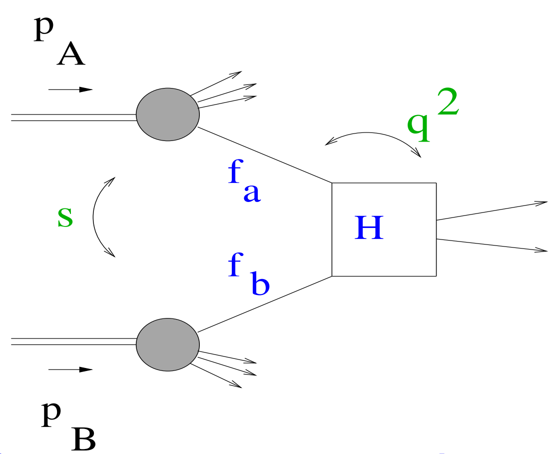
QCD coherence effects in multi-jet final states

F. Hautmann (Oxford)

Cracow Epiphany Conference
Hadronic Interactions at the Dawn of the LHC
Dedicated to the Memory of Jan Kwiecinski
January 2009

- A study of coherent gluon radiation in parton showers at very high energy

Motivation: multi-scale hard processes at the LHC



▷ coherence effects for small longitudinal momentum fractions x:

- not included in standard shower Monte Carlo generators
- included partially in NLO multi-jet calculations
- present to all orders and enhanced by logs of $\sqrt{s}/E_T$

OUTLINE

I. Introduction: parton showers and multi-gluon emission

II. Coherence in space-like showers at high energies

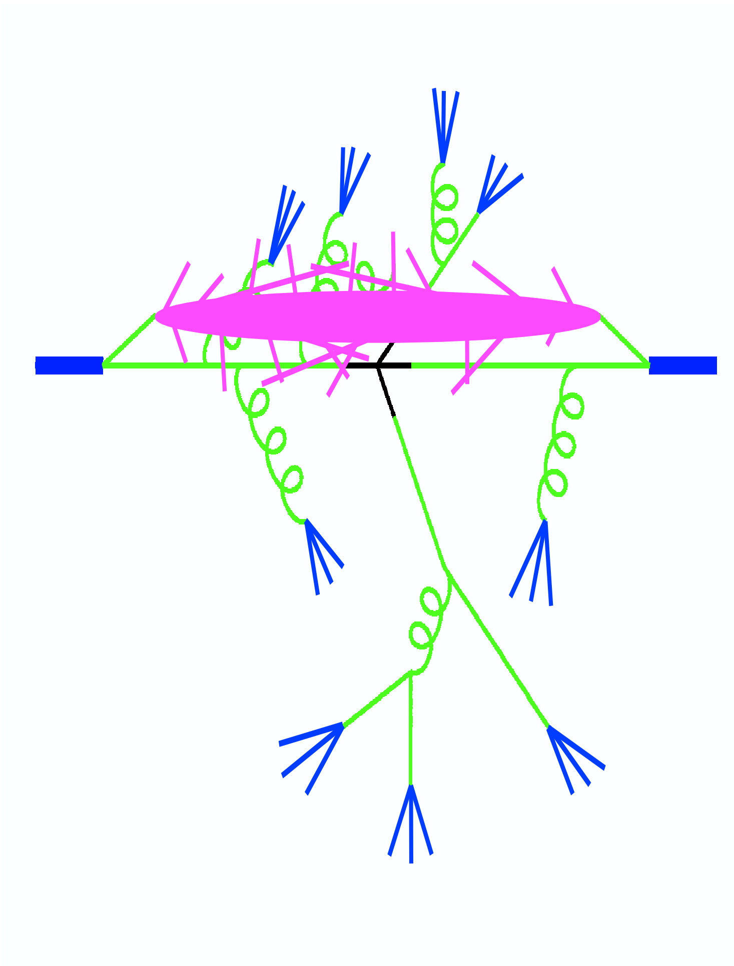
III. Issues on unintegrated parton distributions

IV. Angular correlations and jet production data

V. Summary and prospects for LHC final states

I. INTRODUCTION

Structure of LHC events:



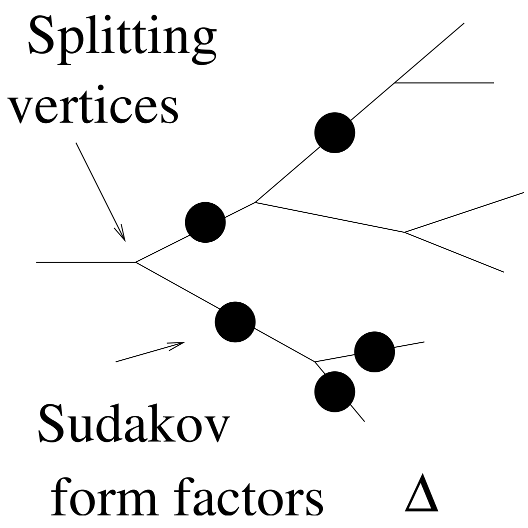
Hard process

Parton shower

Hadronization

Underlying event

PARTON SHOWER



$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \; \alpha_S \; P(z) \; \Delta(q^2, q_0^2)$$

• based on dominance of collinear evolution of jets developing (both "forwards" and "backwards") from hard event

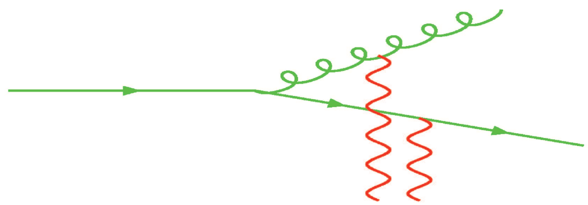
• Factorization of QCD cross sections in collinear limit
$\longrightarrow$ probabilistic (Markov) picture

• summation of logarithmically enhanced radiative contributions

$$(\alpha_S \; \ln p_T/\Lambda)^n$$

SOFT GLUON EMISSION

▷ soft gluons radiated over long times $\longrightarrow$ quantum interferences



$\swarrow$ factorization in soft limit

$$|M_{n+1}^{a_1\dots a_n a}(p_1,p_n,q)\rangle = \mathbf{J}^a |M_n^{a_1\dots a_n}(p_1,p_n)\rangle \ , \quad \mathbf{J}^{a\mu} = \sum_i \mathbf{Q}_i^a \frac{p_i^\mu}{p_i\cdot q} \ , \ \mathbf{Q} = \text{color charge}$$

interference terms $\downarrow$

$$d\sigma_{n+1} = d\sigma_n \ \frac{d^3q}{(q^0)^3} \ \sum_{i,j} \mathbf{Q}_i\cdot\mathbf{Q}_j \ w_{ij} \quad , \qquad w_{ij} = \frac{(q^0)^2 \ p_i\cdot p_j}{(p_i\cdot q)(p_j\cdot q)}$$

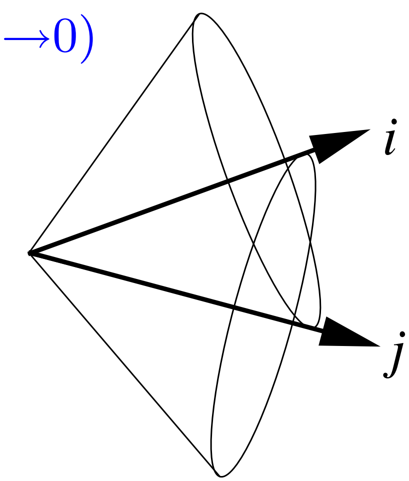
$\longrightarrow$ spoils probabilistic picture? NO, owing to soft-gluon coherence $\hookrightarrow$

• single-emission: separate singularities along emitters' directions

$$\frac{(q^0)^2 p_i \cdot p_j}{(p_i \cdot q)(p_j \cdot q)} \equiv \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}}$$

$$= \frac{1}{2}\left(\frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} - \frac{1}{\zeta_{jq}} + \frac{1}{\zeta_{iq}}\right) + \frac{1}{2}\left(\frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} - \frac{1}{\zeta_{iq}} + \frac{1}{\zeta_{jq}}\right)$$

where $\zeta_{nk} \equiv \frac{p_n \cdot p_k}{p_n^0 p_k^0} \simeq 1 - \cos\theta_{nk} \quad (m \to 0)$



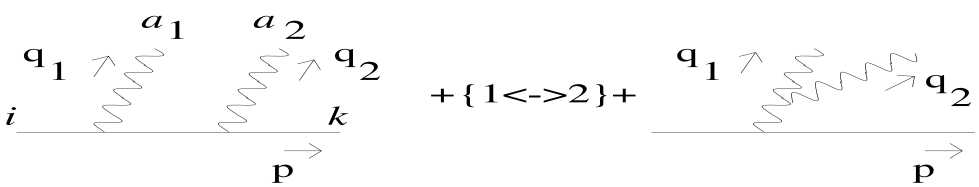
$\longrightarrow$ by azimuthal average

$$\langle \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} \rangle = \frac{1}{\zeta_{iq}}\Theta(\zeta_{ij} - \zeta_{iq}) + \frac{1}{\zeta_{jq}}\Theta(\zeta_{ij} - \zeta_{jq})$$

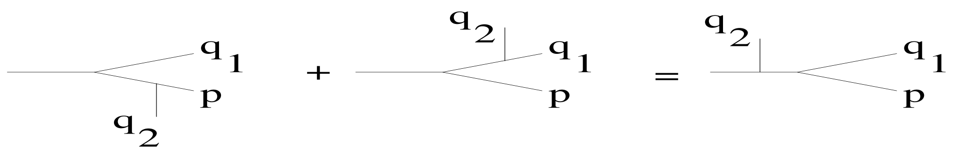
◇ large-angle emissions of soft gluons sum coherently outside angular-ordered cones

- multiple emission: $(q_1,\ q_2$ with $q_2^0 \ll q_1^0)$

$$\mathbf{J}_1^{\mu a_1} = \mathbf{Q}_p^{a_1} \frac{p^\mu}{p \cdot q_1} \quad , \quad \mathbf{J}_2^{\mu a_2} = \mathbf{Q}_p^{a_2} \frac{p^\mu}{p \cdot q_2} + \mathbf{Q}_{q_1}^{a_2} \frac{q_1^\mu}{q_1 \cdot q_2}$$



(a)



(b)

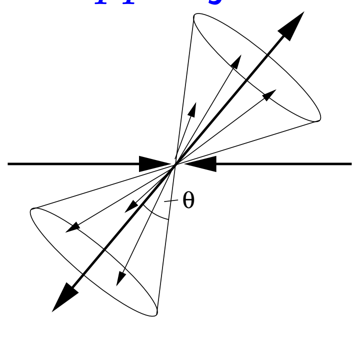
$$\mathcal{M}_{ki}^{a_1 a_2} = g_s^2 \ \langle a_1\ k|\ \mathbf{J}_2 \cdot \varepsilon_2\ |a'\ i'\rangle\ \langle\ i'\ |\mathbf{J}_1 \cdot \varepsilon_1|\ i\ \rangle$$

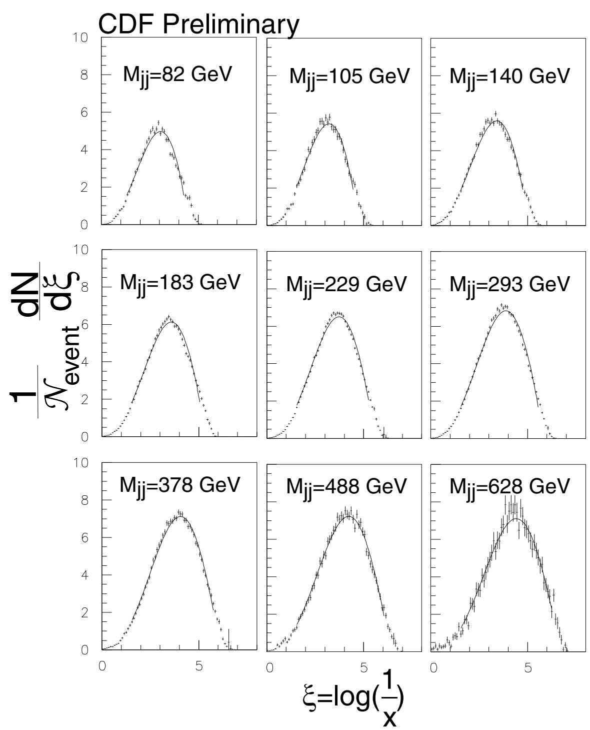
$$= g_s^2\ \frac{p \cdot \varepsilon_1}{p \cdot q_1} \left(\frac{p \cdot \varepsilon_2}{p \cdot q_2}\ t^{a_2} t^{a_1} + \frac{q_1 \cdot \varepsilon_2}{q_1 \cdot q_2}\ [t^{a_1}, t^{a_2}] \right)_{ki}$$

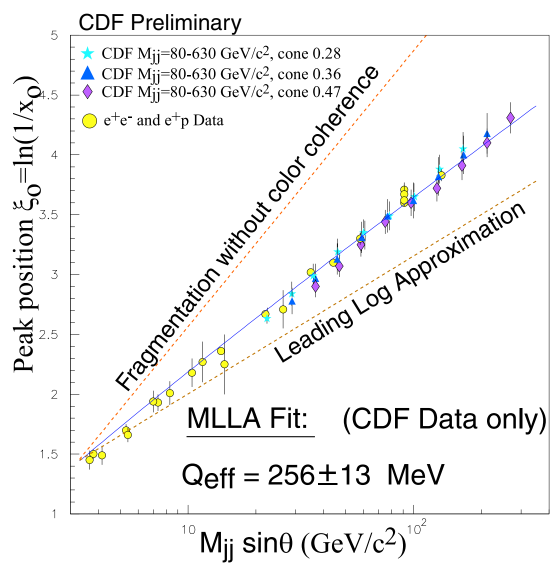
- small angle: bremsstrahlung cones
- large angle $(\theta_{pq_2} \gg \theta_{pq_1})$: sees total charge $\mathbf{Q}_p + \mathbf{Q}_{q_1}$

- Extensive collider data studies emphasize phenomenological relevance.

Example: $p\bar{p}$ dijets



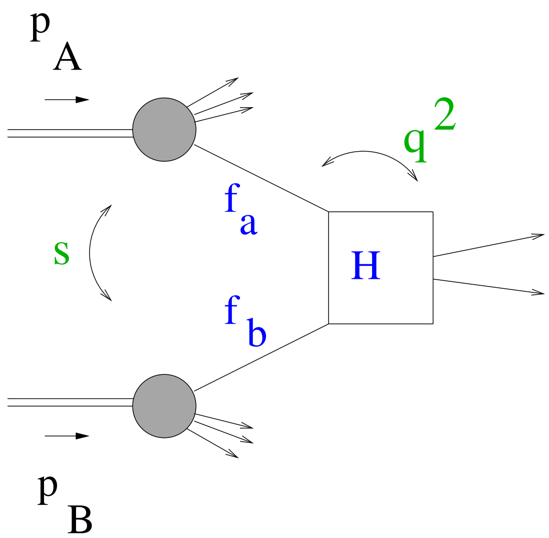




[B. Webber, CERN seminar, 2008]

II. COHERENCE IN HIGH-ENERGY, SMALL-X PARTON SHOWER

- above argument relies on soft vector emission current from external legs: leading IR singularities,
  - appropriate in single-scale hard processes



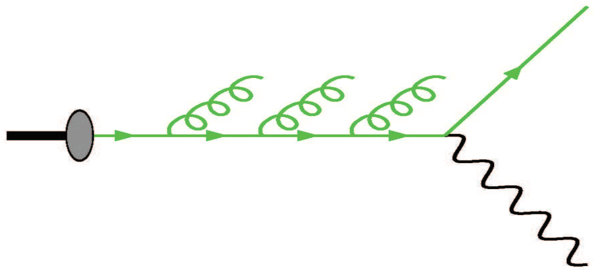
LHC multi-scale hard processes:

$$s = q_1^2 \gg \cdots \gg q_n^2 \gg \Lambda^2$$

$$\Downarrow$$

- initial-state branchings not collinearly-ordered potentially non-negligible
- emissions from internal legs become leading for $x \ll 1$
  $\Rightarrow$ associated coherence effects

▷ internal-emission current also factorizable at high-energy



$$|M_{n+1}\rangle = \mathbf{J}^a |M_n\rangle$$

▷ BUT: • $\mathbf{J}$ depends on total transverse momentum transmitted

⇒ matrix elements and pdf at fixed $k_\perp$ ("unintegrated")

• virtual corrections not all in Δ form factor

⇒ modified branching probability $P(z, k_\perp)$

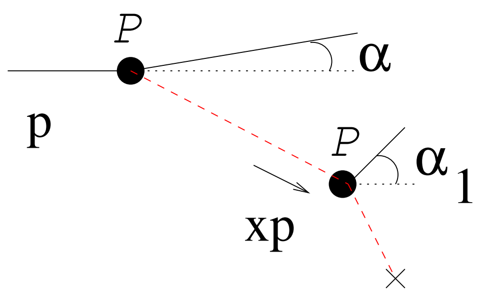
♢ radiative corrections enhanced by $\alpha_S^k \ln^m s/p_T^2$

♢ superleading logs cancel in fully inclusive quantities (space-like anomalous dimensions)

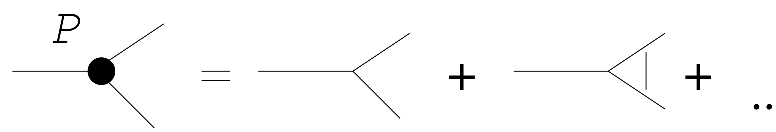
STRUCTURE OF $K_\perp$-DEPENDENT PARTON SHOWERS

- ME by perturbative computation; unintegrated pdf determined from data
- implement all-order summation of $(\alpha_S \ln s/p_T^2) \oplus$ IR $x \to 1$ behavior

$$\text{branching eq.}: \mathcal{A}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(\mu - zq) \times \underbrace{\Delta(\mu, zq)}_{\text{Sudakov}} \underbrace{\mathcal{P}(z, q, k_T)}_{\text{unintegr. splitting}} \mathcal{A}\left(\frac{x}{z}, k_T + (1-z)q, q\right)$$



,

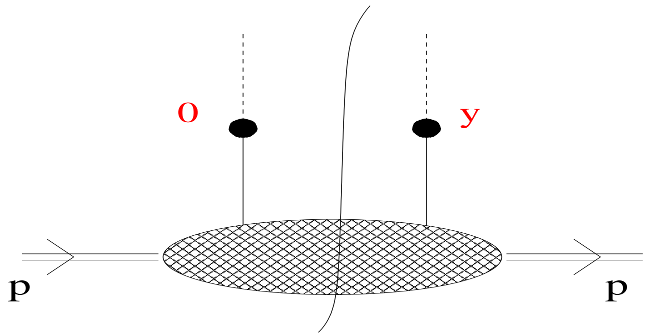


(left) Coherent radiation in the space-like parton shower for $x \ll 1$;
(right) the unintegrated splitting function $\mathcal{P}$, including small-x virtual corrections.

$$\alpha/x > \alpha_1 > \alpha \quad (\text{small} - \text{x coherence region})$$

III. UNINTEGRATED PARTON DISTRIBUTIONS

Example 1: Ordinary (integrated) pdf



p = (p$^+$, m^2 / 2 p$^+$, 0$_\perp$)

$$f_q(x,\mu) = \int \frac{dy^-}{2\pi}\, e^{-ixp^+y^-}\ \widetilde{f}(y)$$

correlation of quark fields at lightlike separation $y = (0, y^-, 0_\perp)$:

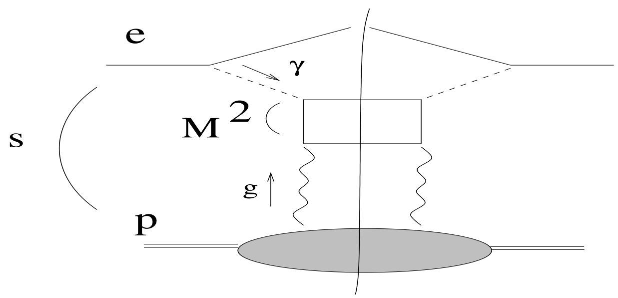
$$\widetilde{f}(y) = \langle\ P \mid \overline{\psi}(y)\ V_y^\dagger(n)\ \gamma^+\ V_0(n)\ \psi(0) \mid P\ \rangle \quad ,$$

$V_y(n) = \mathcal{P} \exp\left(i g_s \int_0^\infty d\tau\ n \cdot A(y + \tau\ n)\right)$ eikonal line in direction $n = (0, 1, 0_\perp)$

- μ-dependence from renormalization of operator product
    - gauge-invariant Wilson line matrix elements

How to characterize u-pdf gauge-invariantly?

Example 2: Unintegrated (TMD) pdf from high energy factorization:



◇ single gluon polarization dominates $s \gg M^2 \gg \Lambda^2_{\rm QCD}$

$\hookrightarrow$ gauge invariance rescued (despite gluon off-shell)

[Ciafaloni; Catani, Hautmann; ...]

◇ energy evolution equations / corrections down by $1/\ln s$ rather than $1/Q$

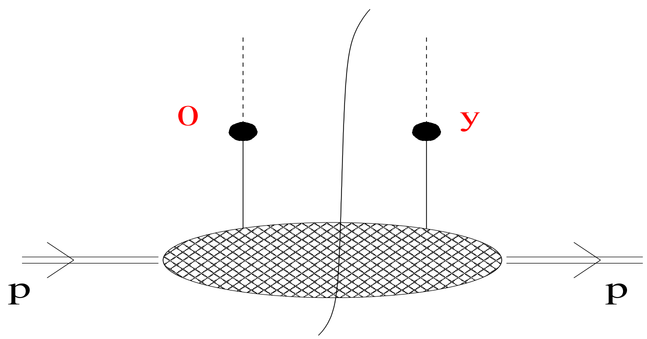
$\hookrightarrow$ BFKL (+ its variants)

◇ Note: can match on to arbitrarily high $k_\perp$ in the UV $\Rightarrow$

- suitable for simulations of jet physics at the LHC
- well-defined summation of higher-order radiative corrections

◇ Gauge-invariant characterization over whole phase space is more difficult

Example 3: Generalize ordinary (lightcone) pdf to non-lightlike distances:



$$p = (p^+ , m^2/2p^+ , 0_\perp)$$

$$\widetilde{f}(y) = \langle \, P \mid \overline{\psi}(y) \; V_y^\dagger(n) \; \gamma^+ \; V_0(n) \; \psi(0) \mid P \, \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

$V_y(n) = \mathcal{P} \exp \left(i g_s \int_0^\infty d\tau \; n \cdot A(y + \tau \; n) \right)$ eikonal Wilson line in direction n

- works at tree level [Mulders, 2002; Belitsky et al., 2003; ...]
- subtler at level of radiative corrections [Collins; Hautmann; Cherednikov, ...

  ↪ endpoint $x \to 1$ behavior ⇒ regulariz. method
- spectator interactions possibly non-decoupling (non-abelian Coulomb phase)

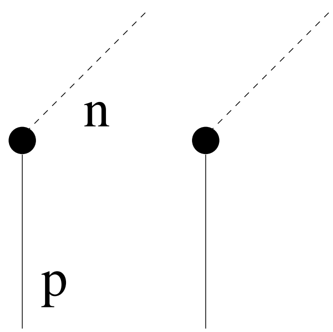
  [Mulders, Bomhof; Collins, Qiu; Brodsky, ...]

CUT-OFF REGULARIZATION

▷ cut-off in Monte-Carlo generators using u-pdf's

CASCADE	www.quark.lu.se/~hannes/cascade
SMALLX	Marchesini & Webber, 90's
LDCMC	www.thep.lu.se/~leif/ariadne

▷ cut-off from gauge link in non-lightlike direction n:



$\eta = (p \cdot n)^2 / n^2$

Collins, Rogers & Stasto, arXiv:0708.2833

Ji, Ma & Yuan, 2005, 2006

earlier work from 80's and 90's

finite $\eta \Rightarrow$ singularity is cut off at $1 - x \gtrsim \sqrt{k_\perp / 4\eta}$

- Lightcone limits $y^2 \to 0$ and $n^2 \to 0$ do not commute $\Rightarrow$

$$\Rightarrow \int dk_\perp \; f(x, k_\perp, \mu, \eta) = F(x, \mu, \eta) \neq \text{ ordinary } \text{ pdf}$$

∗ Note: Subtractive regularization is possible alternative to cut-off [Collins & H, 2001]

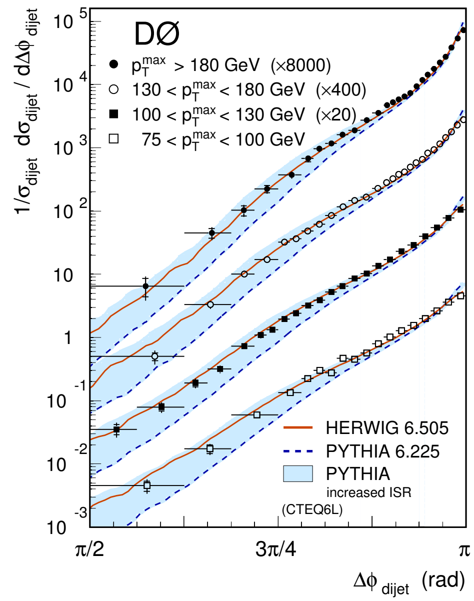
IV. APPLICATIONS TO MULTI-JET FINAL STATES

▷ jet correlations provide sensitive probes of QCD multiple-radiation effects

Ex.: azimuthal $\Delta\phi$ correlation (between two hardest jets)

▷ Tevatron $\Delta\phi$ dominated by leading-order processes

- distribution well described by HERWIG
- used for MC parameter tuning in PYTHIA



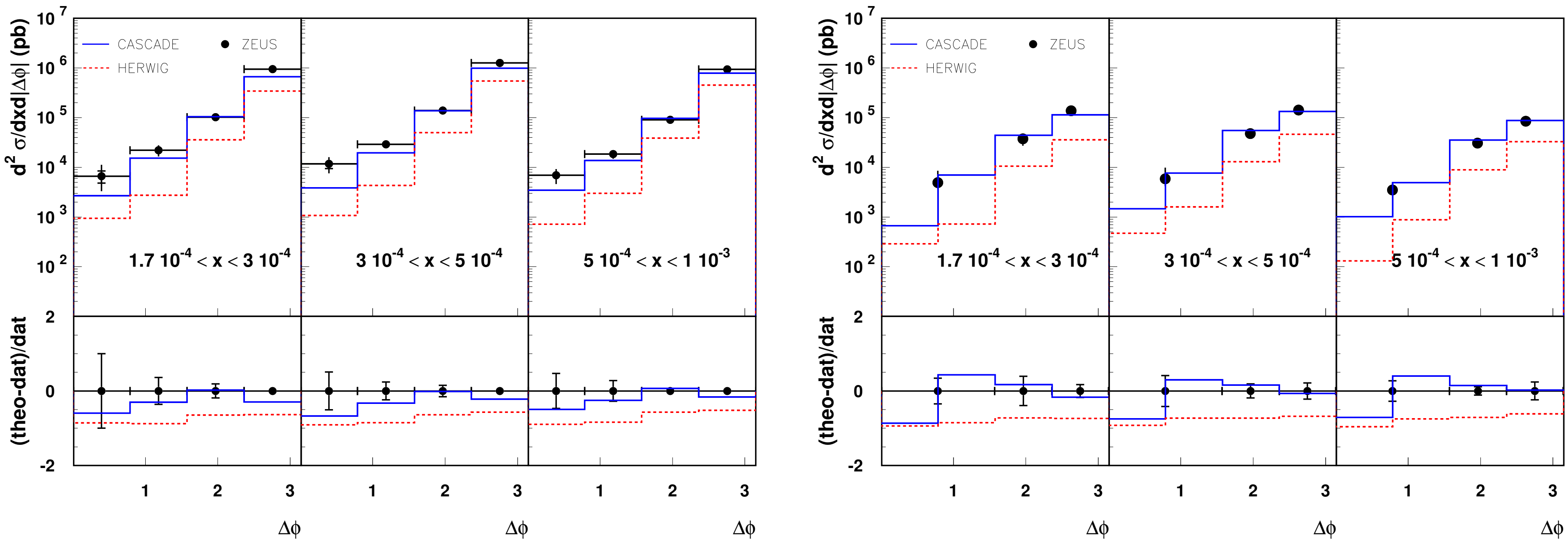
▷ HERA $\Delta\phi$ not well described by standard MC

↪ see next

▷ accessible at the LHC relatively early

↪ probe coherence effects in high-energy spacelike showers

ANGULAR JET CORRELATIONS FROM $K_\perp$-SHOWER (CASCADE) AND COLLINEAR-SHOWER (HERWIG)

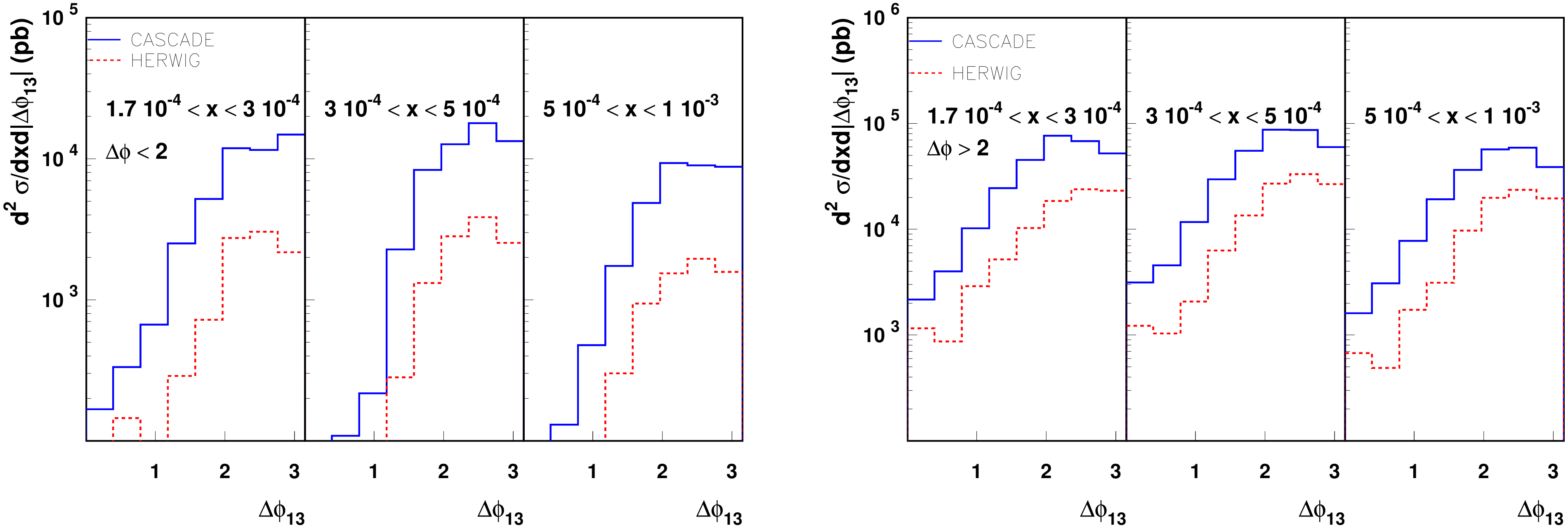


(left) di-jet cross section; (right) three-jet cross section

Jung & H, arXiv:0712.0568 [hep-ph]

- different shapes from the two MC
- largest differences at small $\Delta\phi$
- good description of measurement by $k_\perp$-shower

AZIMUTHAL DISTRIBUTION OF THE THIRD JET

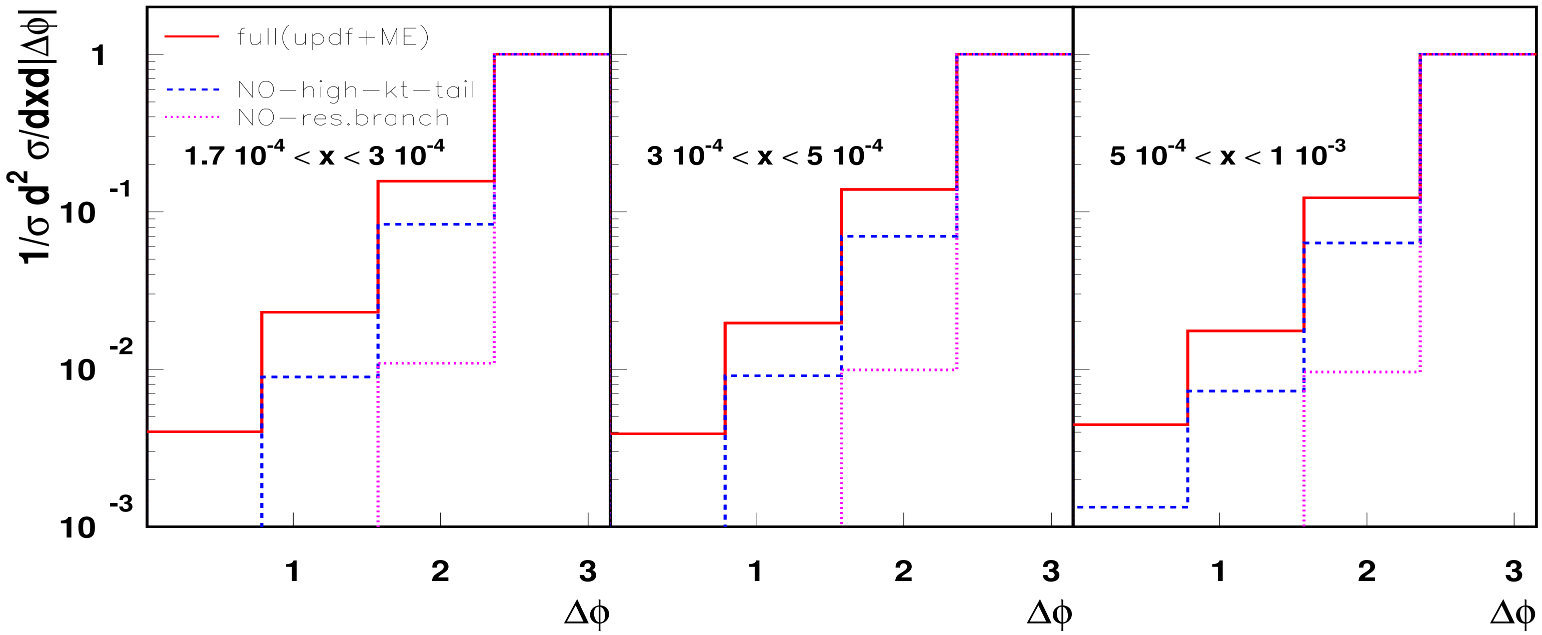


Cross section in the azimuthal angle between the hardest and the third jet for small (left) and large (right) azimuthal separations between the leading jets

Jung & H, arXiv:0805.1049 [hep-ph]

- small $\Delta\phi \Rightarrow$ non-negligible initial $k_\perp \Rightarrow$ larger corrections to collinear ordering
- curves become closer at large $\Delta\phi$

Normalize to the back-to-back cross section:



updf $\oplus$ ME

- - - updf $\oplus$ ME$_{collin.}$: $\mathcal{M} \to \mathcal{M}_{collin.}(k_T) = \mathcal{M}(0_\perp)\ \Theta(\mu - k_T)$

· · · · · · no resolved branching : $\mathcal{A} \to \mathcal{A}_{no-res.}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, Q_0)\ \Delta(\mu, Q_0)$

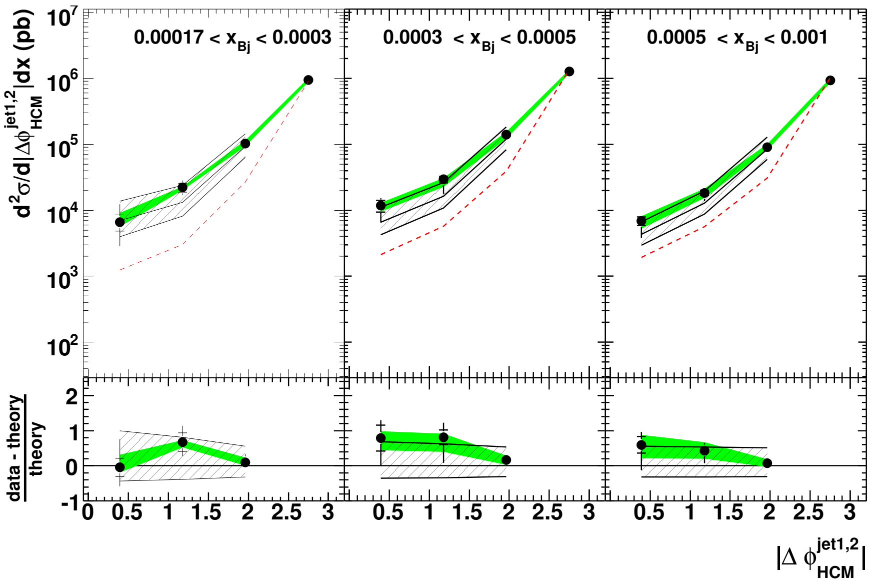
▷ high-k$_\perp$ component in ME essential to describe correlation at small $\Delta\phi$

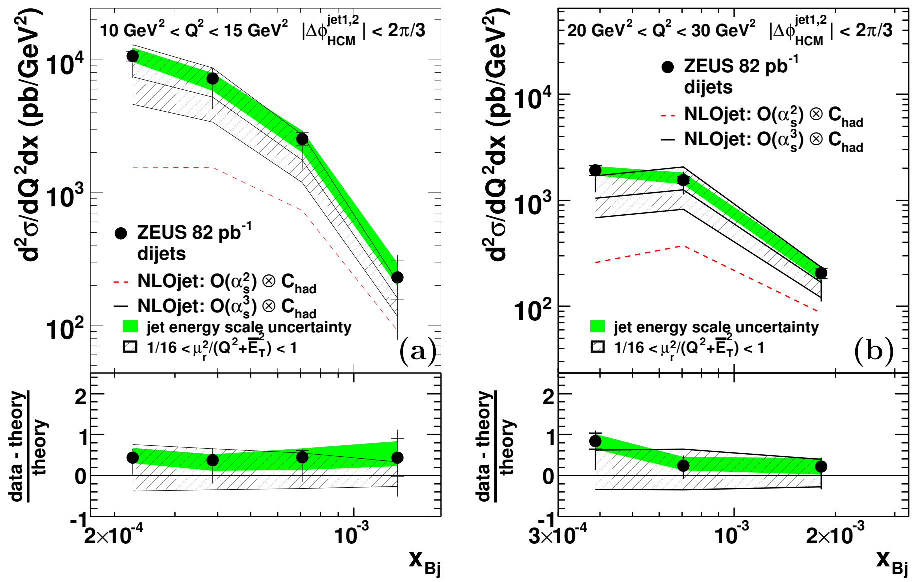
▷ k$_\perp$-dependence in u-pdf alone not sufficient

(cfr., e.g., MC by Höche, Krauss & Teubner, arXiv:0705.4577:

u-pdf but no ME correction)

COMPARISON WITH NLO RESULTS





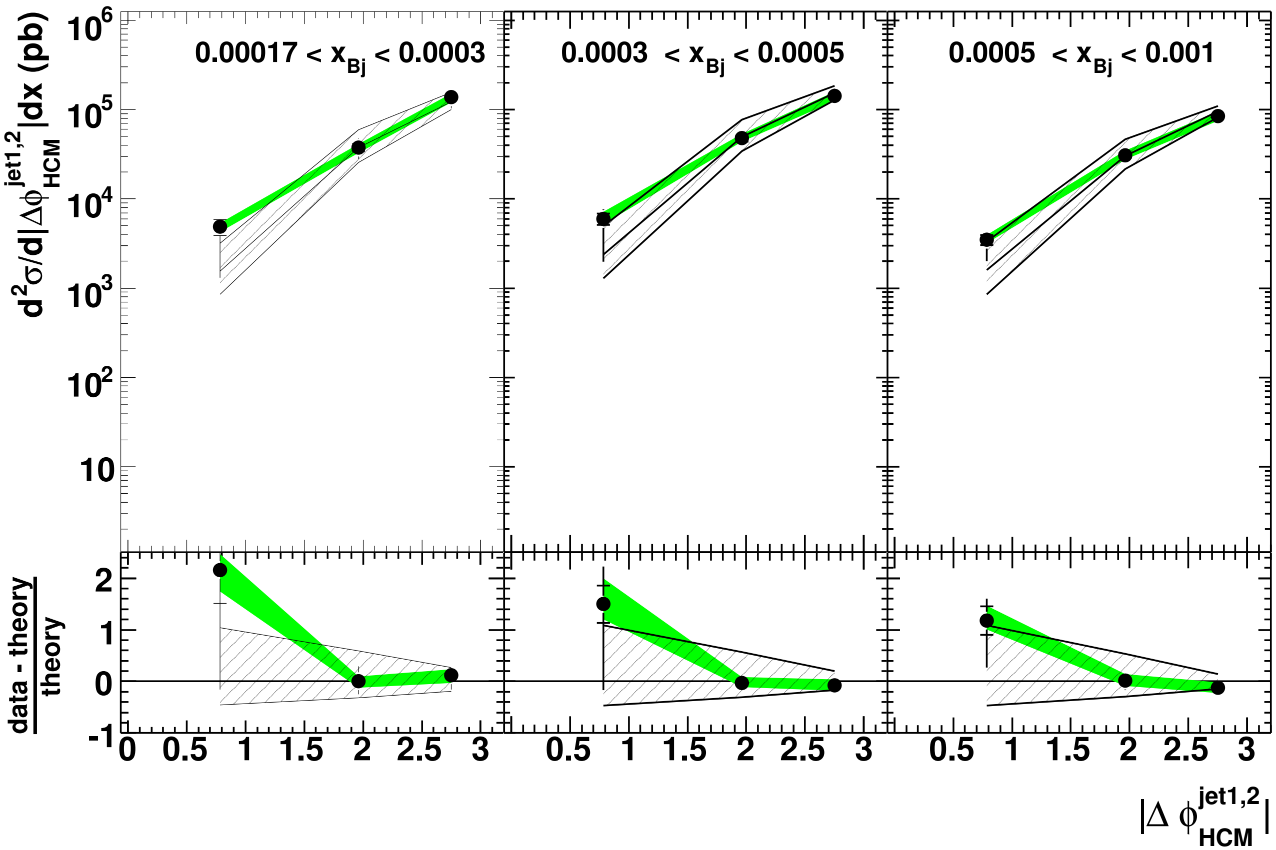
(left) Azimuth dependence and (right) Bjorken-x dependence of di-jet distributions

$$Q^2 > 10 \text{ GeV}^2 \quad , \qquad 10^{-4} < x < 10^{-2}$$

$\diamondsuit$ large variation from order-α_s^2 to order-α_s^3 prediction as $\Delta\phi$ and x decrease

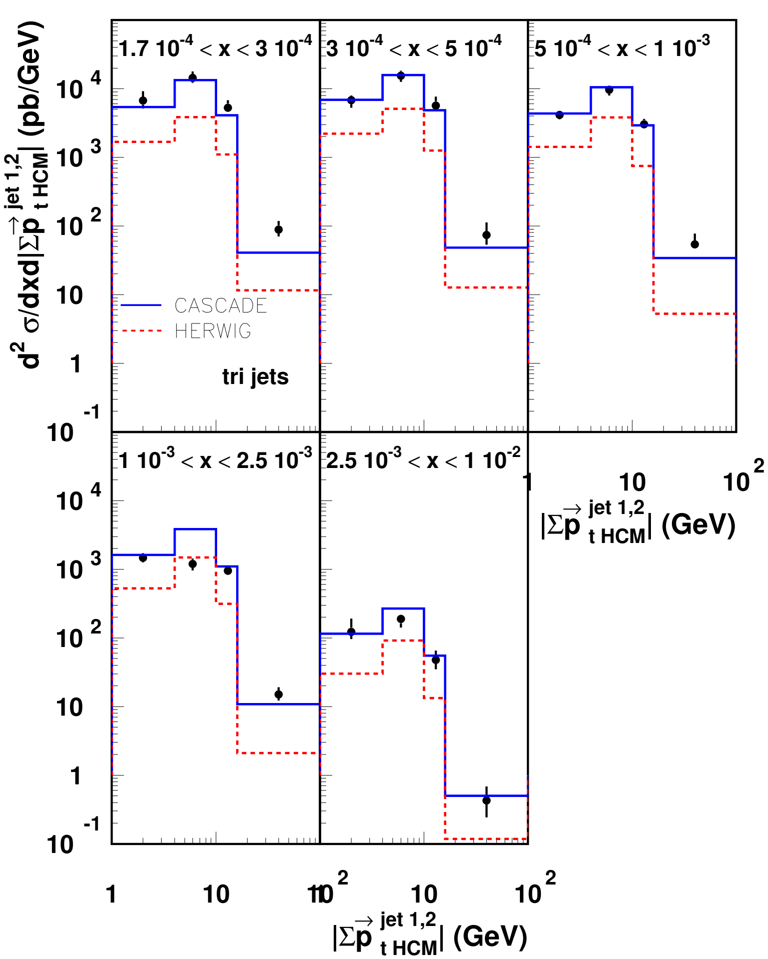
$\Rightarrow$ sizeable theory uncertainty at NLO (underestimated by "μ error band")

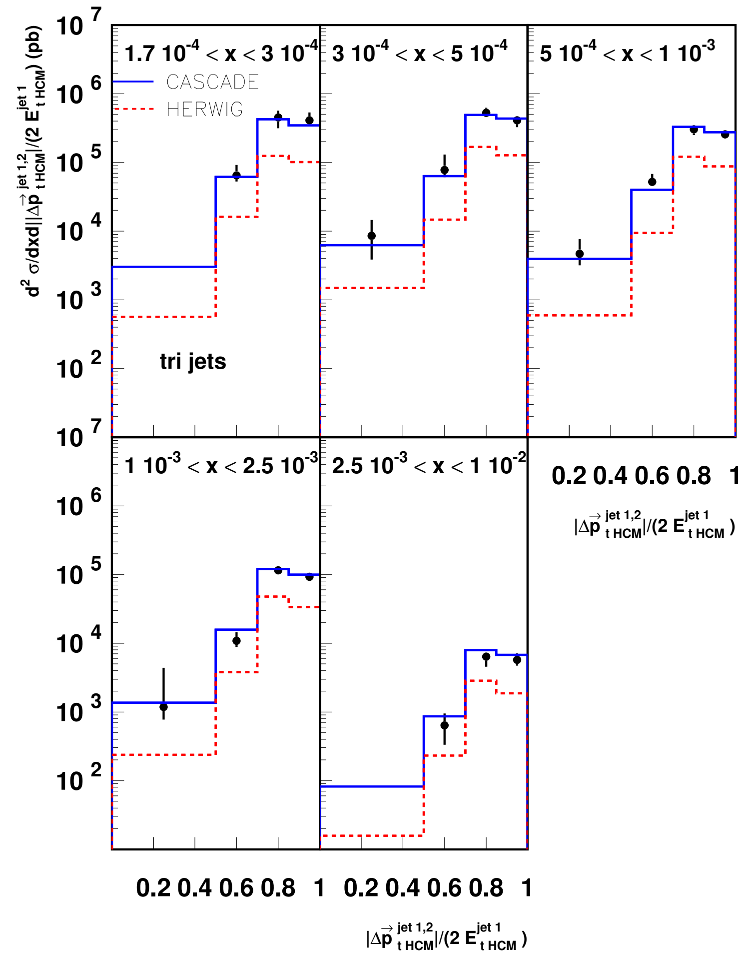
azimuthal distribution in 3-jet cross section [Zeus, 2007]



- besides angular correlations, sizeable NLO uncertainties in other associated distributions
  - NLO results are much more stable for inclusive jet cross sections

MOMENTUM CORRELATIONS [Jung & H, arXiv:0805.1049]





- correlations in the transverse momentum imbalance between the leading jets

Summary on 3-jet

▷ U-pdfs $\oplus$ $k_\perp$-dependent hard MEs describe

multi-jet measurements including correlations.

▷ coherence effects in angular distributions non-negligible

at high energy (small x) and small $\Delta\phi$

(near large $\Delta\phi$, Coulomb/radiative mixed terms also possibly relevant)

▷ Furthermore:

- Results similar to HERWIG if reduced to $k_\perp$-ordered phase space
  - Similar to fixed NLO where corrections are not large

▷ Non-forward jets ⇒ results less dependent on details of

u-pdf evolution models

V. PROSPECTS FOR LHC FINAL STATES AND CONCLUSIONS

♢ MC and radiative corrections to gluon fusion processes:

- production of b, c — what size NLO uncertainties at LHC energies?

  [see MC@NLO; Nason et al.]

  ▷ sizeable corrections from $g \to b\bar{b}$ coupling to spacelike jet

  ▷ coherence effects to $b\bar{b} + 2\ jets$ for $m_b \ll p_T^{(b\bar{b})} \ll p_T^{(jet)}$

  - multi-scale effects in $b\bar{b} + W/Z$ production

  - $k_\perp$-shower vs. MC@NLO for top-antitop pair production

    (↪ see p_T spectrum)

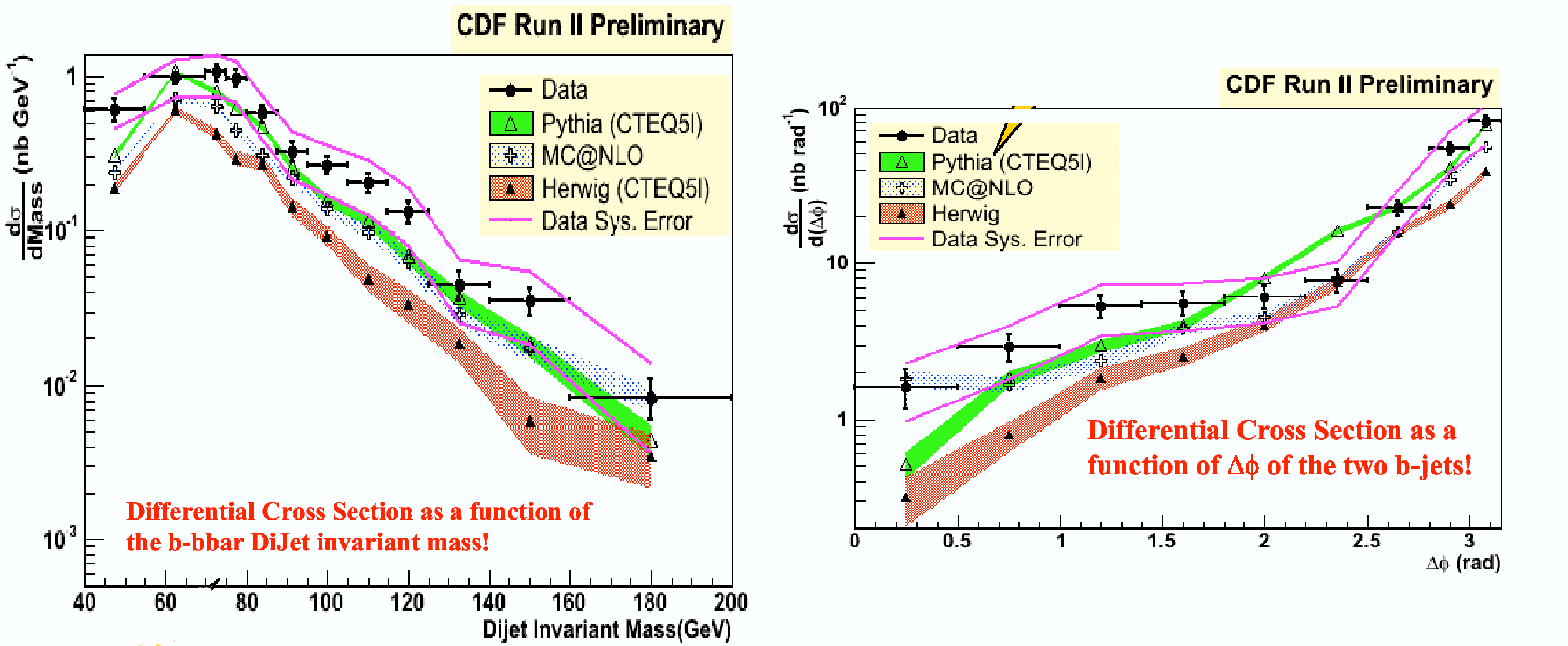
- final states with Higgs

  → possibly 10 ÷ 20 % effects in p_T spectrum from $x \ll 1$ terms?

  [Kulesza, Sterman & Vogelsang, 2004]

Tevatron b-jets correlations

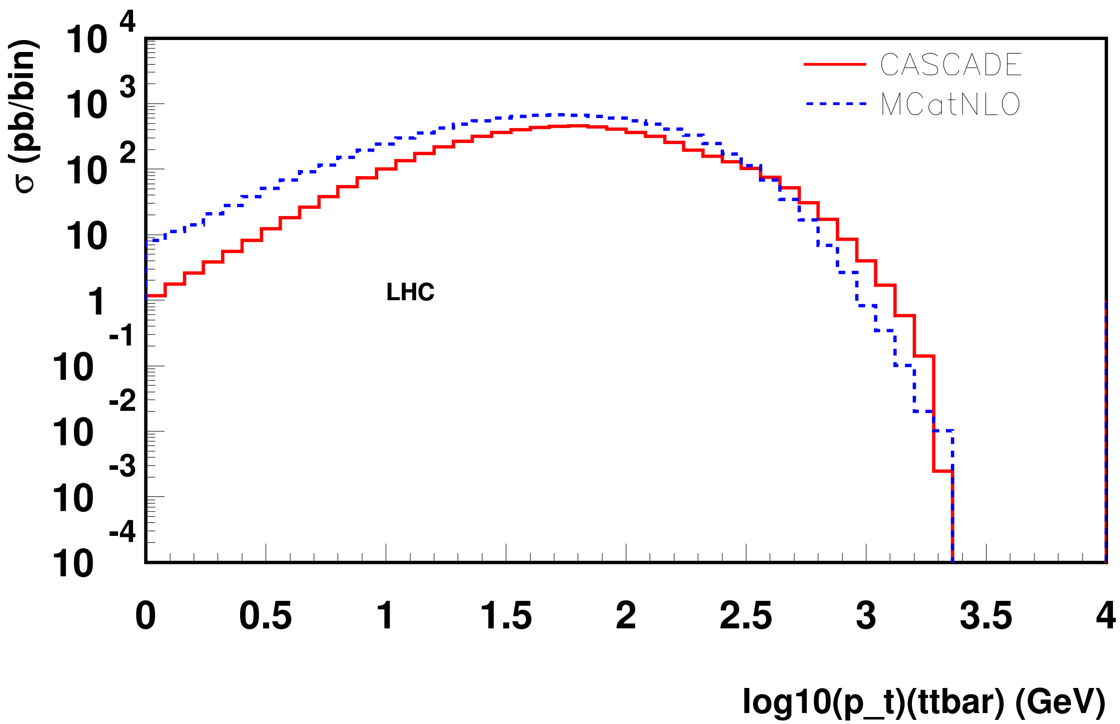
[B. Webber, CERN Lectures, 2008]



- collinear-shower descriptions not fully satisfactory
- phenomenological studies of $k_\perp$-showers potentially interesting
  - may affect underlying event description

p_T distribution of top-antitop pairs
from the $k_\perp$-shower CASCADE and from MC@NLO at LHC energies

[Jung & H, in progress]



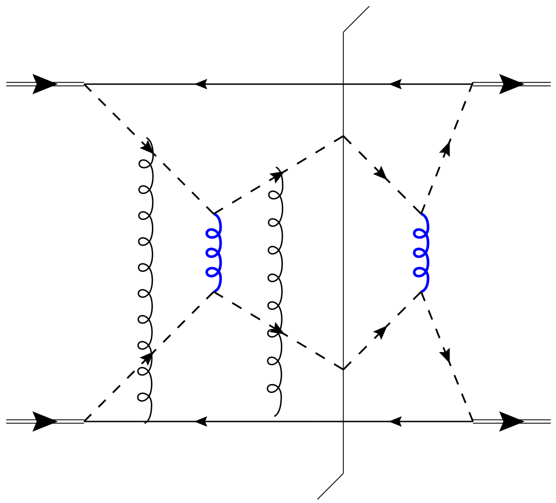
• small-x effects not large in this case

• probe shower in region of finite x and large virtualities on the order of m_{top}

FURTHER ISSUES AT HIGHER ORDER

- soft gluon exchange with spectator partons

  $\Rightarrow$ factorization breaking in higher loops?



Collins, arXiv:0708.4410

Vogelsang and Yuan, arXiv:0708.4398

Bomhof and Mulders, arXiv:0709.1390

$\diamondsuit$ likely suppressed for small-x, small-$\Delta\phi$

$\diamondsuit$ could affect physical picture near back-to-back region

- Note: Coulomb/radiative mixing terms also appear to break coherence in di-jet cross sections with gap in rapidity

Forshaw, Kyrieleis & Seymour, 2006

CONCLUSIONS

- When the LHC turns on, realistic Monte-Carlo are needed to analyze complex multi-particle final states with many hard scales

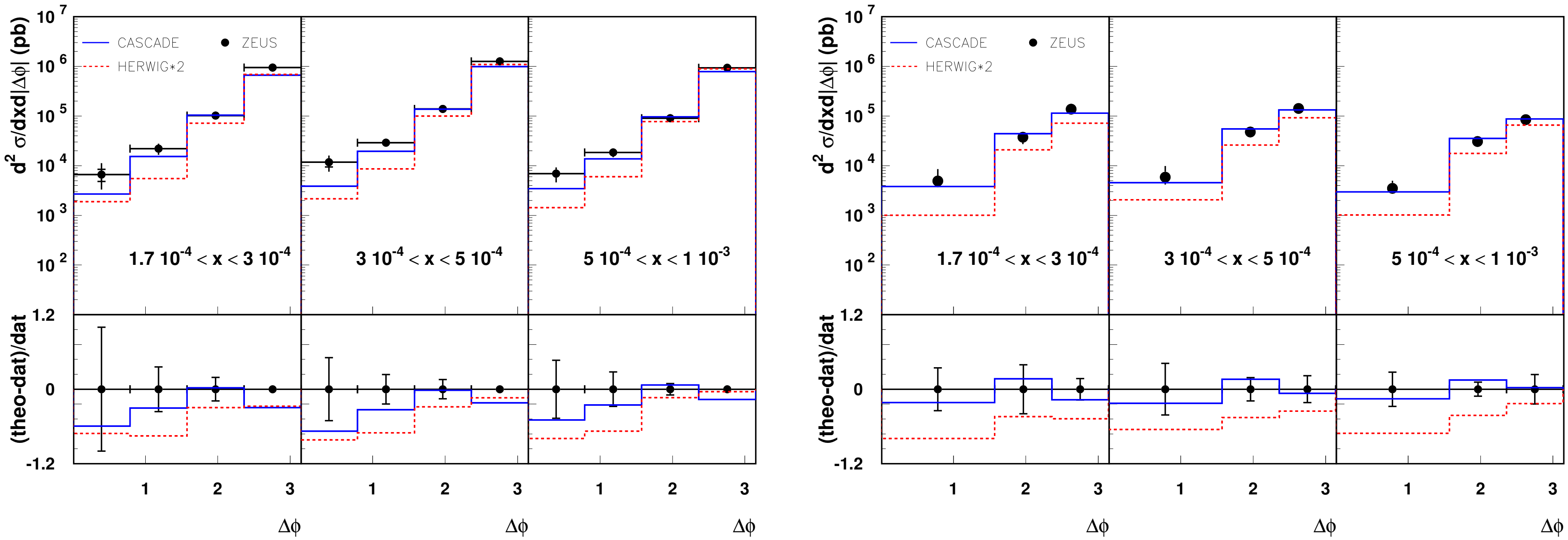
- Branching methods based on u-pdfs and $k_\perp$-MEs useful to
  - ▷ simulate high-energy parton showers
  - ▷ investigate possibly new effects from QCD physics

- Systematic theoretical studies of u-pdf's ongoing

▷ relevant to turn these Monte-Carlo's into general-purpose tools

EXTRA SLIDES

HERWIG K-factor of 2 (from two-jet region)

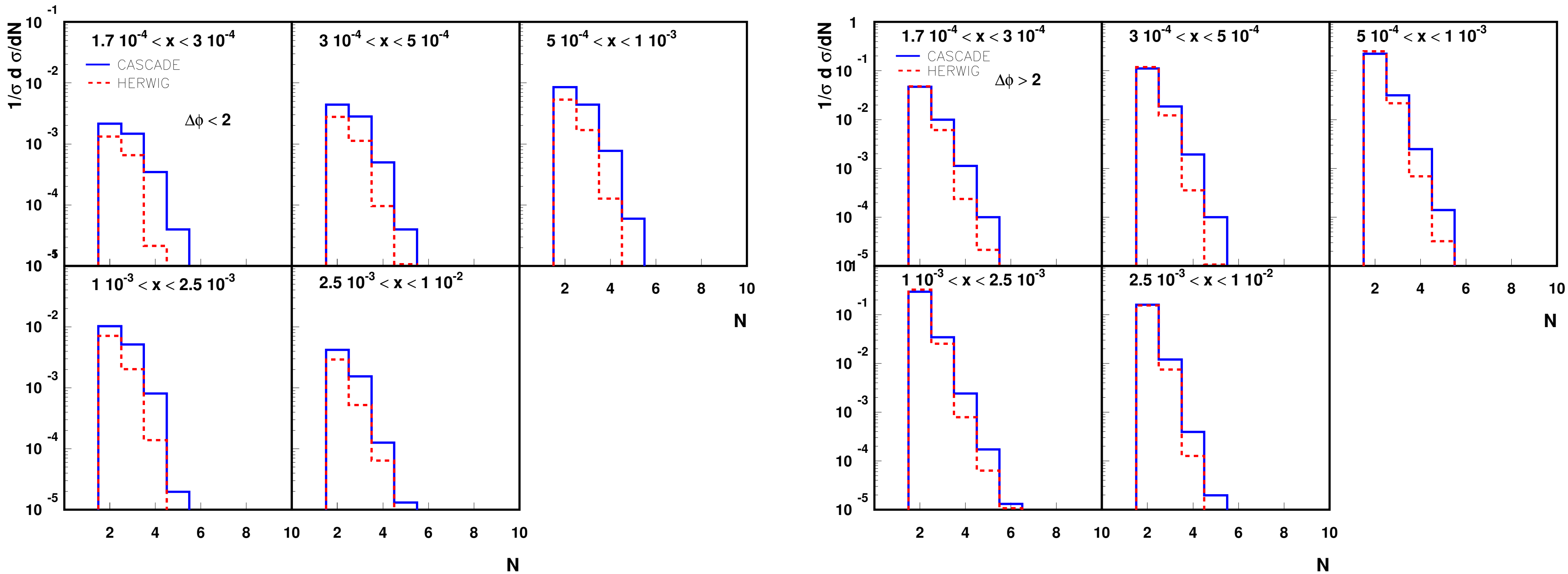


(left) di-jet cross section; (right) three-jet cross section

Jung & H, arXiv:0805.1049 [hep-ph]

- different shapes from the two MC
- small $\Delta\phi$ not well described by HERWIG
- good description of 3-jet by $k_\perp$-shower but not by HERWIG

JET MULTIPLICITIES



(left) $\Delta\phi < 2$; (right) $\Delta\phi > 2$

[Jung & H, arXiv:0805.1049]

- larger contribution from high multiplicity in the MC with u-pdf

$$E_{T,HCM}^{\rm jet-1} > 7 \text{ GeV} \quad , \qquad E_{T,HCM}^{\rm jet-2,3} > 5 \text{ GeV} \quad , \qquad -1 < \eta_{lab} < 2.5$$

- Jet clustering and hadronization:
  - ▷ moderate hadronization corrections from jet algorithm used by Zeus and H1 [arXiv:0705.1931 [hep-ex]; hep-ex/0310019]
    - ▷ jet clustering free of non-global logarithms [Dasgupta et al., hep-ph/0610242]
    - ▷ asymmetric jet cuts to avoid double logs in minimum p_T [Banfi and Dasgupta, hep-ph/0312108]
  - ▷ nonperturbative corrections in inverse powers of Q moderate for $Q^2 > 10 \text{GeV}^2$

- Radiative effects at higher order:
  - ♢ fixed-order beyond NLO is outside present reach for multi-jets in ep and pp
    - ♢ enhanced (soft/collinear) higher orders from near back-to-back region Y.Delenda et al., arXiv:0706.2172; arXiv:0804.3786; HERWIG
      - ♢ largest effects seen at small $\Delta\phi$ (3 well-separated hard jets)

Implementations:

Höche, Krauss and Teubner, arXiv:0705.4577 (KMR)

Golec, Jadach, Placzek, Stephens, Skrzypek, hep-ph/0703317 (CCFM)

LDCMC Lönnblad & Sjödahl, 2005; Gustafson, Lönnblad & Miu, 2002 (LDC)

CASCADE Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)

SMALLX Marchesini & Webber, 1992 (CCFM)

Advantages over standard Monte-Carlo like PYTHIA or HERWIG:

- better treatment of high-energy logarithmic effects
- likely more suitable for simulating underlying event's $k_\perp$

Current limitations:

- radiative terms associated to $x \sim 1$ not automatically included
- procedure to correct for this not yet systematic

  ↪ e.g.: LO-collin. evolution in Höche et al

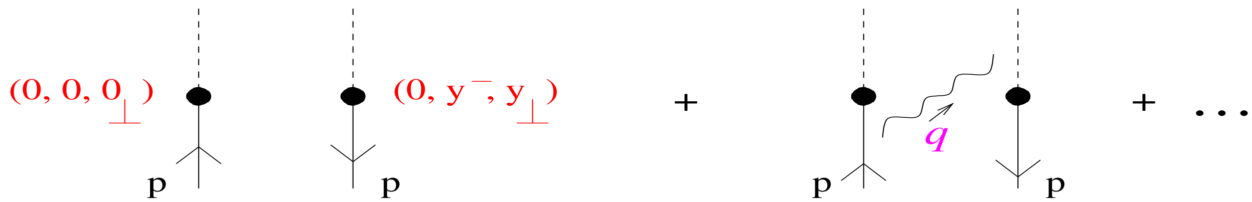
- quark contributions in initial state included partially

  ↪ see also: $k_\perp$ kernel for sea-quark evolution [Catani & H]

- limited knowledge of u-pdf's [Jung et al., arXiv:0706.3793; J. R. Andersen et al., 2006]

◇ Suppose a gluon is absorbed or emitted by eikonal line:

$n = (0, 1, 0_\perp)$



$$f_{(1)} = P_R(x, k_\perp) - \delta(1-x)\,\delta(k_\perp)\int dx'\,dk'_\perp\,P_R(x', k'_\perp)$$

$$\text{where} \qquad P_R = \frac{\alpha_s\,C_F}{\pi^2}\left[\frac{1}{1-x}\,\frac{1}{k_\perp^2+\rho^2} + \{\text{regular at } x\to 1\}\right] \qquad \rho\text{=IR regulator}$$

↑ *endpoint singularity* $(q^+ \longrightarrow 0,\ \forall\ k_\perp)$

◇ Physical observables:

$$\mathcal{O} = \int dx\ dk_\perp\ f_{(1)}(x, k_\perp)\ \varphi(x, k_\perp)$$

$$= \int dx\ dk_\perp\ [\varphi(x, k_\perp) - \varphi(1, 0_\perp)]\,P_R(x, k_\perp)$$

inclusive case: φ independent of $k_\perp \Rightarrow 1/(1-x)_+$ from real + virtual

general case: endpoint divergences (incomplete KLN cancellation)

UPDF's WITH SUBTRACTIVE REGULARIZATION

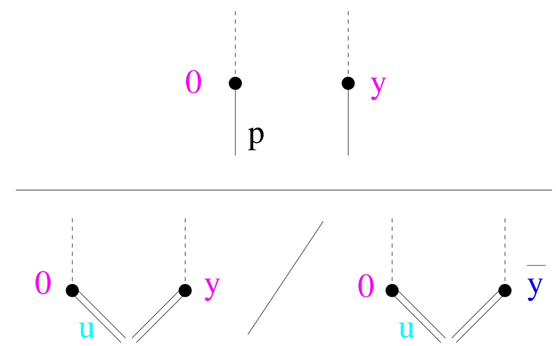
- Endpoint divergences $x \to 1$ from incomplete KLN cancellation

Subtractive method: more systematic than cut-off. Widely used in NLO calculations.
Formulation suitable for eikonal-operator matrix elements: Collins & H, 2001.

- gauge link still evaluated at n lightlike, but multiplied by "subtraction factors"

$$\widetilde{f}^{(\mathrm{subtr})}(y^-, y_\perp) =$$

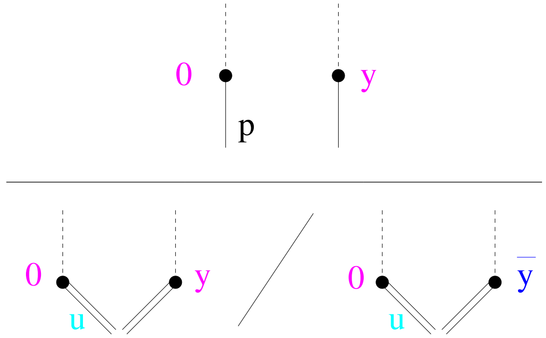
$$\frac{\overbrace{\langle P|\overline{\psi}(y)V_y^\dagger(n)\gamma^+V_0(n)\psi(0)|P\rangle}^{\text{original matrix element}}}{\underbrace{\langle 0|V_y(u)V_y^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle \;/\; \langle 0|V_{\bar{y}}(u)V_{\bar{y}}{}^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle}_{\text{counterterms}}}$$



$\bar{y} = (0, y^-, 0_\perp)$; u = auxiliary non-lightlike eikonal $(u^+, u^-, 0_\perp)$

H, arXiv:0708.1319

♢ u serves to regularize the endpoint; drops out of distribution integrated over $k_\perp$



One loop expansion:

$$f_{(1)}^{(\text{subtr})}(x,k_\perp) \quad = \quad P_R(x,k_\perp) - \delta(1-x)\,\delta(k_\perp) \int dx'\,dk'_\perp\, P_R(x',k'_\perp) \quad (\leftarrow \text{from numerator})$$

$$- \quad W_R(x,k_\perp,\zeta) + \delta(k_\perp) \int dk'_\perp\, W_R(x,k'_\perp,\zeta) \quad (\leftarrow \text{from vev's})$$

with $P_R = \alpha_s C_F/\pi^2 \left\{ 1/[(1-x)\ (k_\perp^2 + m^2(1-x)^2)] + \ldots \right\}$ = real emission prob.

$W_R = \alpha_s C_F/\pi^2 \left\{ 1/[(1-x)\ (k_\perp^2 + 4\zeta(1-x)^2)] + \ldots \right\}$ = counterterm

- ζ-dependence cancels upon integration in $k_\perp$ $[\zeta = (p^{+2}/2)u^-/u^+]$

$$\Rightarrow \mathcal{O} \quad = \quad \int dx\ dk_\perp\ f_{(1)}^{(\text{subtr})}(x,k_\perp)\ \varphi(x,k_\perp)$$

$$= \quad \int dx\ dk_\perp\ \{P_R\ [\varphi(x,0_\perp) - \varphi(1,0_\perp)] + (P_R - W_R)\ [\varphi(x,k_\perp) - \varphi(x,0_\perp)]\}$$

- first term: usual $1/(1-x)_+$ distribution
- second term: singularity in P_R cancelled by W_R

Order-α_s analysis [H, hep-ph/0702196]

- ▷ Expand Wilson-line matrix element to one loop
- ▷ Gauge link at infinity does not contribute in covariant gauge
- ▷ $d = 4 - 2\varepsilon$ for UV divergences

(a) (b)

$$\widetilde{f}_{(a)+(b)}(y) = \frac{\alpha_s C_F}{4^{d/2-2}\pi^{d/2-1}} p^+ \int_0^1 dv \, \frac{v}{1-v} \left[e^{ip\cdot yv} \, 2^{d/2-1} \left(\frac{\rho^2}{\mu^2}\right)^{d/4-1} \right.$$

$$\left. \times \quad \frac{1}{(-y^2\mu^2)^{d/4-1}} K_{d/2-2}(\sqrt{-\rho^2 y^2}) - e^{ip\cdot y} \, \Gamma(2-\frac{d}{2}) \, (\frac{\mu^2}{\rho^2})^{2-d/2} \right]$$

K = modified Bessel function; Γ = Euler gamma function

$$\rho^2 = (1-v)^2 m^2 + v\lambda^2$$

- $v \rightarrow 1$: endpoint singularity
- can relate result to ordinary pdf by expanding in y^2

$\longrightarrow$ separate long-distance terms in $\ln(\mu^2/\rho^2)$

and short-distance terms in $\ln(y^2\mu^2)$

$$\widetilde{f}_{(a)+(b)} \simeq \frac{\alpha_s C_F}{4^{d/2-2}\pi^{d/2-1}} \; p^+ \int_0^1 dv \; \frac{v}{1-v} \; \Big\{ \Big[e^{ip\cdot yv} - e^{ip\cdot y} \Big] \; \Gamma(2-\frac{d}{2}) \; (\frac{\mu^2}{\rho^2})^{2-d/2}$$

$$+ \; e^{ip\cdot yv} \; 4^{d/2-2} \; \Gamma(\frac{d}{2}-2) \; (-y^2\mu^2)^{2-d/2}$$

$$+ \; \sum_{k=1}^{\infty} \frac{\Gamma(2-d/2) \; \Gamma(d/2-1)}{k! \; 4^k \; \Gamma(k+d/2-1)} \; e^{ip\cdot yv} \; (\frac{\rho^2}{\mu^2})^{d/2+k-2} (-y^2\mu^2)^k$$

$$+ \; \sum_{k=1}^{\infty} \frac{4^{d/2-2-k} \; \Gamma(d/2-2) \; \Gamma(3-d/2)}{k! \; \Gamma(k+3-d/2)} \; e^{ip\cdot yv} \; (\frac{\rho^2}{\mu^2})^k (-y^2\mu^2)^{2-d/2+k} \Big\}$$

$v \longrightarrow 1$ endpoint singularity

- cancels for ordinary pdf (first term in rhs)
- present, even at $d \neq 4$ and finite ρ, in subsequent terms