QCD factorization at fixed $Q^2(1-x)$

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ANALYTICITY AND A FINITE-ENERGY SUM RULE FOR THE REGGEON-PARTICLE AMPLITUDE IN
$a + b \rightarrow c + d + e$

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Outline of talk

Hard inclusive processes have interesting features at high x (x_B, x_F)

Bloom-Gilman duality in DIS $eN \to eN^*$

Angular distribution of muon pairs in Drell-Yan $\pi^- N \to \mu^+\mu^- X$

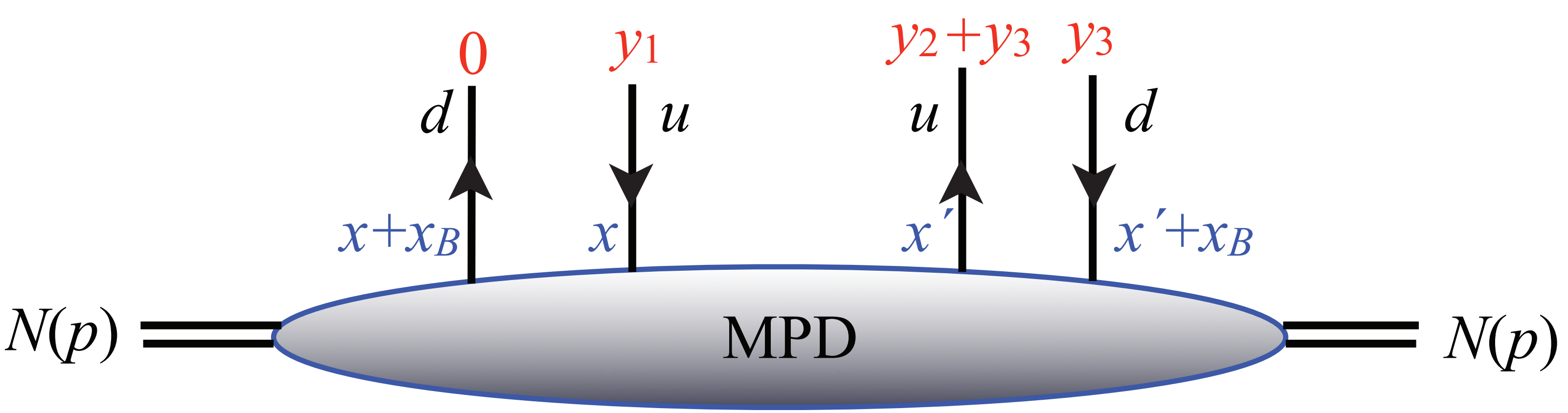
Large single spin asymmetries $p^\uparrow p \to \pi + X$, ...

Higher twist contributions are suppressed by $1/Q^2(1-x)$

$\Rightarrow$ Consider QCD factorization in a new limit: $Q^2 \to \infty$ with $Q^2(1-x_F)$ fixed

The hard subprocess is coherent with the entire projectile Fock state

Soft matrix element is a forward multi-parton distribution:



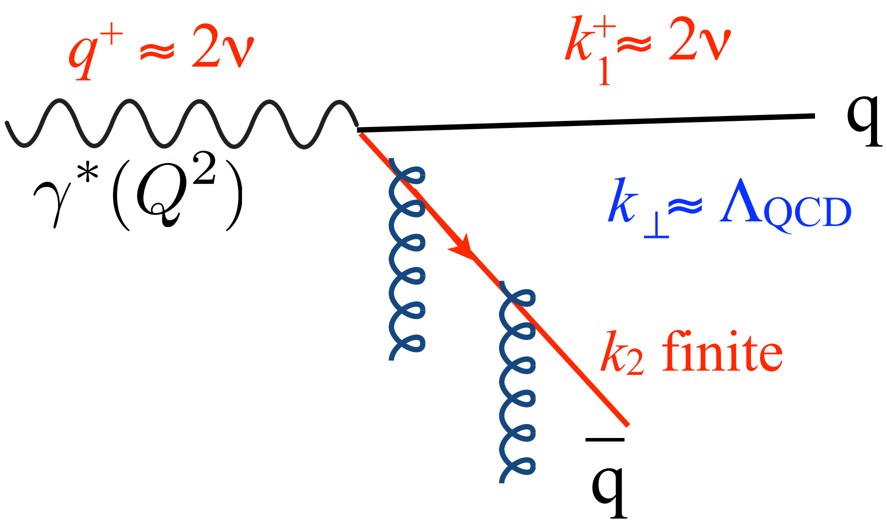
Coherence between hard and soft partons as $x \to 1$

Example: DIS in the "target rest frame" (Light-Cone gauge)

$\gamma^* \to q(z) + \bar{q}(1-z)$

$2\nu(1-z) \sim \Lambda_{QCD}$

Soft (re)scattering of antiquark in target releases virtual $\gamma^*(Q^2) \to q\bar{q}$ fluctuation



Landshoff, Polkinghorne and Short (1971)

Possible, since the life-time of the antiquark is similar to that of the virtual photon

$$x^+_{\bar{q}} \sim 1/k_2^- \sim \text{finite}$$

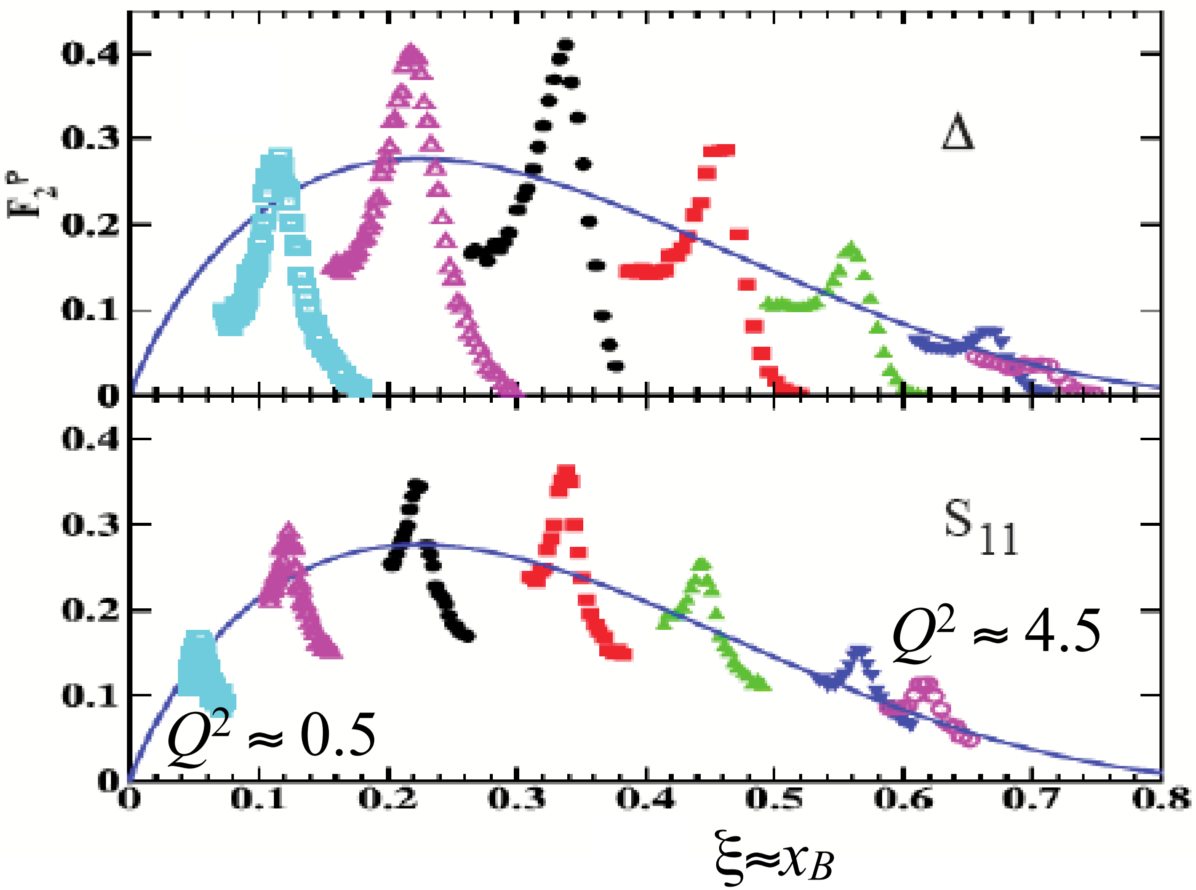
$$x^+_{\gamma^*} \sim 2\nu/Q^2 = 1/m x_B$$

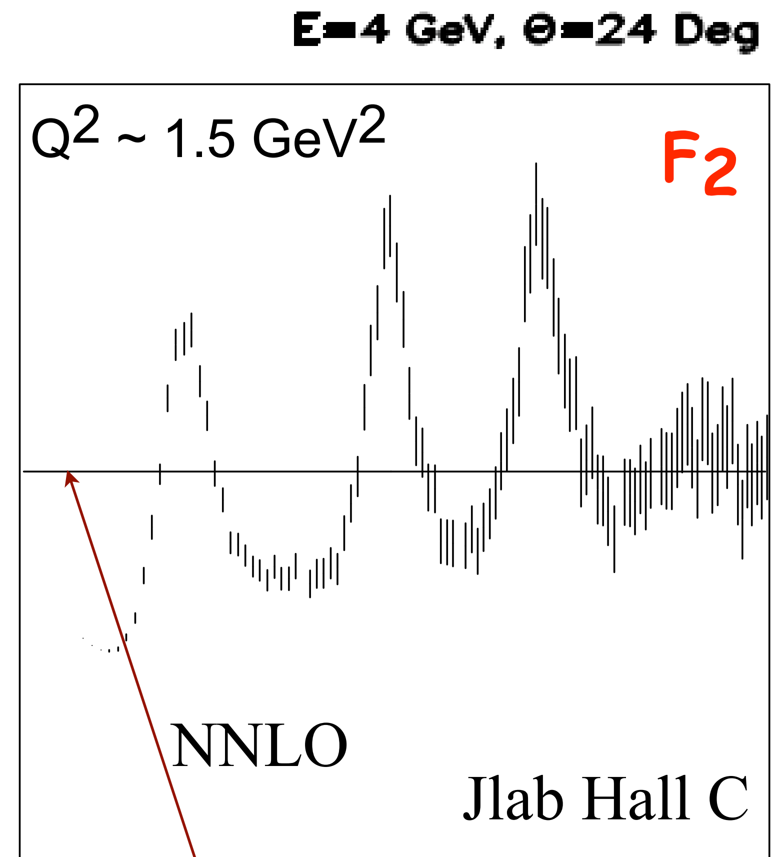
Lesson: Partons which take nearly all the momentum, $x \sim 1$ can be coherent with the soft partons, $1 - x \sim 0$

$$x^\pm = x^0 \pm x^3$$

Bloom – Gilman duality

Jlab Hall C





S. Alekhin, PRD 68 (2003) 014002

Smooth curves: Scaling F_2 at large Q^2 and fixed x_B Bj limit

Peaks: Δ and S_{11} contributions for increasing Q^2 and fixed $(1-x_B)\, Q^2$ BB limit

Duality shows that the Bj and BB limits are simultaneously valid

Photon polarization in Drell-Yan at high x_F

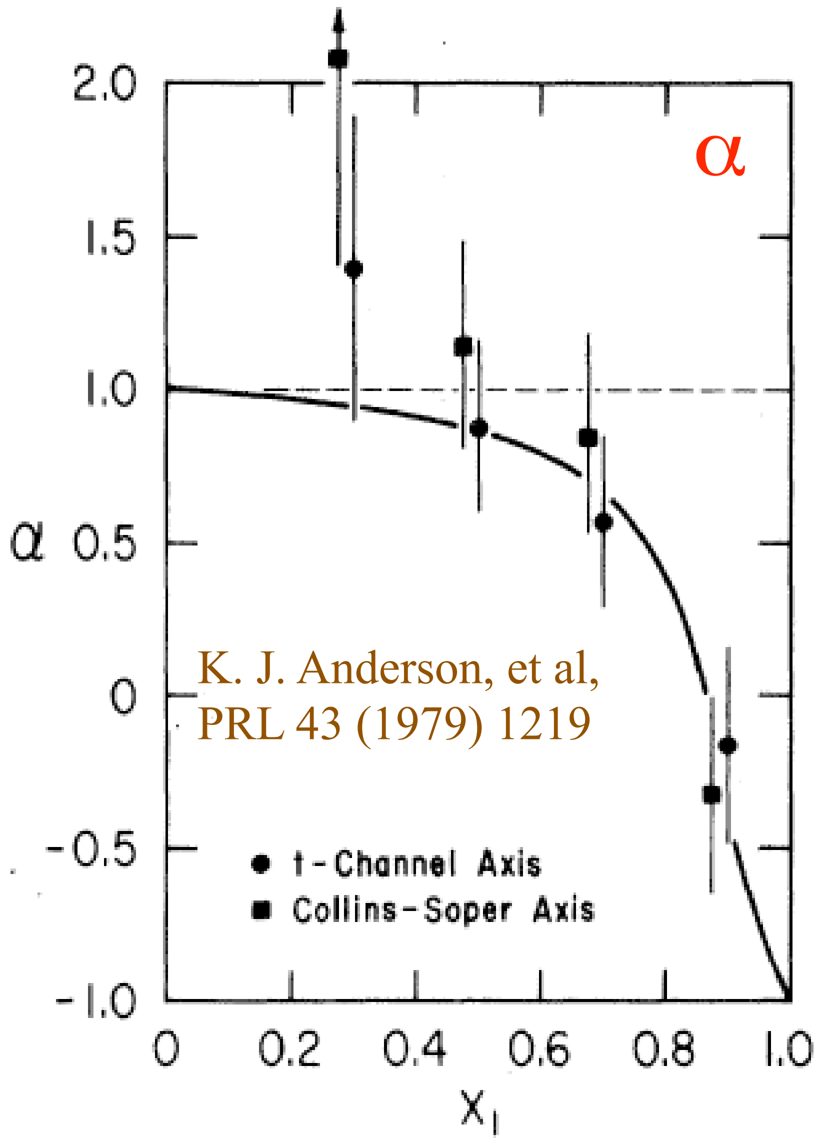


FIG. 2. The dependence of α on x_1 for data with $M > 4$ GeV. The dashed line is the expected result for the naive Drell-Yan model. The solid curve is the QCD prediction of Berger and Brodsky (Ref. 8).

E615: $\pi^- N \to \mu^+\mu^- + X$

$p_{lab} = 263$ GeV/c, $Q > 4$ GeV

$$\frac{d\sigma}{d\cos\theta^*} \propto 1 + \alpha\cos^2\theta^*$$

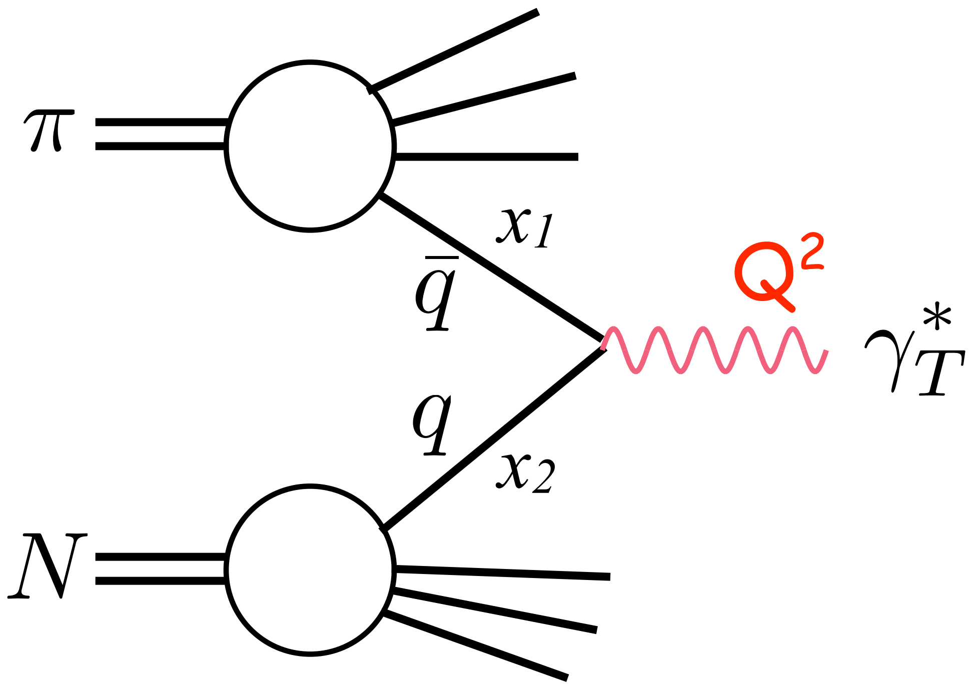
$$\to \sin^2\theta^* \text{ as } x_F \to 1$$

Berger and Brodsky, PRL 42 (1979) 940

⇒ colliding quark(s) are far off-shell: new subprocess

How does the hard subprocess factorize from the soft matrix element?

Drell-Yan in the Bj limit: $Q^2 \to \infty$ at fixed x



$$Q^2 = x_1 x_2 s \to \infty$$

$x_1, x_2\,;\, x_F = x_1 - x_2$ fixed

Transversely polarized photon, since quarks are ~ on-shell

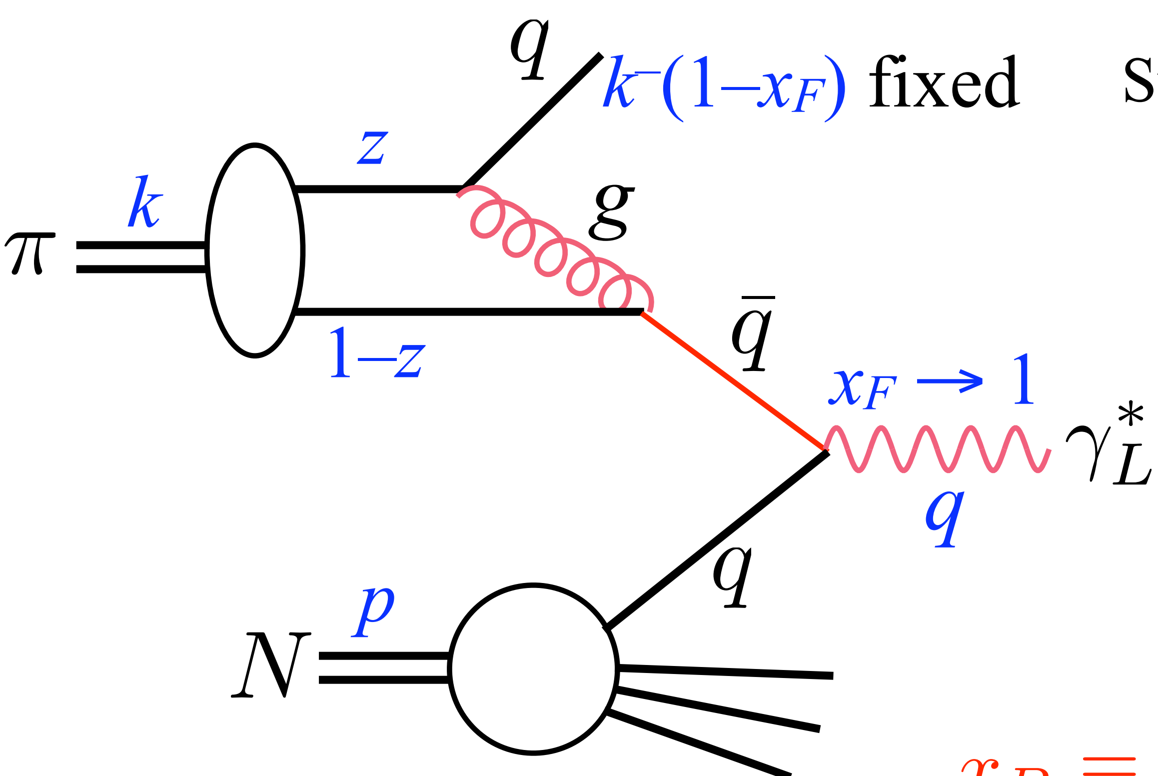
Leading twist: One active parton in beam and target hadrons

Spectators are incoherent with the hard subprocess

Factorization: $\sigma = f_{\bar{q}/\pi}(x_1) f_{q/N}(x_2)\hat{\sigma}(\bar{q}q \to \gamma^*)$

Higher twist corrections are of order $\dfrac{1}{Q^2}\dfrac{1}{1-x}$

Drell-Yan in the BB limit: $Q^2 \to \infty$ at fixed $Q^2(1-x_F)$



Stopped parton coherent with γ^*

$\bar{q}, g$ virtualities of order Q^2

$\Rightarrow$ *higher twist process*

γ^*_L : Longitudinal polarization

$$x_B \equiv \frac{q^+}{p^+} = \frac{Q^2}{2q \cdot p} = \frac{Q^2}{s} \quad \text{fixed}$$

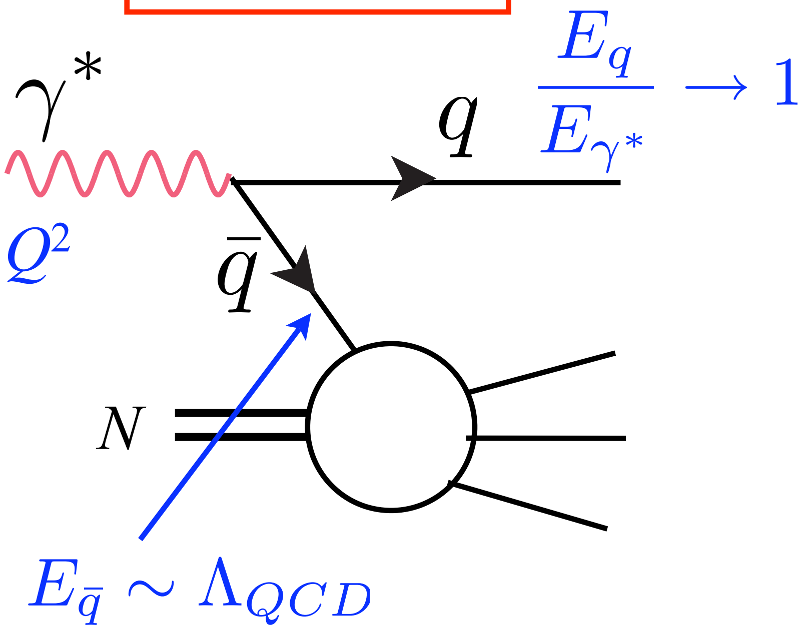
The hadronic mass

$$M_X^2 \equiv (k+p-q)^2 \simeq (1-x_B)[s(1-x_F)+m_N^2] \quad \text{fixed}$$

Stopped quark is comoving with the target.
Its interactions in the target affect the hard subprocess.

Analogy to DIS: "Target rest frame" vs. Handbag

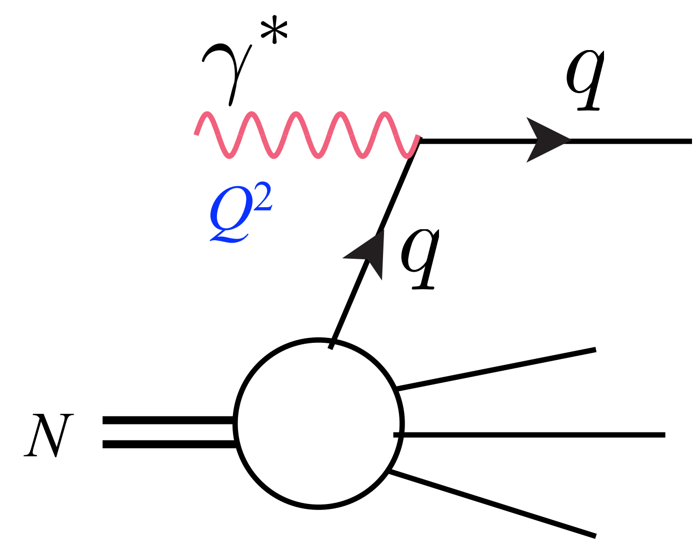
$$\gamma^* \to q\bar{q}$$



DIS viewed as photon splitting:

The antiquark is comoving with the target.
Its scattering in the target determines DIS σ

$$\gamma^* q \to q$$



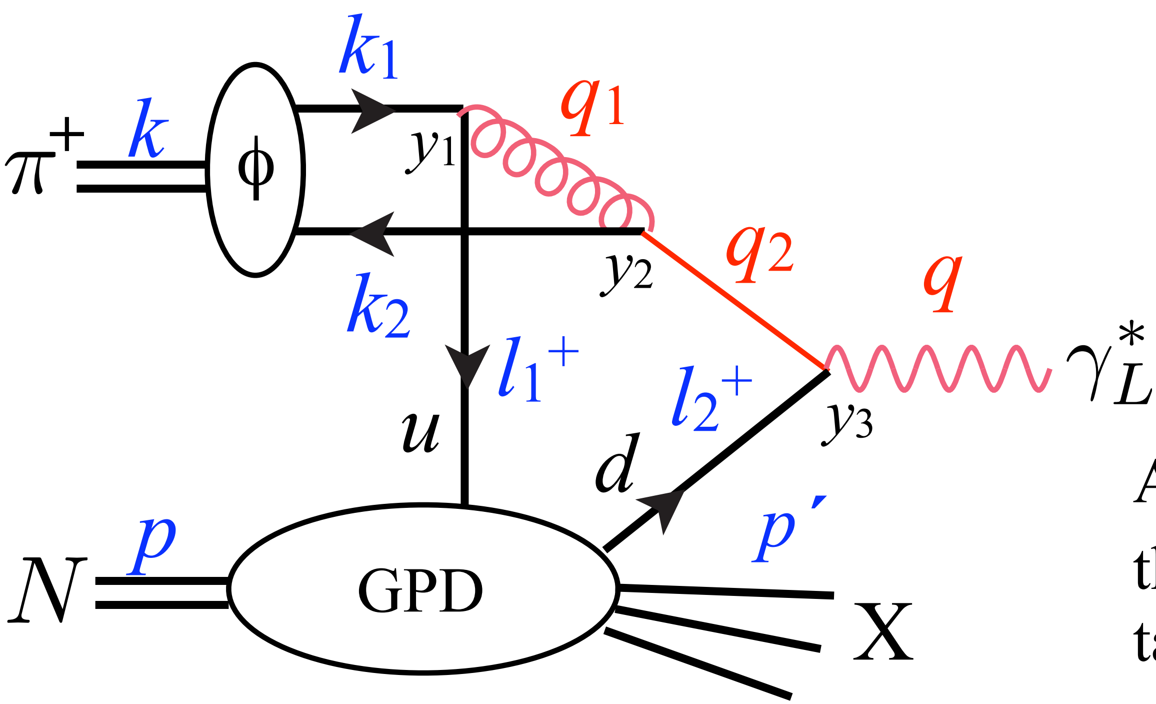
DIS viewed as photon scattering:

The DIS cross section determined by the "probability of finding the quark in the target"

In either case, the large photon virtuality Q^2 arises from the difference in longitudinal momenta of the quarks, not from their transverse momentum

Hence the stopped quark should be connected to the target:

$k_1 = (0^+, zk^-, \boldsymbol{k}_\perp)$
$k_2 = (0^+, (1-z)k^-, -\boldsymbol{k}_\perp)$



Since $q_1^2 \approx -zk^- l_1^+ \to \infty$

the pion wave function contributes through its *distribution amplitude* ϕ

Also q_2^2, q_1^-, $q_2^- \to \infty$, hence the space-time separation of the target interaction points y_1, y_3 is

$$|\boldsymbol{y}_{1\perp} - \boldsymbol{y}_{3\perp}| = \mathcal{O}(1/Q) \to 0$$
$$|y_1^+ - y_3^+| = \mathcal{O}(1/Q^2) \to 0$$
$$|y_1^- - y_3^-| = \mathcal{O}(1/\ell_1^+) \text{ finite}$$

For each final state X the target matrix element is given by a GPD with skewness

$$l_2^+ - l_1^+ = q^+ = x_B\, p^+$$

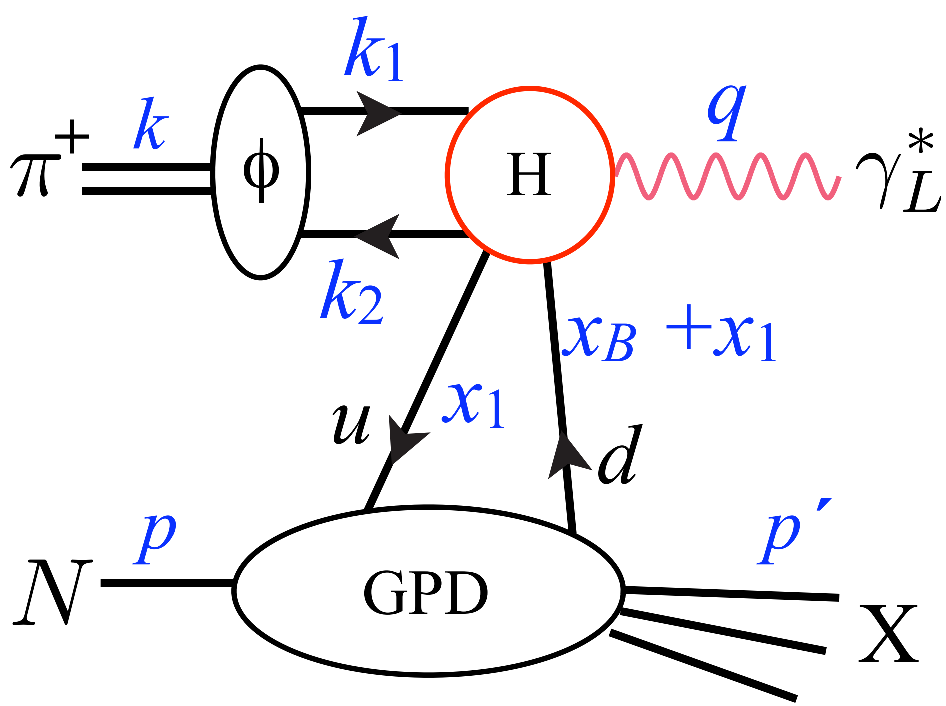
Using perturbative propagators for the gluon q_1 and d-quark q_2 and adding three more diagrams we get

$$T(\pi^+ N \to \gamma_L^* X) = \frac{-ieg^2\, C_F}{2\pi Q\sqrt{2N_c}} \int dx_1 C(x_B, x_1)$$

$$z = k_1^-/k^-$$
$$x_B = q^+/p^+$$
$$x_1 = l_1^+/p^+$$

$$\times \int dy_1^-\, e^{-iy_1^- x_1 p^+/2} \langle X(p')|\bar{\psi}_u(y_1)\gamma^+\gamma_5\, \psi_d(0)|p\rangle_{y_1^+ = y_{1\perp} = 0}$$

where $$C(x_B, x_1) = \int dz\, \phi_\pi(z) \left(\frac{e_d}{z} \frac{1}{x_1 - i\varepsilon} + \frac{e_u}{1-z} \frac{1}{x_B + x_1 + i\varepsilon} \right)$$



For $X = p$ we recognize the time-reversed amplitude for deeply virtual pion production, $\gamma^* + p \to \pi^+ + n$

For $X \neq p$ we have a "transition" GPD L.L. Frankfurt *et al,* PRD **60** (1999) 014010

Before summing over all *X*, consider photon helicity (same for each *X*)

The photon helicity

Intuitively: In the BB limit of $\pi^+ N \to \gamma^* + X$ the photon carries the helicity of the pion ($\lambda = 0$), since the process is coherent on the pion wave function

But: In p $N \to \gamma^* + X$ the photon helicity differs from the proton helicity by $|\Delta\lambda| \geq 1/2$, whether it is longitudinal or transverse. What is then the photon polarization?

Helicity systematics in the BB limit is a consequence of the low transverse momenta, $q_\perp \sim \Lambda_{QCD}$ and the conservation of $J_z = L_z + S_z$

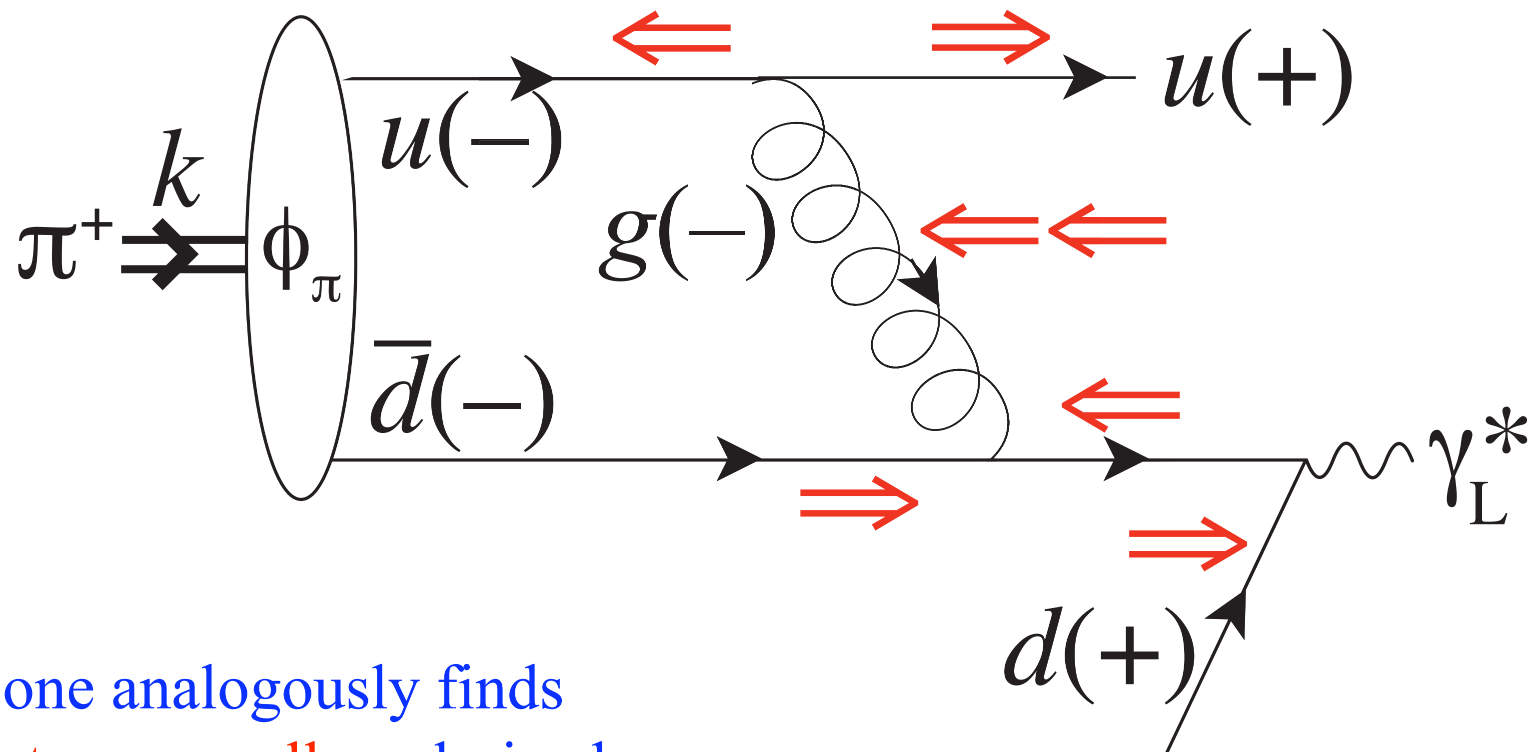
Each unit of L_z brings a suppression $q_\perp/Q \sim \Lambda_{QCD}/Q$

$\Rightarrow$ Leading contribution obtained by setting all $L_z = 0$ (when possible)

Conservation of S_z

Up to terms $\sim m_q/Q$ the helicity of the quark line is conserved:

Contribution to S_z ($\Rightarrow$ or $\Leftarrow$)determined by direction of motion ($\pm$), giving $S_z = 0$ of the photon in $\pi^+ N \to \gamma^* + X$.



In p $N \to \gamma^* + X$ one analogously finds that the photon is transversally polarized.

Consistent with pN DY data
(J. C. Peng, private comm.)

Exclusive → Inclusive

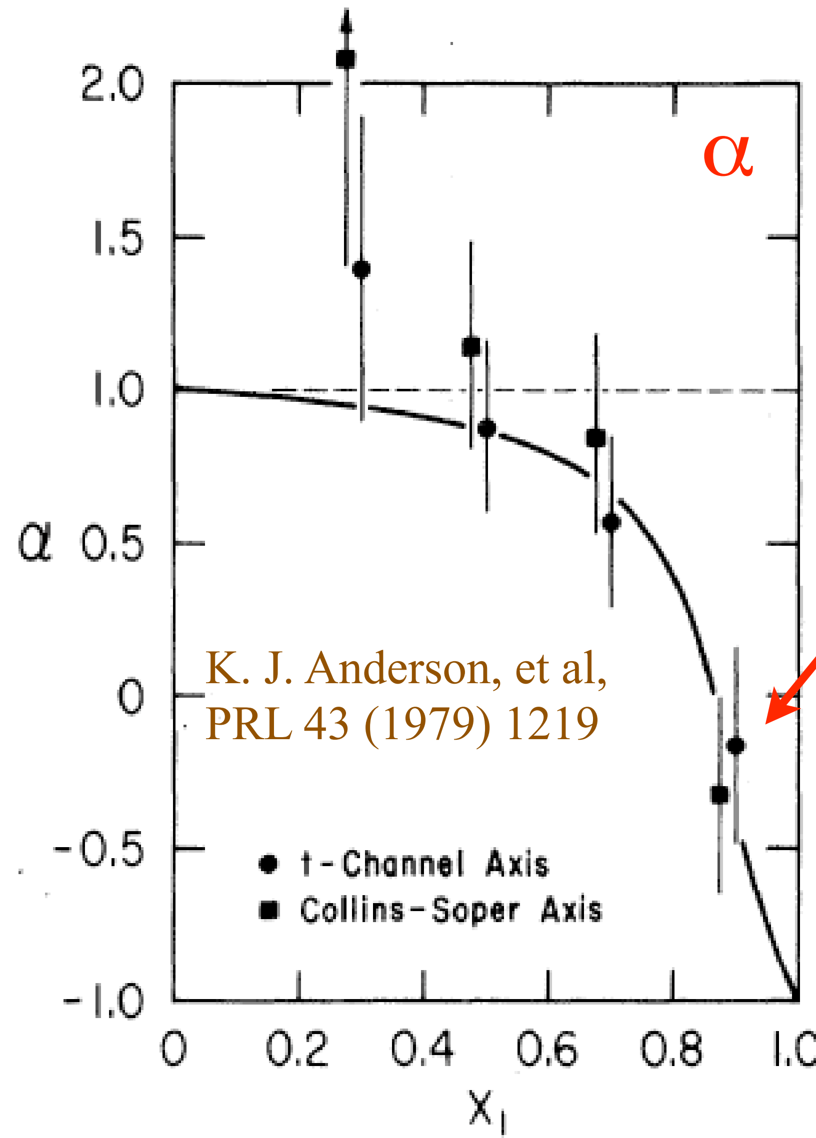
$\pi + N \to \gamma^* + X$

The dominance of longitudinal photons is known for exclusive meson production,

$\gamma^* + p \to \pi^+ + n$

The change of polarization in inclusive DY occurs for sizeable missing mass

Next we sum over X to obtain the inclusive DY cross section



$M_X \approx 7$ GeV for $x_F = 0.9$

FIG. 2. The dependence of α on x_1 for data with $M > 4$ GeV. The dashed line is the expected result for the naive Drell-Yan model. The solid curve is the QCD prediction of Berger and Brodsky (Ref. 8).

The $\pi^+ N \to \gamma^* + X$ cross section

$$\sigma(\pi^+ N \to \gamma^*_L X) = \frac{1}{2s}\sum_X \int \frac{dq^- d^2\boldsymbol{q}_\perp}{(2\pi)^3 2q^-} |T(\pi^+ N \to \gamma^*_L X)|^2 (2\pi)^4 \delta^4(k+p-q-p')$$

The completeness sum over X includes summing over $p_X \equiv p'$

$$\sum_X |X\rangle\langle X| \equiv \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} \frac{d^3\boldsymbol{p}_i}{(2\pi)^3 2E_i} |\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n\rangle\langle \boldsymbol{p}_1,\ldots,\boldsymbol{p}_n| = 1$$

and must not be constrained by the momentum conserving δ-functions.

Integrating the DY cross section over $\boldsymbol{q}_\perp = -\boldsymbol{p}'_\perp$
eliminates the transverse momentum constraint on p'.

The longitudinal δ-functions can be incorporated into the GPD matrix element via two position integrals:

$$\langle N(p)|\bar{\psi}_d(0)\gamma^+\gamma_5\,\psi_u(y_2)|X(p')\rangle\, 2(2\pi)^2\delta(p^+ - q^+ - p'^+)\delta(k^- + p^- - q^- - p'^-)$$
$$= \frac{1}{2}\int dy_3^+\, dy_3^-\, \langle N(p)|\bar{\psi}_d(y_3)\gamma^+\gamma_5\,\psi_u(y_2+y_3)|X(p')\rangle \exp\left[iy_3\cdot(k-q)\right]$$

After Σ_X the inclusive cross is given by a forward multiparton distribution depending on the fractional \`+´ momenta

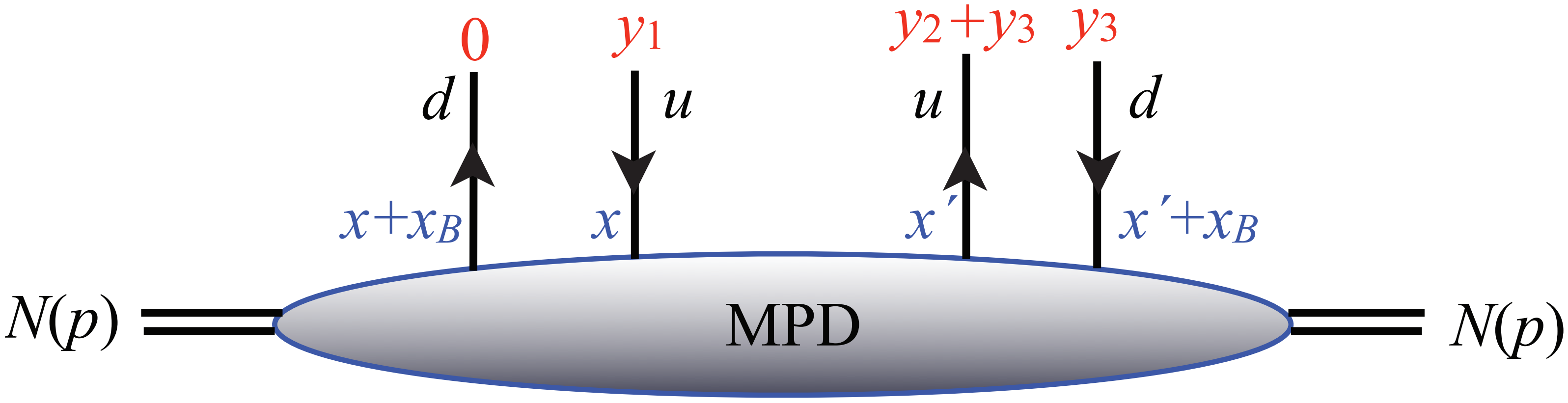
$$x_B = q^+/p^+,\; x = l_1^+/p^+,\; x' = l_1'^+/p^+$$

and the fractional \`−´ momentum x_M transferred to the inclusive system

$$x_M = k^-(1-x_F)/p^- \qquad \frac{x_B + \boldsymbol{q}_\perp^2/m_N^2}{1-x_B} \leq x_M \leq \infty$$

which is conjugate to y_3^+. At fixed x_F we would have $x_M \to \infty$ and $y_3^+ \to 0$.

The MultiParton Distribution



$$f_{d\bar{u}/p}(x_B, x_M; x, x') =$$

$$= \frac{1}{4(4\pi)^3} \int dy_1^- dy_2^- dy_3^- dy_3^+ \exp\left\{\tfrac{1}{2} i \left[-y_1^- l_1^+ + y_2^- l_1^{+\prime} - y_3^- q^+ + y_3^+ x_M p^-\right]\right\}$$

$$\times \langle N(p)|\bar{\psi}_d(y_3)\gamma^+\gamma_5\, \psi_u(y_2+y_3)\, \bar{\psi}_u(y_1)\gamma^+\gamma_5\, \psi_d(0)|N(p)\rangle_{y_{i\perp}=0;\; y_1^+=y_2^+=0}$$

The '−' momentum fraction x_M determines the inclusive mass,

$$M_X^2 = m_N^2(1-x_B)(1+x_M) - \boldsymbol{q}_\perp^2$$

The $\pi^+ N \to \gamma^* + X$ cross section in the BB limit

$$\frac{d\sigma(\pi^+ N \to \gamma_L^* X)}{dM_X^2} = \frac{2(eg^2C_F)^2}{Q^2 s^2 (1-x_B) N_c}$$

$$\times \int dx\, dx'\, C(x_B, x) C^*(x_B, x')\, f_{d\bar{u}/p}(x_B, x_M; x, x')$$

where $$C(x_B, x) \equiv \int_0^1 dz\, \phi_\pi(z) \left(\frac{e_u}{1-z} \frac{1}{x_B + x + i\varepsilon} + \frac{e_d}{z} \frac{1}{x - i\varepsilon} \right)$$

The dependence on x_M (*i.e.*, $y_3^+ \neq 0$) distinguishes $f_{d\bar{u}/p}(x_B, x_M; x, x')$ from the higher twist multiparton distributions considered by Jaffe

R. L. Jaffe, Nucl. Phys. B229 (1983) 205

$$H_{abcd}(\alpha_1, \alpha_2, \alpha_3) = \left(\frac{p^+}{2\pi}\right)^3 \int dx_1^- \, dx_2^- \, dx_3^- \, e^{ip^+(\alpha_1 x_1^- - \alpha_2 x_2^- - \alpha_3 x_3^-)}$$

$$\times \langle p | T\left(\phi_d^*(x_2^-)\phi_c^*(x_3^-)\phi_b(0)\phi_a(x_1^-)\right)| p \rangle_C$$

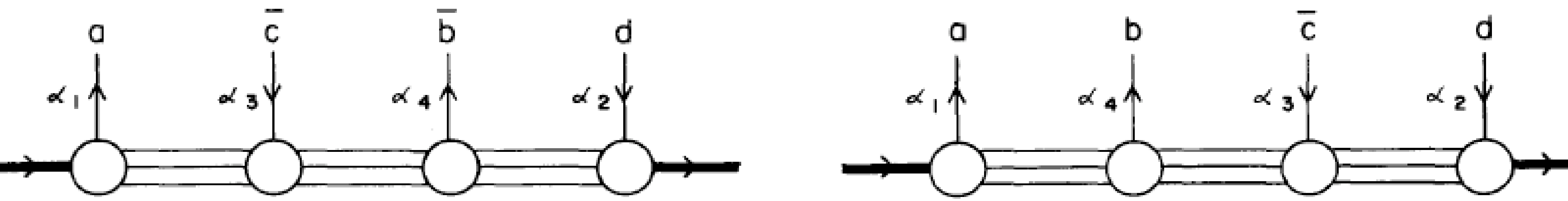
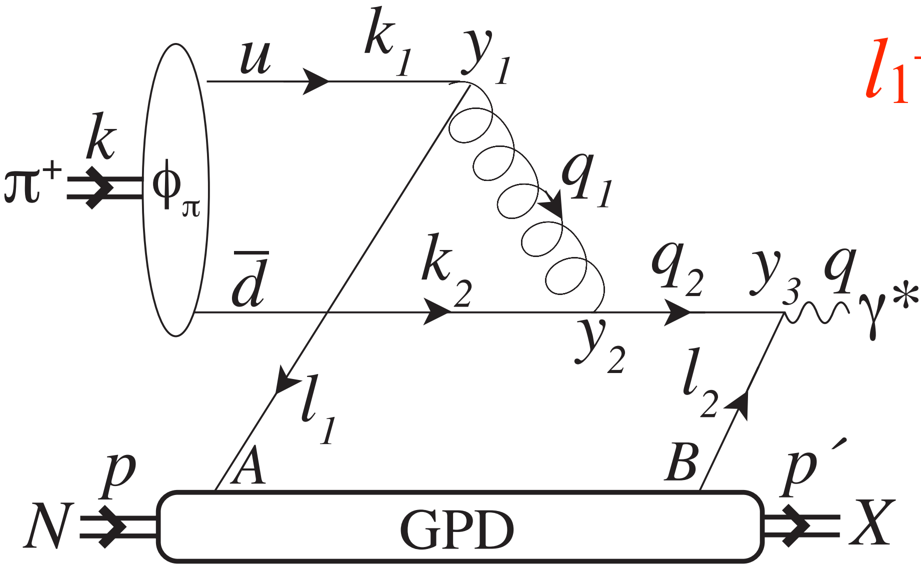


Fig. 10. Parton representations of a four-parton distribution function (a) $\alpha_1, \alpha_2 > 0$, $\alpha_3, \alpha_4 < 0$; (b) $\alpha_1, \alpha_4, \alpha_2 > 0$, $\alpha_3 < 0$.

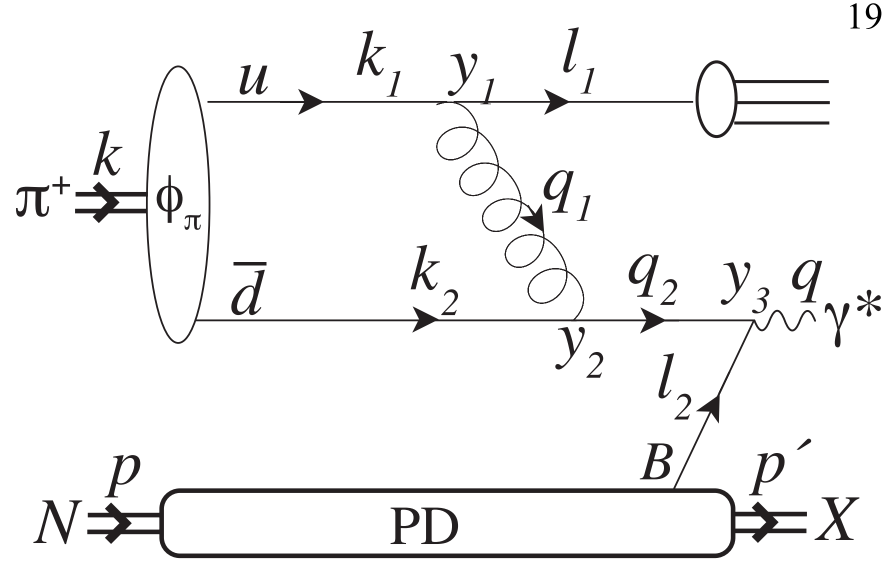
With $y_i^+ = 0$ all (anti)commutators vanish on the light-front.

Conversely, in $f_{d\bar{u}/p}(x_B, x_M; x, x')$ there can be contractions which in the $x_M \to \infty$, $y_3^+ \to 0$ limit corresponds to the quark in the pion with large $x_M p^- = (1-x_F)k^-$ forming a separate jet in the final system X.



$l_1^- \to \infty$

$\Rightarrow$



In the $y_3^+ \to 0$ limit the *u*-quark propagator becomes LF dominated, reducing the GPD to the standard *d*-quark PDF:

$$f_{d\bar{u}/p} \to \delta(l_1^+ - l_1'^+)\,\frac{l_1^+}{4\pi}\,\theta(l_1^+)\,f_{d/p}(l_2^+/p^+)$$

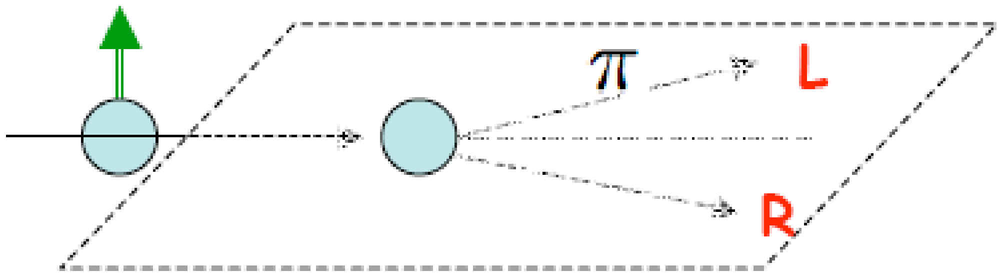
The DY cross section then becomes

Berger and Brodsky,
PRL 42 (1979) 940

$$\frac{d\sigma(\pi^+ N \to \gamma_L^* X)}{dM_X^2} = \frac{(e e_d g^2 C_F)^2}{Q^2 s^2 (1-x_B) N_c} \int \frac{dl_1^+}{2\pi l_1^+}\,\theta(l_1^+) \left(\int \frac{dz}{z}\phi_\pi(z)\right)^2 f_{d/p}(l_2^+/p^+)$$

Transverse Single-Spin Asymmetries (A_N)

$$p^{\uparrow} p \to \pi(x_F, k_T) + X$$



$$A_N = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

SSA requires helicity flip and a dynamical phase in a subprocess which is coherent with the high $k_\perp$ parton

$$A_N \sim \mathcal{I}m(\, M_+ \, M_-^* \,)$$

In the Bjorken limit: $A_N \sim \alpha_s m_q / p_T$

$A_N \sim 0.001$ for $p_T \sim 2$ GeV/c

Kane, Pumplin and Repko

PRL **41** (1978) 1689

SSA in $p^{\uparrow}p \to \pi(x_F, k_\perp) + X$

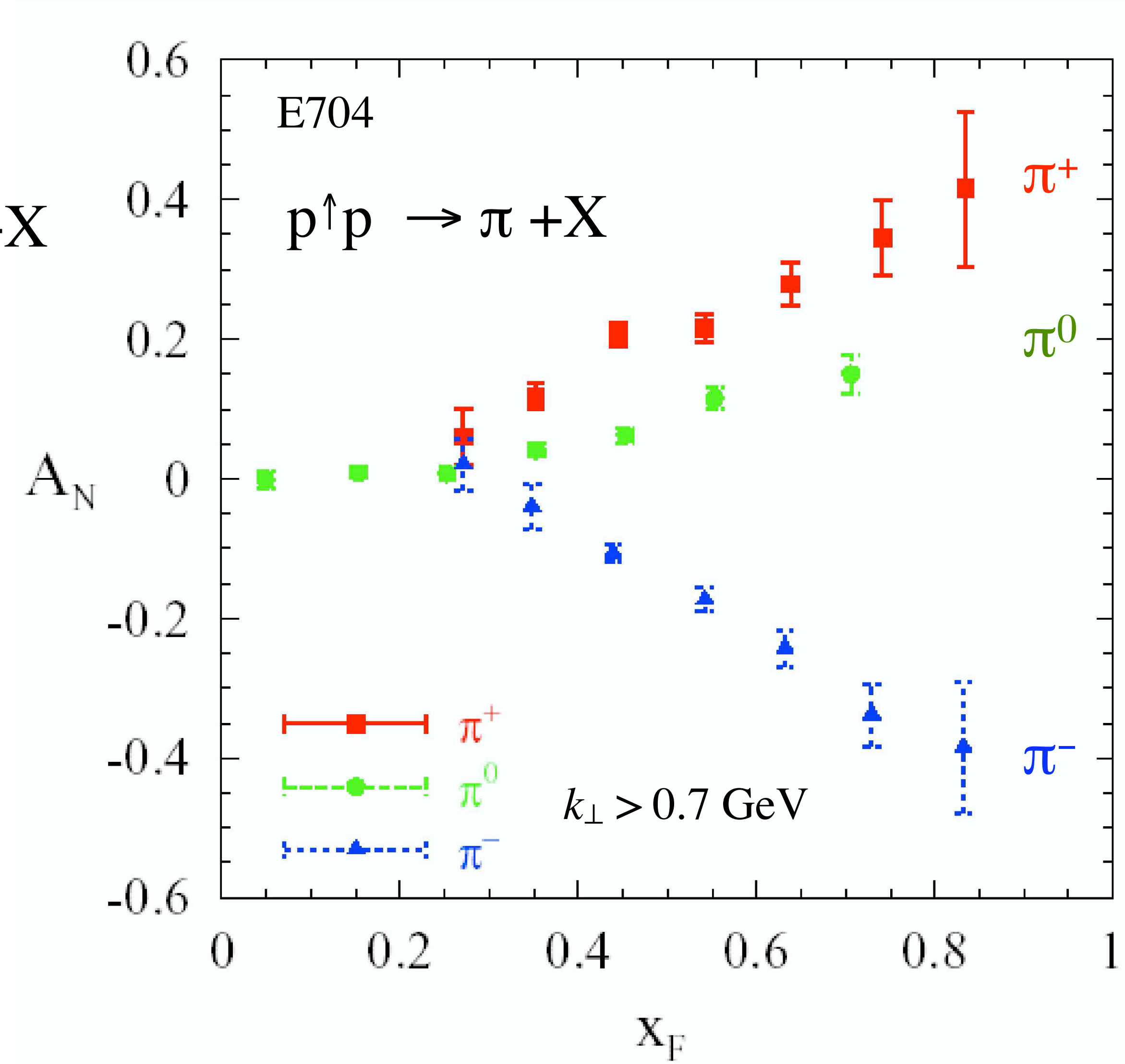
- $A_N \approx 0.4$ at $x_F = 0.8$
  Almost all beam momentum and spin is transferred to π, suggesting coherence over entire $p^{\uparrow}$ Fock state even at high $k_\perp$

- Leading twist analysis underestimates the $p^{\uparrow}p \to \pi + X$ cross section at high x_F

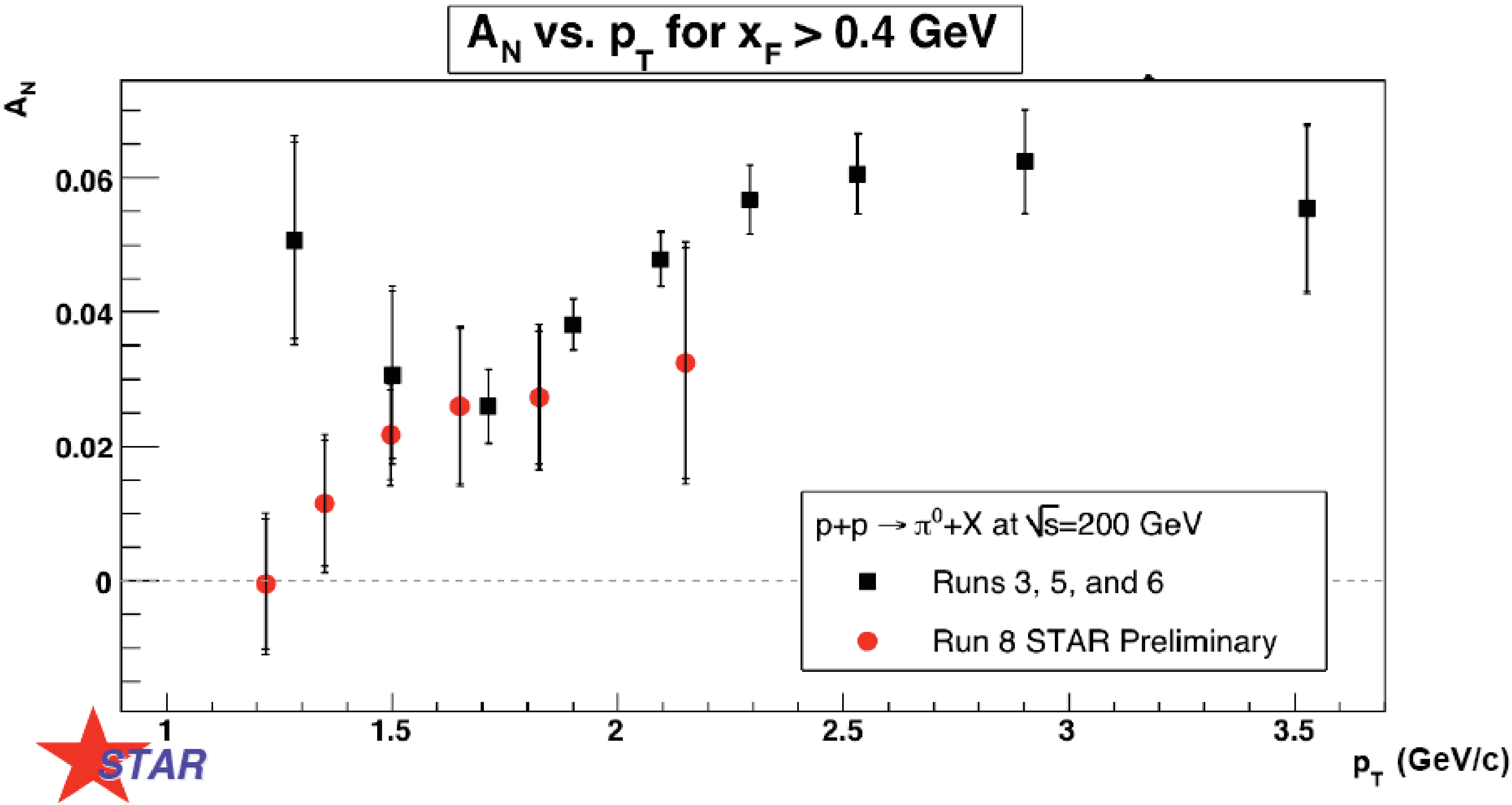
$$x_F = x_{q/p}\, z_{\pi/q}$$

$$\boxed{0.8 = 0.9 \cdot 0.9}$$

C. Bourrely and J. Soffer, Eur. Phys. J. C **36** (2004) 371



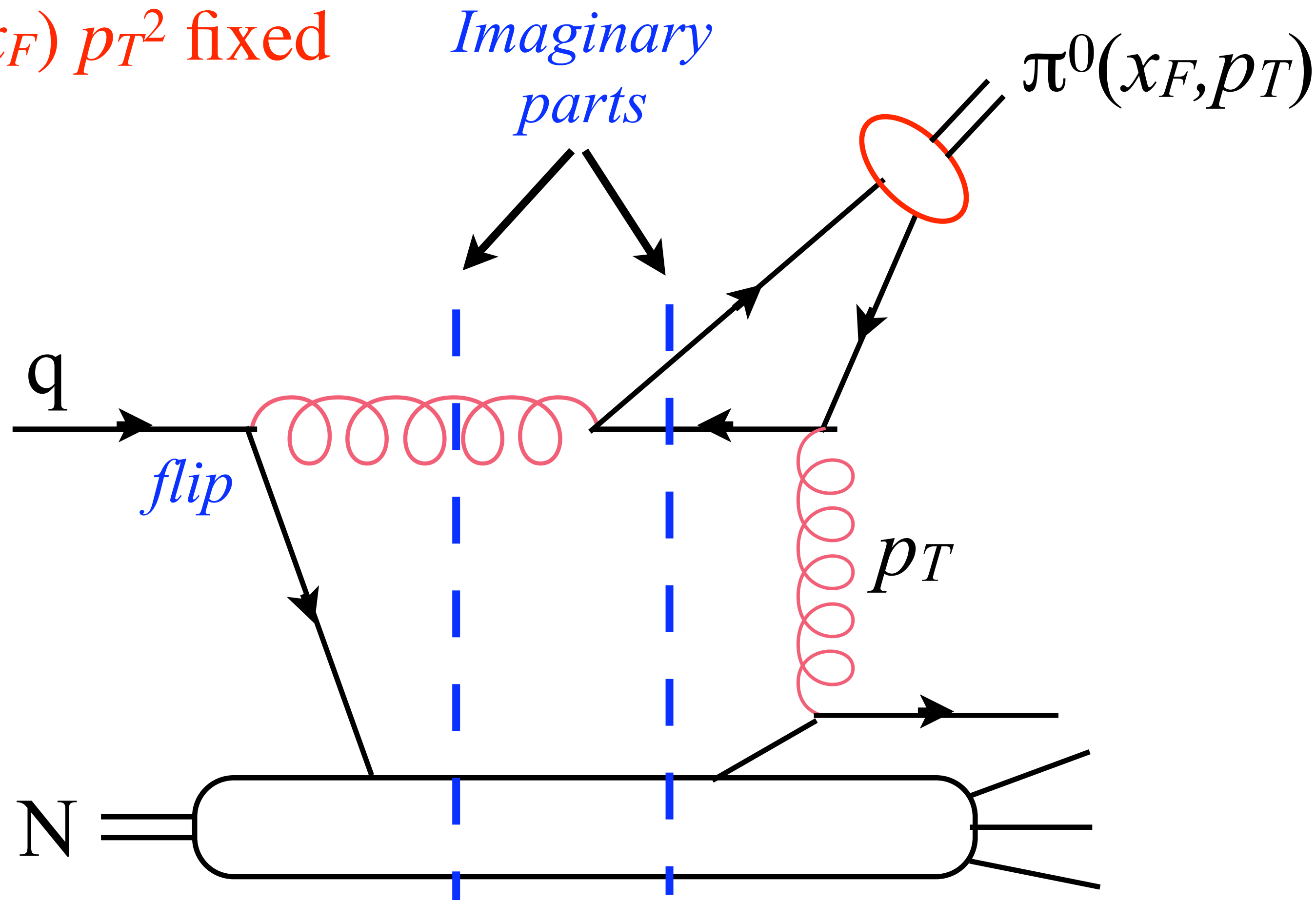
- A_N generated at twist 3 or via Sivers effect obeys $A_N \propto 1/k_\perp$ which contradicts data.



STAR Collaboration
PRL.101:222001,2008
arXiv:0801.2990

Tentative scheme for generating an SSA through helicity flip in a soft process which is coherent with the large p_T scattering

$p_T \to \infty$ with $(1-x_F)\, p_T^2$ fixed



If successful, would indicate that soft processes where a quark loses most of its momentum may influence hard scattering in, e.g., form factors.

Summary

Data reveals qualitatively new features of scattering at high x

- Longitudinal photons in πN Drell-Yan
- Large single spin asymmetries
- Bloom-Gilman duality

This suggests to consider hard processes at fixed $Q^2(1-x)$ "BB" limit

- Hard subprocess coherent with entire wave function of projectile
- Approaching exclusive processes from better understood region

DY cross section factorized in terms of a forward multi-parton distribution

- Finite LF time difference y_3^+ between T and T^*

Application to SSA in $p^\uparrow p \to \pi + X$ under consideration.

Possibly relevant also for an understanding of Bloom-Gilman duality and the dynamics of exclusive form factors.