Perturbative gauge theory from strings in $AdS_5 \times S^5$

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Outline

1. **Motivation**
   - The AdS/CFT correspondence
   - Anomalous dimensions
   - Direct 4-loop perturbative computation for the Konishi operator
2. **Anomalous dimensions from strings in $AdS_5 \times S^5$**
3. **The Konishi computation from strings**
4. **Twist two operators with arbitrary spin**
5. **Conclusions**

Motivation

$\mathcal{N} = 4$ Super Yang-Mills theory	$\equiv$	Superstrings on $AdS_5 \times S^5$
strong coupling		(semi-)classical strings
nonperturbative physics		or supergravity
very difficult		'easy'
weak coupling		highly quantum regime
'easy'		very difficult

New ways of looking at nonperturbative gauge theory physics
but very difficult to test...

Goal: Interpolate from strong to weak coupling to reach perturbative results staying on the string theory side of the correspondence

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  1. gluons (ordinary Yang-Mills) + specific matter content:
  2. 4 fermions in the adjoint representation
  3. 6 scalars in the adjoint representation
  4. appropriate interactions (Yukawa+quartic)
- The theory is *exactly* conformal (scale invariant) even on the quantum level
- In this theory one can perform quite rigorous computations at strong coupling using the AdS/CFT correspondence
- *$\mathcal{N} = 4$ SYM may be the 'harmonic oscillator' of four dimensional gauge theories* – D. Gross
- Use $\mathcal{N} = 4$ SYM as a theoretical laboratory for studying nonperturbative (and *perturbative*) gauge theory physics
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Anomalous dimensions

- Since $\mathcal{N}=4$ SYM is exactly conformal anomalous dimensions may be defined simply through two-point correlation functions

$$\langle O(x)O(y)\rangle = \frac{const}{|x-y|^{2\Delta}}$$

- The dimension Δ depends in a nontrivial way on the coupling $g^2=\lambda/16\pi^2$ where $\lambda \equiv g^2_{YM}N_c$ is kept fixed in the limit $N_c \to \infty$
- When computing anomalous dimensions from two point functions there are two types of graphs:

and

- The first class is contained in the so-called Asymptotic Bethe Ansatz of Beisert and Staudacher
- The second class are 'wrapping interactions' which start to appear at order g^{2L} (these are not contained in the Asymptotic Bethe Ansatz)

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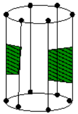

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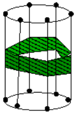

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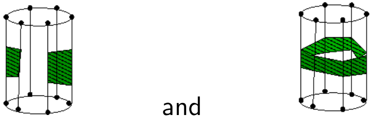

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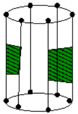

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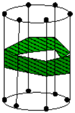

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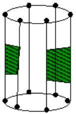

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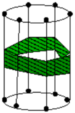

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The Konishi operator

- Simplest operator which is not protected by supersymmetry — the Konishi operator

$$\mathrm{tr}\,\Phi_i^2 \quad \longleftrightarrow \quad \mathrm{tr}\,Z^2X^2 + \ldots \quad \longleftrightarrow \quad \mathrm{tr}\,ZD^2Z + \ldots$$

- Its anomalous dimension should be given by the ABA exactly up to 3 loops:

$$E_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \ldots$$

- The true result is

$$E = E_{Bethe} + \Delta_{wrapping}E$$

  with $\Delta_{wrapping}E$ appearing first at 4 loops
- Recently a 4-loop perturbative computation was completed by F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon
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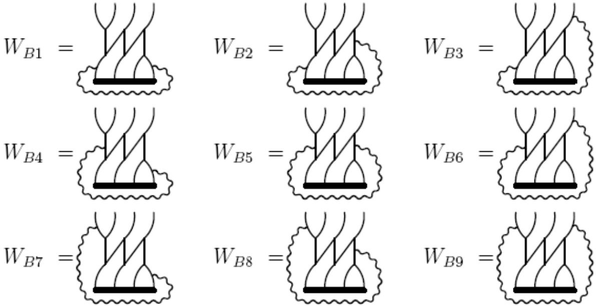


Figure C.1: Wrapping diagrams with chiral structure $\chi(1, 2, 3)$

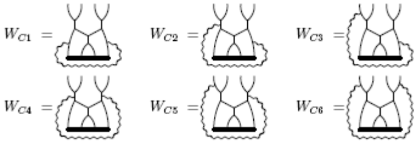


Figure C.2: Wrapping diagrams with chiral structure $\chi(1,3,2)$

$W_{C1} \rightarrow *$	1	$W_{C4} \rightarrow$ finite	
$W_{C2} \rightarrow *$	2	$W_{C5} \rightarrow -W_{C3}$	
$W_{C3} \rightarrow -W_{C5}$		$W_{C6} \rightarrow$ finite	

Table C.2: Results of D-algebra for diagrams with structure $\chi(1,3,2)$

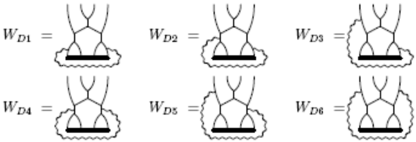
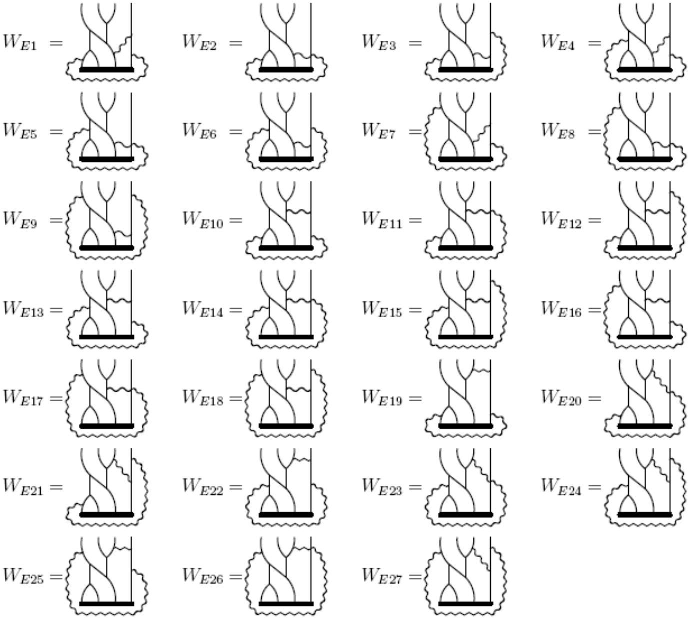


Figure C.3: Wrapping diagrams with chiral structure $\chi(2,1,3)$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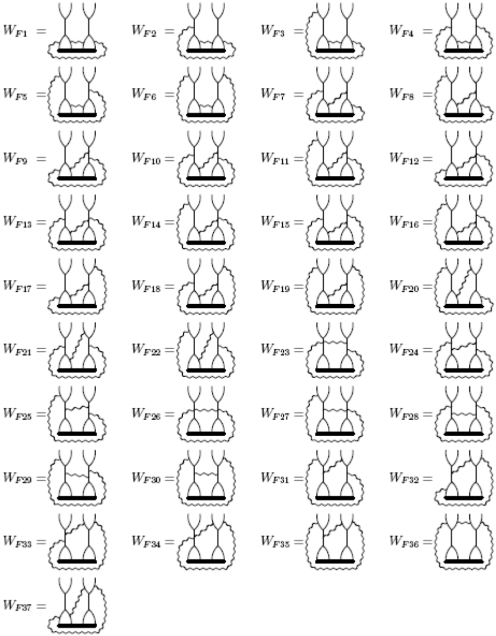
$W_{E27} =$


$W_{F1} =$
$W_{F2} =$
$W_{F3} =$
$W_{F4} =$
$W_{F5} =$
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$W_{F7} =$
$W_{F8} =$
$W_{F9} =$
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$W_{F24} =$
$W_{F25} =$
$W_{F26} =$
$W_{F27} =$
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$W_{F29} =$
$W_{F30} =$
$W_{F31} =$
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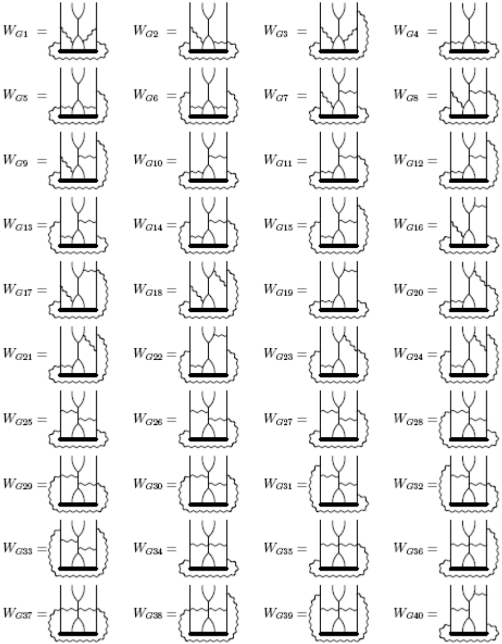
$W_{F36} =$
$W_{F37} =$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W_G2 =
W_G3 =
W_G4 =
W_G5 =
W_G6 =
W_G7 =
W_G8 =
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W_G10 =
W_G11 =
W_G12 =
W_G13 =
W_G14 =
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W_G17 =
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W_G39 =
W_G40 =

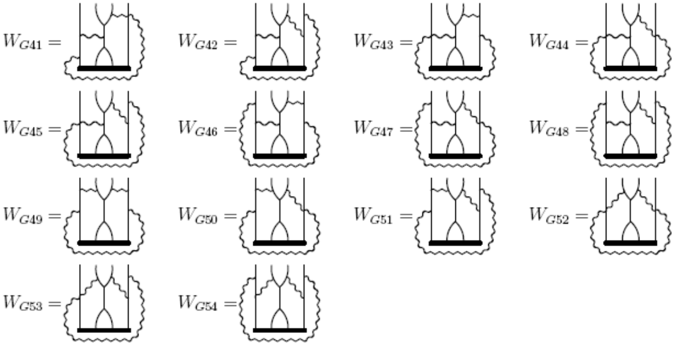
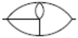


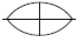
Figure C.6: Wrapping diagrams with chiral structure $\chi(1)$ (continued)

$I_1 = J_1 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{24\varepsilon^4} + \frac{1}{4\varepsilon^3} - \frac{19}{24\varepsilon^2} + \frac{5}{4\varepsilon}\Big)$

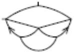
$I_2 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{24\varepsilon^4} + \frac{1}{4\varepsilon^3} - \frac{19}{24\varepsilon^2} + \frac{1}{\varepsilon}\Big(\frac{5}{4} - \zeta(3)\Big)\Big)$

$I_3 = J_5 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{12\varepsilon^4} + \frac{1}{3\varepsilon^3} - \frac{5}{12\varepsilon^2} - \frac{1}{\varepsilon}\Big(\frac{1}{2} - \zeta(3)\Big)\Big)$

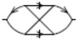
$I_4 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{6\varepsilon^4} + \frac{1}{3\varepsilon^3} + \frac{1}{3\varepsilon^2} - \frac{1}{\varepsilon}(1 - \zeta(3))\Big)$

$I_5 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}5\zeta(5)$

$I_6 =$  $= \frac{1}{(4\pi)^8}\Big(\frac{1}{12\varepsilon^2} - \frac{7}{12\varepsilon}\Big)$ $\quad I_7 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}(-\zeta(3))$

$I_8 =$  $= \frac{1}{(4\pi)^8}\Big(\frac{1}{4\varepsilon^2} - \frac{11}{12\varepsilon}\Big)$ $\quad I_9 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(\frac{1}{2}\zeta(3) - \frac{5}{2}\zeta(5)\Big)$

$I_{10} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{2} - \frac{1}{2}\zeta(3) + \frac{5}{2}\zeta(5)\Big)$

$I_{11} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{4} - \frac{3}{2}\zeta(3) + \frac{5}{2}\zeta(5)\Big)$

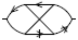
$I_{12} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{8} - \frac{1}{4}\zeta(3) + \frac{5}{4}\zeta(5)\Big)$

Table C.8: Loop integrals for 4-loop wrapping diagrams. The arrows of the same type indicate contracted spacetime derivatives

Perturbative 4-loop result for the Konishi

- The final result for the anomalous dimension of the Konishi operator is

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{\left(-2496 + 576\zeta(3) - 1440\zeta(5)\right) g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

($-2584 \longrightarrow -2496$ after the appearance of our paper)

- The wrapping part is thus

$$\Delta_{wrapping} E = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- Later this result was confirmed by an independent perturbative gauge theory computation using ordinary Feynman graphs by V. Velizhanin (total number of four loop diagrams: 131015)
- Transcendental numbers start to appear...
- $\zeta(3)$ was expected to appear, but $\zeta(5)$ was initially a surprise

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Our goal:

Compute the same 4-loop anomalous dimension from string theory

How to describe strings in $AdS_5 \times S^5$?

- Consider a closed string in $AdS_5 \times S^5$:

- The embedding coordinates of the point (τ, σ) are *quantum fields* $X^\mu(\tau, \sigma)$ on the worldsheet which has the geometry of a cylinder
- String theory in $AdS_5 \times S^5$ $\equiv$ a specific two dimensional quantum field theory defined on a cylinder (worldsheet QFT)
- It turns out that this worldsheet QFT is *integrable*

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Anomalous dimensions from strings in $AdS_5 \times S^5$

- Anomalous dimensions correspond to energies of string states in $AdS_5 \times S^5$
- Energies of string states are just energy levels of the two dimensional worldsheet QFT
- The Konishi operator has the same anomalous dimension as $\mathrm{tr}\, ZXZX - \mathrm{tr}\, Z^2X^2$
- We have to identify the corresponding string state...
- number of X's $\equiv$ number of particles on the string worldsheet
- number of Z's $\equiv$ size of the cylinder
- We have to compute the energy of a two particle state on a cylinder of size $J = 2$

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Answer:

- Leading part is identical to the Asymptotic Bethe Ansatz
- On top of this there are virtual corrections (Lüscher corrections generalized to multiparticle states) — these correspond to *wrapping interactions/graphs*
- These may be summarized by a single graph contributing to the desired order ($\mathcal{O}\left(g^8\right)$):

$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left(\frac{z^-}{z^+}\right)^2 \sum_b (-1)^{F_b} \left[S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm)\right]_{b(11)}^{b(11)}$$

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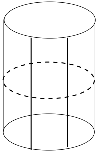

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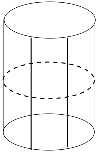

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- We have

$$\left(\frac{z^-}{z^+} \right)^2 = \frac{16 g^4}{(Q^2+q^2)^2} + \dots$$

- The scalar part gives

$$S_{Q-1}^{scalar,sl(2)} = \frac{3q^2 - 6iQq + 6iq - 3Q^2 + 6Q - 4}{3q^2 + 6iQq - 6iq - 3Q^2 + 6Q - 4} \cdot \frac{16}{9q^4 + 6(3Q(Q+2)+2)q^2 + (3Q(Q+2)+4)^2}$$

- The matrix part (summed over b) evaluates to

$$S_{Q-1}^{matrix,sl(2)} = \frac{5184 Q^2 (3q^2 + 3Q^2 - 4)^2 g^4}{(q^2+Q^2)^2((3q - 3iQ + 3i)^2 - 3)^2}$$

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$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left(\frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- We have

$$\left(\frac{z^-}{z^+} \right)^2 = \frac{16 g^4}{(Q^2+q^2)^2} + \dots$$

- The scalar part gives

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- The integral over q can be carried out analytically by residues
- The result is

$$\sum_{Q=1}^{\infty}\left\{-\frac{num(Q)}{(9Q^4-3Q^2+1)^4(27Q^6-27Q^4+36Q^2+16)}+\frac{864}{Q^3}-\frac{1440}{Q^5}\right\}$$

where

$$num(Q)=7776Q(19683Q^{18}-78732Q^{16}+150903Q^{14}-134865Q^{12}+ \\ +1458Q^{10}+48357Q^8-13311Q^6-1053Q^4+369Q^2-10)$$

- Two last terms give at once $864\,\zeta(3)-1440\,\zeta(5)$
- The remaining rational function remarkably sums up to an integer giving finally

$$\Delta_{wrapping}E=(324+864\zeta(3)-1440\zeta(5))g^8$$

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Generalization:

Compute wrapping corrections for twist two operators with any M from string theory

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Answer:

$$\gamma_8^{wrapping}(M) = -640 S_1^2(M)\zeta(5) - 512 S_1^2(M) S_{-2}(M)\zeta(3) + C_7(M)$$

where

$$C_7(M) = 256 S_1^2 \left(-S_5 + S_{-5} + 2S_{4,1} - 2S_{3,-2} + 2S_{-2,-3} - 4S_{-2,-2,1}\right)$$

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- LO and NLO BFKL give a prediction for $\gamma_8(M)$ analytically continued to $M = -1 + \omega$

$$\gamma_8(\omega) \sim -256 \left(\frac{4\zeta(3)}{\omega^4} + \frac{\frac{5}{4}\zeta(4)}{\omega^3} + \mathcal{O}\left(\frac{1}{\omega^2}\right) \right)$$

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Conclusions

- The agreement of the Konishi computation with the 4-loop weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!
- The computation of the finite size effects through Lüscher corrections is of a distinctly (2D) quantum field theoretical nature – so string theory is essential here
- The result came from a single diagram – in contrast to direct perturbative computations in gauge theory which are much more complex
- This suggests that one can use string theory methods of AdS/CFT as an efficient calculational tool also at *weak coupling*
- The calculations have been extended to general twist two operators at four loops (no complete gauge theory computation so far!) [Bajnok,RJ,Łukowski]
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