Fundamental problems with hadronic and leptonic interactions

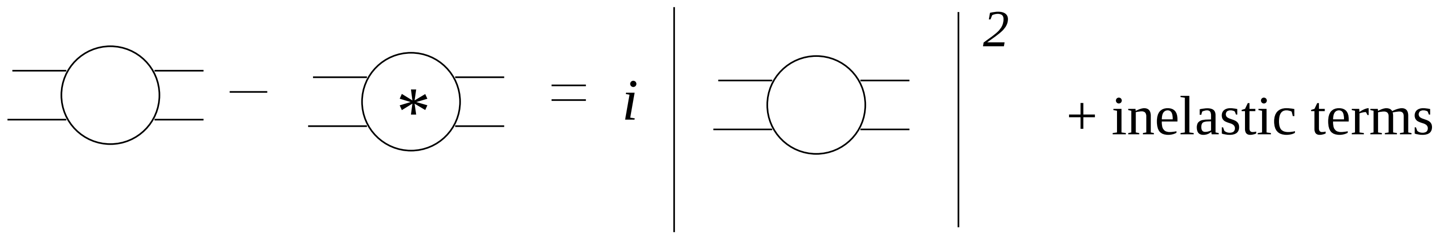
- LHC will probe physics under extreme conditions: past physical intuition may be unreliable

- Should ask whether accepted folklore has a sound theoretical basis

Unitarity DGLAP

$$\boldsymbol{\sigma^{\mathrm{LHC}} = 125 \pm 35\ \mathrm{mb}}$$

Unitarity



$$\text{Im}\ a_\ell(s) = |a_\ell(s)|^2\ +\ \text{inelastic terms}$$

so that

$$|a_\ell(s)| < 1$$

Gives the Froissart–Lukaszuk–Martin bound:

$$\sigma^{\text{TOT}}(s) < \frac{\pi}{m_\pi^2}\ \log^2(s/s_0)$$

Several barns!

Pumplin bound: $\sigma^{\text{ELASTIC}} < \frac{1}{2}\,\sigma^{\text{TOTAL}}$

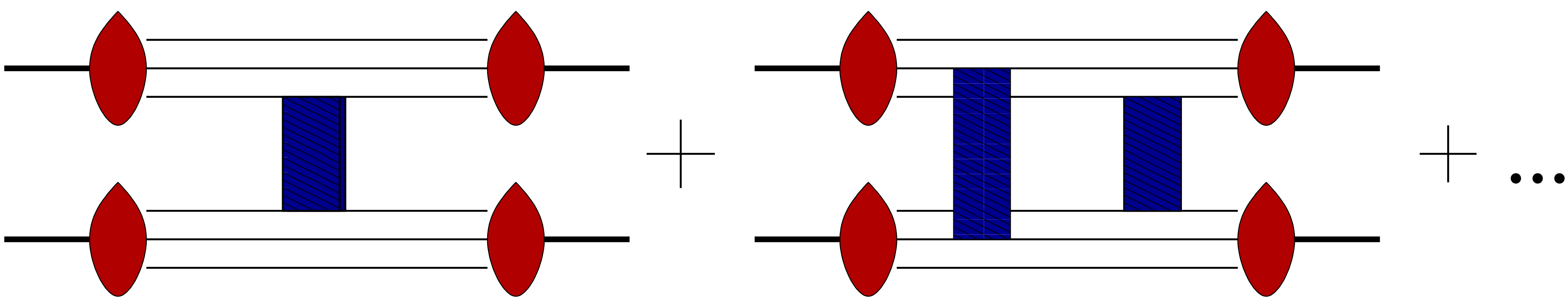
Pomeron exchange alone gives

$$\sigma^{\text{TOTAL}} \sim s^{\epsilon} \qquad \epsilon \approx 0.08$$

$$\text{and} \quad \left.\frac{d\sigma}{dt}^{\text{ELASTIC}}\right|_{t=0} \sim s^{2\epsilon}$$

Violates the bound at large s

Must sum single-$I\!\!P$, double-$I\!\!P$, ... exchanges:



But we do not know how!

Eikonal model

Write the amplitude as a 2-dimensional Fourier integral

$$A(s,-\mathbf{q}^2) = 4\int d^2b\, e^{-i\mathbf{q}.\mathbf{b}}\tilde{A}(s,\mathbf{b}^2)$$

Define $\chi(s,b) = -\log(1+2i\tilde{A}/s)$ so that

$$\tilde{A}(s,\mathbf{b}^2) = \tfrac{1}{2}is(1-e^{-\chi(s,b)})$$

Then

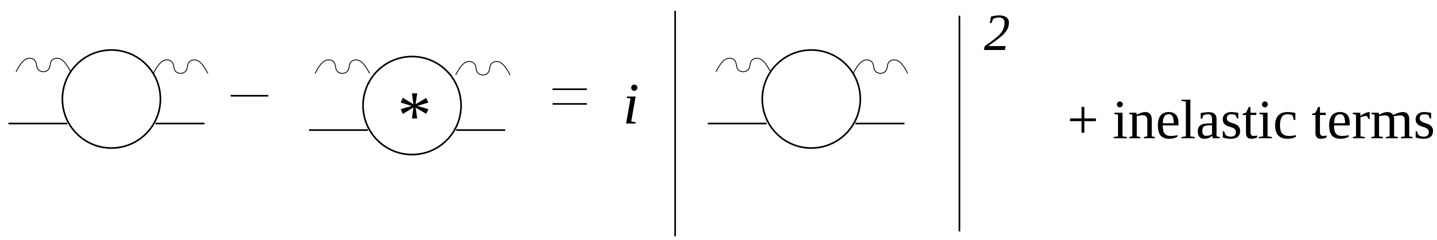
$$|a_\ell(s)| < 1 \quad \text{implies} \quad \text{Re}\;\chi(s,b) \geq 0$$

$$\begin{aligned} A(s,-\mathbf{q}^2) &= 2is\int d^2b\, e^{-i\mathbf{q}.\mathbf{b}}(1-e^{-\chi(s,b)}) \\ &= 2is\int d^2b\, e^{-i\mathbf{q}.\mathbf{b}}\left(\chi - \frac{\chi^2}{2!} + \frac{\chi^3}{3!}\dots - \frac{(-\chi)^n}{n!}\dots\right) \end{aligned}$$

Approximate first term by single-$I\!\!P$ exchange. The second has the correct general structure of double-$I\!\!P$ exchange, etc.

But this model is definitely <u>wrong</u>!

Deep inelastic scattering



is not true!

There is no unitarity bound on $F_2(x, Q^2)$!

This is the case even for real photons:

$\sigma^{\gamma p}$ might get very big at high energy

DGLAP

The singlet DGLAP eqation is:

$$\frac{\partial}{\partial t}\mathbf{u}(x,Q^2) = \int_x^1 dz\,\mathbf{P}(z,\alpha_s(Q^2))\,\mathbf{u}(x/z,Q^2) \qquad \mathbf{u}(x,Q^2) = \begin{pmatrix} q(x,Q^2) \\ g(x,Q^2) \end{pmatrix}.$$

The terms of the perturbation expansion of $\mathbf{P}(z,\alpha_s(Q^2))$ diverge like $1/z$ at $z=0$.
It is wrong to expand it in powers of α_s — unless $\mathbf{u}(x,Q^2)$ rises steeply with $1/x$.

Approximate $\mathbf{u}(x, Q^2)$ in some range of Q^2:

$$\mathbf{u}(x, Q^2) \sim \mathbf{f}(Q^2) x^{-\epsilon}$$

Then

$$\frac{\partial}{\partial t} \log \mathbf{f}(Q^2) = \tilde{\mathbf{P}}(N = \epsilon, \alpha_s(Q^2)) - \int_0^x dz z^\epsilon \mathbf{P}(z, \alpha_s(Q^2))$$

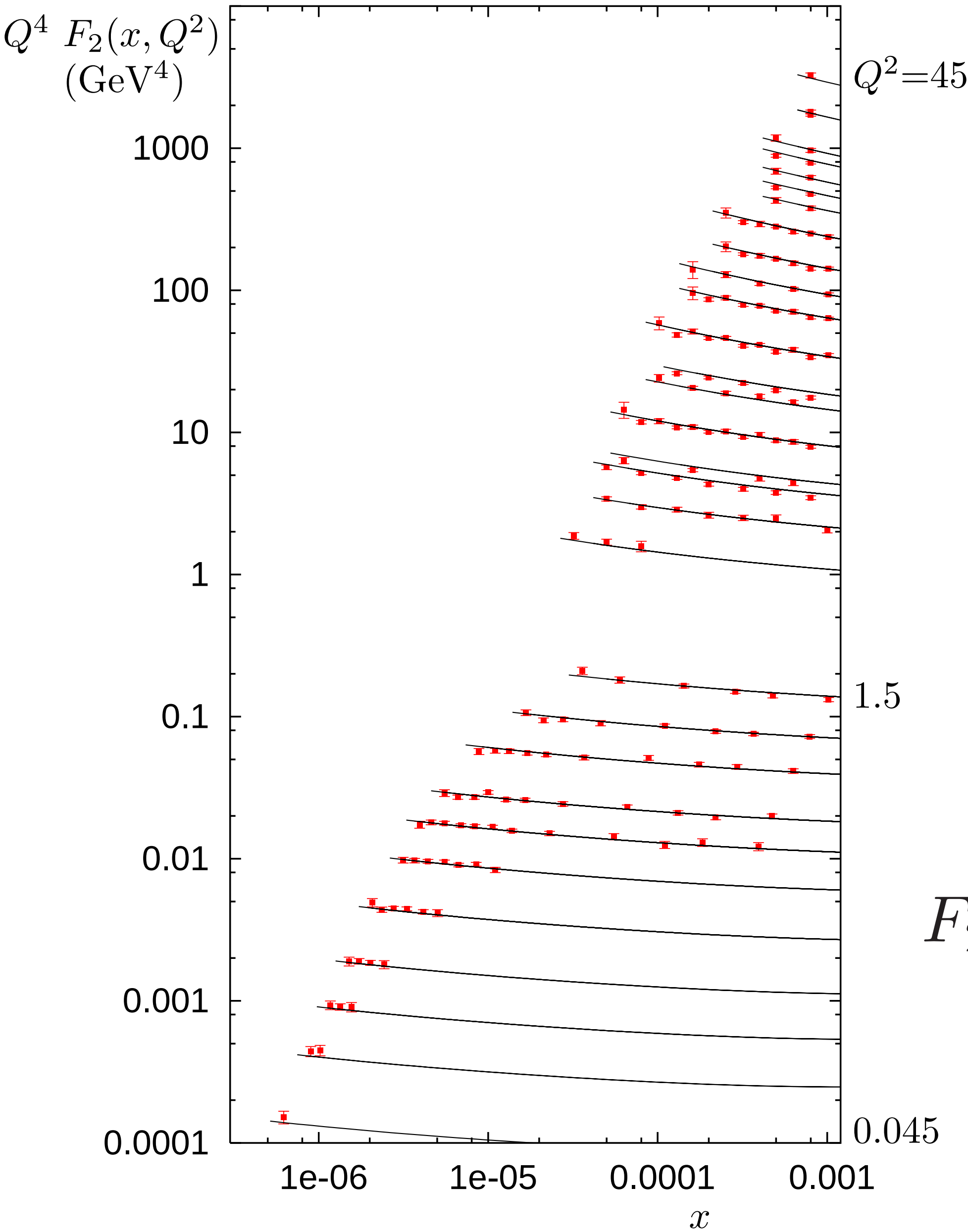
with $\tilde{\mathbf{P}}$ the Mellin transform of $\mathbf{P}$.

The last term $\sim x^\epsilon$ and is negligible at small x if ϵ is some way above 0.

Then also it is safe to expand the first term in powers of α_s.

Simplest fit to F_2 at small x

Donnachie + PVL

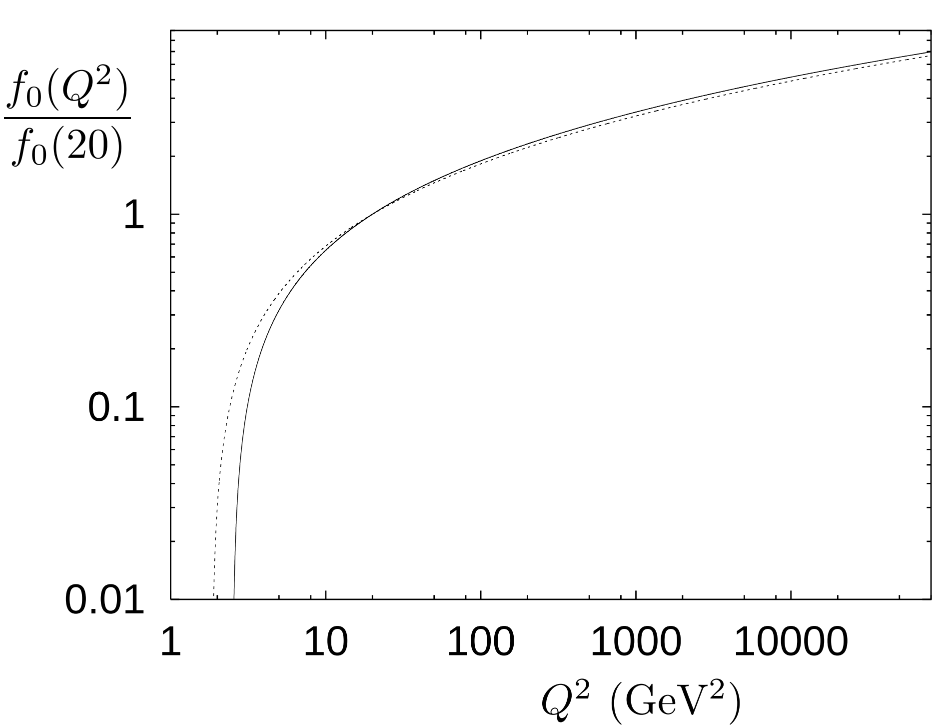


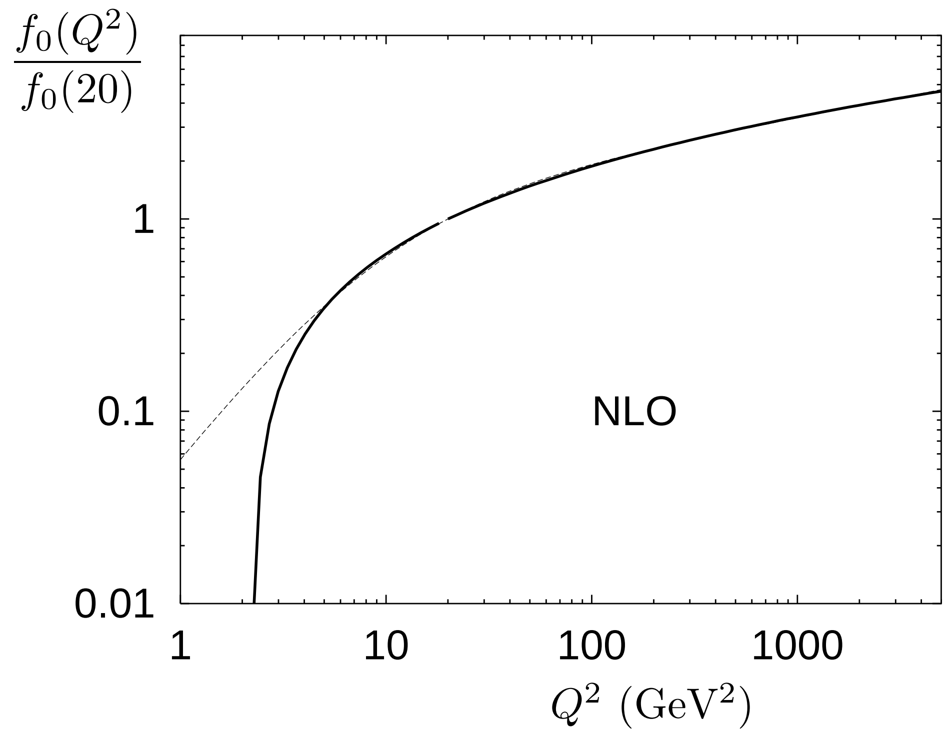
$$F_2(x,Q^2) = f_0(Q^2)x^{-\epsilon_0} + f_1(Q^2)x^{-\epsilon_1}$$

$$\epsilon_0 \approx 0.4 \quad \epsilon_1 = 0.0808$$

5 free parameters, including ϵ_0

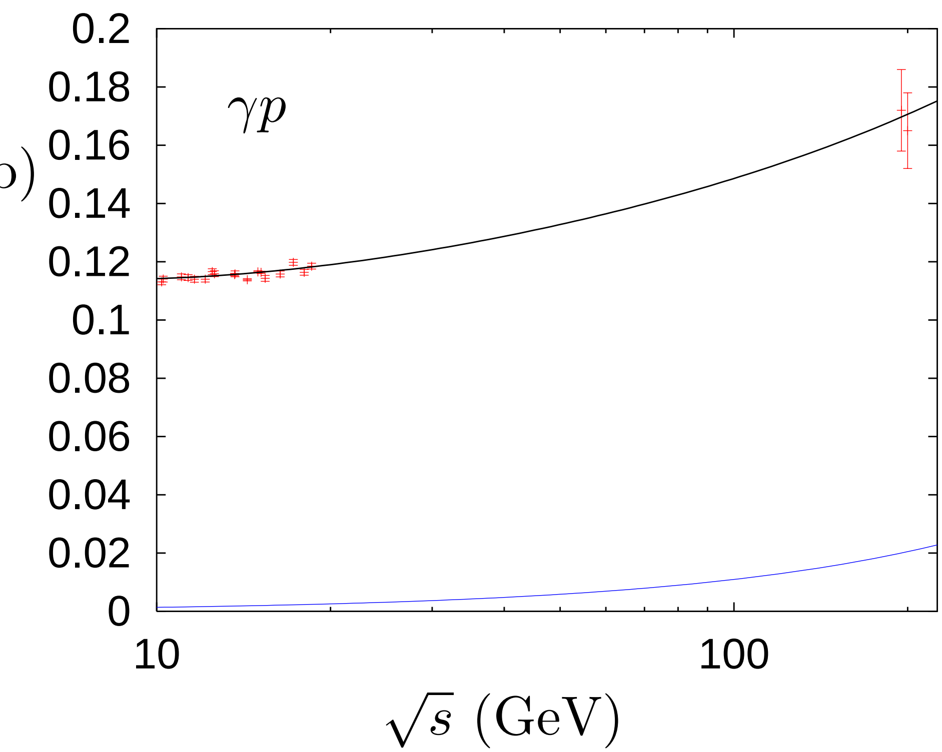
- The variation of $f_0(Q^2)$ with Q^2 obeys DGLAP

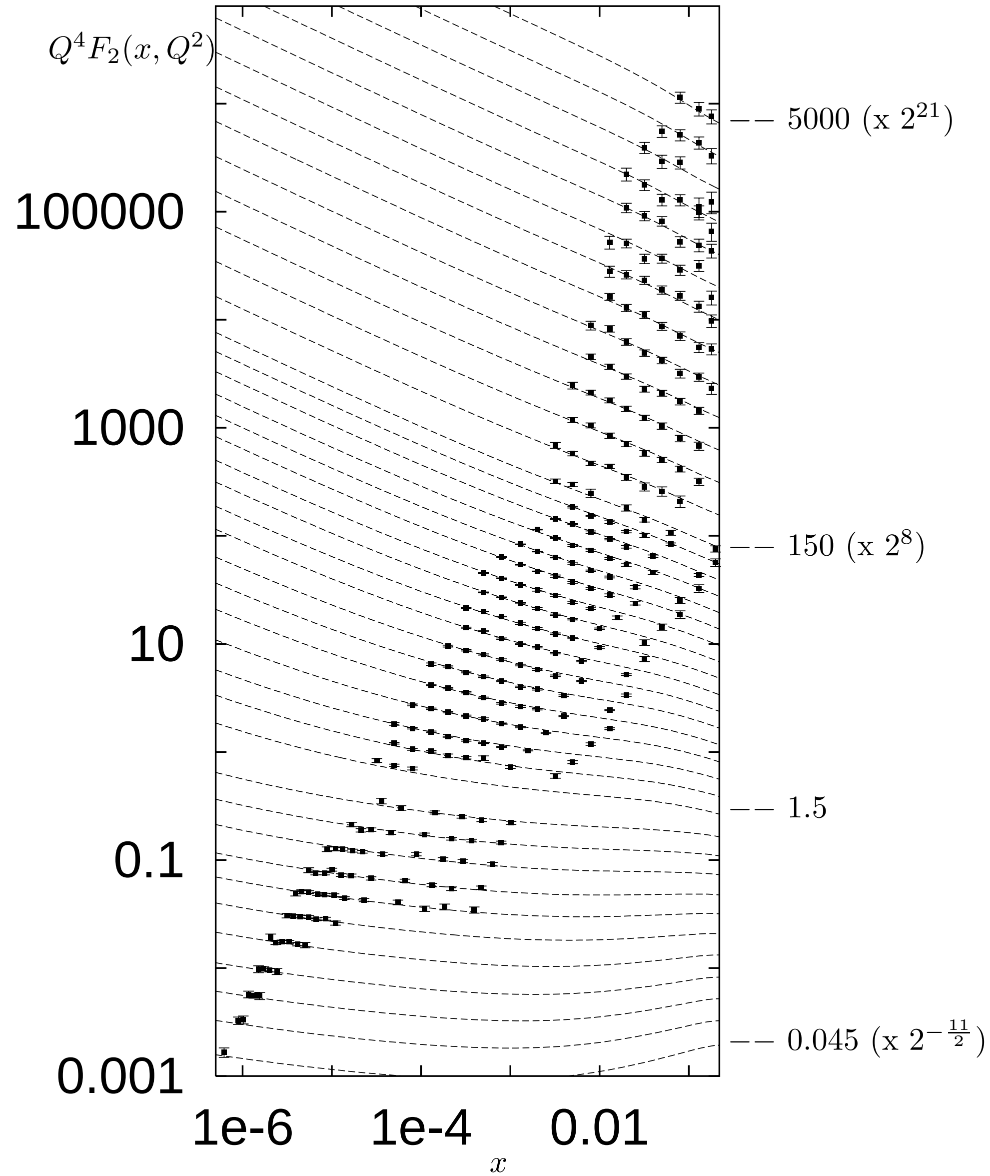




- pQCD breaks down for $f_1(Q^2)$
- pQCD breaks down for Q^2 less than about 5 GeV^2

To go to larger x, add in Regge term $x^{-\epsilon_R}$ with $\epsilon_R \approx \frac{1}{2}$ and introduce powers of $(1-x)$ given by dimensional counting.

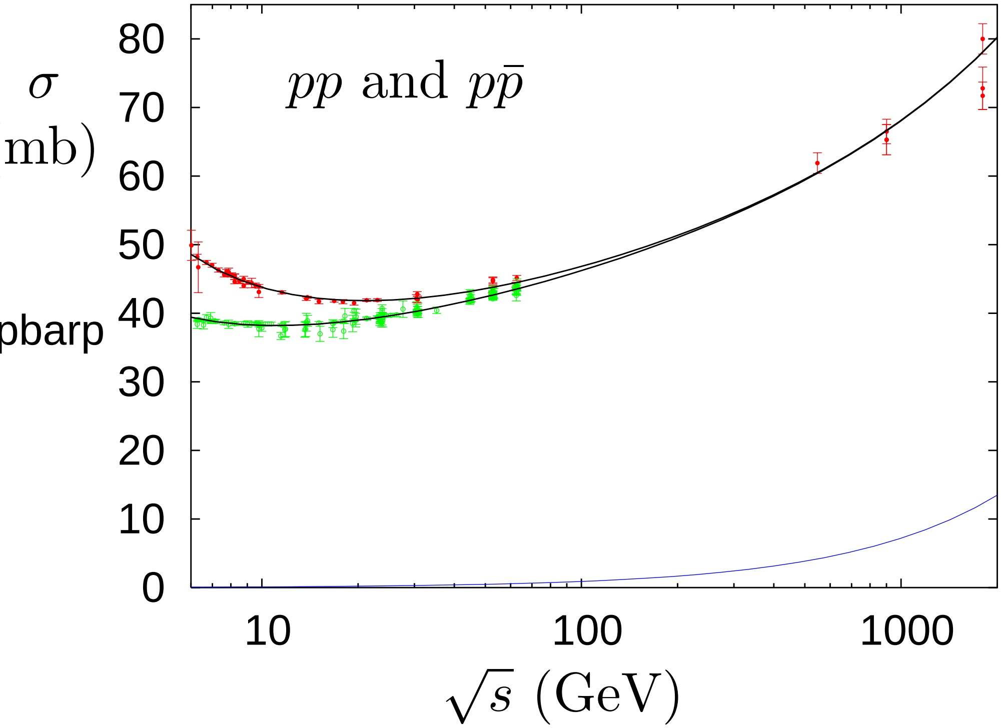


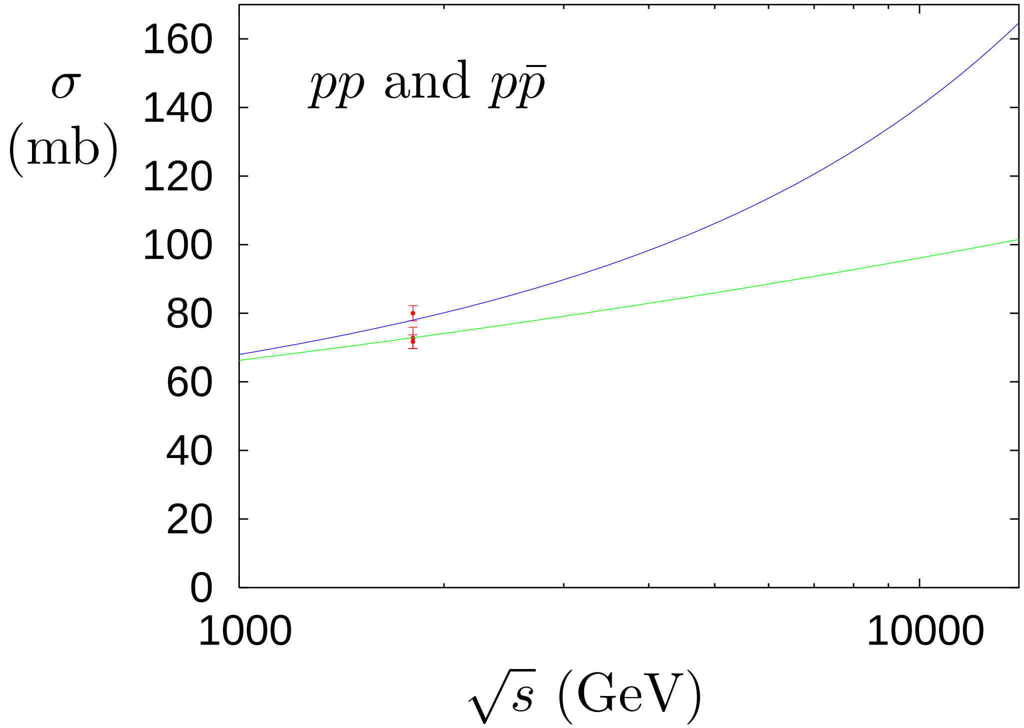


Include hardpom also in pp and $p\bar{p}$

Use hardpom, softpom and reggeon exchange
$\sigma(pp), \sigma(p\bar{p}), \sigma(\gamma p)$:

$$\sigma = X_0 s^{\epsilon_0} + X_1 s^{\epsilon_1} + Y s^{\epsilon_R}$$





Must <u>try</u> to take account of unitarity

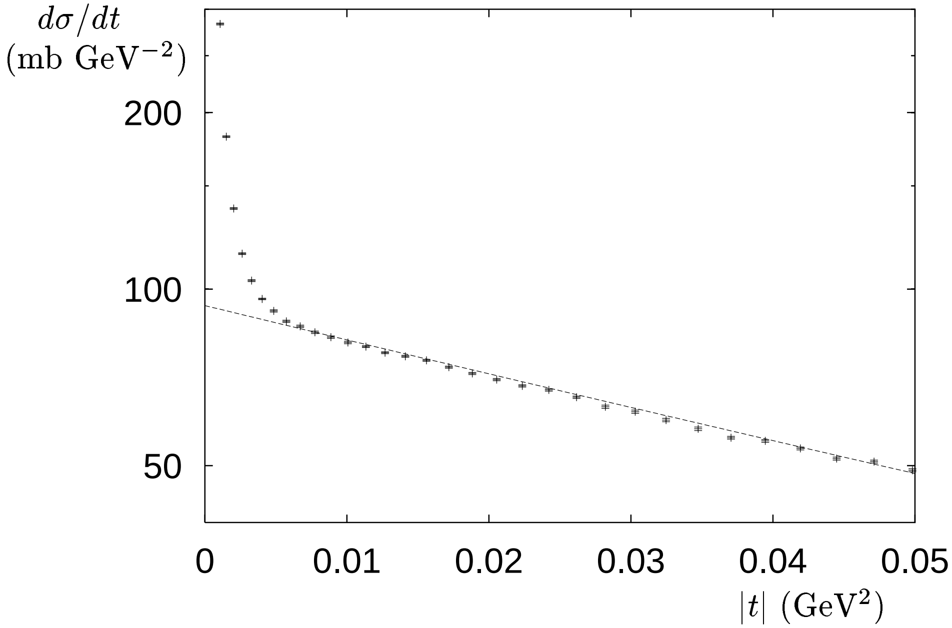
Elastic scattering

At present energies, soft-pomeron exchange dominates:

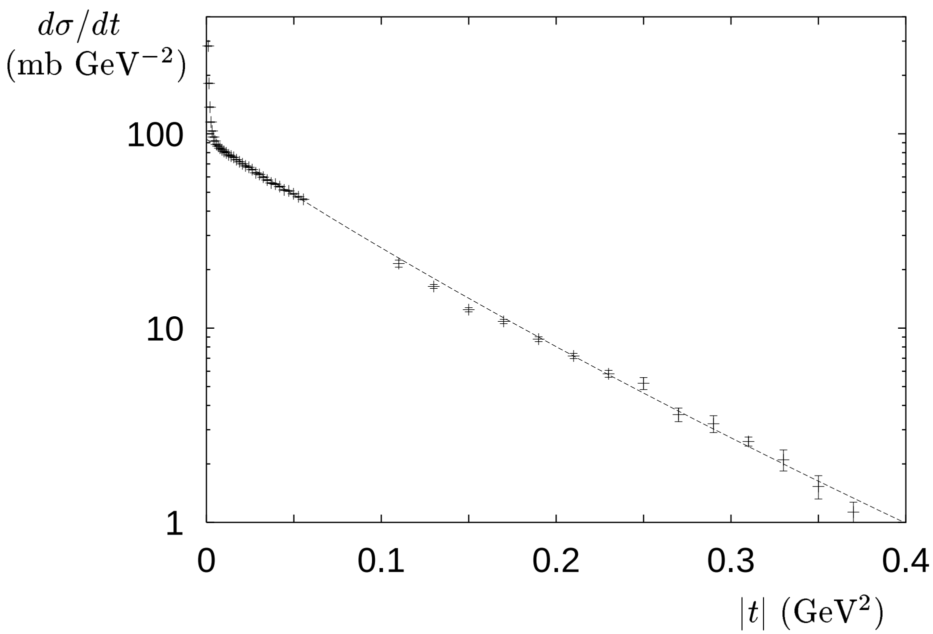
$$\frac{d\sigma}{dt} = \frac{[3\beta_1 F_1(t)]^4}{4\pi}(\alpha_1' s)^{2(\epsilon_1+\alpha_1' t)}$$

β_1 and ϵ_1 are known from σ^{TOT}

Data fix only free parameter α_1' to be 0.25 GeV^{-2} — Jaroskiewicz (1970)



(a)

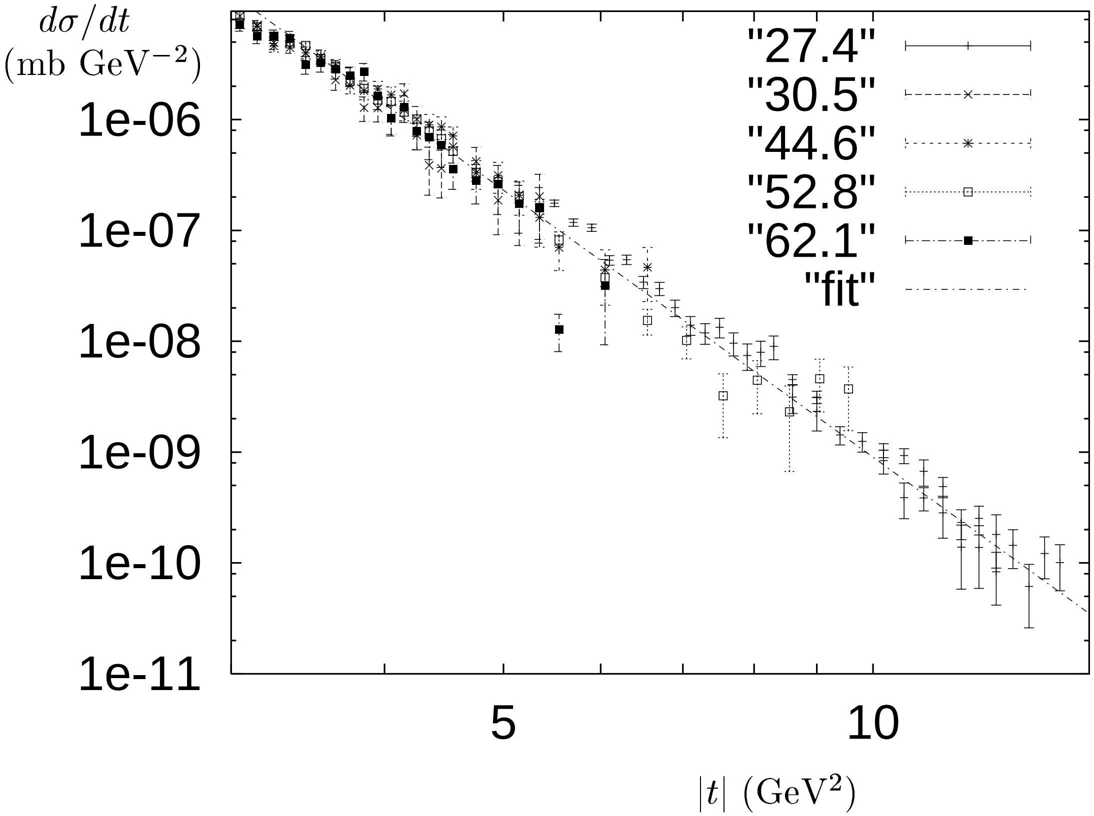


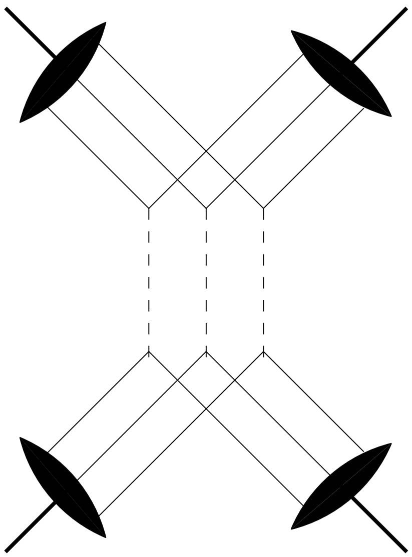
(b)

Curves do not include photon exchange

pp elastic scattering data at $\sqrt{s} = 53$ GeV

Large t

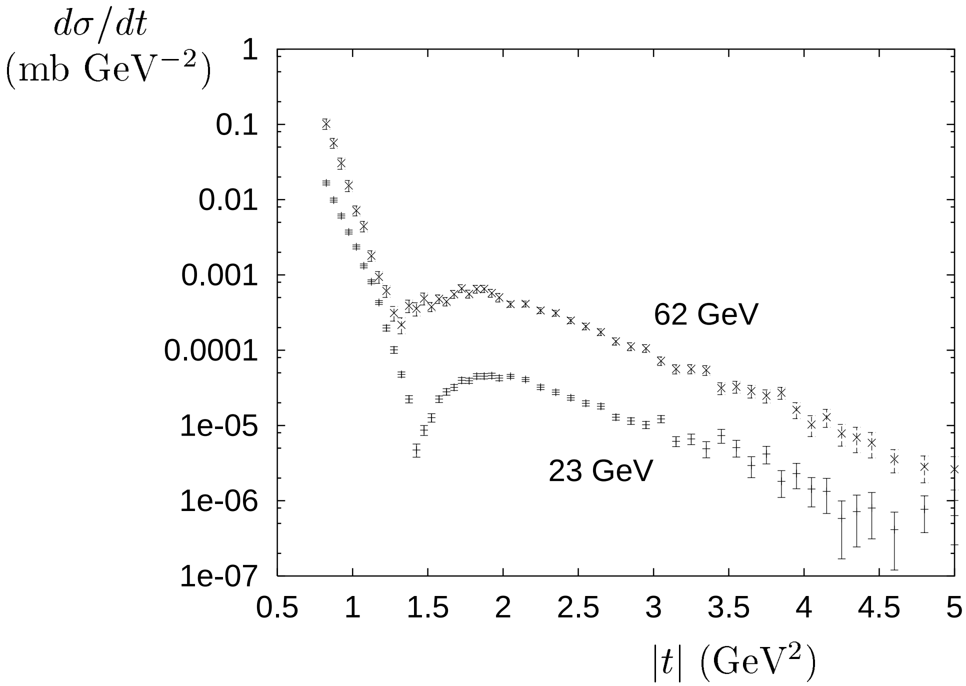




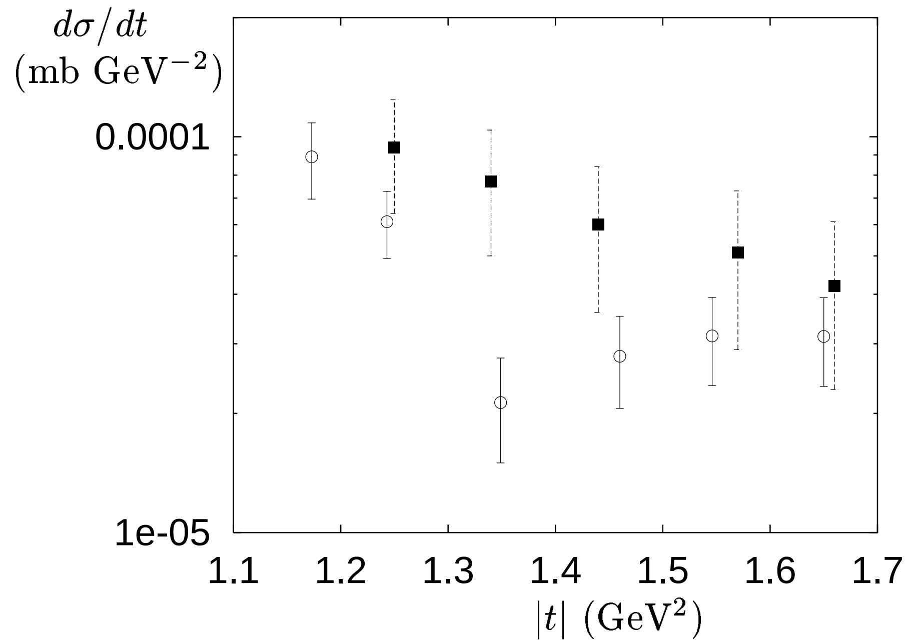
$d\sigma/dt = 0.09\,t^{-8}$ independent of s, as for triple-gluon exchange

What if replace gluons with hard pomerons??

Smaller values of t



62 GeV data are multiplied by 10

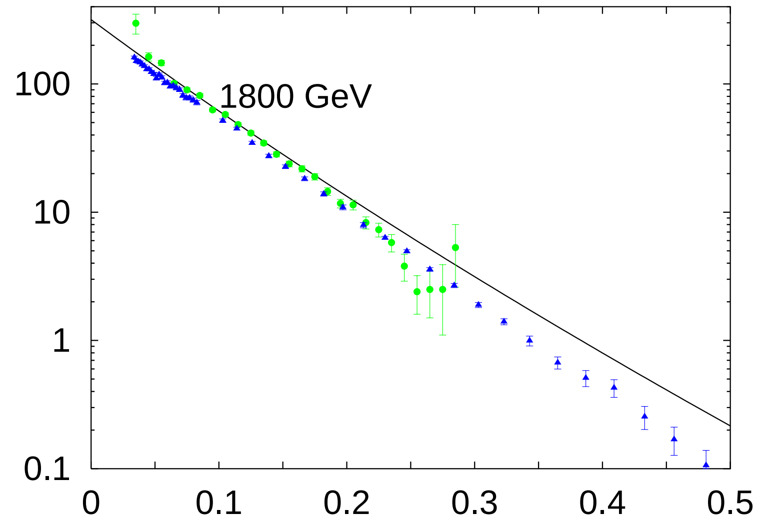


$\sqrt{s}$=53 GeV

Upper points $\bar{p}p$ Lower points pp

- It is not easy to generate a dip: Re A and Im A must be very small at the same t
- Need 3 contributions: probably $I\!P$, $I\!P I\!P$ and ggg
- Predicts dip absent in $\bar{p}p$ (DL)

pp elastic without $I\!\!P I\!\!P$ exchange:



$I\!\!P I\!\!P$ exchange pulls $d\sigma/dt$ down at larger t.

But nobody knows how to calculate it!

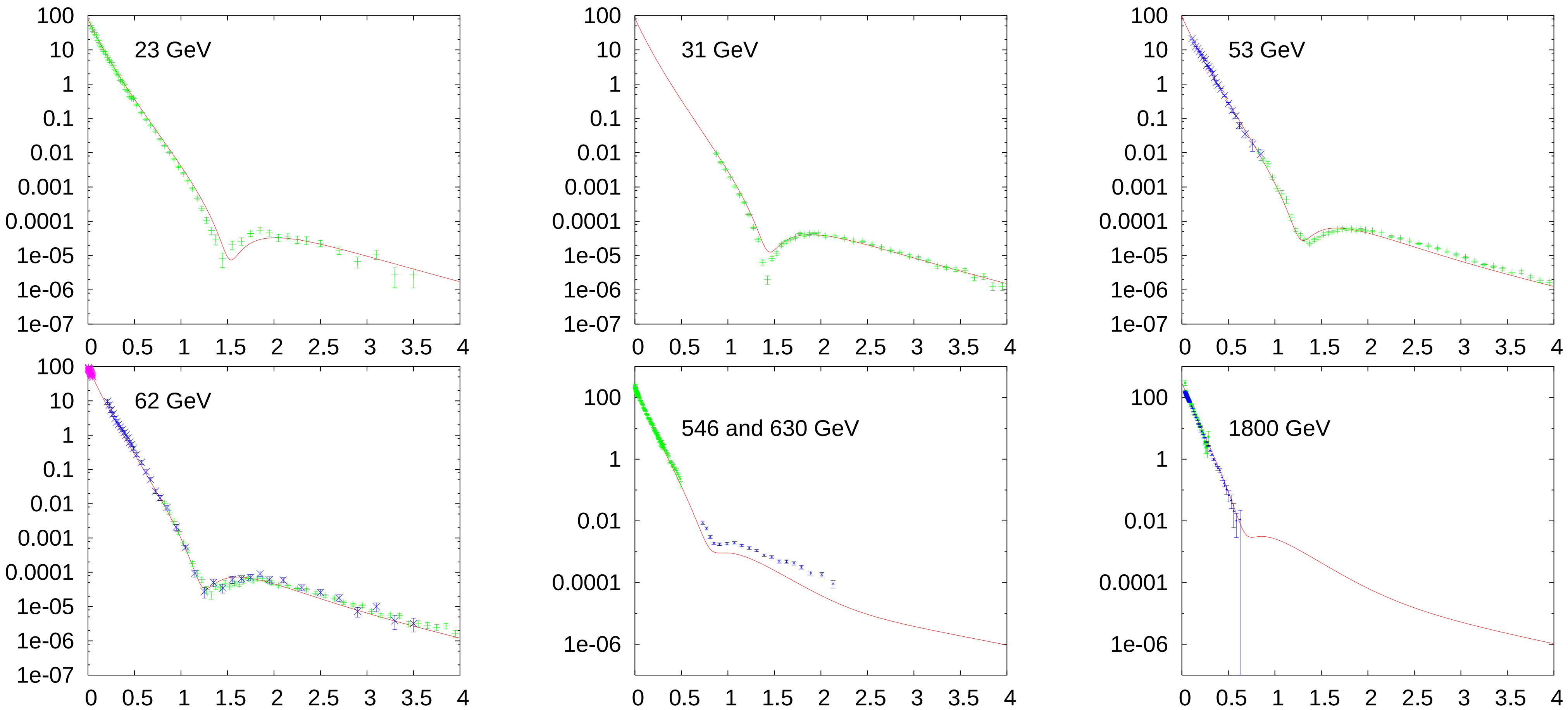
Very crude model:

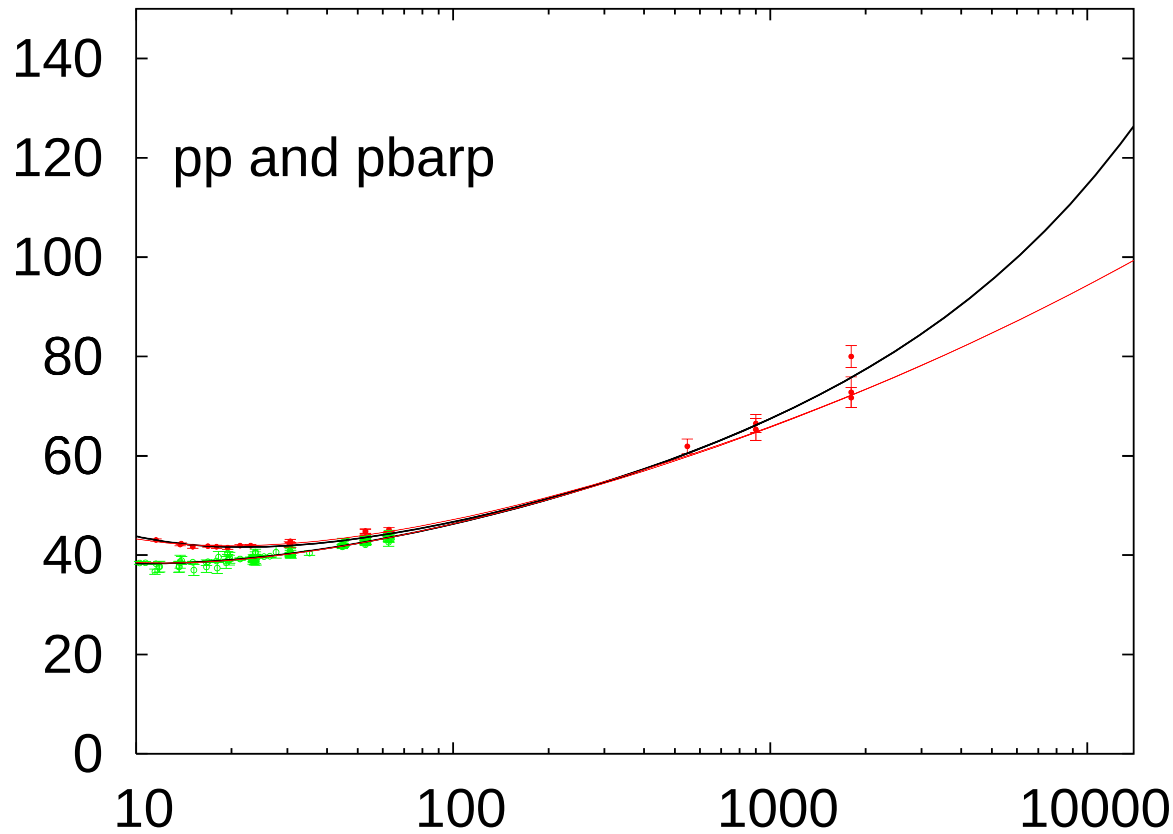
Calculate b-space amplitude for $I\!\!P$ exchange, square it:

$$\tilde{A}(s,b) = 2is\Big(\chi(s,b) - \lambda[\chi(s,b)]^2\Big)$$

Choose λ to get pp dips at the right t

Fit pp and $\bar{p}p$ elastic data with $I\!P + I\!P I\!P + ggg$





Conclusion $\sigma(\text{LHC}) = 125 \pm 25$ mb

NOTE: If the CDF Tevatron cross section is correct, something dramatic must happen — independently of any theory!

Summary

- $\sigma^{\rm LHC} = 125 \pm 35$ mb
- We do not know how usefully to impose unitarity — eikonal-type models are surely wrong
- Unitarity does not constrain lepton-induced cross sections, nor photon
- We still cannot calculate $I\!\!P I\!\!P$ exchange – even after more than 45 years
- There are severe mathematical problems with DGLAP at small x
- DGLAP cannot be used below $Q^2 = 5$ GeV^2
- Regge fits to $F_2(x, Q^2)$ are the simplest — and probably the most correct
- Elastic scattering at the LHC at large t may be surprisingly large
- If the CDF Tevatron cross section is correct, something dramatic must happen — independently of any theory!