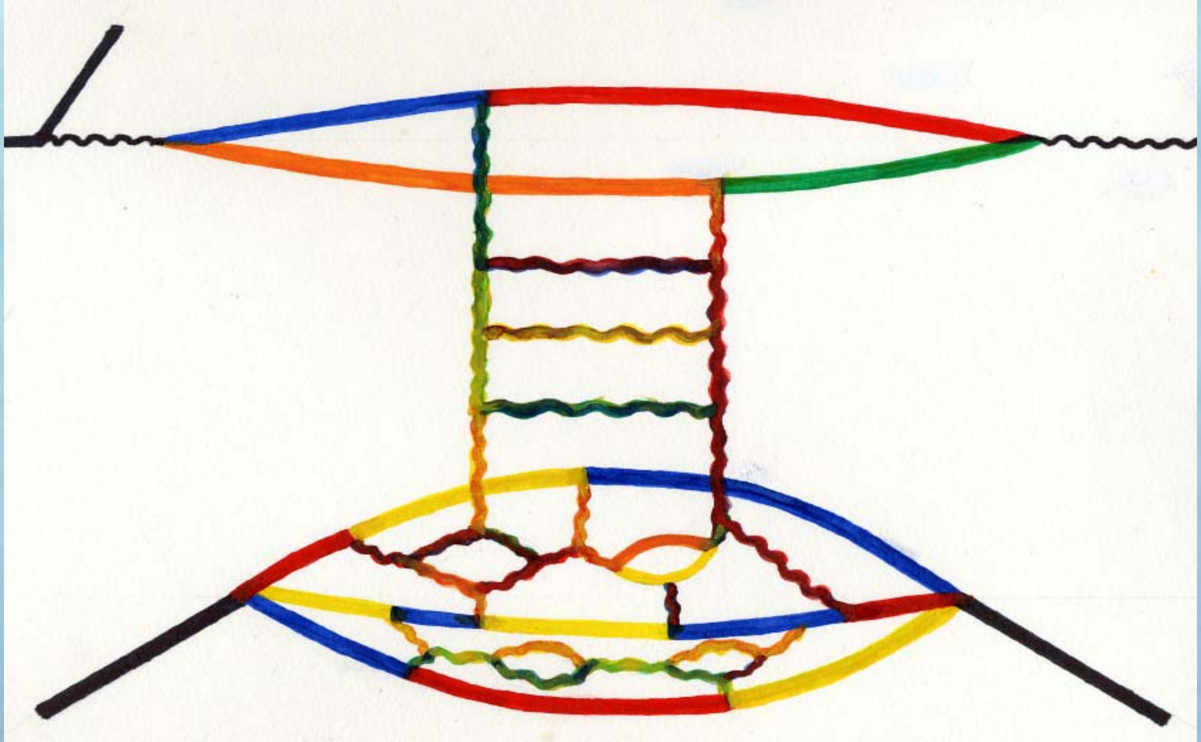
The Dipole Model & the DAF-BFKL Pomeron

Henri Kowalski



EPIPHANY 2009

Krakow, 7th of January 2009

Outline of the talk:

Short review of low x HERA data, Dipole Picture,
Diffractive Jets
DVCS & VM in the Dipole Picture

Saturation, oomph factor and all that

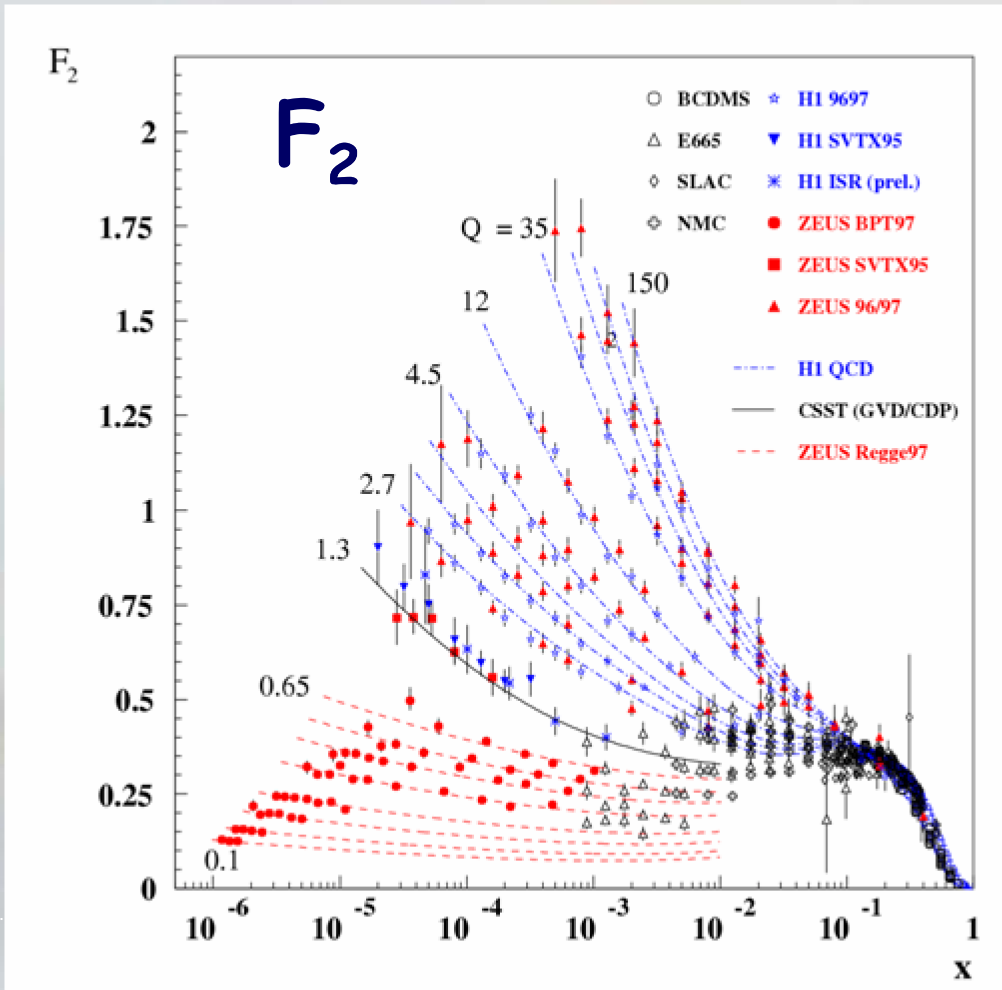
Why Pomeron at HERA?,
What is DAF-BFKL Pomeron
Evidence for DAF-Pomeron from HERA data

Relation with DGLAP
MRST ←→ EKR

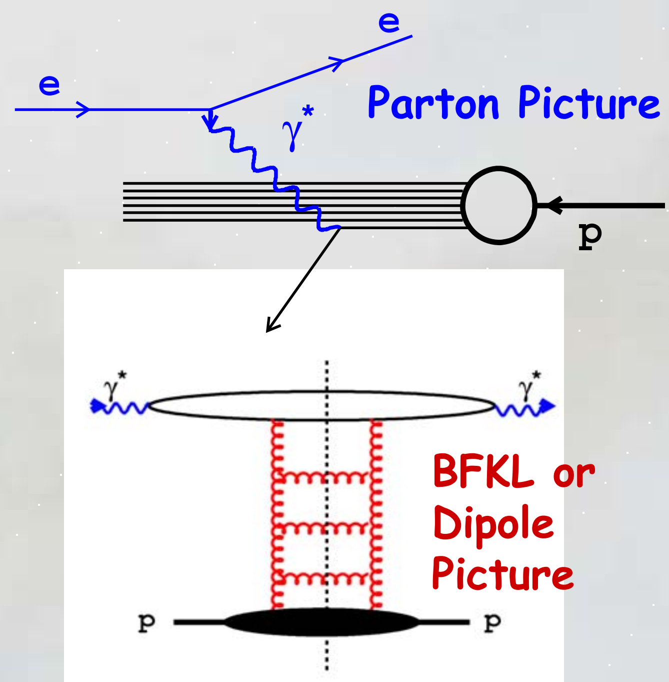
Pomeron-Graviton Correspondence

Consequences for LHC, EIC, LHeC

Low-x Physics @ HERA



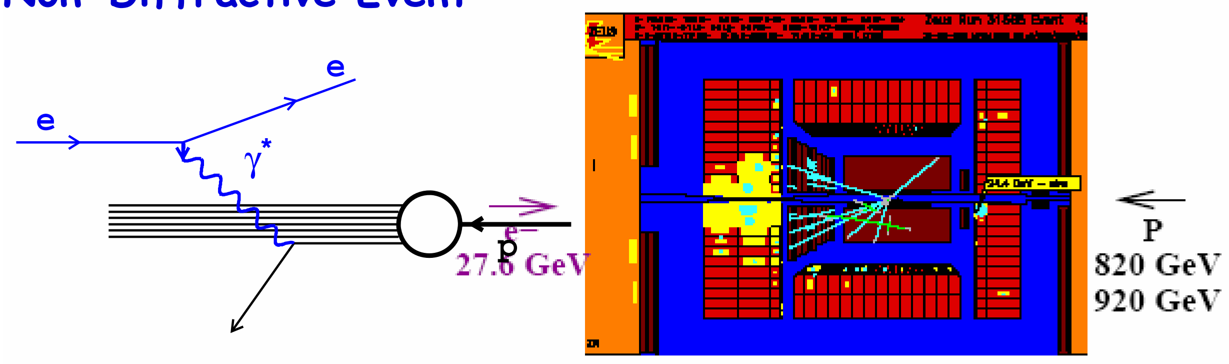
- At low x and high Q^2, steep rise in structure function

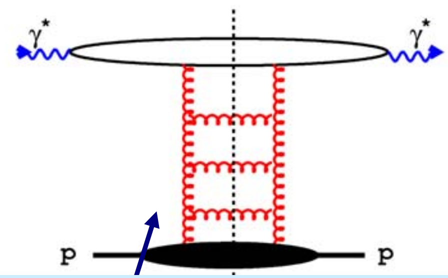


Behavior of F_2 is dominated by gluon density at small-x

Hard Diffraction – the HERA surprise

Non-Diffractive Event

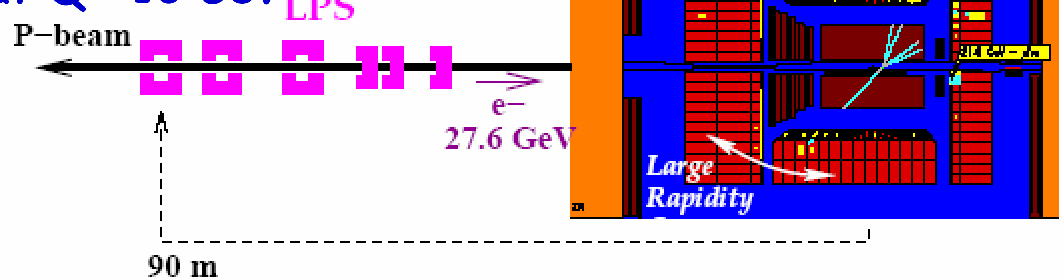


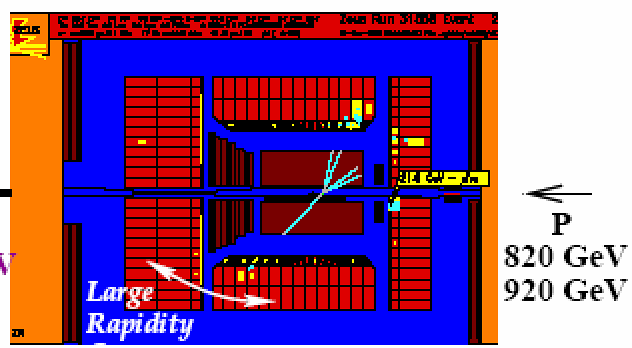


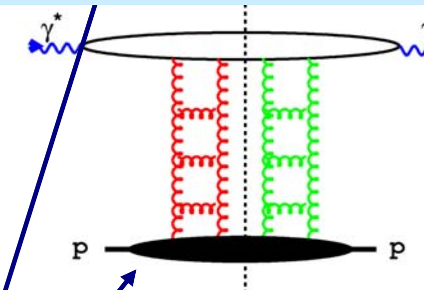
$$\tau_{qq} \approx \frac{1}{\Delta E} \approx \frac{1}{m_p x} \approx 10 - 1000 \quad \mathrm{fm}$$

Diffractive Event

expected before HERA
<0.01%, seen over 10%
at Q^2=10 GeV^2







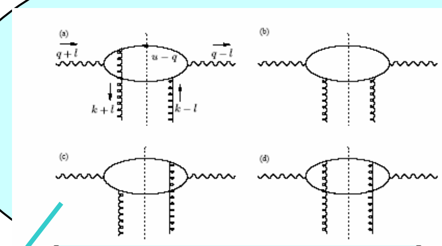
Diffraction at HERA is so large because it is a shadow of DIS (i.e. inelastic processes) ➔ dipole picture

$$\sigma_{tot}^{\gamma^* p} = \frac{1}{W^2} Im A_{el}(W^2, t = 0)$$

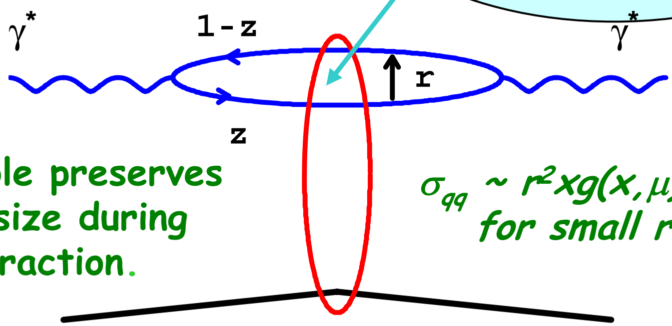
Dipole description of DIS

equivalent to Parton Picture in the perturbative region

momentum space



configuration space



dipole preserves its size during interaction.

$\sigma_{qq} \sim r^2 xg(x,\mu)$ for small r

$$\left|\Psi_T^f\right|^2 = \frac{3\alpha_{em}}{2\pi^2} e_q^2 \{[z^2+(1-z)^2]\varepsilon^2 K_1^2(\varepsilon r) + m_q^2 K_0^2(\varepsilon r)\}$$

$$\left|\Psi_L^f\right|^2 = \frac{3\alpha_{em}}{2\pi^2} e_q^2 \{4Q^2 z^2 (1-z)^2 K_0^2(\varepsilon r)\}$$

$$\varepsilon^2 = z(1-z)Q^2 + m_q^2$$

$\varepsilon r << 1$

$Q^2 \sim 1/r^2$

Optical T

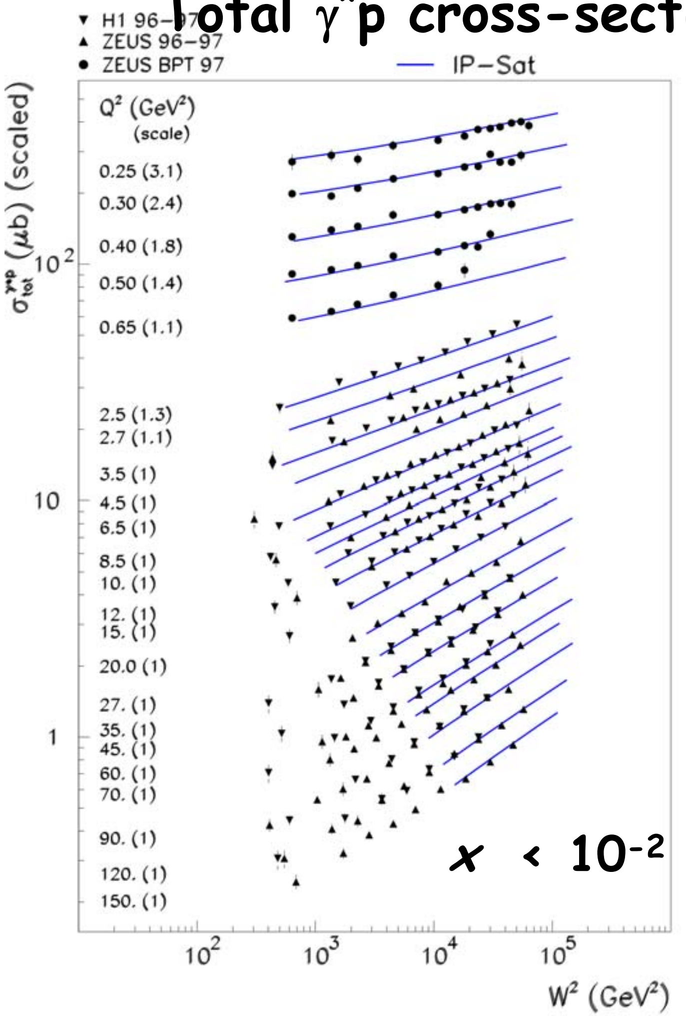
Mueller, Nikolaev, Zakharov

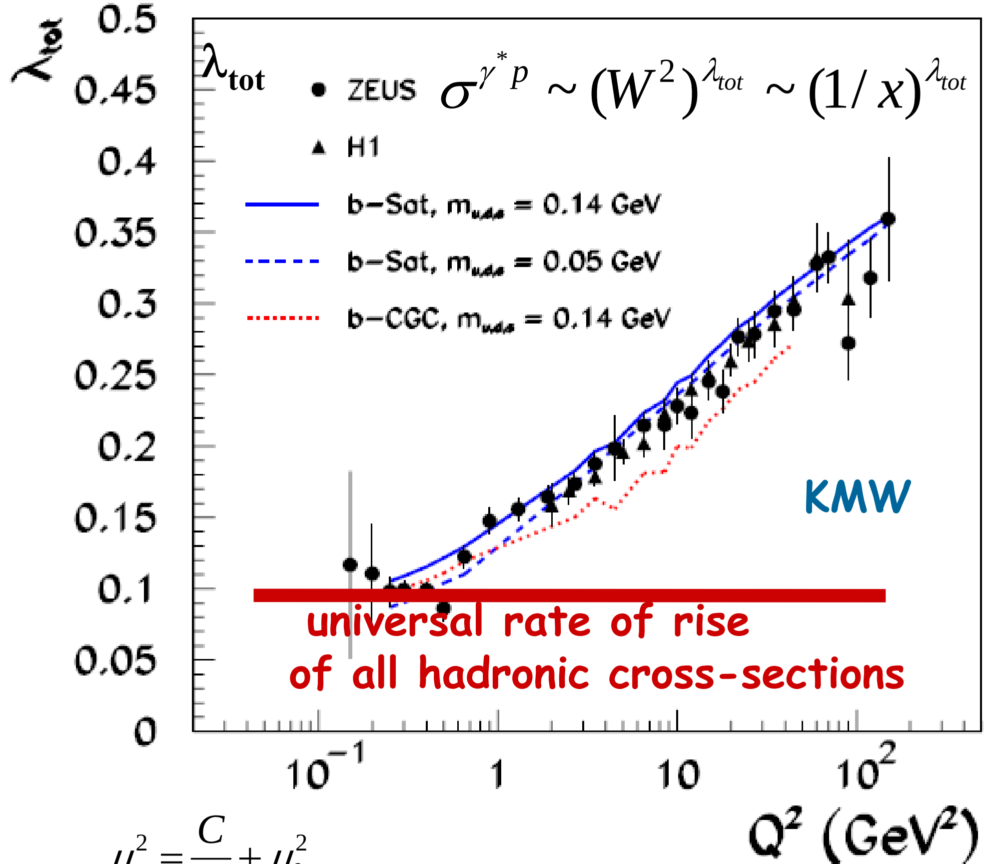
$$\sigma_{tot}^{\gamma^* p} = \int d^2\hat{\vec{r}} \int_0^1 dz\, \Psi^* \sigma_{q\bar{q}}(x, r^2)\Psi$$

$$\frac{d\sigma_{VM}^{\gamma^* p}}{dt}\Big|_{t=0} = \frac{1}{16\pi} \left|\int d^2\vec{r} \int_0^1 dz \Psi_{VM}^*(Q^2, z, \vec{r})\ \sigma_{q\bar{q}}(x, r^2)\Psi(Q^2, z, \vec{r})\right|^2$$

$$\frac{d\sigma_{diff}^{\gamma^* p}}{dt}\Big|_{t=0} = \frac{1}{16\pi} \int d^2\vec{r} \int_0^1 dz\, \Psi^* \sigma_{q\bar{q}}^2(x, r^2)\Psi$$

Total $\gamma^* p$ cross-section





$$\sigma^{\gamma^* p} \sim (W^2)^{\lambda_{tot}} \sim (1/x)^{\lambda_{tot}}$$

$$\mu^2 = \frac{C}{r^2} + \mu_0^2$$

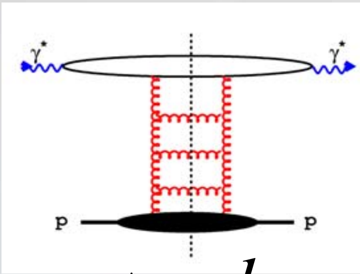
$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2 \boldsymbol{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) x g(x,\mu^2) T(b)\right)\right] \quad xg(x,\mu_0^2) = A_g \left(\frac{1}{x}\right)^{\lambda_g} (1-x)^{5.6}$$

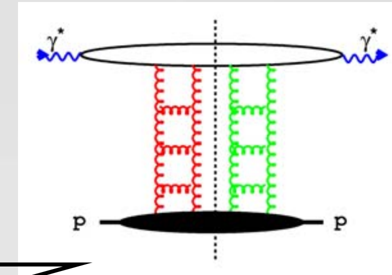
b-Sat

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2 \boldsymbol{b}} \equiv 2\mathcal{N}(x,r,b) = 2 \times \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s + \frac{1}{\kappa\lambda Y} \ln \frac{2}{rQ_s}\right)} & : \quad rQ_s \le 2 \\ 1 - \mathrm{e}^{-A \ln^2(BrQ_s)} & : \quad rQ_s > 2 \end{cases}$$

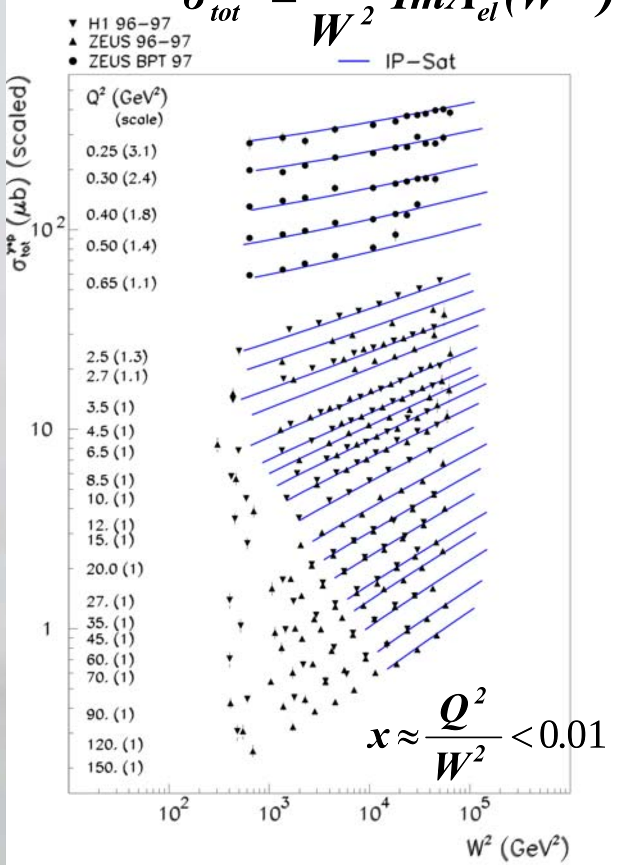
b- CGC
IIM+KMW

Low-x Physics @ HERA





$$\sigma_{tot}^{\gamma^* p} = \frac{1}{W^2} Im A_{el}(W^2)$$



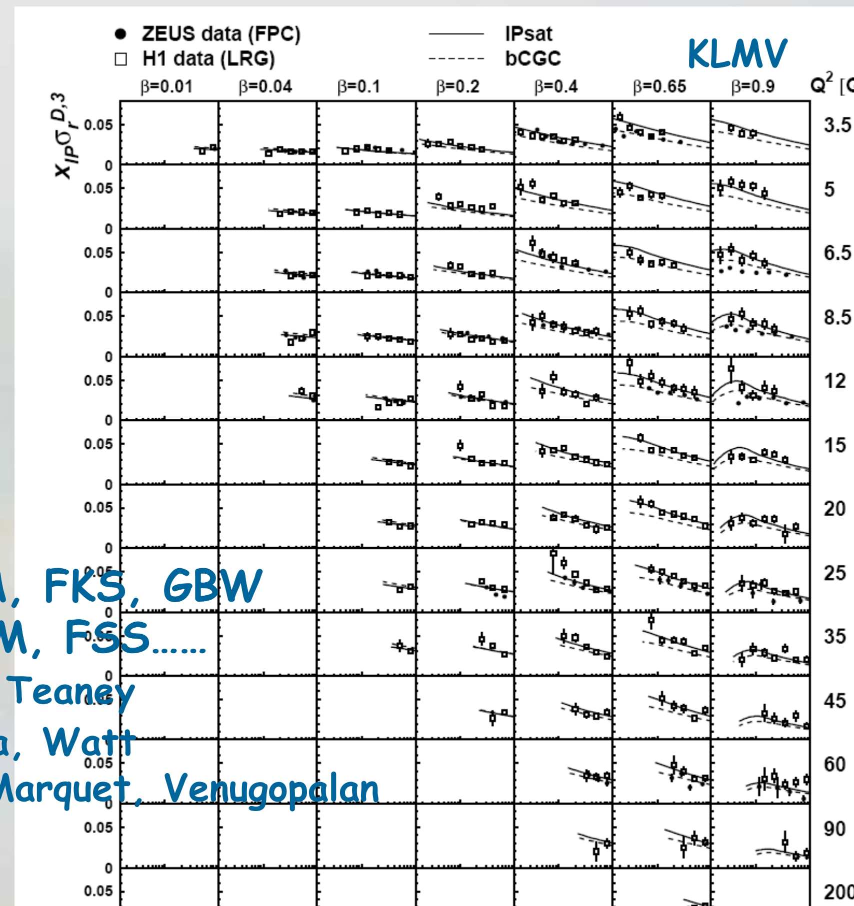
$$x \approx \frac{Q^2}{W^2} < 0.01$$

$$\sigma_{tot}^{\gamma^* P}(W,Q^2) = \frac{4\pi^2 \alpha_{em}}{Q^2} \cdot F_2(x,Q^2)$$

Diffraction at HERA is a shadow of DIS

➔

dipole picture, equivalent to LO p-QCD for small dipoles, Q~1/r



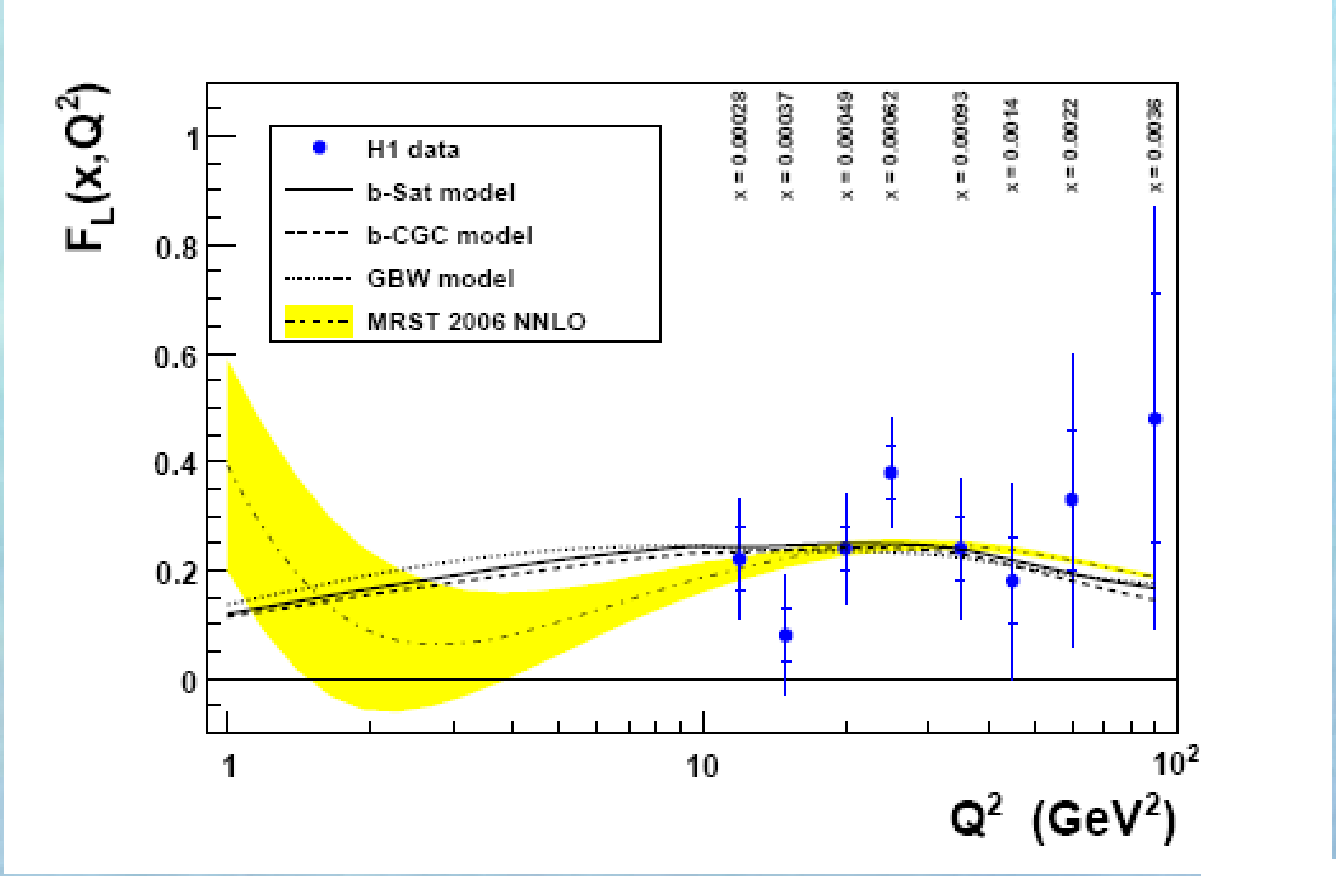
NNPZ, AM, GLM, FKS, GBW

DGKP, BGBK, IIM, FSS……

KT - Kowalski, Teaney

KMW - K, Motyka, Watt

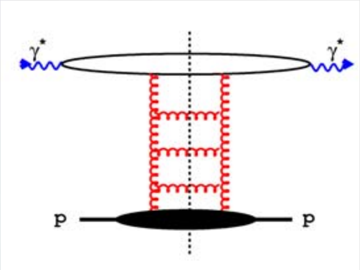
KLMV – K, Lappi, Marquet, Venugopalan



$F_L(x,Q^2)$
H1 data
b-Sat model
b-CGC model
GBW model
MRST 2006 NNLO
x = 0.00028
x = 0.00037
x = 0.00049
x = 0.00062
x = 0.00093
x = 0.0014
x = 0.0022
x = 0.0036
1
0.8
0.6
0.4
0.2
0
1
10
10^2
Q^2 (GeV^2)

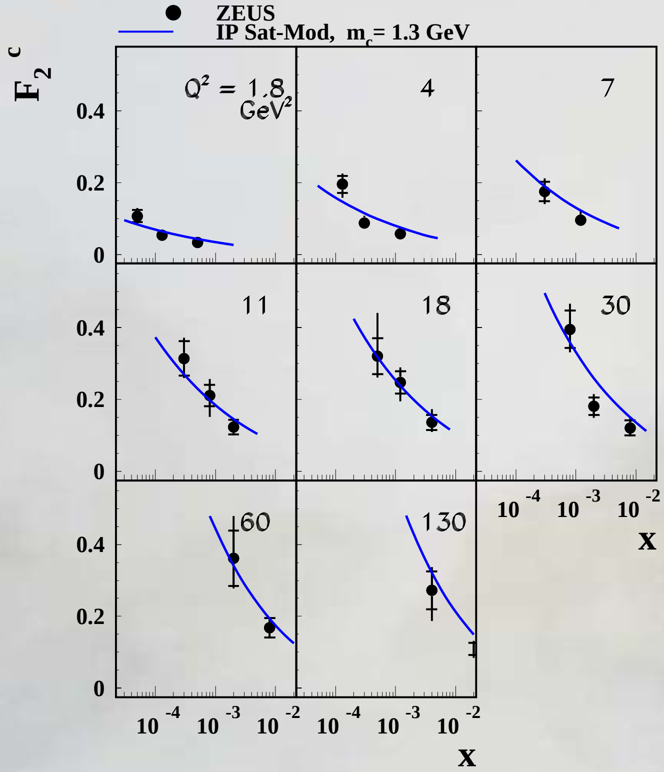
K+Watt

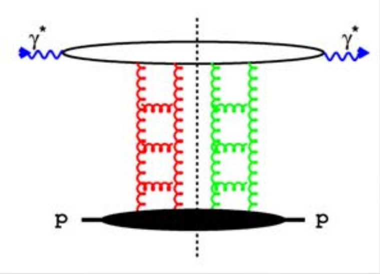
Dipole Picture-gluon density convoluted with the dipole wave functions ➔ simultaneous prediction/description of many reactions



KT

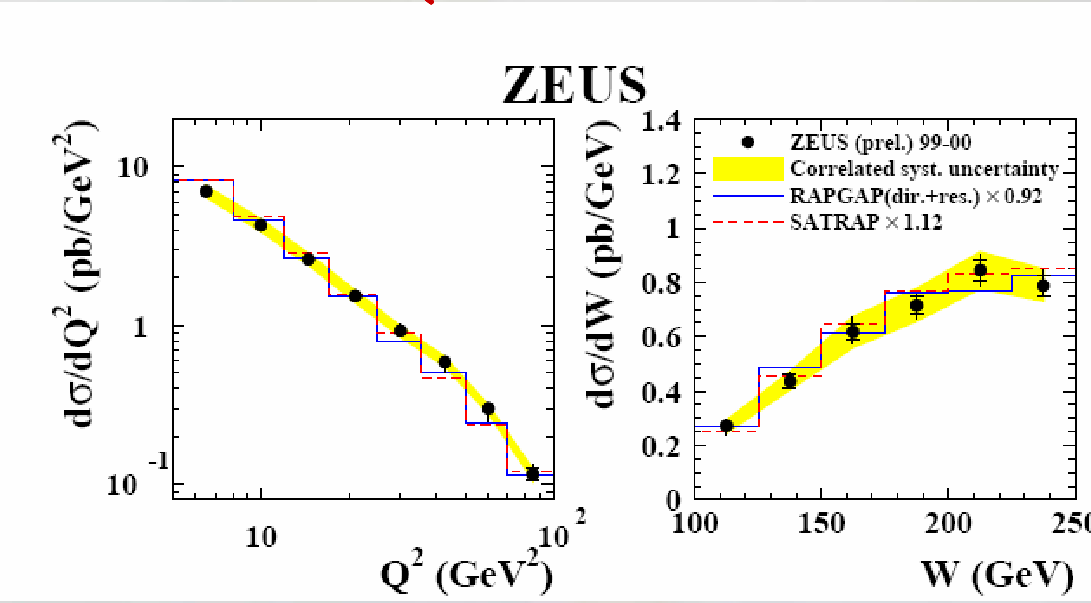
$F_2^{\,c}$



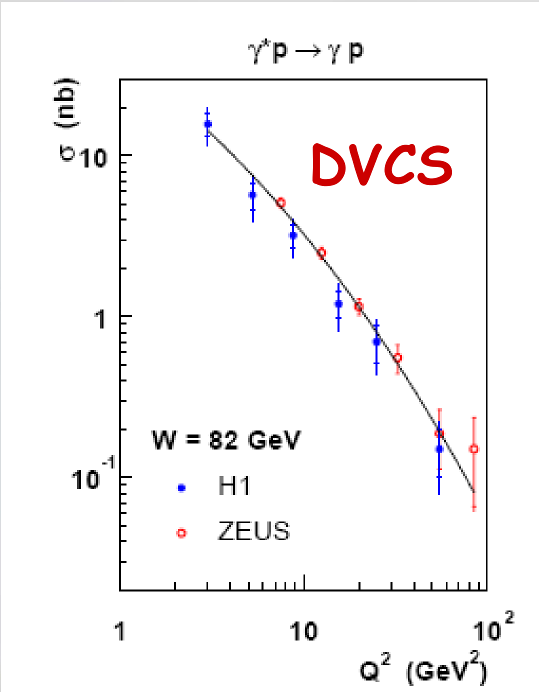


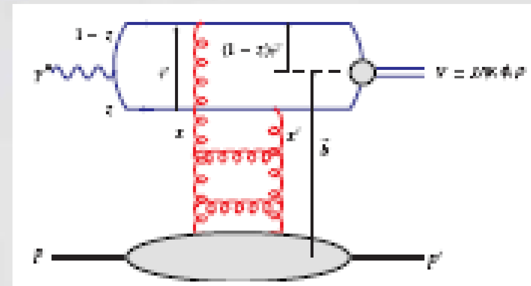
Diffractive Di-jets

$Q^2 > 5\ GeV^2$



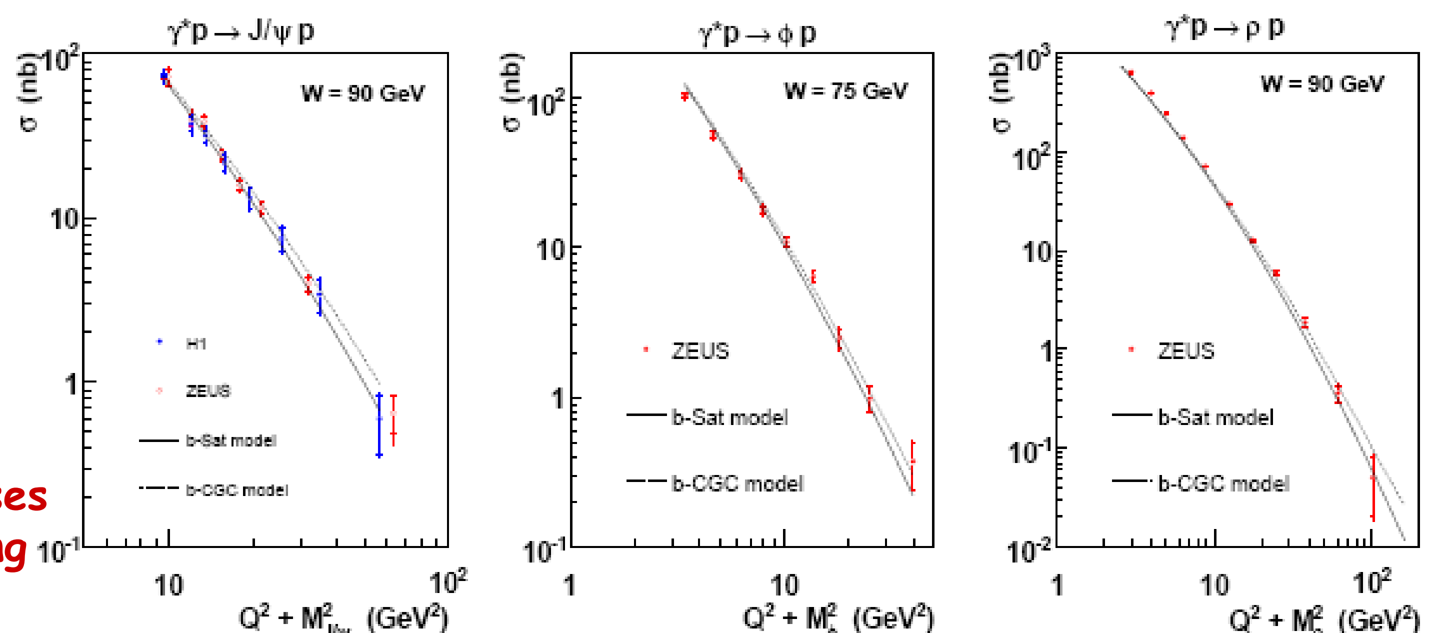
Dipole Picture-gluon density convoluted with the dipole wave functions → simultaneous prediction/description of many reactions





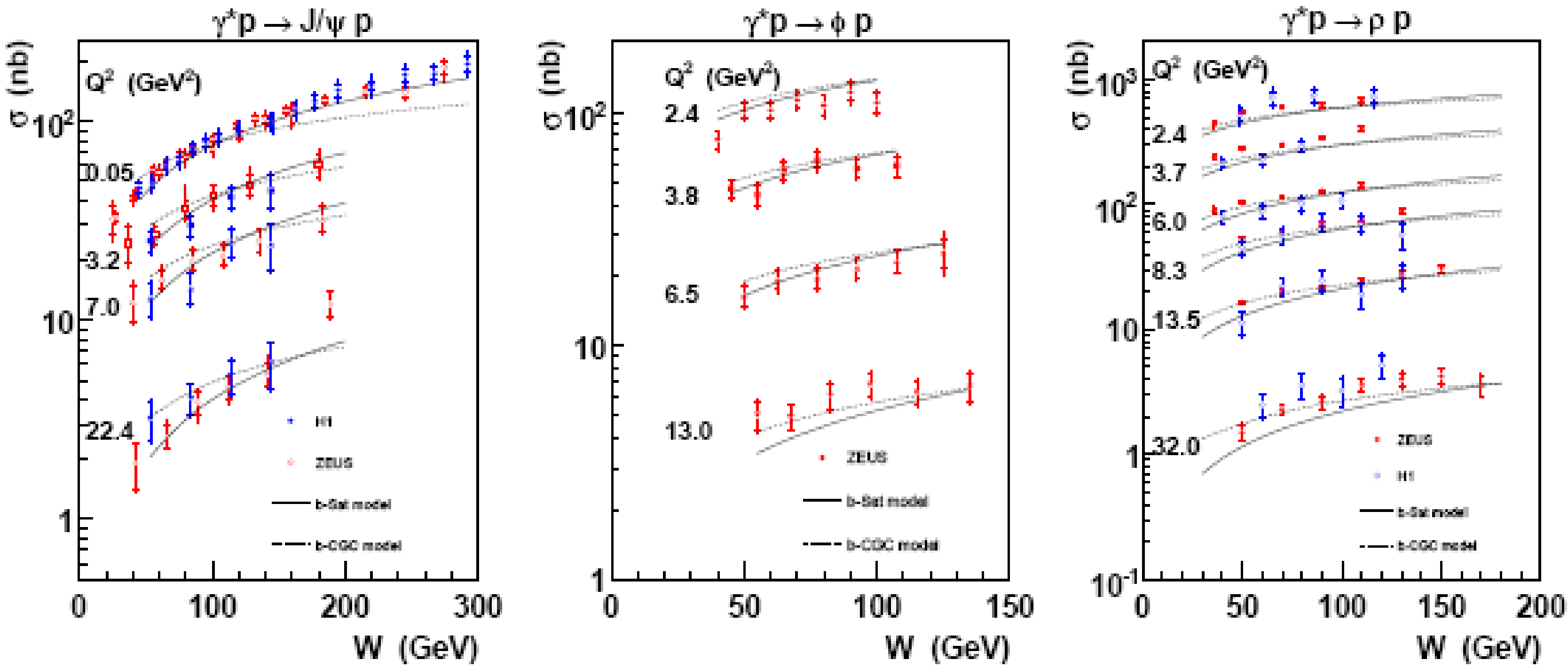
Vector Mesons

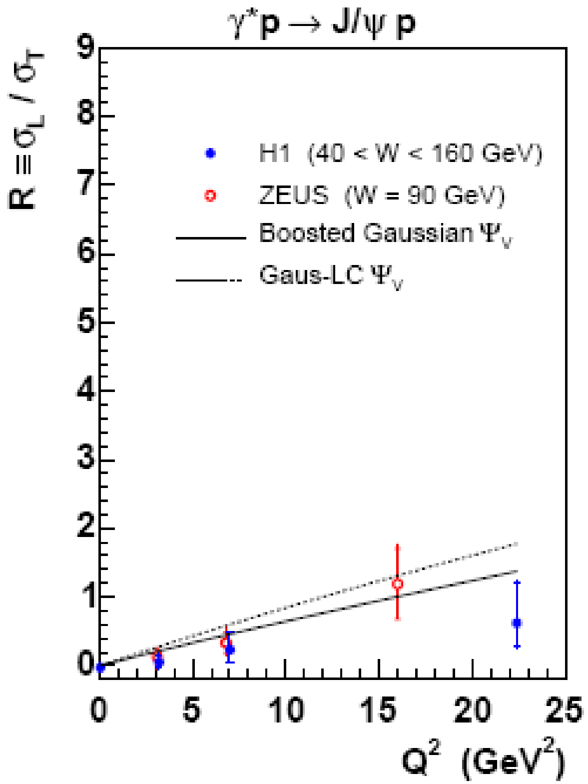
KMW

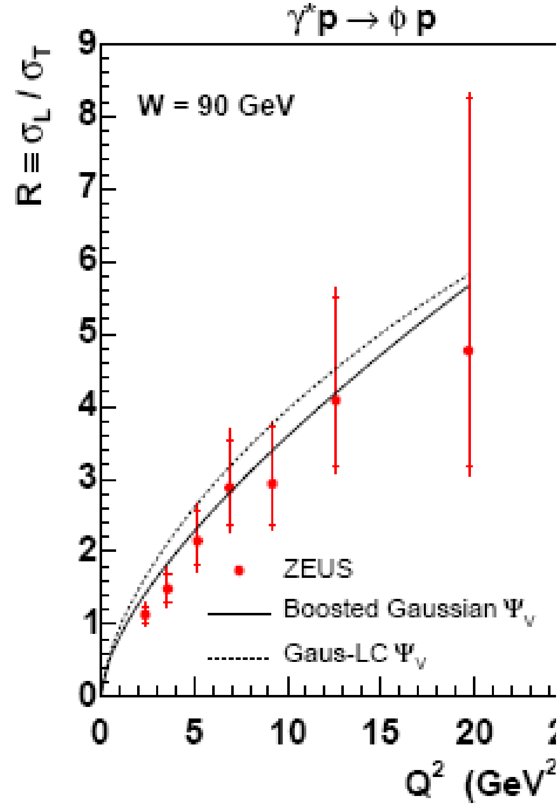


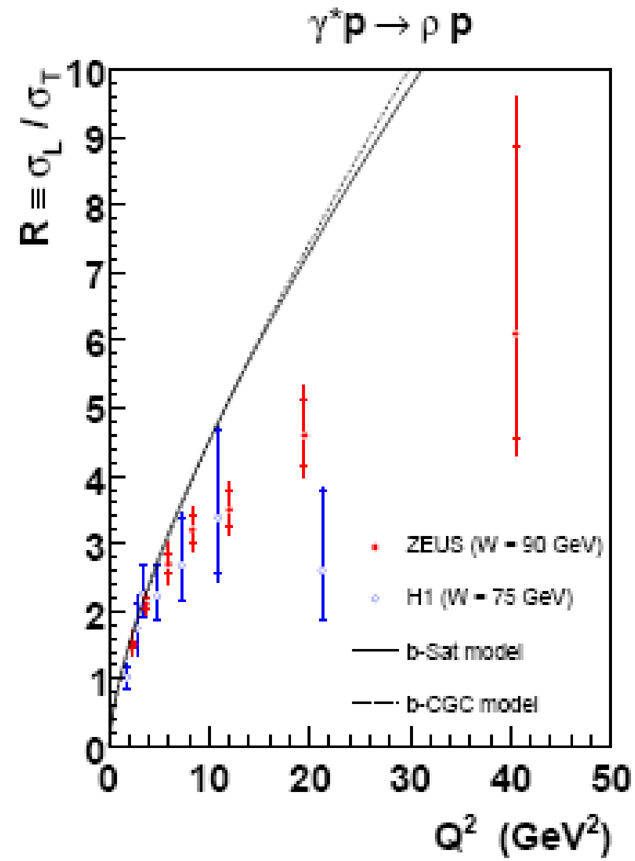
Note: educated guesses for VM wf are working very well

KMW


γ*p → J/ψ p
σ (nb)
Q² (GeV²)
0.05
3.2
7.0
22.4
H1
ZEUS
b-Sat model
b-CGC model
W (GeV)
γ*p → φ p
σ (nb)
Q² (GeV²)
2.4
3.8
6.5
13.0
ZEUS
b-Sat model
b-CGC model
W (GeV)
γ*p → ρ p
σ (nb)
Q² (GeV²)
2.4
3.7
6.0
8.3
13.5
32.0
ZEUS
H1
b-Sat model
b-CGC model
W (GeV)



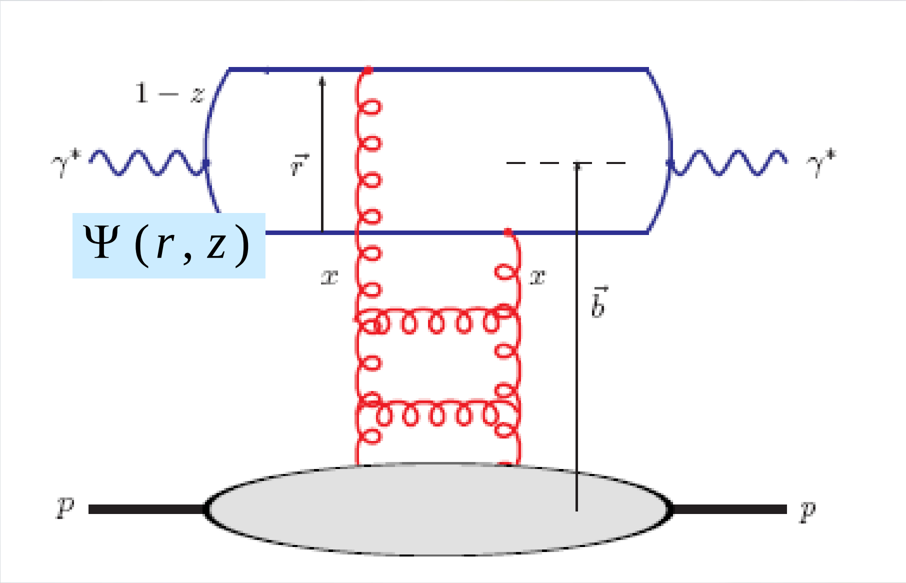


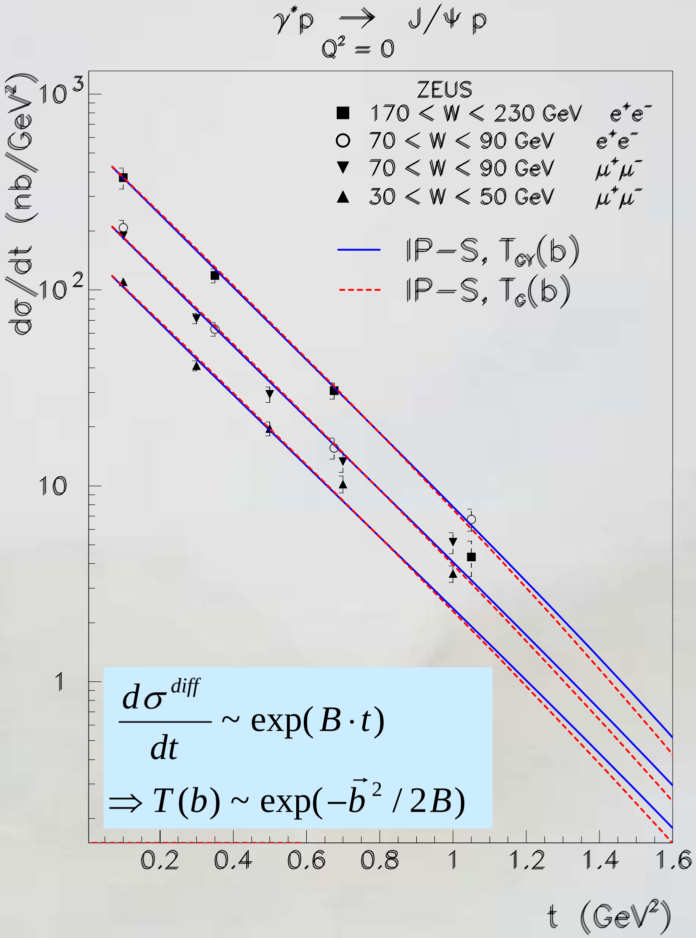


Note: educated guesses for J/Ψ and ϕ wave functions are working very well

More work to do for ρ meson wave function

Extracting Proton Vertex using Dipole Models





Can use vector meson production to extract proton profile:

$$\frac{d\sigma_{VM}^{\gamma^* p}}{dt} = \frac{1}{16\pi} \left| \int d^2\vec{r} \int d^2 b\, e^{-i\vec{b}\cdot\vec{\Delta}} \int_0^1 dz \Psi_{VM}^* \; 2\left\{1-\exp\left(-\frac{\Omega}{2}\right)\right\}\Psi \right|^2$$

KT, KMW

$$\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2)\, xg(x,\mu^2)\, T(b)$$

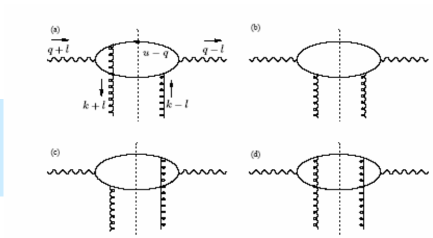
T(b)-proton shape

Description of the size of interaction region B_D

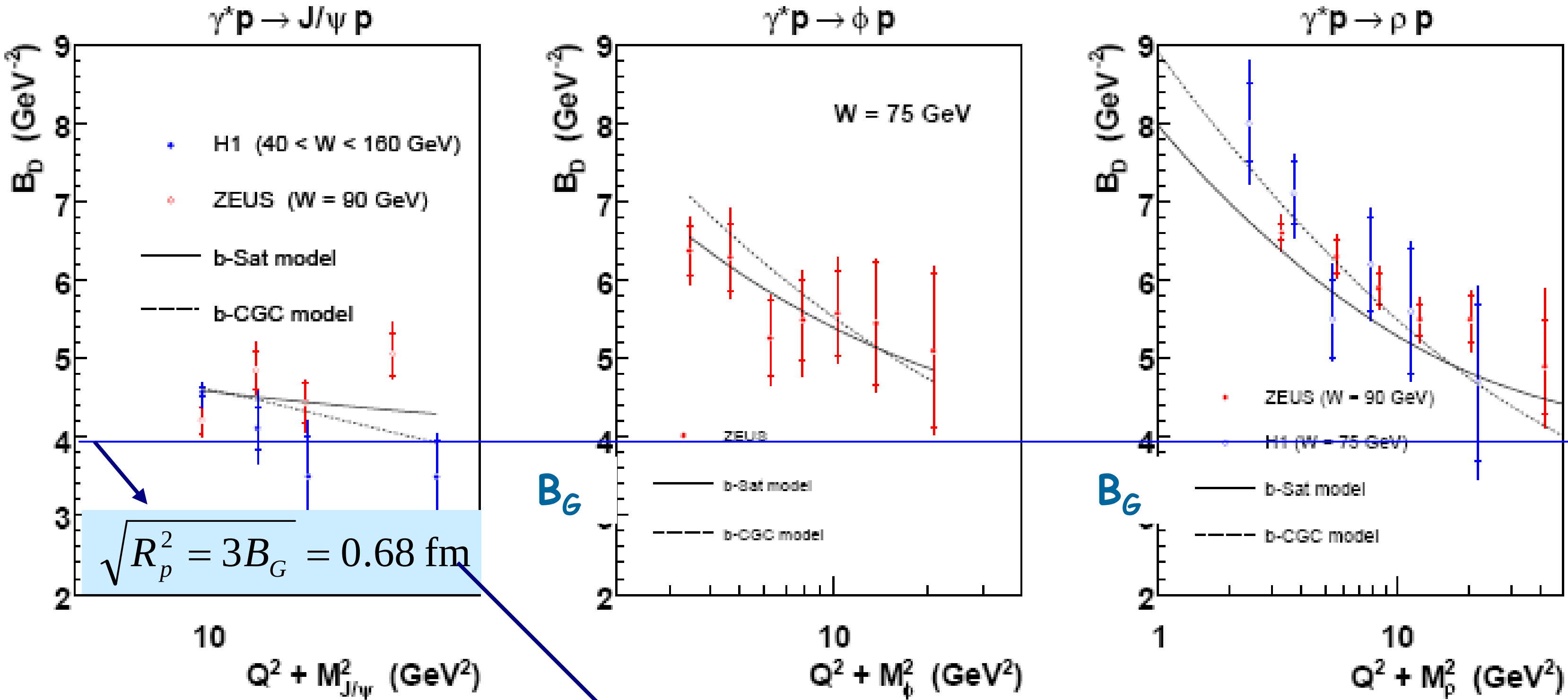
$$\frac{d\sigma^{diff}}{dt} \sim \exp(B_D \cdot t) \qquad \Rightarrow T(b) \sim \exp(-\vec{b}^2 / 2B_G)$$

Modification by Bartels, Golec-Biernat, Peters

$$e^{i\vec{b}\cdot\vec{\Delta}} \rightarrow e^{i(\vec{b}+(1-z)\vec{r})\cdot\vec{\Delta}}$$



KMW



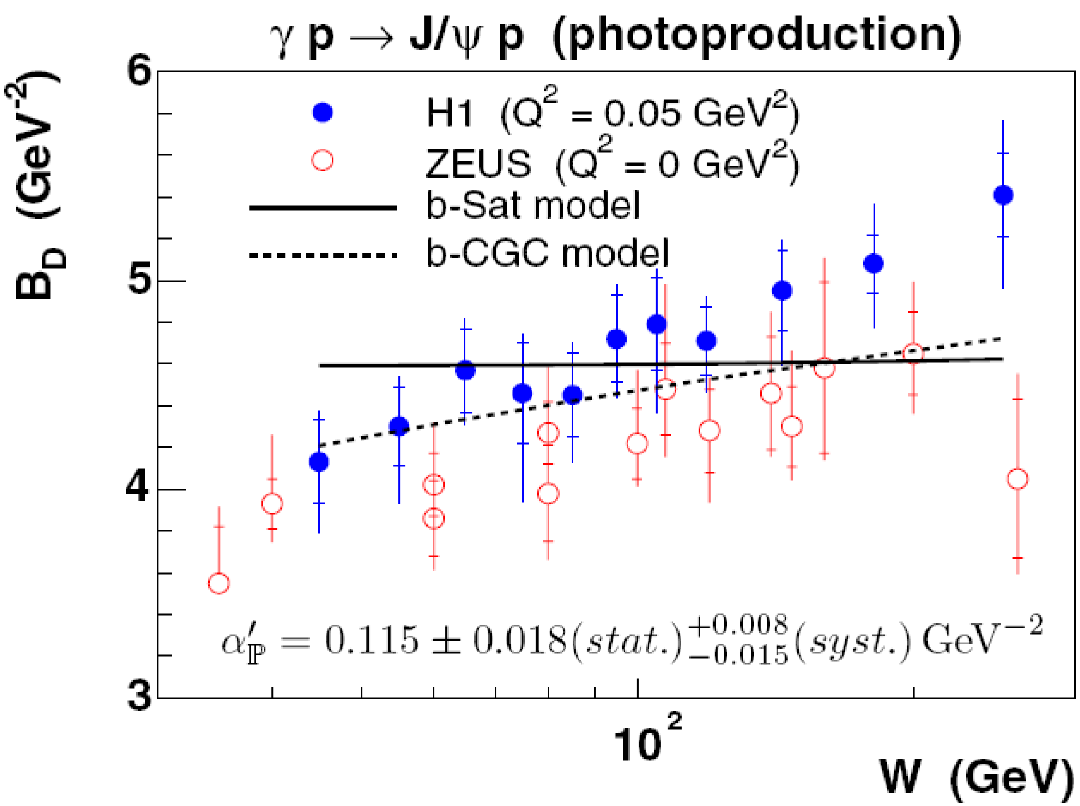
$$\sqrt{R_p^2 = 3B_G} = 0.68 \text{ fm}$$

$$R_p = 0.870 \pm 0.008 \text{ fm}$$

$$\Rightarrow B_G = 6.48 \text{ GeV}^2$$

the gluonic proton radius smaller than the quark radius

measurement of α’



α’ measurement suggests that $B_G \sim 3$ GeV^{-2}

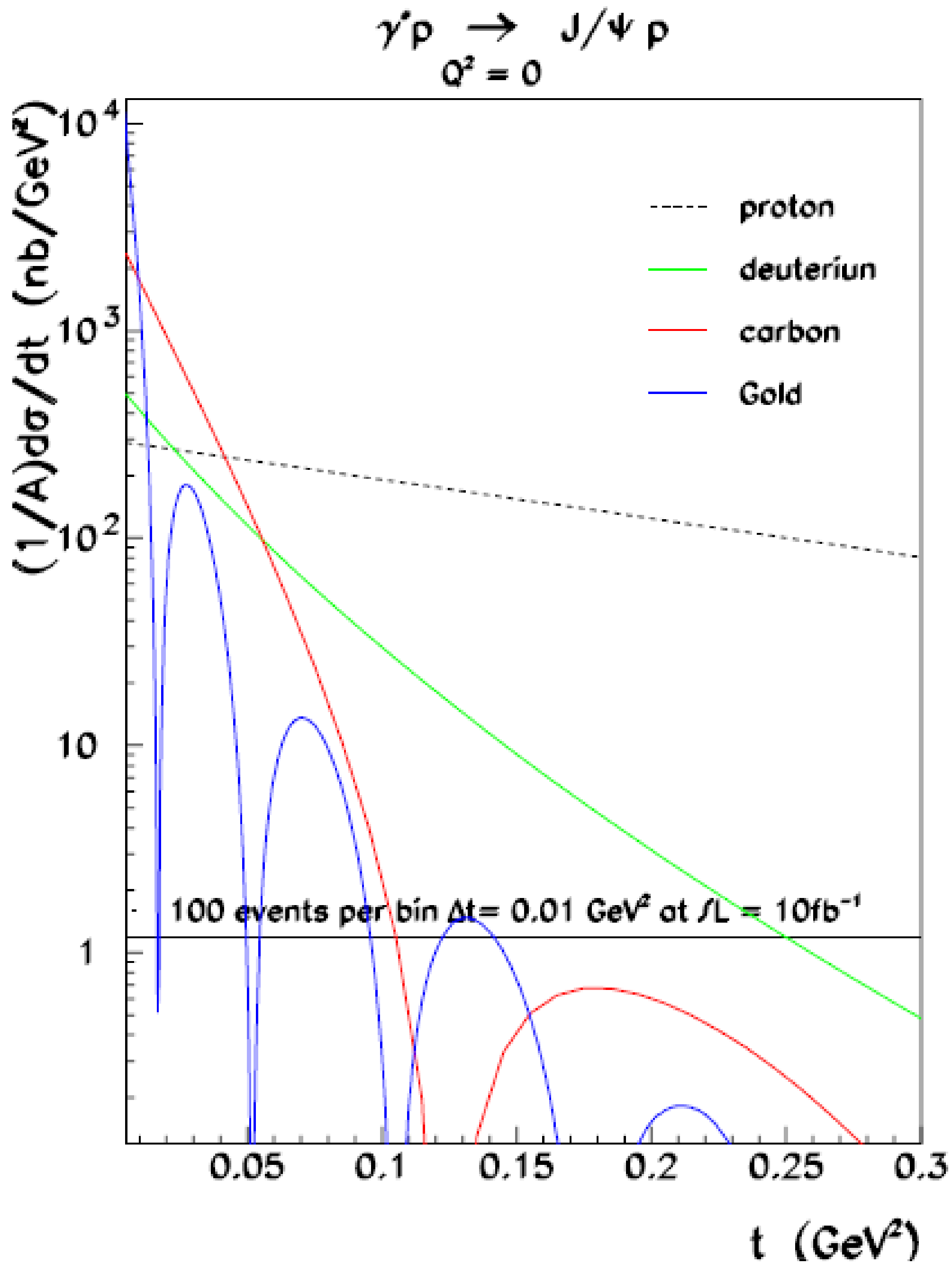
➔ $R_P \sim 0.6$ fm

t-distributions for coherent (ie exclusive) diffractive meson production on proton and nuclei at EIC

KLMV

first estimate of the expected measurement precision:

$\Delta p_T < 30$ MeV, $t \sim p_T^2$
$\Delta t < 0.01\ GeV^2$
for proton and light nuclei

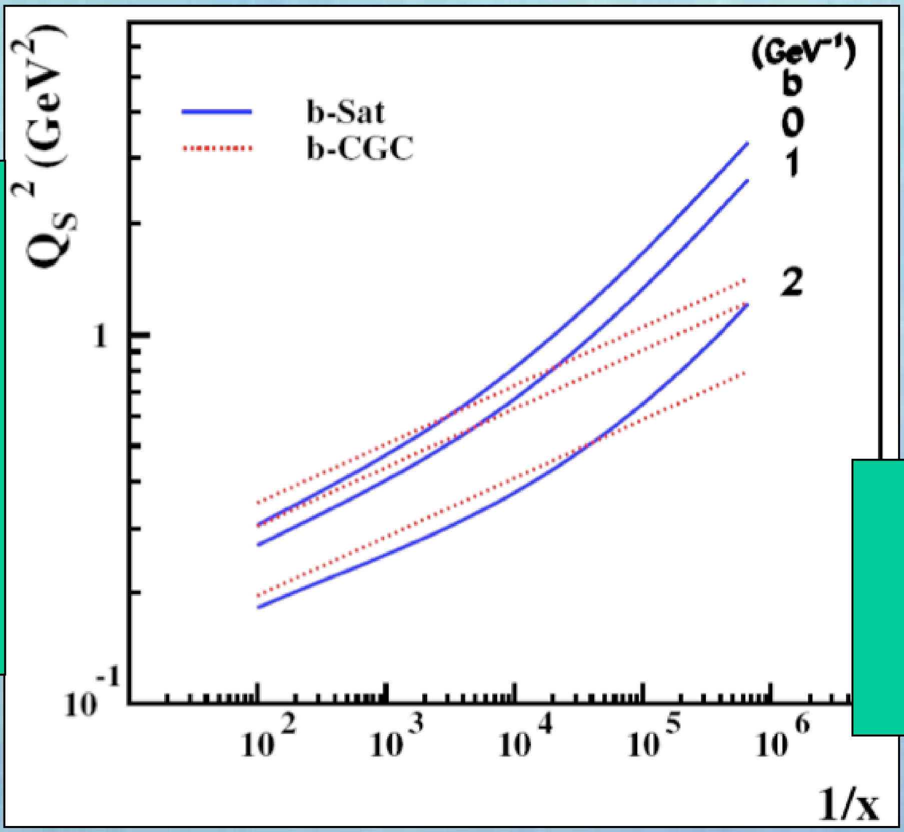


Saturation

Q_S: Measure of gluon density for which a dipole, r_S, is absorbed by a proton with 1-1/e probability: $Q_S=2/r_S$

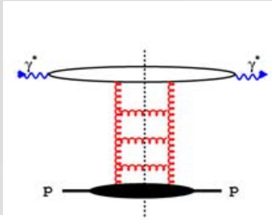
BGBK➔ Saturation shapes data in a similar way as DGLAP

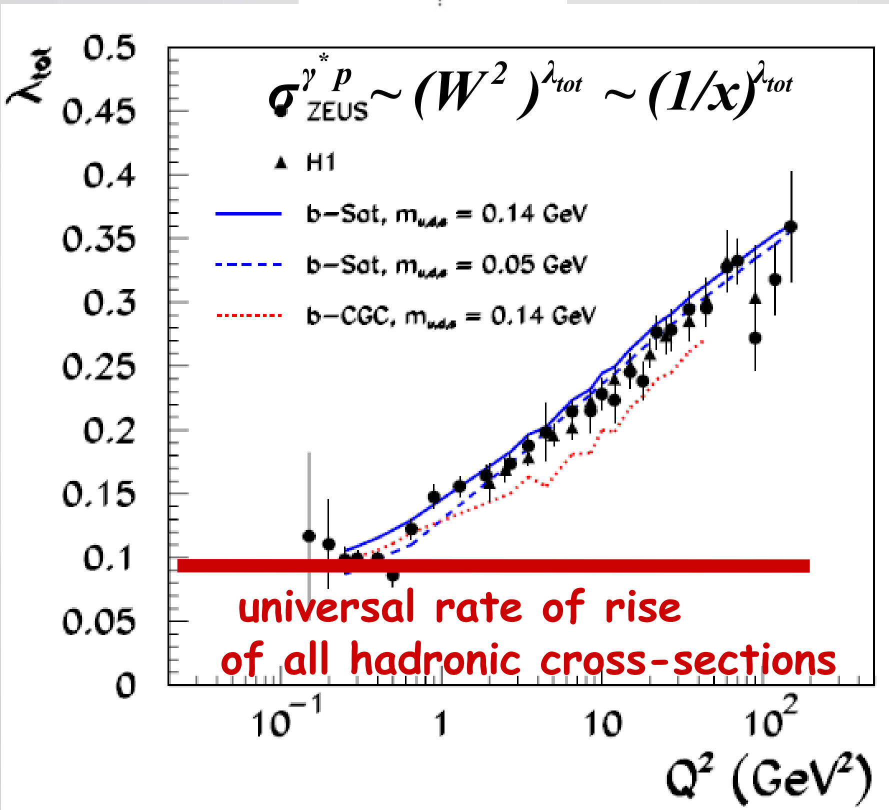
➔ Difficult to distinguish at HERA

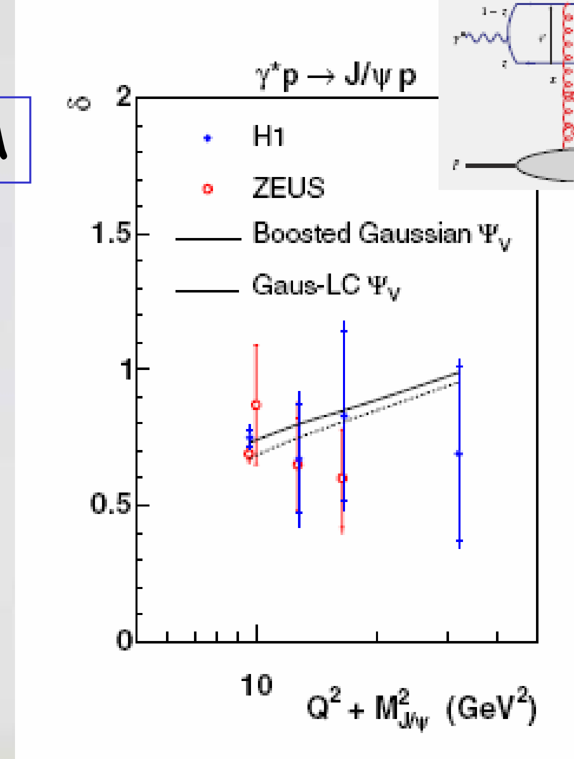


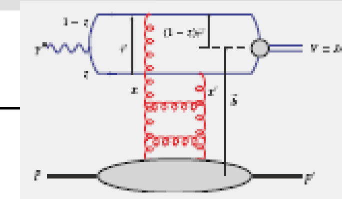
➔ Oomph factor Increase saturation by going to nuclei

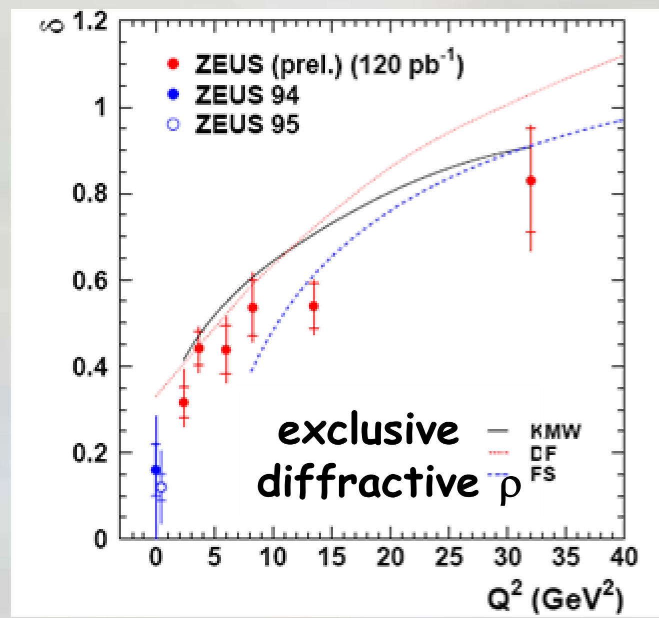
Discovery of HERA









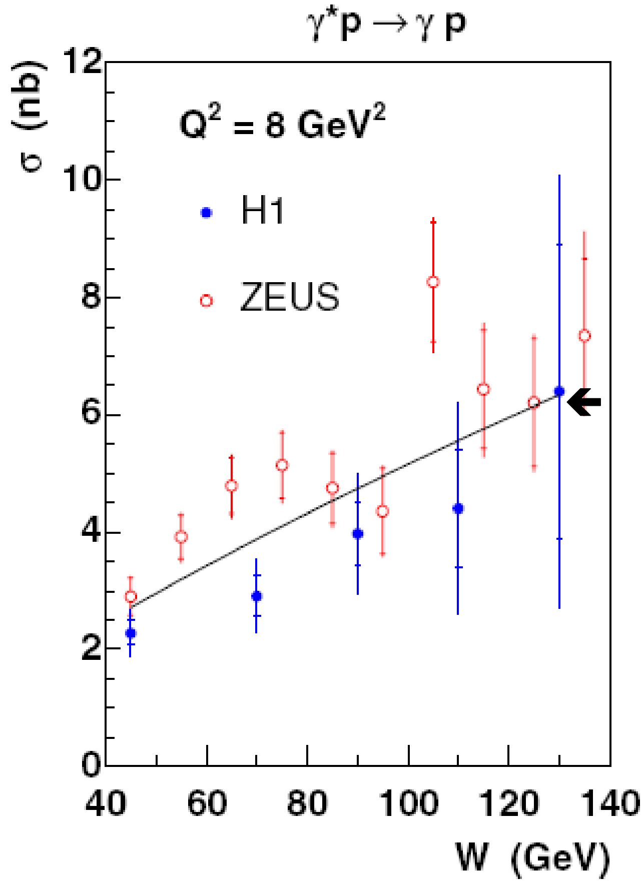


Universality of the observed intercepts

➔ Universal, “Pomeron like” QCD object
soft and hard Pomeron join together

Pomeron at work

Rise of the DVCS cross-sections



b-Sat or IP-Sat

At EIC (LHeC) it should be possible to reduce the errors by a large factor,

➔ detailed study of the Pomeron possible

first paper

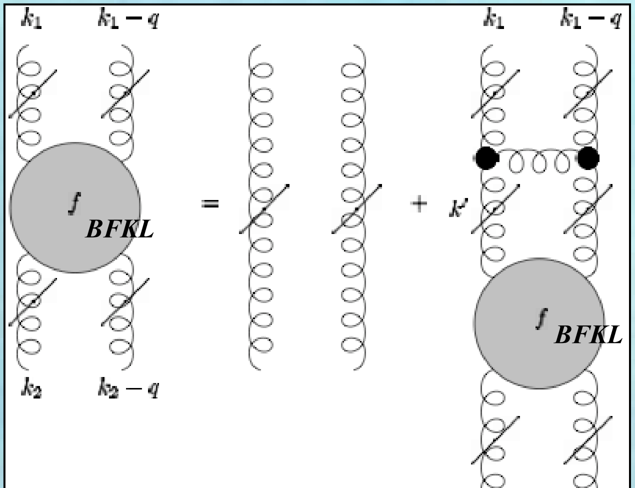
Physics Letters B 668 (2008) 51–56

Evidence for the discrete asymptotically-free BFKL Pomeron from HERA data

J. Ellis [a], H. Kowalski [b], D.A. Ross [a,c]

Ongoing Investigation

Basics of BFKL



BFKL equation at q=0

Conformal invariance

solved by finding a complete set of eigenfunctions

$$\omega \tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) = \delta^2(\mathbf{k_1} - \mathbf{k2}) + \frac{\alpha C_A}{\pi^2} \int \frac{d^2 \mathbf{k}'}{(\mathbf{k_1} - \mathbf{k}')^2} \left[\tilde{f}(\omega, \mathbf{k}', \mathbf{k_2}) - \frac{\mathbf{k_1}^2}{\mathbf{k}'^2 + (\mathbf{k}' - \mathbf{k_1})^2} \tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) \right]$$

Eigen-functions

$$f_\omega(k^2) = \frac{(k^2)^{i\nu}}{\sqrt{k^2}}$$

Characteristic function

$$\omega = \bar{\alpha}_s \chi(\nu)$$

$$\chi(\nu) = -2\gamma_E - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)$$

ψ is the Digamma function

Green function

$$f_{BFKL}(\omega, k_1, k_2) = \int_{-\infty}^{\infty} d\nu \left(\frac{k_1^2}{k_2^2} \right)^{i\nu} \frac{1}{2\pi^2 k_1 k_2} \frac{1}{(\omega - \bar{\alpha}_s \chi(\nu))}$$

Green function

$$f_{BFKL}(\omega, k_1, k_2) = \int_{-\infty}^{\infty} d\nu \left(\frac{k_1^2}{k_2^2}\right)^{i\nu} \frac{1}{2\pi^2 k_1 k_2} \frac{1}{(\omega - \bar{\alpha_s}\chi(\nu))}$$

usually approximated by:

$$\chi(\nu) = 4 \ln 2 - 14\zeta(3)\nu^2 + \ldots$$

$$f(\sqrt{s}, \mathbf{k_1}, \mathbf{k_2}) \sim \frac{1}{\mathbf{k_1 k_2}} s^{4\overline{\alpha_s}\ln(2)} \frac{1}{\sqrt{\ln(s)}} \exp\left\{\frac{-\ln^2(\mathbf{k_1}/\mathbf{k_2})}{14\zeta(3)\overline{\alpha_s}\ln(s)}\right\}$$

not used for DAFP

NLO BFKL with running α_s

NLO

$$\omega \equiv \chi(\alpha_s, \nu) = \bar{\alpha}_s (1 - A\bar{\alpha}_s) \chi_0 \left(\frac{1}{2} + \bar{\alpha}_s B + i\nu + \frac{\omega}{2}\right) + \bar{\alpha}_s^2 \chi_1(\nu).$$

Fadin, Lipatov
G. Salam
resummation

running coupling

$$\omega = \chi\left(\alpha_s(k), \nu_\omega(k)\right).$$

definition of k_{crit}

$$\omega = \chi(\alpha_s(k_{\rm crit}), 0).$$

property of χ:
largest ω at $\nu=0$

Airy functions are solving BFKL eq. around $k \sim k_{crit}$

$$\left[\frac{d^2}{d\ln(k^2/k_0^2)} + \frac{\beta_0}{2\pi} \frac{\dot{\chi}(\alpha_s(k_{\rm crit}), 0)}{\chi''(\alpha_s(k_{\rm crit}), 0)} \ln\left(\frac{k^2}{k_0^2}\right)\right] \overline{f_\omega}(k) = 0,$$

$$\bar{f}_\omega(k) = \left(k^2\right)^{i\nu},$$

away of k_{crit}

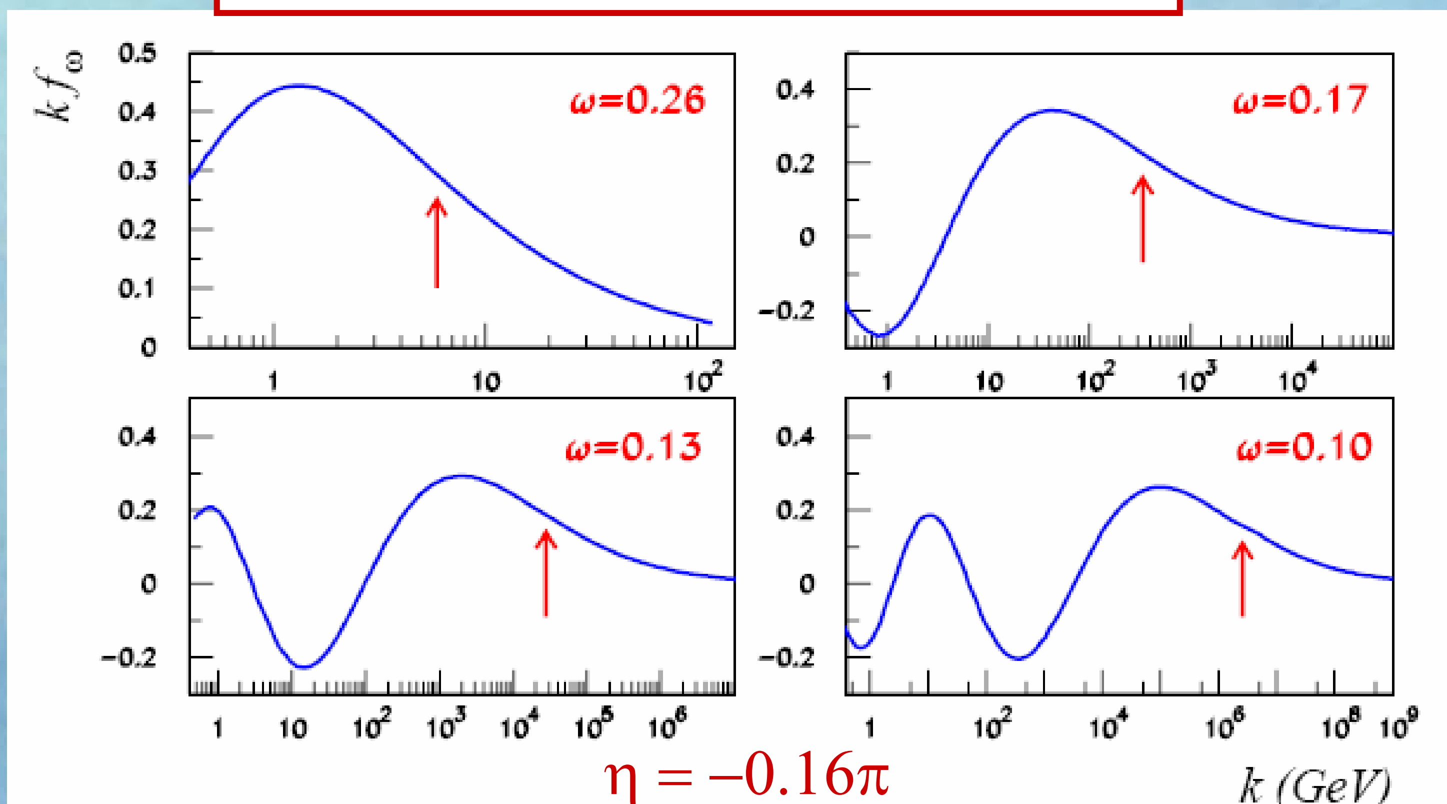
$$\overline{f_\omega}(k) \;=\; e^{\pm i\varphi_\omega(k)},$$

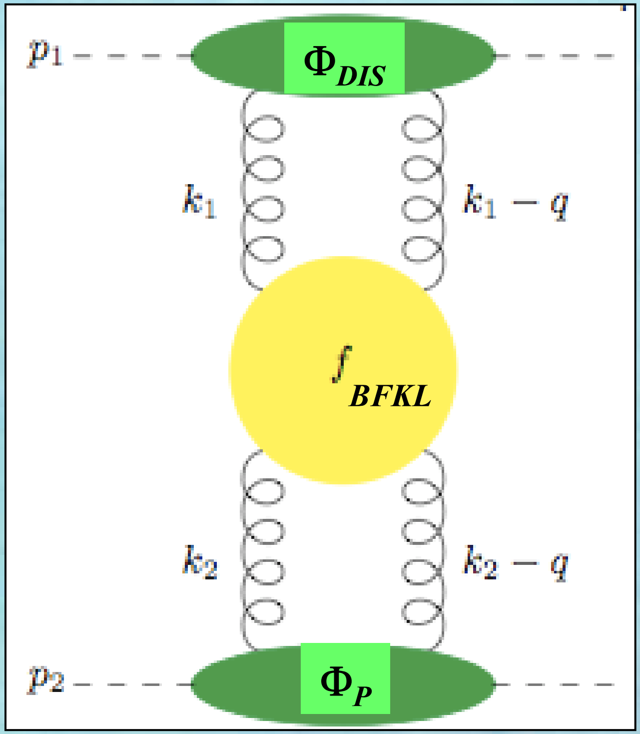
$$\varphi_\omega(k) = 2\int_{k}^{k_{\rm crit}} \frac{dk'}{k'}\left|\nu_\omega(k)\right.$$

Matching the solutions at $k=k_{crit}$ determines the phase of oscil.= $\pi/4$

Lipatov 86 ➔ encode the infrared behaviour of QCD by assuming a fixed phase η at k_0

$$\varphi_\omega(k_0) \;\equiv\; 2\int_{k_0}^{k_{\rm crit}} \frac{dk'}{k'}\,|\nu_\omega(k)| \;=\; \left(n-\frac{1}{4}\right)\pi+\eta,$$





Φ_{DIS} *known in QCD*

Φ_P *barely known*

Structure functions in DIS

$$F_2(x, Q^2) = \int_x^1 dz \int \frac{dk}{k} \Phi_{\mathrm{DIS}}(z, Q, k) xg\left(\frac{x}{z}, k\right),$$

unintegrated gluon density

$$xg(x,k) = \sum_n \int \frac{dk'}{k'} \Phi_p(k') \left(\frac{k'x}{k}\right)^{-\omega_n} k^2 f^*_{\omega_n}(k') f_{\omega_n}(k),$$

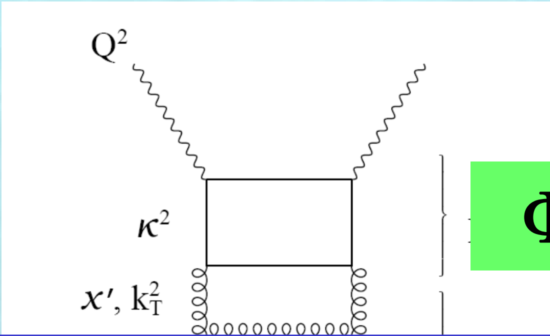
enhancement of leading eigenfun.by $(1/x)^{\omega}$

$$xg(x,k) = \sum_n a_n x^{-\omega_n} k^{(2+\omega_n)} f_{\omega_n}(k).$$

HERA
LHeC

no enhancement of leading eigenfun.

$$\Phi_p(k) = \sum_n a_n k^{(2-\omega_n)} f_{\omega_n}(k),$$



Φ_{DIS}

Kwiecinski, Martin Stasto

$$S_q(x,Q^2) \;=\; \frac{Q^2}{4\pi^2}\int\frac{dk^2}{k^4}\int_0^1 d\beta\int d^2\kappa'\alpha_S\left\{[\beta^2+(1-\beta)^2]\left(\frac{\boldsymbol{\kappa}}{D_{1q}}-\frac{\boldsymbol{\kappa}-\boldsymbol{k}}{D_{2q}}\right)^2\right.$$

$$\left.+\;[m_q^2+4Q^2\beta^2(1-\beta)^2]\left(\frac{1}{D_{1q}}-\frac{1}{D_{2q}}\right)^2\right\} f\left(\frac{x}{z},k^2\right)\Theta\left(1-\frac{x}{z}\right)$$

$\kappa' = \kappa - (1-\beta)k$ and

$$D_{1q} \;=\; \kappa^2 + \beta(1-\beta)Q^2 + m_q^2$$

$$D_{2q} \;=\; (\boldsymbol{\kappa}-\boldsymbol{k})^2 + \beta(1-\beta)Q^2 + m_q^2$$

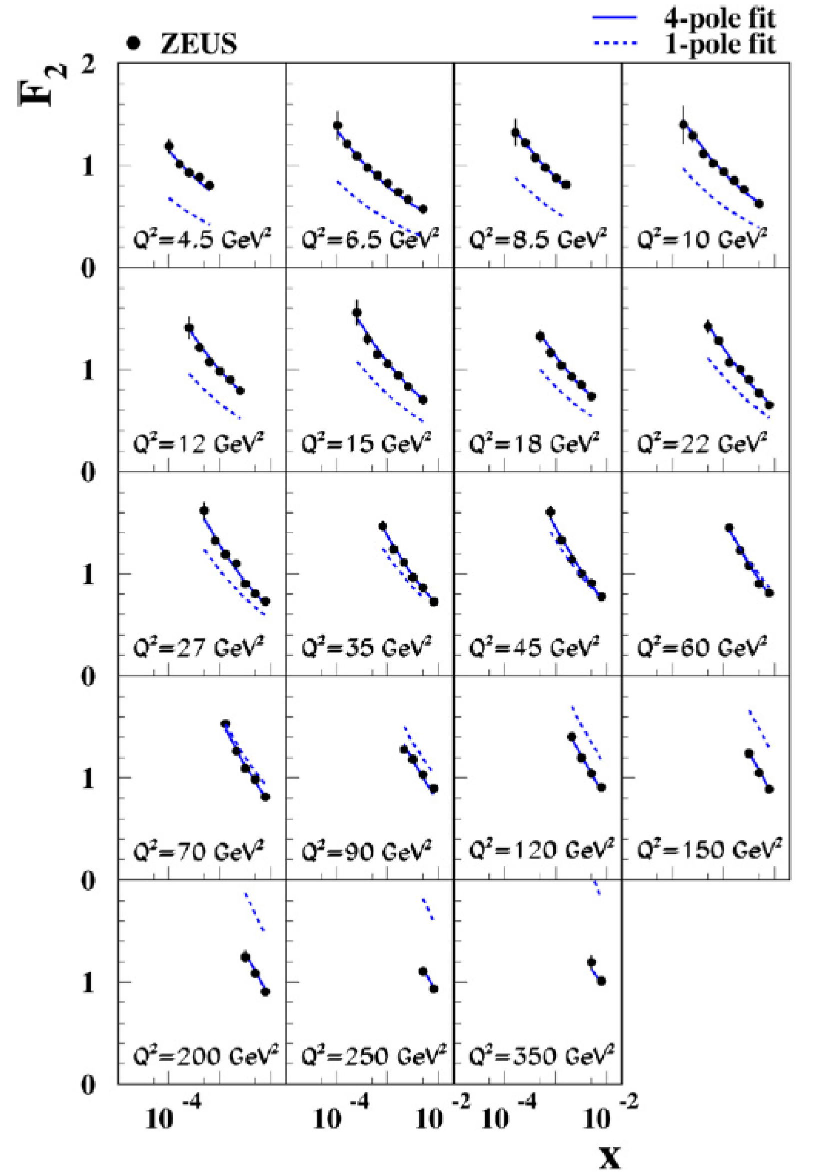
$$z \;=\; \left[1+\frac{\kappa'^2+m_q^2}{\beta(1-\beta)Q^2}+\frac{k^2}{Q^2}\right]^{-1}.$$

$$F_2 \;=\; \sum_q e_q^2(S_q+V_q),$$

Fit with charm

Correct qualitative behaviour from leading singularity

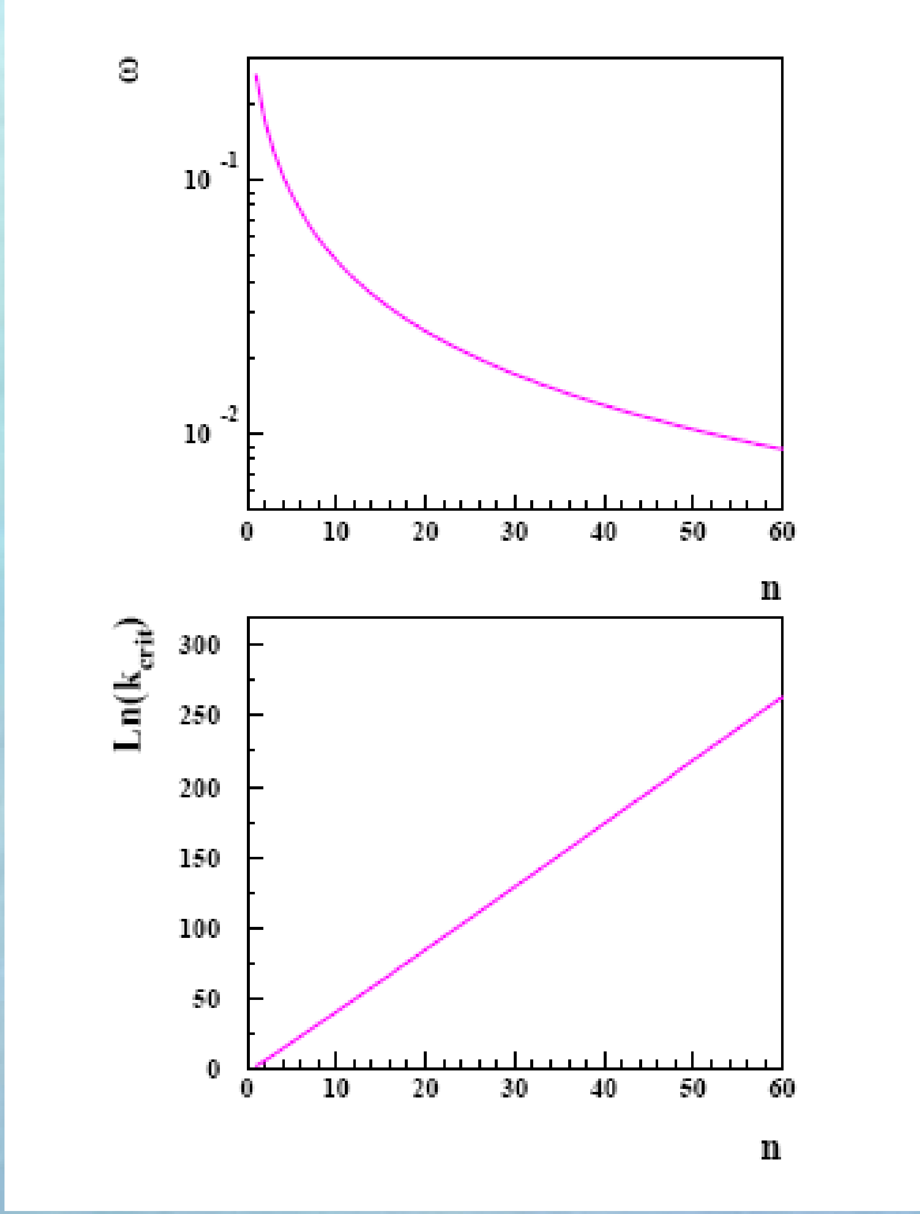
Excellent fit to data for $x < 10^{-2}$ with 4 poles



The qualities of fits using up to 4 poles, and the corresponding pole residues, assuming $\eta = -0.16\pi$ at $k_0 = 0.3$ GeV

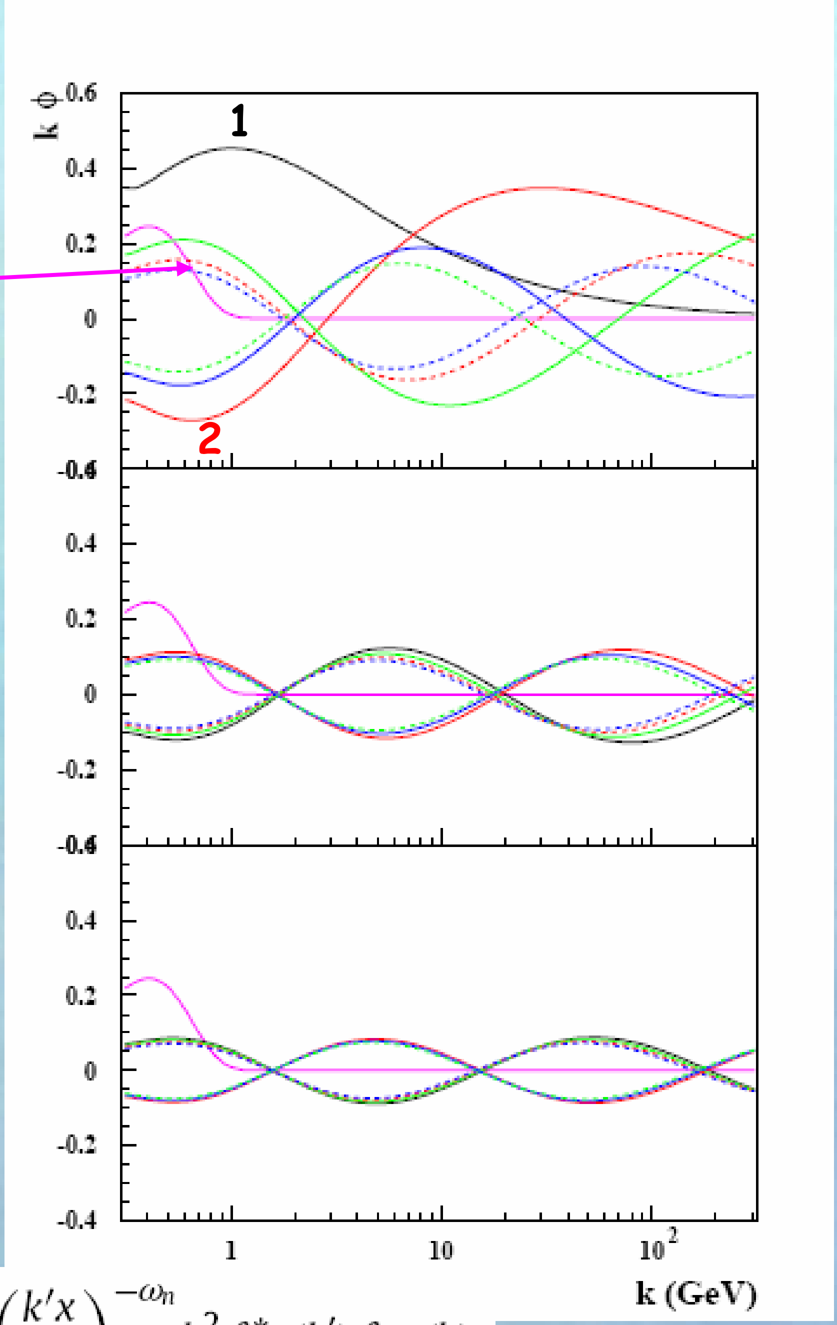
Number of poles	χ^2/N_{df}	a_1	a_2	a_3
1	11 894/101	0.478	–	–
2	1157/100	0.566	−0.98	–
3	167/99	0.707	0.87	3.70
4	83.3/98	0.483	−6.32	−26.0

ω_n and $\log(k_{crit})$



DAF Pomeron fit with

$\Phi_p = k^2 \exp(-bk^2)$



Eigenfunctions

1-7

8-14

15-21

$$xg(x,k) = \sum \int \frac{dk'}{k'} \Phi_p(k') \left(\frac{k'x}{k}\right)^{-\omega_n} k^2 f^*_{\omega_n}(k') f_{\omega_n}(k),$$

DAF Pomeron fit with

$\Phi_p = k^2 \exp(-bk^2)*(x_0)^{\omega}$

$\eta = -0.20*\pi$

$b = 1 - 10\ GeV^{-2}$

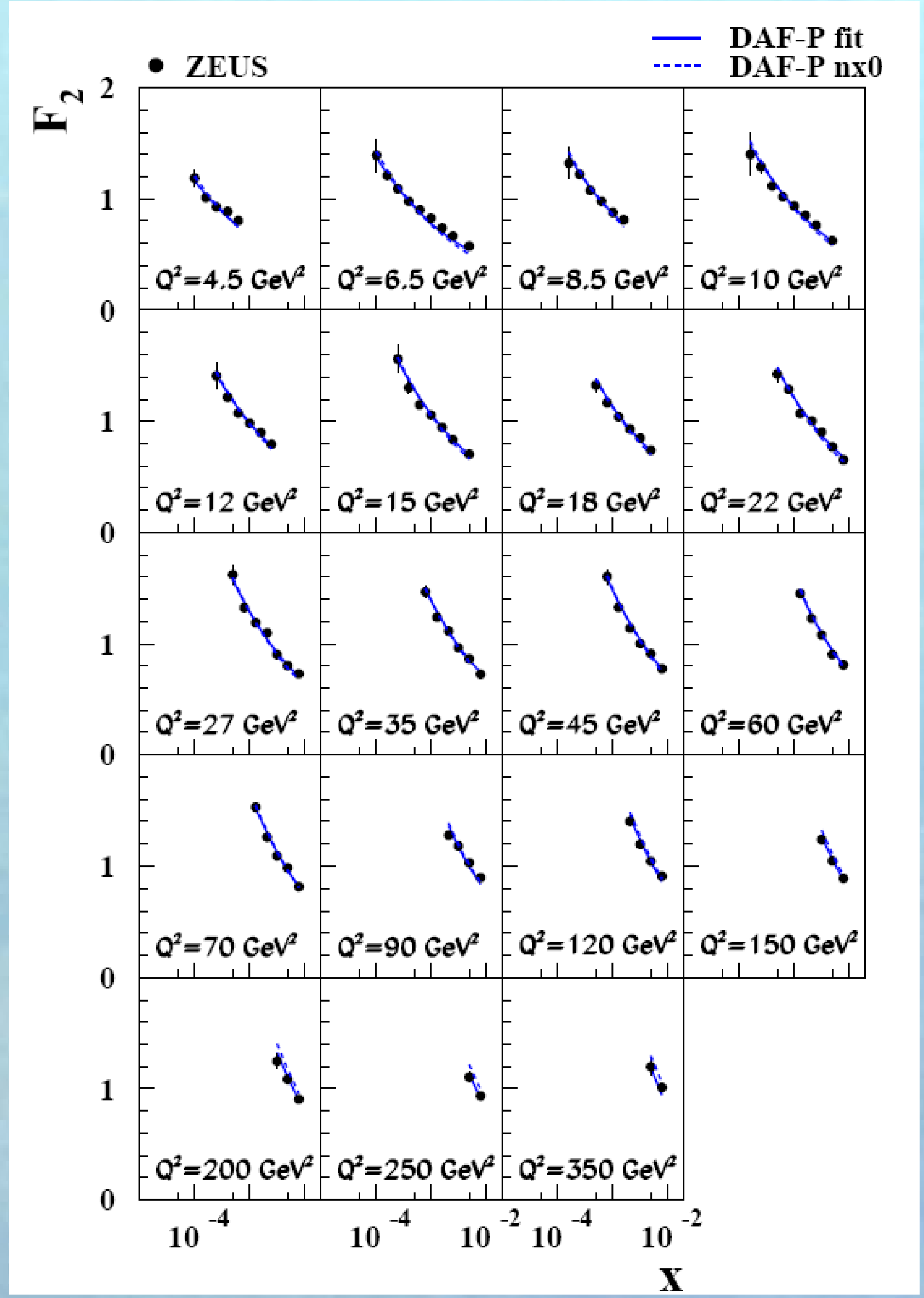
$x_0 \sim 0.2\text{-}0.4$

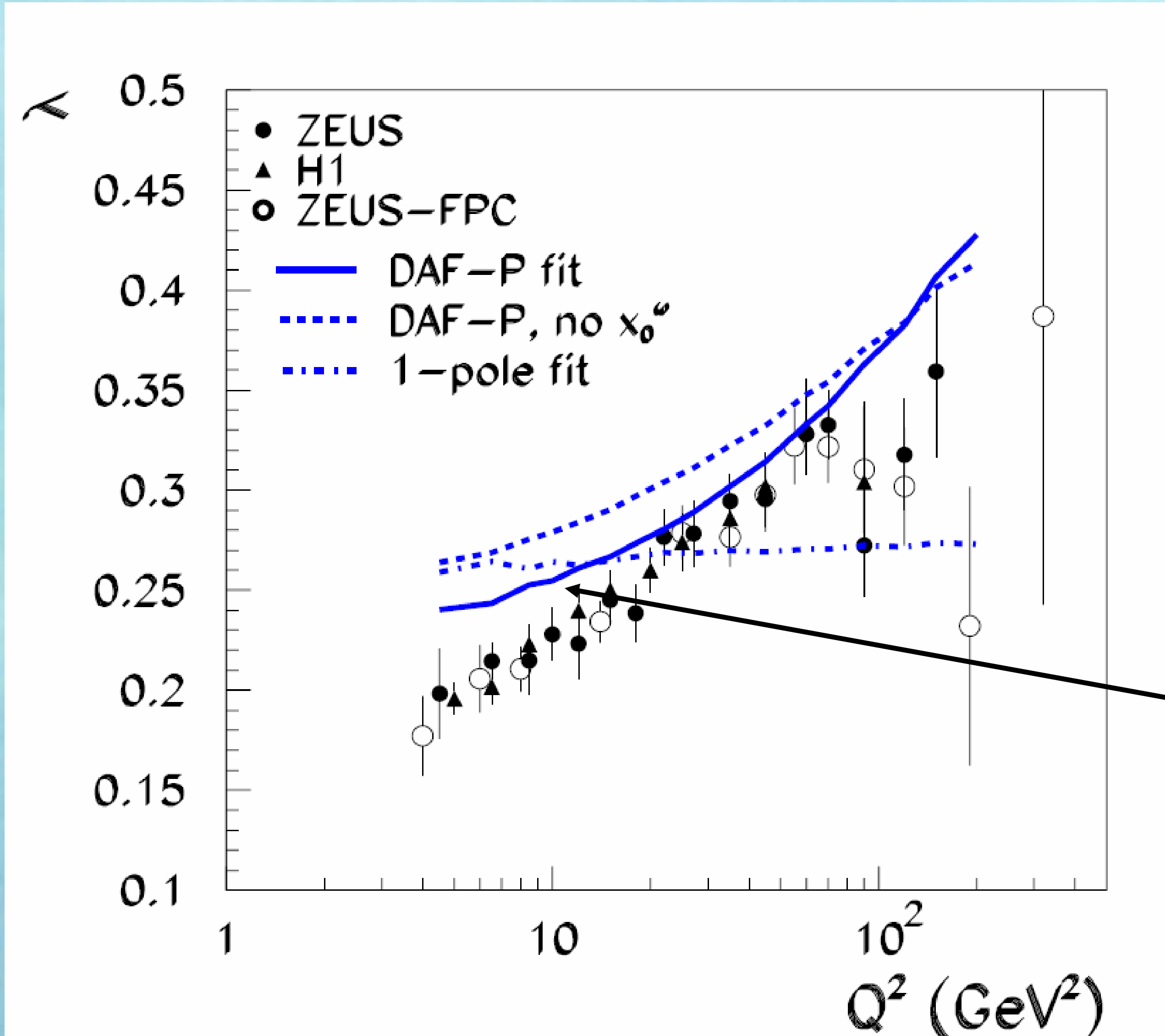
for $x < 0.001$

$\chi^2/ndf = 18/30$

for $x < 0.01$

$\chi^2/ndf = 115/100$

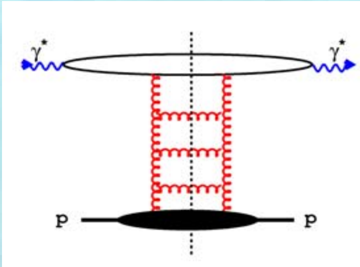




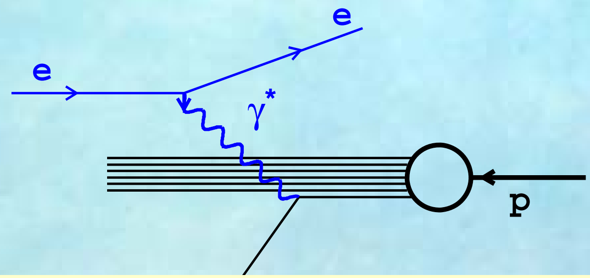
Sum of contributions with small eigenvalues can give a larger rate of rise than the leading eigenvalue !!!

GBW model tells that such overshoot at low Q2 can be cured by saturation effects

Does this mean that we have more saturation than usually assumed? Wait until we finish the investigation

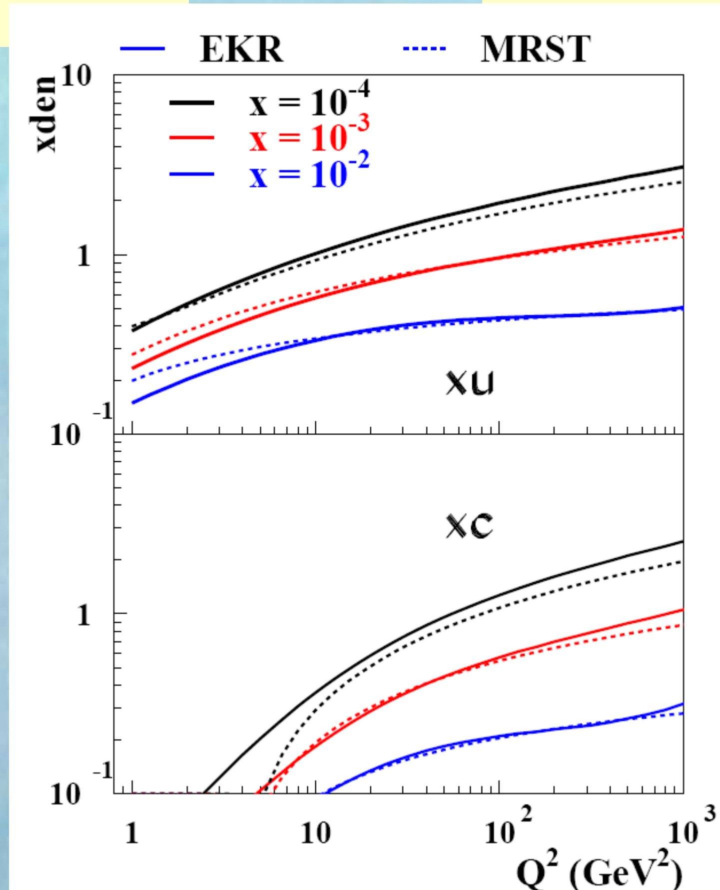


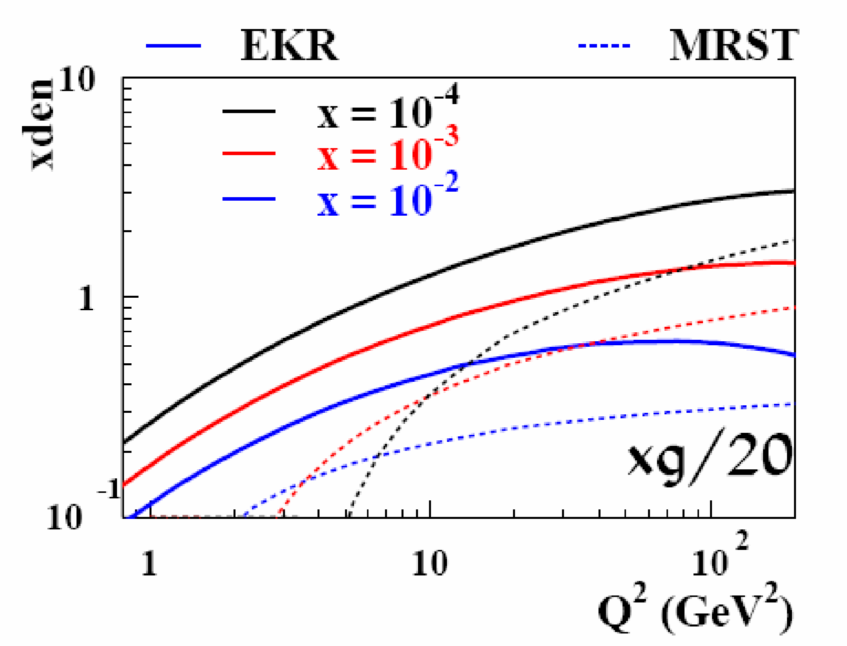
$$F_2(x,Q^2) = x\sum_q e_q^2 q(x,Q^2)$$



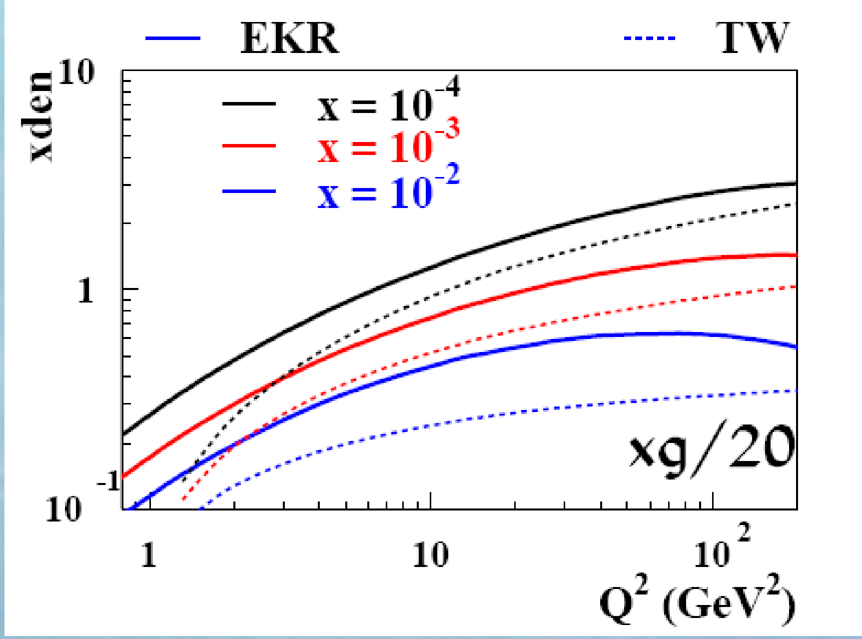
$$q(x,Q) = \int_0^Q \frac{dk}{k}\Phi_{DIS}(Q,k)xg(x,k)$$

$$q(x,\mu^2) = q_0(x) + \frac{\alpha_s}{\pi}\int_x^1 \frac{d\xi}{\xi} q_0(\xi)\left\{P_{qq}\left(\frac{x}{\xi}\right)\ln\mu/\kappa + \ldots\right.$$





EKR
MRST
xden
x = 10^{-4}
x = 10^{-3}
x = 10^{-2}
xg/20
10
1
10^{-1}
1
10
10^{2}
Q^2 (GeV^2)



EKR
TW
xden
x = 10^{-4}
x = 10^{-3}
x = 10^{-2}
xg/20
10
1
10^{-1}
1
10
10^{2}
Q^2 (GeV^2)

Where Do BFKL and DGLAP Meet

Lipatov, private communication

Unintegrated BFKL gluon density (LO, no running α_s)

$$xg(x,k^2) = \int d\gamma \Phi_p(\gamma) \left(\frac{k^2}{\mu^2}\right)^\gamma x^{-\bar{\alpha_s}\chi(\gamma)} = \int d\gamma \Phi_p(\gamma) \exp(F(\gamma))$$

$\gamma = ½ + i\nu$

Saddle point

$$(F(\gamma))' = (\gamma \ln(k^2/\mu^2) + \bar{\alpha_s} \ln(1/x)\chi(\gamma))' = 0$$

$$\chi(\gamma) \;=\; \frac{1}{\gamma} - 2\zeta(3)\gamma^2 + \cdots \quad \Longrightarrow \quad \gamma^2 \;=\; \frac{\bar{\alpha}\ln(1/x)}{\ln(k^2/\mu^2)}$$

$$\omega \approx \bar{\alpha_s}/\gamma = \sqrt{\frac{\bar{\alpha_s}\ln(k^2/\mu^2)}{\ln(1/x)}}$$

valid if $\bar{\alpha}(k^2)\ln(1/x) \;\ll\; 1,$

! not fulfilled for HERA or even Higgs at LHC !

equal to DLL limit of DGLAP (LO, no running α_s)

Pomeron and Gauge/String Duality

Brower, Polchinski, Strassler, and Tan, hep-th/0603115

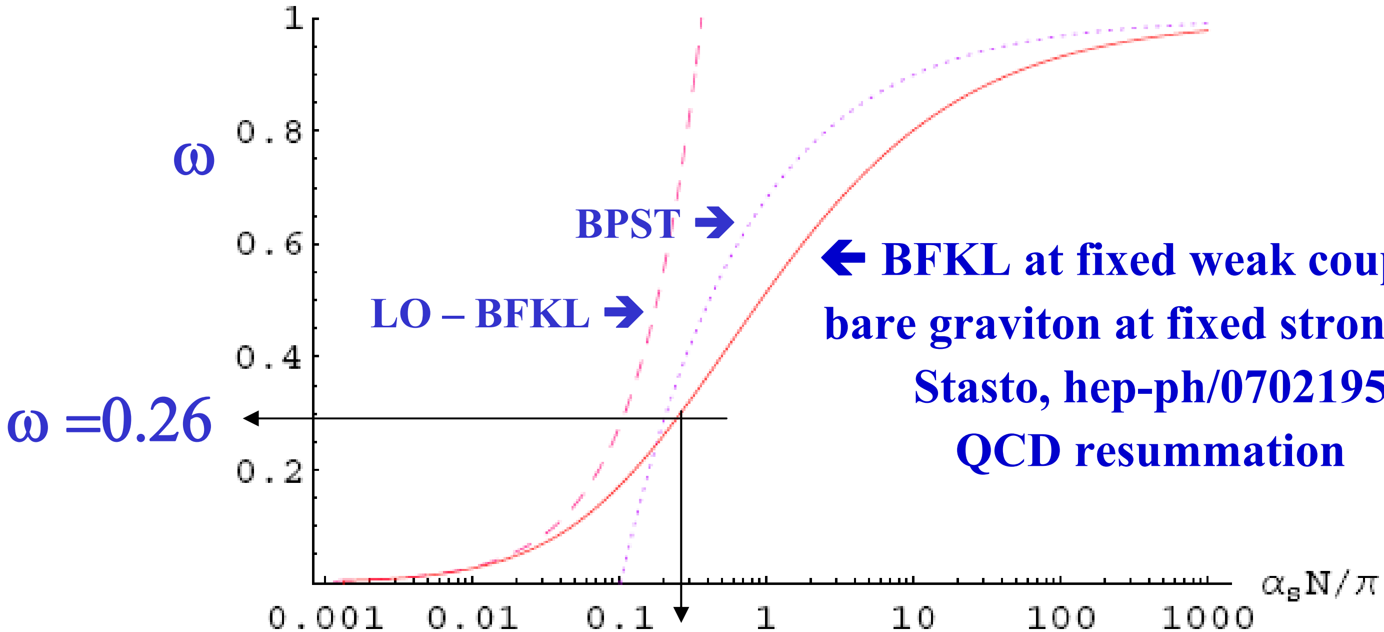
Pomeron is a coherent color-singlet object, build from gluons, with universal properties; it is a closed string propagating in ADS space, when the conformal symmetry is broken at some infrared point in the fifth dimension

$$1+\omega = 2 - \frac{2}{\sqrt{4\pi\alpha_s N}}$$

in ADS/CFT

in N=4 YM SuSy QCD

Kotikov, Lipatov, Onishchenko, Velizhanin, Physt. Lett. B 632, 754 (2006)



← BFKL at fixed weak coupling

bare graviton at fixed strong coupling

Stasto, hep-ph/0702195

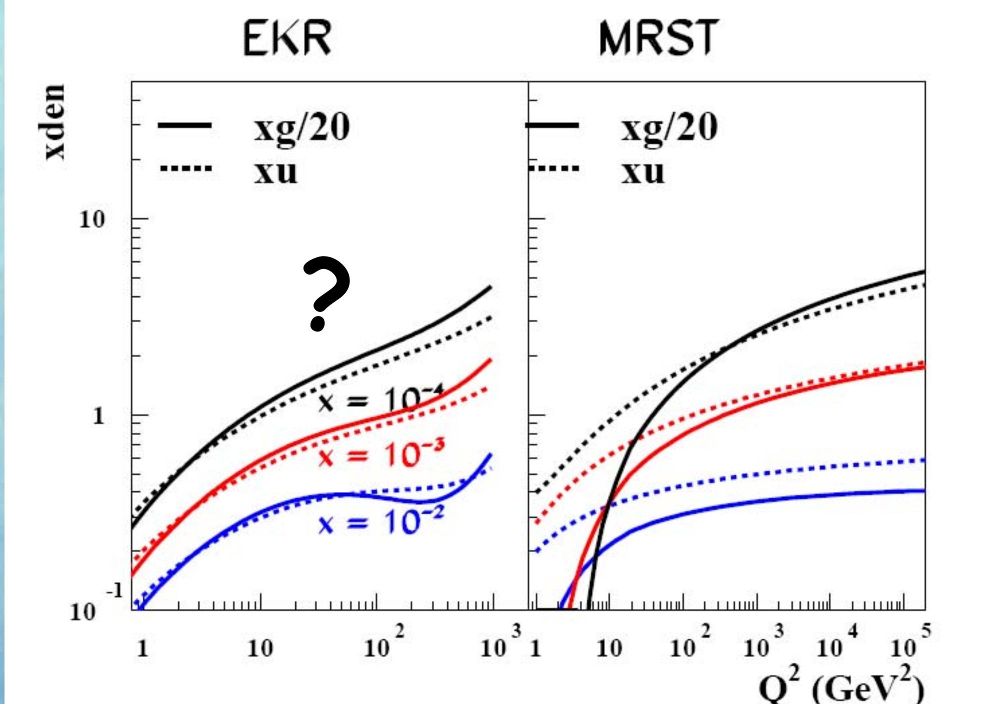
QCD resummation

Consequences for LHC

Good knowledge of gluon density around $x \sim 10^{-2}$ and $Q^2 \sim 10000$ GeV^2 is essential for LHC physics (Higgs region)

Large effort is going into precise measurement of W and Z inclusive X-sections ➔ precise determination of sea-quark distributions
➔ precise gluon density

Is the sea-quark ←➔ gluon density relation the same in the DGLAP-like picture (MRST/CTEQ) and DAF-Pomeron?



sea-quark←➔ gluon relation can be checked by the jets with p_T around 50 GeV

Instead of Conclusions

Study of Gluon Density are very important because it is the analog of Black Body Radiation in QED

It seemed hopeless to study pure Gluon Radiation since it is never free. However, it is becoming free for a short moment in HEP reactions

HERA has shown that physics processes at low-x are completely dominated by pure Gluon Density,

Investigation of Gluon Density has a chance to become as fundamental as Black Body radiation