansion,
  - ▶ Boost inv. broken by quantum fluctuations
    - ▶ **subject of this talk**
- Coherent initial state
  - ▶ thermalization, isotropization **?**
- Locally isotropic, boost invariant hydrodynamical expansion ?



Coordinate system: Proper time $\tau = \sqrt{2x^- x^+}$, Rapidity $\eta = \frac{1}{2} \ln x^+ / x^-$

Glass and Glasma

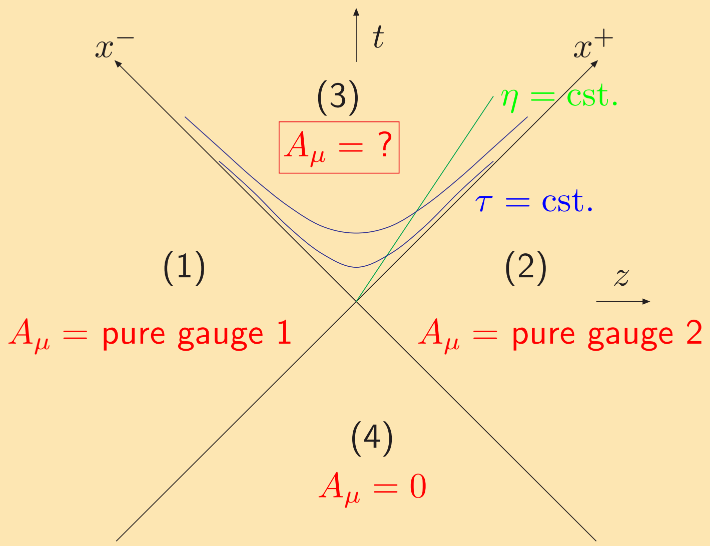
Gluon saturation: At large energies (small x) the hadron/nucleus wavefunction is characterized by saturation scale $Q_s \gg \Lambda_{QCD}$.

At $\mathbf{p}_T \sim Q_s$: strong gluon fields $A_\mu \sim 1/g$ ▶ large occupation numbers $\sim 1/\alpha_s$ ▶ classical field approximation.

CGC: The saturated wavefunction of one hadron/nucleus too many references to mention here

Glasma:[1]

- The coherent, classical field configuration of two colliding sheets of CGC.
- Initial condition for heavy ion collision at $0 < \tau \lesssim 1/Q_s$.



[1] T. Lappi and L. McLerran, *Nucl. Phys.* **A772** (2006) 200 [arXiv:hep-ph/0602189].

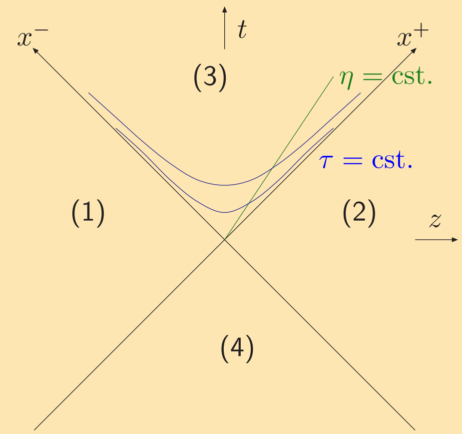
Color Glass Condensate as an effective theory

Separation of scales between small x and large x:

classical field **color charge**

$$[D_\mu, F^{\mu\nu}] \quad = \quad J^\nu$$

$$J^\mu = \delta^{\mu +}\rho_{(1)}(\mathbf{x}_T)\delta(x^-) + \delta^{\mu -}\rho_{(2)}(\mathbf{x}_T)\delta(x^+)$$



What is the charge density $\rho(\mathbf{x}_T)$? A static (**glass!**) stochastic variable, distribution

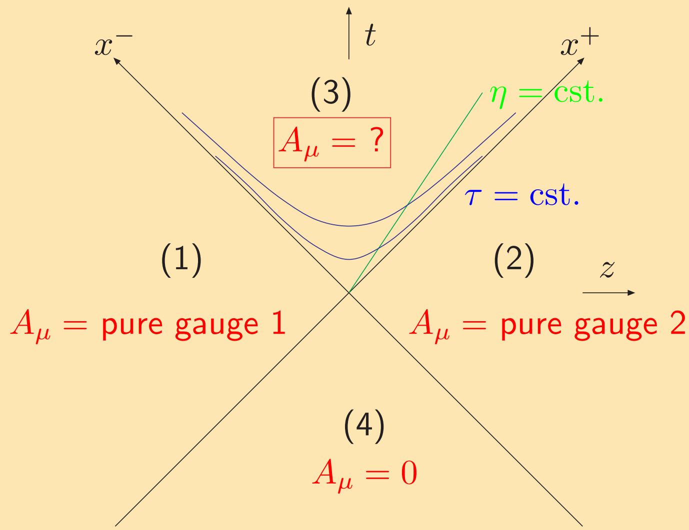
$$\boxed{W_y[\rho(\mathbf{x}_T)]}$$

E.g. MV model [2]: $W[\rho(\mathbf{x}_T)] \sim \exp\left[-\frac{1}{2}\int \mathrm{d}^2\mathbf{x}_T \rho^a(\mathbf{x}_T)\rho^a(\mathbf{x}_T)/g^2\mu^2\right]$

Cannot compute $W_y[\rho(\mathbf{x}_T)]$ from first principles, but can derive evolution equation for $y = \ln 1/x$–dependence: **JIMWLK**. Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

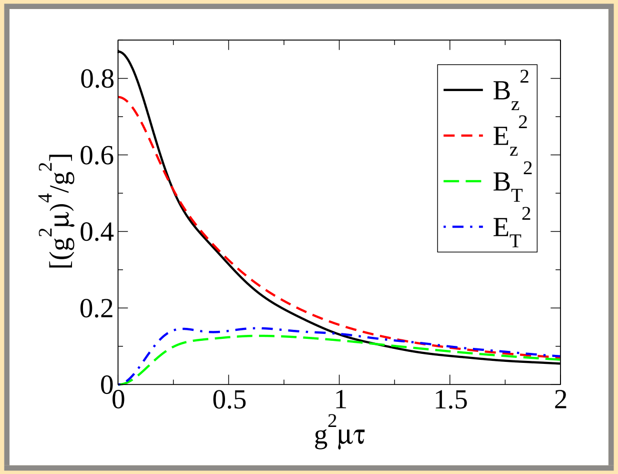
[2] L. D. McLerran and R. Venugopalan, *Phys. Rev.* **D49** (1994) 2233 [arXiv:hep-ph/9309289].

From glass to glasma: initial condition



Analytical solution for (1) & (2): pure gauge

Initial condition (Kovner, McLerran, Weigert[3]) for numerical solution (Krasnitz, Venugopalan[4]) in region (3):



For $Q_s\tau \lesssim 1$ longitudinal fields, negative "longitudinal pressure"

$$T_{zz} = \frac{1}{2}\left(E_T^2 - E_z^2 + B_T^2 - B_z^2\right)$$

no particle interpretation at early times.

[3] A. Kovner, L. D. McLerran and H. Weigert, *Phys. Rev.* **D52** (1995) 3809 [arXiv:hep-ph/9505320].
[4] A. Krasnitz and R. Venugopalan, *Nucl. Phys.* **B557** (1999) 237 [arXiv:hep-ph/9809433].

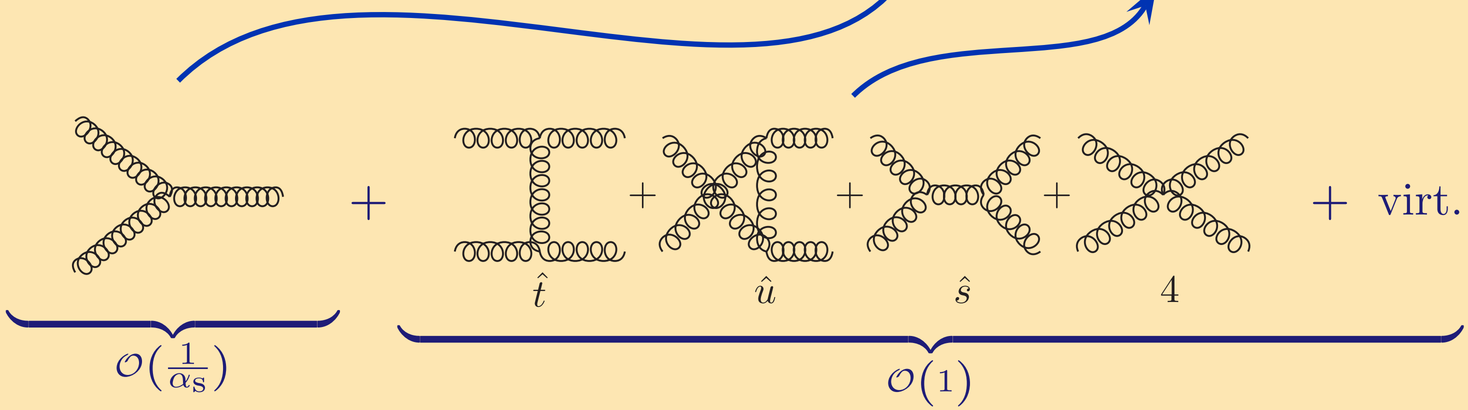
NLO corrections, factorization: BFKL

LO = classical field. What to expect at NLO?

Weak field (small ρ) / BFKL limit:

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_T\,\mathrm{d}y} = \frac{1}{\alpha_\mathrm{s}}\frac{1}{\mathbf{p}_T{}^2}\int \frac{\mathrm{d}^2\mathbf{k}_T}{(2\pi)^2}\phi_y(\mathbf{k}_T)\phi_y(\mathbf{p}_T-\mathbf{k}_T)\left[1+\alpha_\mathrm{s}\int \mathrm{d}y'(\cdots)\right]$$

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_T\,\mathrm{d}y} =$$



Divergence $\Delta y = y - y' \to \infty$ (t–channel) compensated with BFKL evolution of unintegrated pdf's $\phi_y(\mathbf{k}_T)$ ▶ $\mathbf{k}_T$-factorization.

JIMWLK is nonlinear generalization of BFKL: **RGE for** $W_y[\rho(\mathbf{x}_T)]$ (in stead of $\phi_y(\mathbf{k}_T)$)

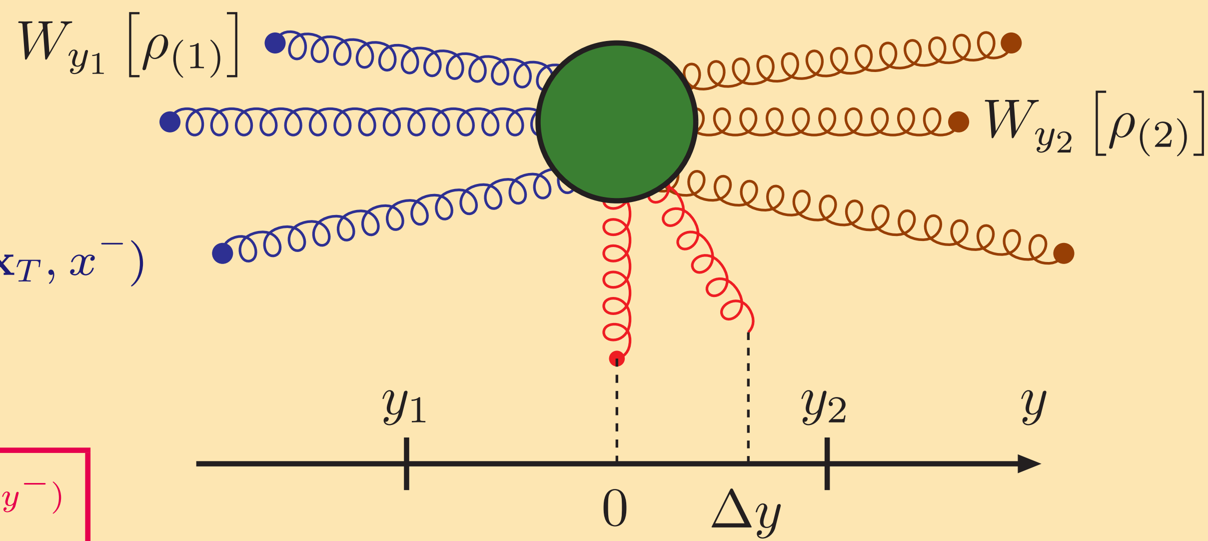
So far **derived** for DIS, but not **proven** to be universal (for gluon production in AA).

NLO corrections: JIMWLK

Restrict $y_1 < \Delta y < y_2$

▼

Physics indep. of $y_1,\ y_2$
(to appropriate order in α_s).



$$\boldsymbol{\nabla}_T{}^2 \mathcal{A}^+(\mathbf{x}_T, x^-) = -g\rho(\mathbf{x}_T, x^-)$$

Wilson line (cf $\sigma_{\rm dip}$ DIS)

$$U(\mathbf{x}_T) = \mathrm{P}e^{i\int \mathrm{d}y^- \mathcal{A}^+(\mathbf{x}_T, y^-)}$$

Sources W evolve with **JIMWLK Hamiltonian**:

$$\mathcal{H} \equiv \frac{1}{2}\int \mathrm{d}^2\mathbf{x}_T\, \mathrm{d}^2\mathbf{y}_T \frac{\delta}{\delta \widetilde{\mathcal{A}}_c^+(\mathbf{y}_T)} \eta^{bc}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \frac{\delta}{\delta \widetilde{\mathcal{A}}_b^+(\mathbf{x}_T)}$$

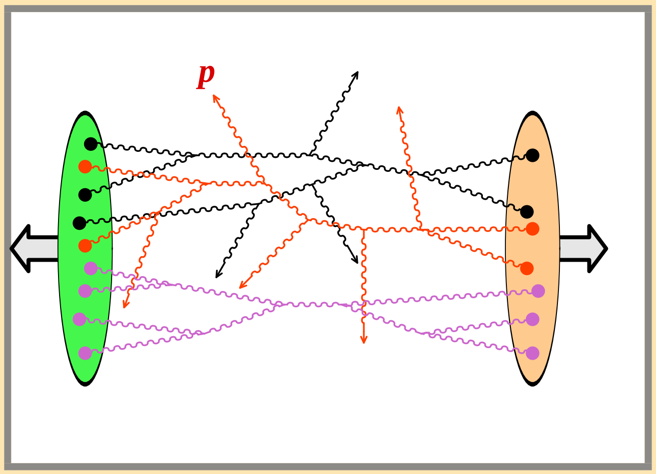
$$\eta^{bc}(\mathbf{x}_T, \mathbf{y}_T) = \frac{1}{\pi}\int \frac{d^2\mathbf{u}_T}{(2\pi)^2} \frac{(\mathbf{x}_T - \mathbf{u}_T)\cdot(\mathbf{y}_T - \mathbf{u}_T)}{(\mathbf{x}_T - \mathbf{u}_T)^2 (\mathbf{y}_T - \mathbf{u}_T)^2}$$
$$\times \left[U(\mathbf{x}_T)U^\dagger(\mathbf{y}_T) - U(\mathbf{x}_T)U^\dagger(\mathbf{u}_T) - U(\mathbf{u}_T)U^\dagger(\mathbf{y}_T) + 1\right]_{bc}$$

Gluon multiplicity as cut vacuum graphs

Particle production with strong external sources Gelis, Venugopalan [5]:

$J^\mu \sim 1/g$ ► All insertions of source at same order

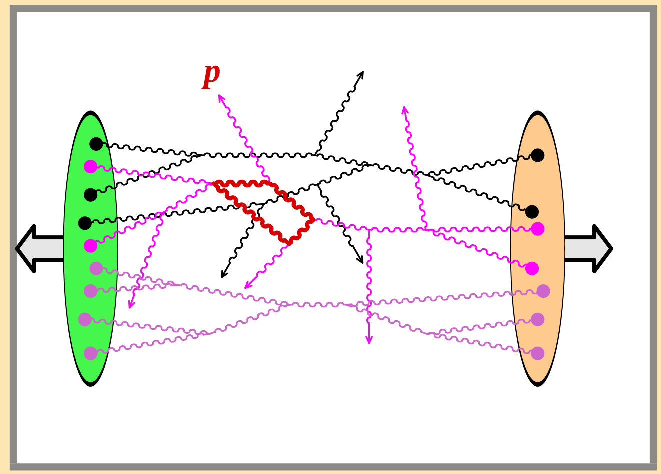
Compute **multiplicity** $$\frac{dN}{d^3\vec{\boldsymbol{p}}} \sim \sum_{n=0}^{\infty} \frac{1}{n!} \int \left[d^3\vec{\boldsymbol{p}}_1 \cdots d^3\vec{\boldsymbol{p}}_n\right] \; \left|\langle \vec{\boldsymbol{p}} \;\; \vec{\boldsymbol{p}}_1 \cdots \vec{\boldsymbol{p}}_n | 0 \rangle\right|^2$$



◄ LO: tree diagrams

NLO: 1 loop ►

Integrate phase space of additional gluons.



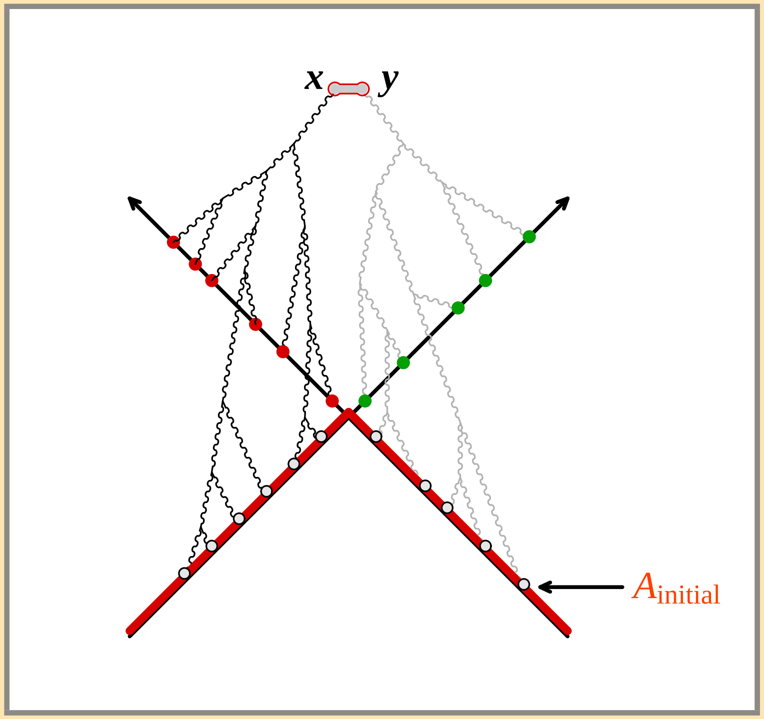
Schwinger-Keldysh formalism, leads to **retarded** propagators.

[5] F. Gelis and R. Venugopalan, *Nucl. Phys.* **A776** (2006) 135 [arXiv:hep-ph/0601209].

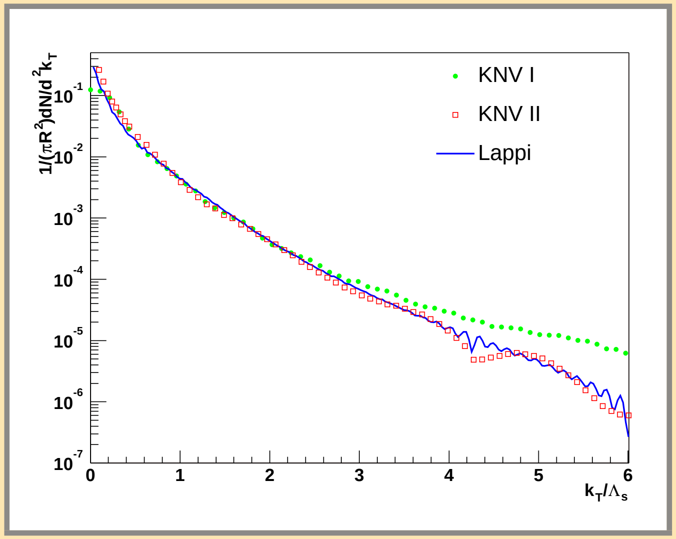
LO is classical field

Leading order multiplicity from **retarded** solution of classical field equations.

$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\mathrm{LO}} = \int_{\vec{\boldsymbol{x}}\vec{\boldsymbol{y}}} e^{i\vec{\boldsymbol{p}}\cdot(\vec{\boldsymbol{x}}-\vec{\boldsymbol{y}})}\,(\cdots)\left[\mathcal{A}^{\mu}(t,\vec{\boldsymbol{x}})\mathcal{A}^{\nu}(t,\vec{\boldsymbol{y}})\right]\Big|_{t\to\infty}$$



Gluon spectrum numerically[6,7]

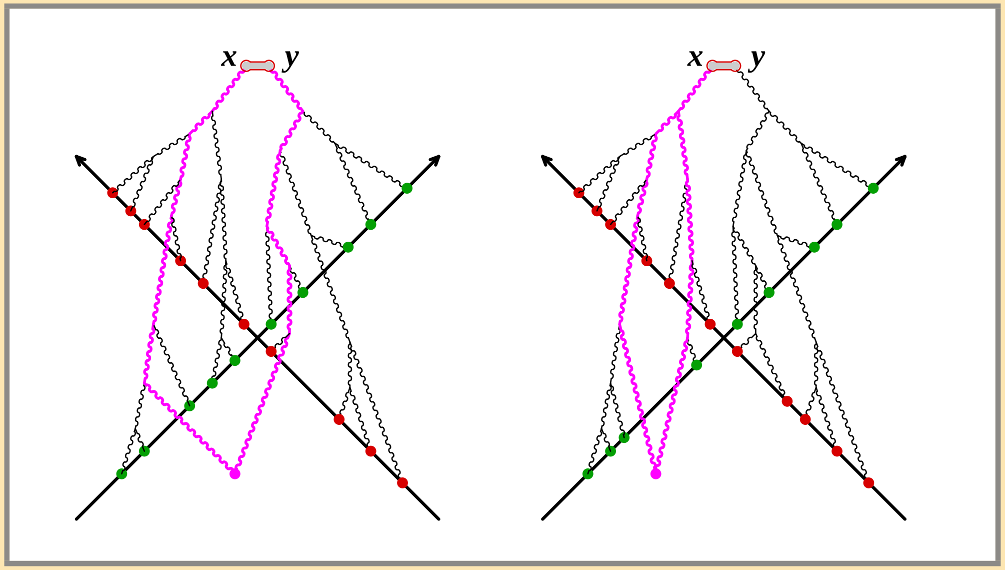


Multiplicity as functional of classical field on initial surface.

[6] A. Krasnitz, Y. Nara and R. Venugopalan, *Phys. Rev. Lett.* **87** (2001) 192302 [arXiv:hep-ph/0108092].
[7] T. Lappi, *Phys. Rev.* **C67** (2003) 054903 [arXiv:hep-ph/0303076].

NLO is 1 loop

"real", pair production



"virtual", loop correction to field at x

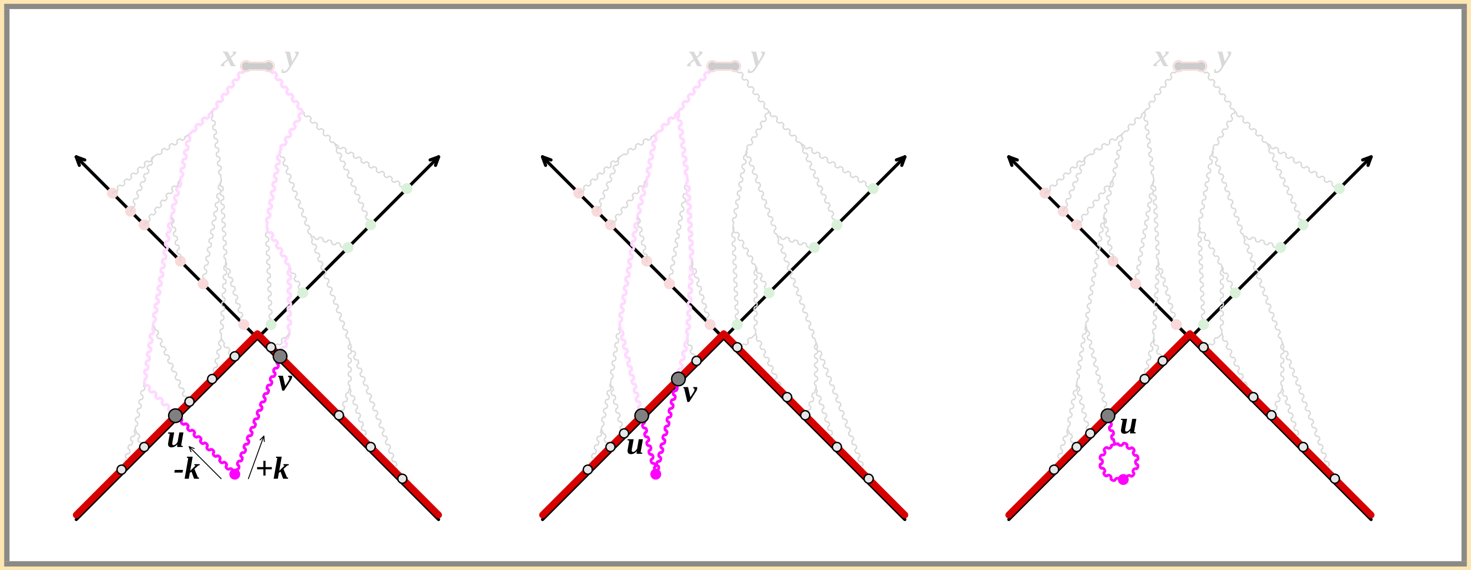
$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm NLO} = \int_{\vec{\boldsymbol{x}}\vec{\boldsymbol{y}}} e^{i\vec{\boldsymbol{p}}\cdot(\vec{\boldsymbol{x}}-\vec{\boldsymbol{y}})} \; (\cdots) \Big[\mathcal{G}^{\mu\nu}(x,y) + \beta^\mu(t,\vec{\boldsymbol{x}})\;\mathcal{A}^\nu(t,\vec{\boldsymbol{y}}) + \mathcal{A}^\mu(t,\vec{\boldsymbol{x}})\;\beta^\nu(t,\vec{\boldsymbol{y}})\Big]\Big|_{t\to\infty}$$

- $\mathcal{G}^{\mu\nu}$ is a 2-point function on top of the classical field: $+-$ Schwinger-Keldysh component, must go on shell.
- β^μ is a small field fluctuation driven by a 1-loop source

Both in terms of retarded propagators ▶ good for eventual real time numerics

Plasma instability of perturbation ?

NLO: propagators as functional derivatives



$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\text{NLO}} = \underbrace{\Big[\frac{1}{2}\int_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in\text{LC}} \mathcal{G}(\mathbf{u}_T,\mathbf{v}_T)\mathbb{T}_{\mathbf{u}_T}\mathbb{T}_{\mathbf{v}_T} + \int_{\vec{\boldsymbol{u}}\in\text{LC}} \beta(\mathbf{u}_T)\mathbb{T}_{\mathbf{u}_T}\Big]}_{\text{below LC}} \underbrace{\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\text{LO}}}_{\text{above LC}}$$

$$a^\mu(x) = \int_{\vec{\boldsymbol{u}}\in\text{LC}} a(\vec{\boldsymbol{u}})\cdot\mathbb{T}_{\mathbf{u}_T}\mathcal{A}^\mu(x) \qquad \mathcal{G}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}) \equiv \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3 2E_{\boldsymbol{k}}}\,\eta_{-\boldsymbol{k}}(u)\,\eta_{+\boldsymbol{k}}(v)$$

Divergence from $\int \frac{\mathrm{d}k^+}{k^+}$ Functional derivative in $\mathbb{T}$ ▶ $\frac{\delta}{\delta\widetilde{\mathcal{A}}_c^+(\mathbf{y}_T)}$ in JIMWLK

Some aspects of the factorization theorem[8]

- High energy kinematics: fixed $Q^2 \sim Q_\mathrm{s}^2$, large $\sqrt{s} \sim e^y$ ▶ weak field limit is BFKL
- Not factorization of pdf's, but color charge distributions
- Power counting: sources $\sim 1/g$
  - Nonperturbative, all orders in classical field
  - NLO in weak coupling/loop expansion (not all orders)
- Work with multiplicities, not cross sections
  - Most natural thing to look at in multiparticle production
  - Retarded propagation
  - Diffractive observables **?**
- Express retarded propagators as functional derivatives wrt. initial condition ▶ relate to functional derivatives in JIMWLK Hamiltonian

[8] F. Gelis, T. Lappi and R. Venugopalan, *Phys. Rev.* **D78** (2008) 054019 [arXiv:0804.2630 [hep-ph]].

Multiple gluon production[9]

Generating functional $\mathcal{F}[z(\boldsymbol{p})] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \Big[\prod_{i=1}^{n} d^3\boldsymbol{p}_i \, (z(\boldsymbol{p}_i) - 1) \Big] \frac{\mathrm{d}^n N_n}{\mathrm{d}^3\boldsymbol{p}_1 \cdots \mathrm{d}^3\boldsymbol{p}_n}$

We develop Taylor coefficients around $z = 1$ to NLO, i.e.

$$\Big\langle N \Big\rangle \quad \Big\langle N(N-1) \Big\rangle \quad \ldots \quad \Big\langle N(N-1)\cdots(N-n+1) \Big\rangle$$

Result:

$$\Big\langle \frac{\mathrm{d}^n P_n}{\mathrm{d}^3\boldsymbol{p}_1 \cdots \mathrm{d}^3\boldsymbol{p}_n} \Big\rangle = \int_{\rho_1,\rho_2} \underbrace{W_Y[\rho_1] W_Y[\rho_2]} \frac{1}{n!} \frac{\mathrm{d}N}{\mathrm{d}^3\boldsymbol{p}_1} \cdots \frac{\mathrm{d}N}{\mathrm{d}^3\boldsymbol{p}_n} e^{-\int \mathrm{d}^3\boldsymbol{p} \frac{\mathrm{d}N}{\mathrm{d}^3\boldsymbol{p}}}$$

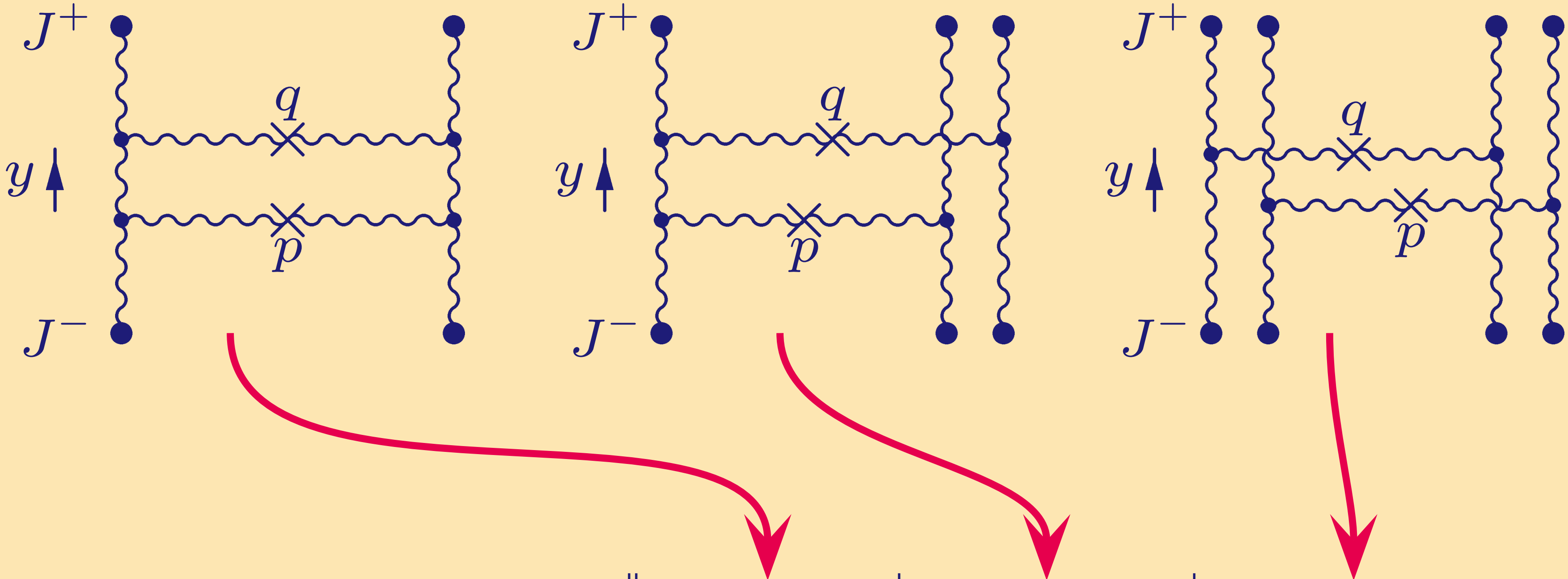
Note: Poissonian-looking form result of choosing to develop and truncate moments that are simple $\langle N \rangle^n$ for Poisson

Nontrivial result is that the LLog corrections factorize into evolution of source.

[9] F. Gelis, T. Lappi and R. Venugopalan, *Phys. Rev.* **D78** (2008) 054020 [arXiv:0807.1306 [hep-ph]].

Power counting: AA is simpler than pA or pp

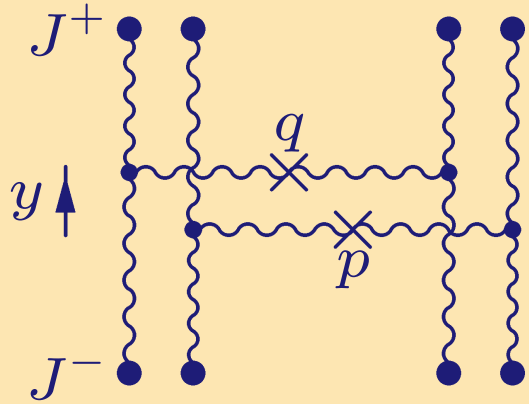
Some of the diagrams for 2-gluon production:



	connected	interference	disconnected
"pp", $J^\pm \sim g$	g^8	g^{10}	g^{12}
"pA", $J^+ \sim g$, $J^- \sim 1/g$	g^4	g^4	g^4
"AA", $J^\pm \sim 1/g$	g^0	g^{-2}	g^{-4}

"AA" classical, "pp" quantum, "pA" both.

Two gluon production

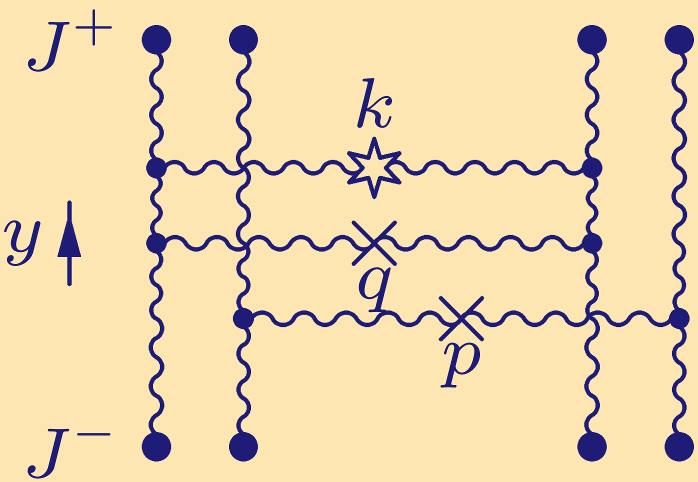


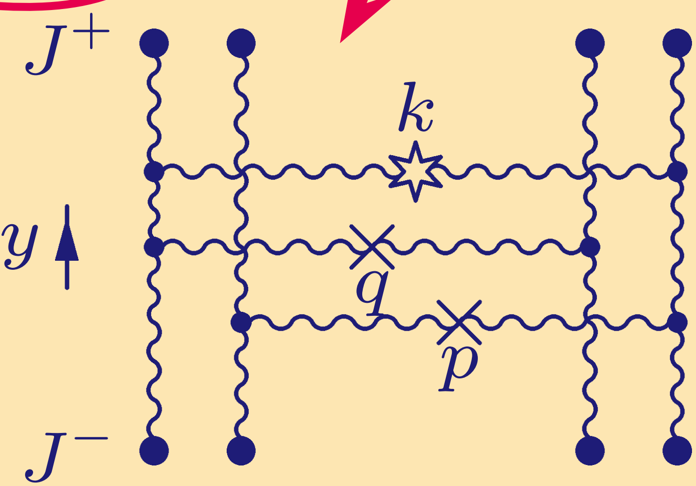
◀ Leading order is disconnected diagram Armesto *et al.*[10]

$$\text{(Recall } a^\mu(x) = \int_{\vec{\boldsymbol{u}}\in\mathrm{LC}} a(\vec{\boldsymbol{u}}) \cdot \mathbb{T}_{\mathbf{u}_T} \mathcal{A}^\mu(x) \text{)}$$

What $\mathbb{T}^2$ does: insert G_{+-}, $\int \frac{\mathrm{d}k^\pm}{k^\pm}$ can have log.

LLog correction: $\mathbb{T}^2$ on LO. The $\mathbb{T}$-operators can act on the same or different disc. part





. . . same also when p and q are separated in rapidity, just divide k-integration!

[10] N. Armesto, L. McLerran and C. Pajares, *Nucl. Phys.* **A781** (2007) 201 [arXiv:hep-ph/0607345].

JIMWLK evolution in Langevin form

Analogy: diffusion equation $$\partial_t P(x,t) = D\partial_x^2 P(x,t)$$

Equivalent to **Langevin equation**

$$\dot{x} = \sqrt{2D}\eta(t), \qquad \langle \eta(t)\eta(t')\rangle = \delta(t-t')$$

Blaizot, Iancu, Weigert [11]: can take $\sqrt{\cdot}$ of JIMWLK Hamiltonian:

$$\mathcal{H} \equiv \frac{1}{2}\int \mathrm{d}^2\mathbf{x}_T\,\mathrm{d}^2\mathbf{y}_T\,\mathrm{d}^2\mathbf{z}_T \frac{\delta}{\delta\widetilde{\mathcal{A}}_c^+(\mathbf{y}_T)} {\mathbf{e}_T}^{ba}(\mathbf{x}_T,\mathbf{z}_T)\cdot {\mathbf{e}_T}^{ca}(\mathbf{y}_T,\mathbf{z}_T)\frac{\delta}{\delta\widetilde{\mathcal{A}}_b^+(\mathbf{x}_T)},$$

$${\mathbf{e}_T}^{ba}(\mathbf{x}_T,\mathbf{z}_T) = \frac{1}{\sqrt{4\pi^3}}\frac{\mathbf{x}_T-\mathbf{z}_T}{(\mathbf{x}_T-\mathbf{z}_T)^2}\Big(1-U^\dagger(\mathbf{x}_T)U(\mathbf{z}_T)\Big)^{ba}$$

The JIMWLK equation can then be written in Langevin form.

$$U_{y+\,\mathrm{d}y}(\mathbf{x}_T) = U_y(\mathbf{x}_T)e^{-i\,\mathrm{d}y\alpha(\mathbf{x}_T,y)} \qquad \alpha^a(\mathbf{x}_T,y) = \sigma^a(\mathbf{x}_T,y)+\int_{\mathbf{z}_T}{\mathbf{e}_T}^{ab}(\mathbf{x}_T,\mathbf{z}_T){\boldsymbol{\eta}_T}^b(\mathbf{z}_T,y)$$

(Time must be discrete for unique interpretation, Itô)

Basis for numerical sulution of JIMWLK Rummmukainen, Weigert[12]

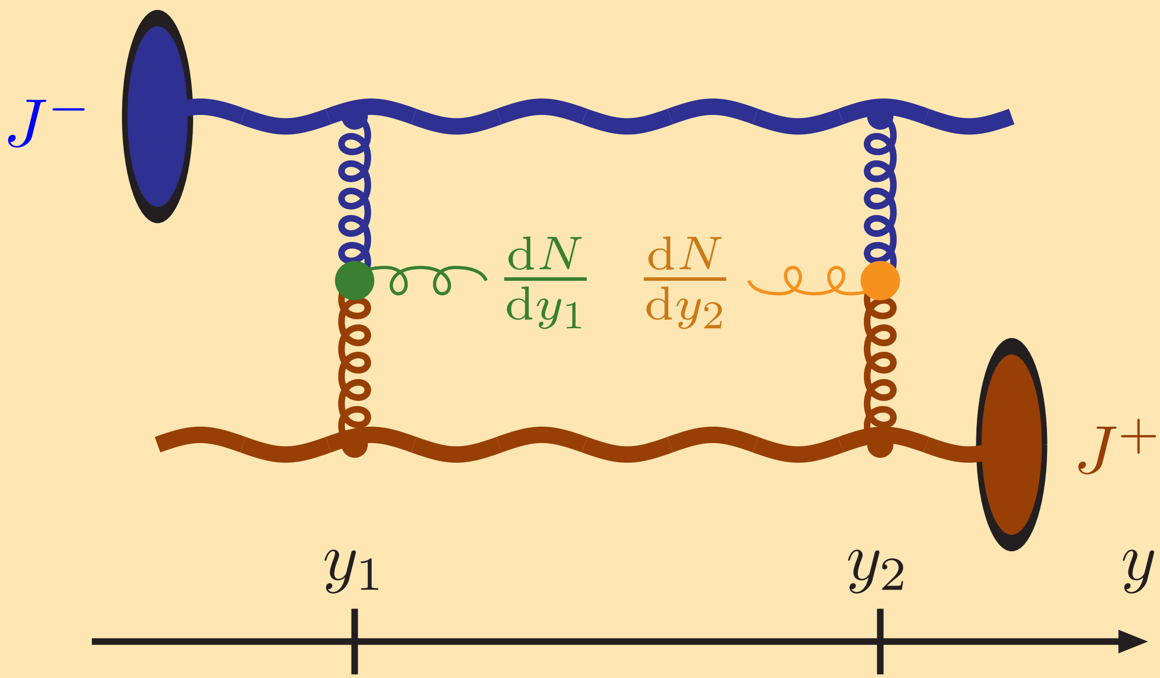
[11] J.-P. Blaizot, E. Iancu and H. Weigert, *Nucl. Phys.* **A713** (2003) 441 [arXiv:hep-ph/0206279].
[12] K. Rummukainen and H. Weigert, *Nucl. Phys.* **A739** (2004) 183 [arXiv:hep-ph/0309306].

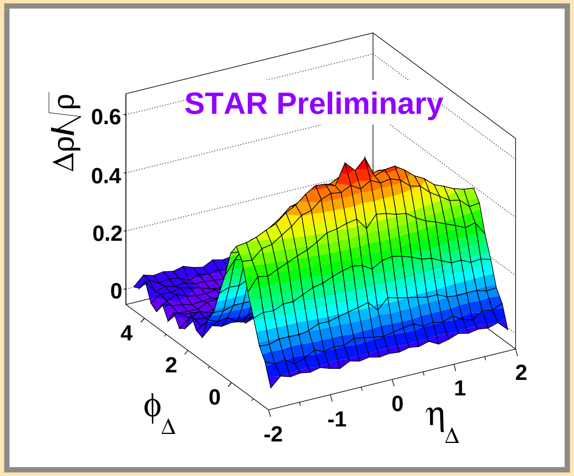
Rapidity correlations from Langevin approach to evolution

Take rapidity dependence in the trajectories seriously

▶ rapidity correlations

One event = one Langevin trajectory





Rapidity correlations in practice[13]

Average over **trajectories** of Langevin equation:

$$\left\langle \mathcal{O} \right\rangle_{\rm LLog} = \int \left[D\Omega_1(y, \mathbf{x}_T) D\Omega_2(y, \mathbf{x}_T)\right] W\left[\Omega_1(y, \mathbf{x}_T)\right] W\left[\Omega_2(y, \mathbf{x}_T)\right]\ \mathcal{O}_{\rm LO}\ .$$

E.g. two gluon correlation with leading log corrections

$$\frac{\mathrm{d}N_2}{\mathrm{d}^2\mathbf{p}_T\,\mathrm{d}y_p\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}y_q} = \int \overbrace{\left[DU_1^p(\mathbf{x}_T) DU_2^p(\mathbf{x}_T) DU_1^q(\mathbf{x}_T) DU_2^q(\mathbf{x}_T)\right]}^{\text{measure}} \overbrace{Z_{y_p}\left[U_1^p\right] Z_{y_q}\left[U_2^q\right]}^{\text{initial distribution}}$$

$$\times \underbrace{G_{y_p,y_q}\left[U_1^p, U_1^q\right] G_{y_q,y_p}\left[U_2^q, U_2^q\right]} \left.\frac{\mathrm{d}N_1\left[U_1^p, U_2^p\right]}{\mathrm{d}^2\mathbf{p}_T\,\mathrm{d}y_p}\right|_{\rm LO} \left.\frac{\mathrm{d}N_1\left[U_1^q, U_2^q\right]}{\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}y_q}\right|_{\rm LO}$$

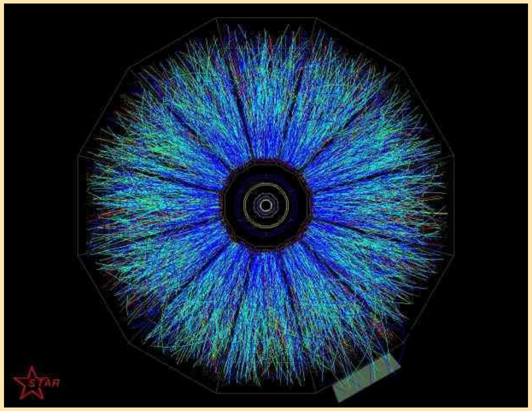
JIMWLK propagator:

$$\partial_{y_p} G_{y_p,y_q}\left[U^p, U^q\right] = \mathcal{H}\left[U^p\right] G_{y_p,y_q}\left[U^p, U^q\right] \qquad \lim_{y_p \to y_q} G_{y_p,y_q}\left[U^p, U^q\right] = \delta\left[U^p - U^q\right]\ .$$

[13] F. Gelis, T. Lappi and R. Venugopalan, arXiv:0810.4829 [hep-ph].

Summary

- CGC and glasma framework for heavy ion collisions
- Factorize leading $\ln 1/x$ 1-loop corrections into RG evolution.
- Multigluon and rapidity correlations



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