CAN WE TRUST SMALL X RESUMMATION?

STEFANO FORTE
UNIVERSITÀ DI MILANO

CRACOW EPIPHANY CONFERENCE

JANUARY 6, 2009

SUMMARY

- WHY AND WHERE SMALL x RESUMMATION IS NECESSARY

- THE THREE INGREDIENTS FOR STABLE RESUMMATION

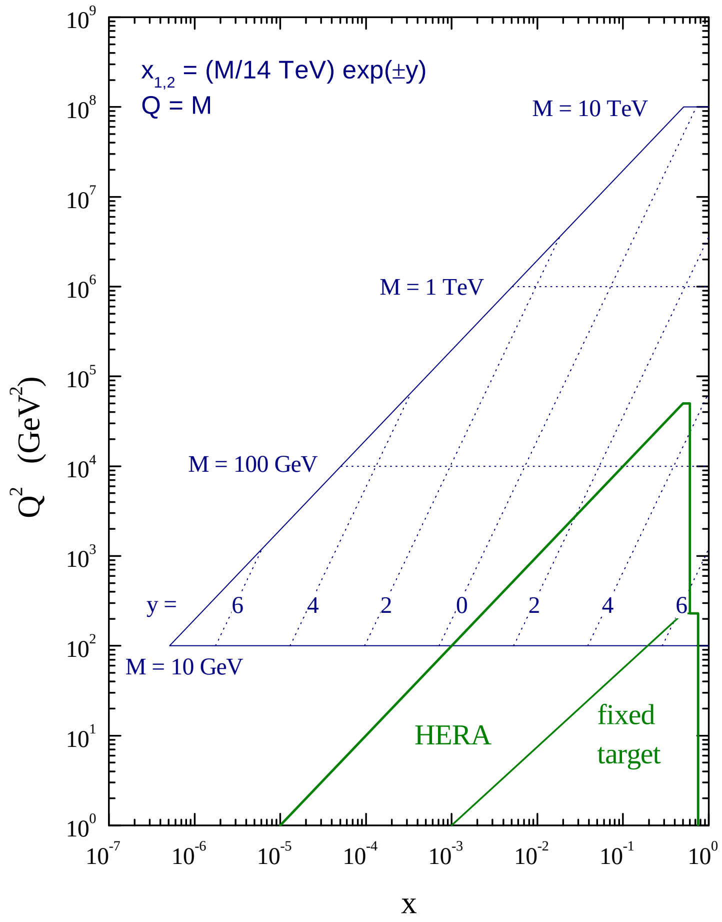
- MATCHING AND PHENOMENOLOGY

BASED ON WORK DONE WITH G. ALTARELLI & R. BALL 1998-2008
some comparison with related work by

M. CIAFALONI, D. COLFERAI, G. SALAM & A. STAŚTO
& R. THORNE AND C. WHITE
BASED ON SEMINAL WORK BY L. LIPATOV, V. FADIN, J. KWIECIŃSKI,
J. COLLINS, T. JAROSZEWICZ, M. CIAFALONI, S. CATANI (1975-1998)

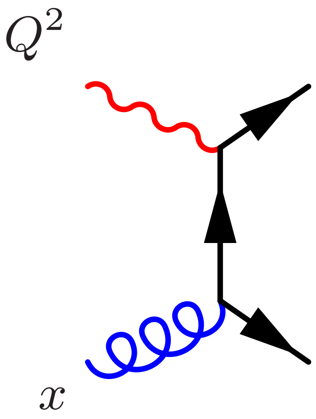
PRECISION QCD: FROM HERA TO LHC

LHC parton kinematics

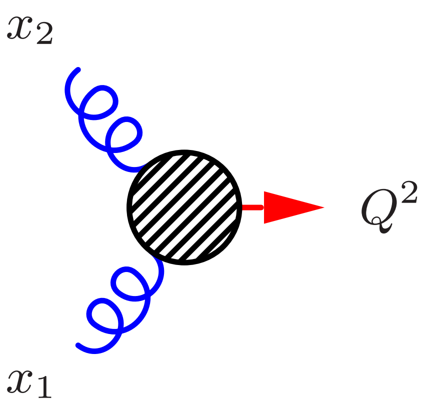


e–p VS. p–p

COLLIDER KINEMATICS



$$x = \frac{Q^2}{2p\cdot q}$$



$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y}$$

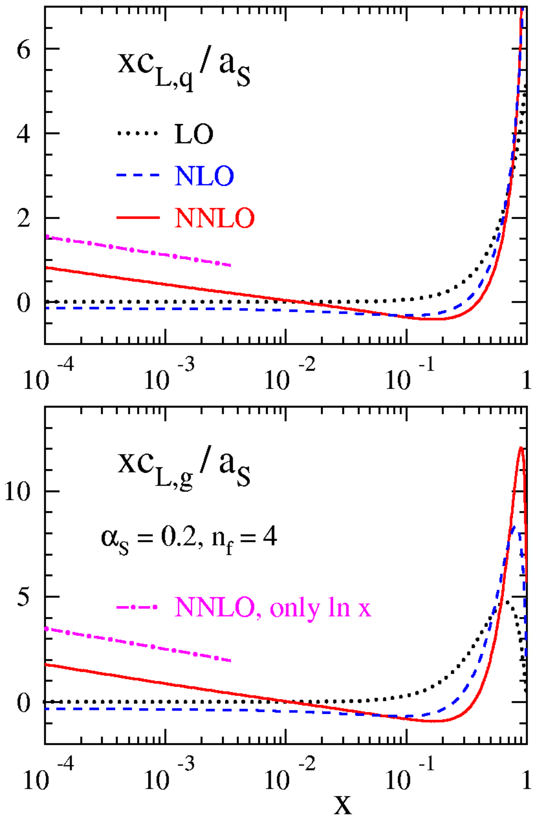
WHY WE SHOULD WORRY ABOUT SMALL X:

THE NNLO CORRECTIONS

THEORY

THE COEFFICIENT FUNCTION C_L

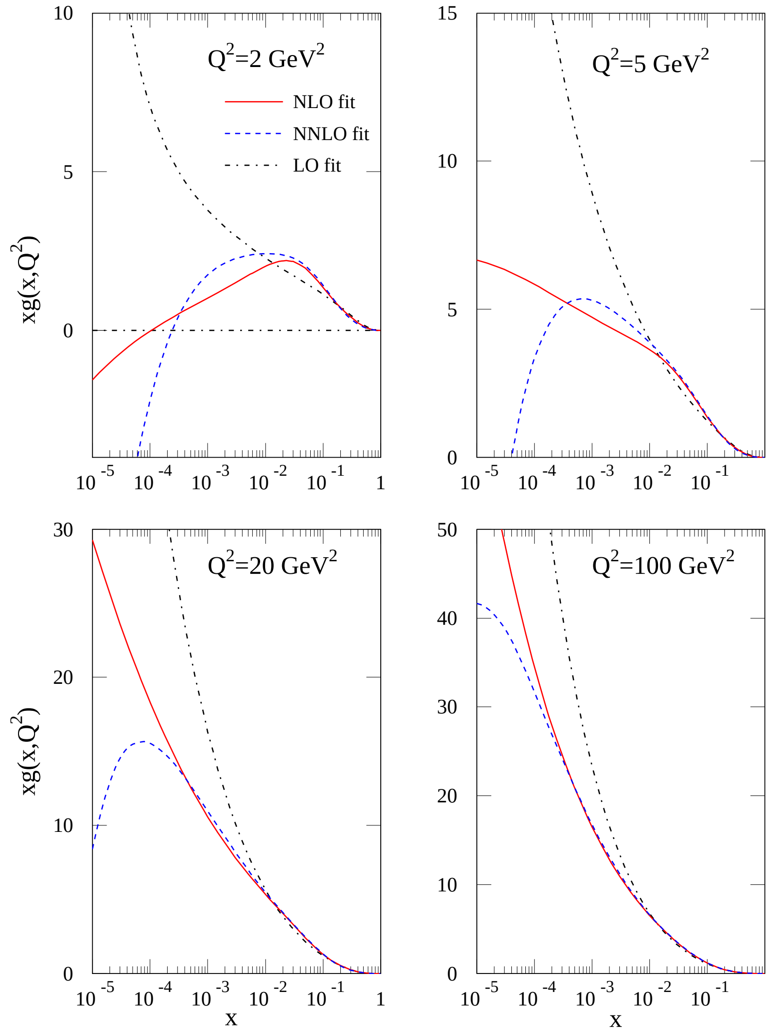
(Moch, Vermaseren, Vogt 2005)



PHENOMENOLOGY

THE BEST–FIT GLUON

(mstw 2008)



- PERTURBATIOMN THEORY UNSTABLE
- LEADING LOG APPROX POOR

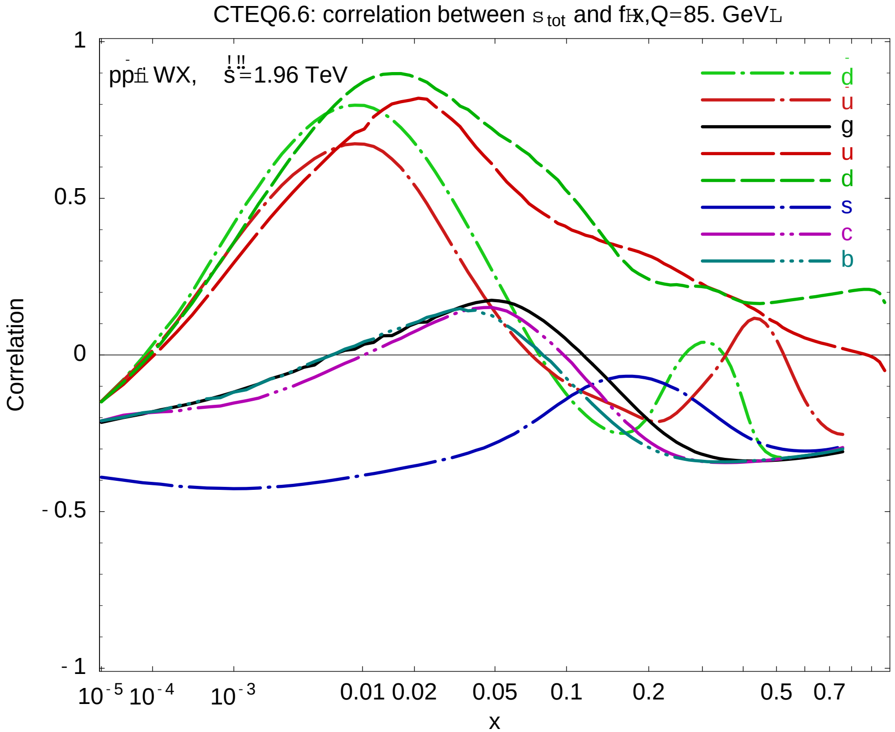
WHY WE SHOULD WORRY ABOUT SMALL X:

THE IMPACT AT LHC

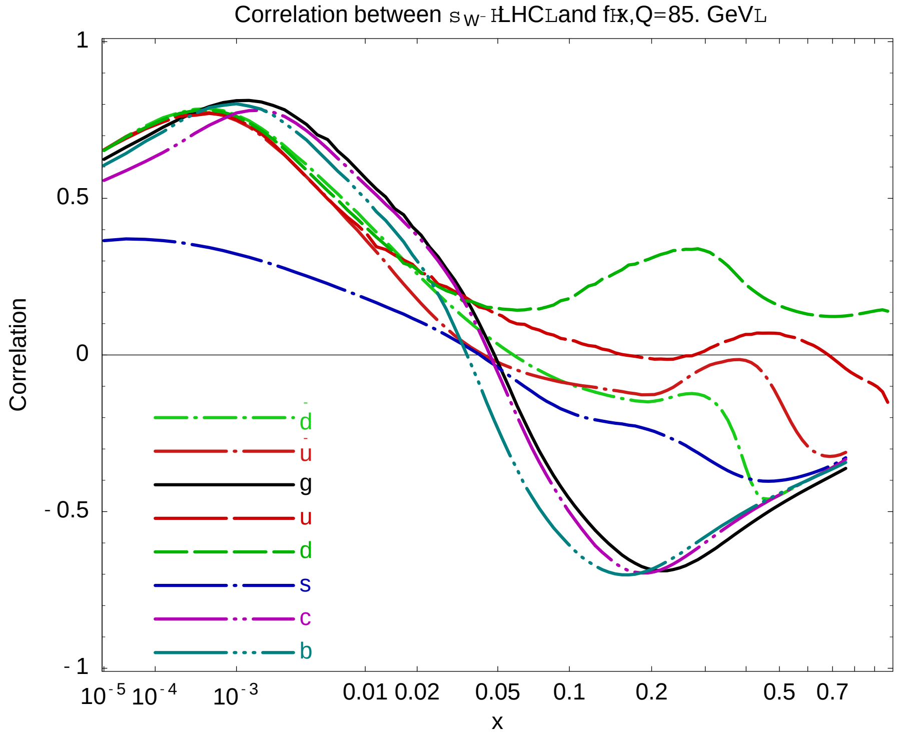
CORRELATION BETWEEN PDFS AND THE W TOTAL CROSS SECTION

(CTEQ 2008)

TEVATRON



LHC



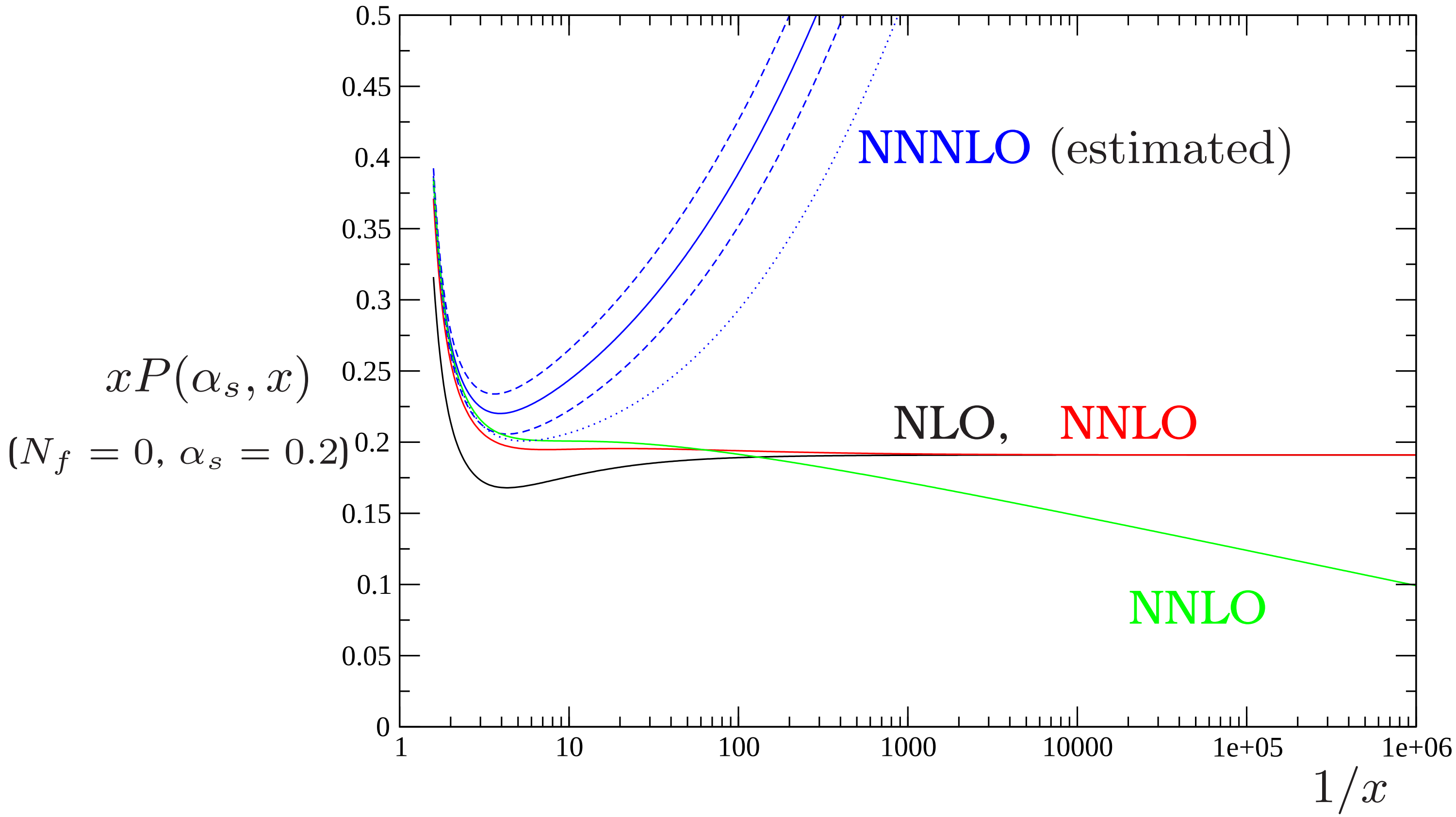
UNCERTAINTIES ON SMALL x PDFS PROPAGATE TO INCLUSIVE OBSERVABLES

PERTURBATIVE INSTABILITY:

THE SINGLET SPLITTING FUNCTION

$$xP(\alpha_s, x) \underset{x\to 0}{\sim}$$

$$\alpha_s c_1^{(1)} + \alpha_s^2 c_2^{(1)} + \alpha_s^3 \left(c_3^{(2)} \ln x + c_3^{(1)} \right) + \alpha_s^4 \left(c_4^{(4)} \ln^3 x + c_4^{(3)} \ln^2 x + c_4^{(2)} \ln x + c_4^{(1)} \right) + \ldots$$



QUESTIONS & ANSWERS

- Q: CAN ONE RESUM LARGE SMALL x CORRECTIONS TO ALL ORDERS

  A:

- Q: CAN ONE COMBINE SMALL x RESUMMATION WITH STANDARD PERTURBATIVE EVOLUTION

  A:

- Q: CAN ONE OBTAIN A STABLE PERTURBATIVE EXPANSION AT THE RESUMMED LEVEL?

  A:

- Q: CAN ONE UNDERSTAND THE SUCCESS OF NLO PERTURBATION THEORY DESPITE LARGE SMALL x TERMS?

  A:

QUESTIONS & ANSWERS

- Q: CAN ONE RESUM LARGE SMALL x CORRECTIONS TO ALL ORDERS

  A: YES, BOTH AT THE LEADING AND SUBLEADING LEVEL FOR ANOMALOUS DIMENSIONS

  $\surd$ BFKL 75-76, FL 98;

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- Q: CAN ONE UNDERSTAND THE SUCCESS OF NLO PERTURBATION THEORY DESPITE LARGE SMALL x TERMS?

  A: YES, IF ONE RESUMS AT THE RUNNING COUPLING LEVEL

  √ Ciafaloni, Colferai 99, ABF 01, Thorne 01

QUESTIONS & ANSWERS II

- Q: DOES THE RESUMMATION INTERFERE WITH THE CHOICE OF FACTORIZATION SCHEME

  A:

- Q: CAN ONE ESTIMATE THE AMBIGUITIES IN THE RESUMMATION AND HOW LARGE ARE THEY

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- Q: ARE RESUMMED HARD COEFFICIENTS AVAILABLE FOR HADROPRODUCTION PROCESSES? AND HOW LARGE ARE THEY

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- Q: DOES THE RESUMMATION INTERFERE WITH THE CHOICE OF FACTORIZATION SCHEME

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  K.Ellis, Ball 01; Marzani, Ball, Del Duca, Forte, Vicini 08; Marzani, Ball 08

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  BUT PHENOMENOLOGY STILL UNAVAILABLE K.Ellis, Ball 01; Marzani, Ball, Del Duca, Forte, Vicini 08; Marzani, Ball 08

THE THREE INGREDIENTS FOR STABLE RESUMMATION

THE FIRST INGREDIENT: DUALITY (fixed coupling)

(T. JAROSZEWICZ, 1982; R. BALL & S.F., 1995)

THE ALTARELLI-PARISI EQN IS AN INTEGRO–DIFFERENTIAL EQUATION $\Rightarrow$ IT CAN BE EQUIVALENTLY VIEWED AS Q^2–EVOLUTION EQUATION FOR x–MOMENTS (usual RG eqn.), OR x–EVOLUTION EQUATION FOR Q^2–MOMENTS(BFKL eqn.)

EVOLUTION IN $t = \ln Q^2$

$$\frac{d}{dt} G(N,t) = \gamma(N,\alpha_s)\; G(N,t)$$

MELLIN x–MOMENTS

$$G(N,t) = \int_0^\infty d\xi\, e^{-N\xi}\; G(\xi,t)$$

EVOLUTION IN $\xi = \ln 1/x$

$$\frac{d}{d\xi} G(\xi,M) = \chi(M,\alpha_s)\; G(\xi,M)$$

MELLIN Q^2–MOMENTS

$$G(\xi,M) = \int_{-\infty}^\infty dt\, e^{-Mt}\; G(\xi,t)$$

THE TWO EQUATIONS HAVE THE SAME SOLUTIONS PROVIDED THE EVOLUTION KERNELS ARE RELATED BY

$$\chi(\gamma(N,\alpha_s),\alpha_s) = N$$
$$\gamma(\chi(M,\alpha_s),\alpha_s) = M$$

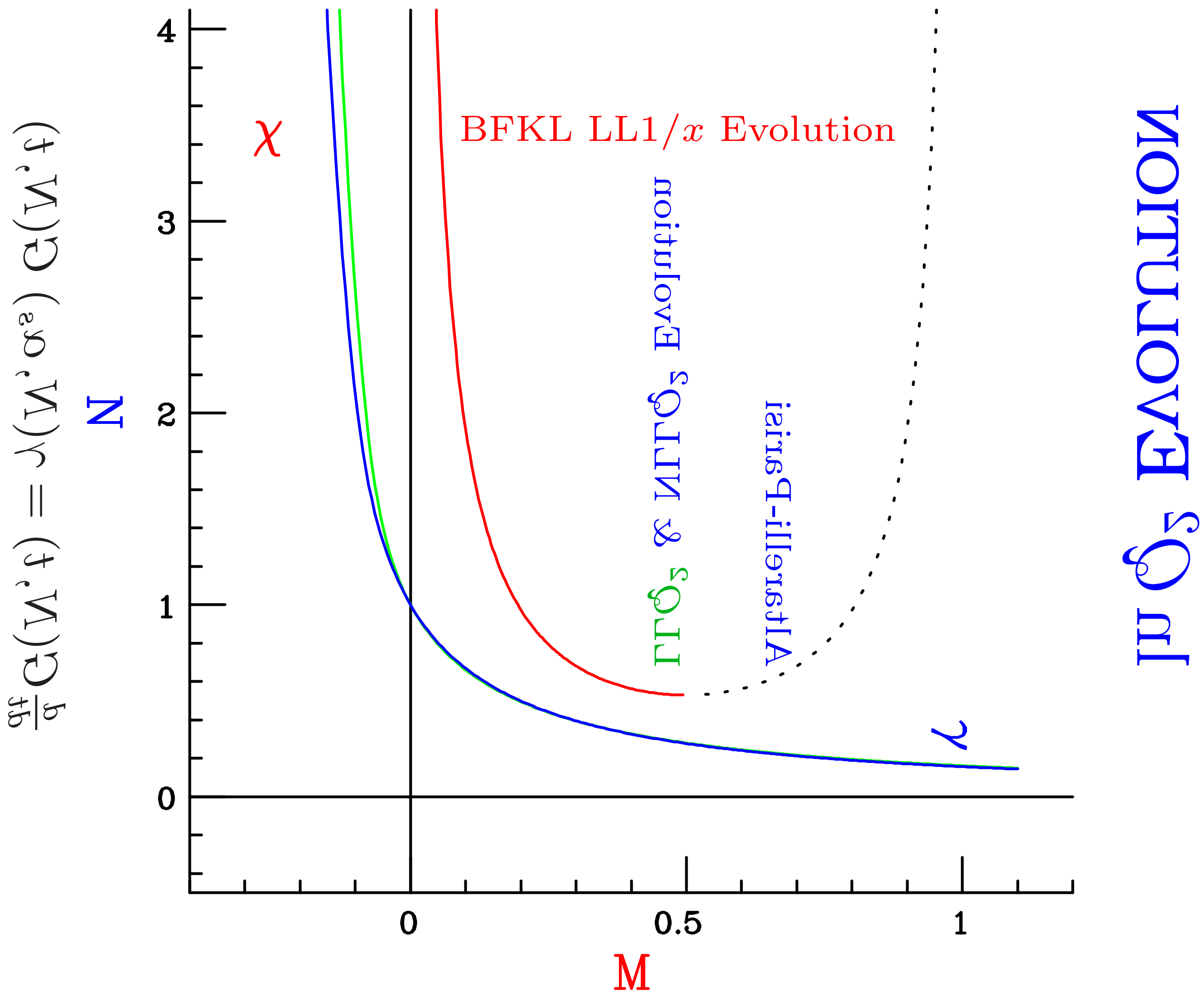
& BOUNDARY CONDITIONS RELATED BY

$$H_0[M] \to G_0(N) = H_0[\gamma(N,\alpha_s)]/\chi'(\gamma(N,\alpha_s))$$

. . . CAN SWITCH FROM LLQ^2 TO LL$1/x$

CHOOSING THE EVOLUTION KERNEL

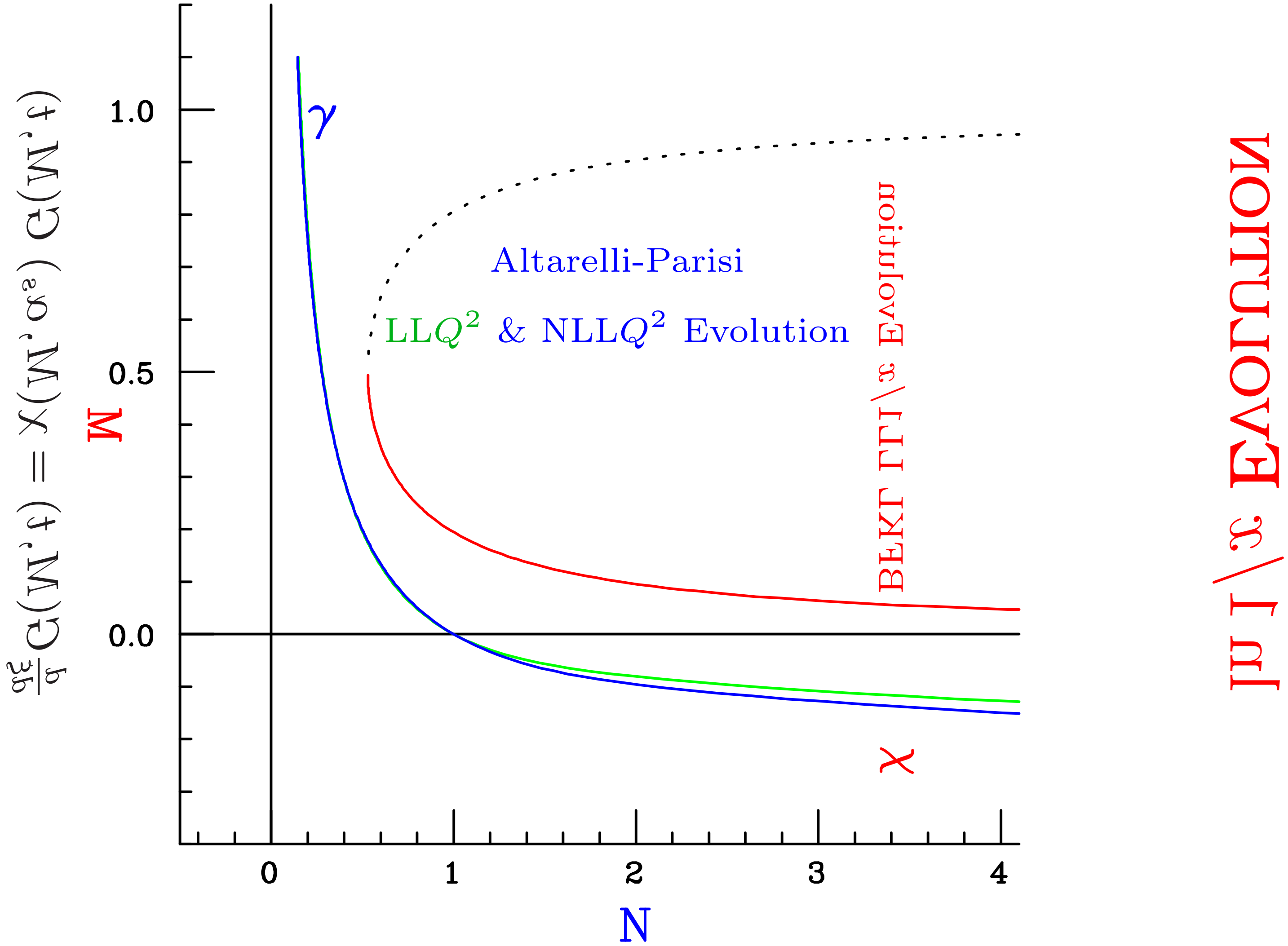
$\ln 1/x$ EVOLUTION



$$\frac{d}{d\xi}G(M,t) = \chi(M,\alpha_s)\; G(M,t)$$

. . . IN EITHER EQUATION!

$\ln Q^2$ Evolution



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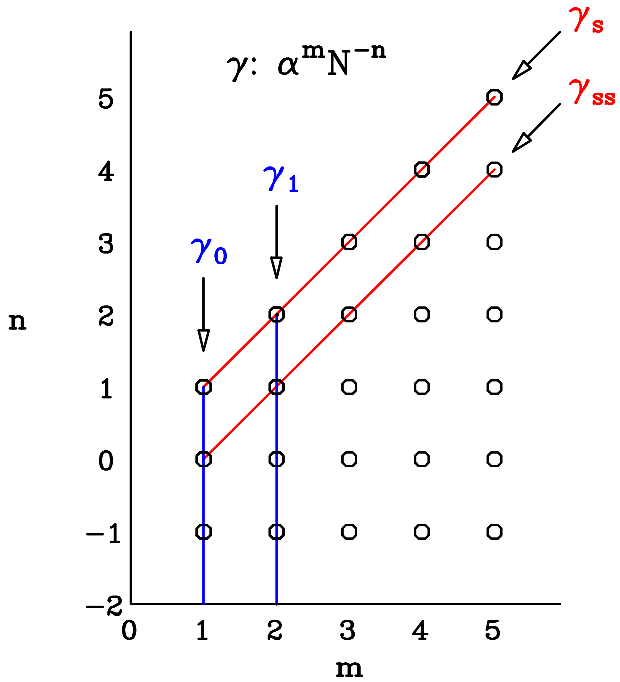
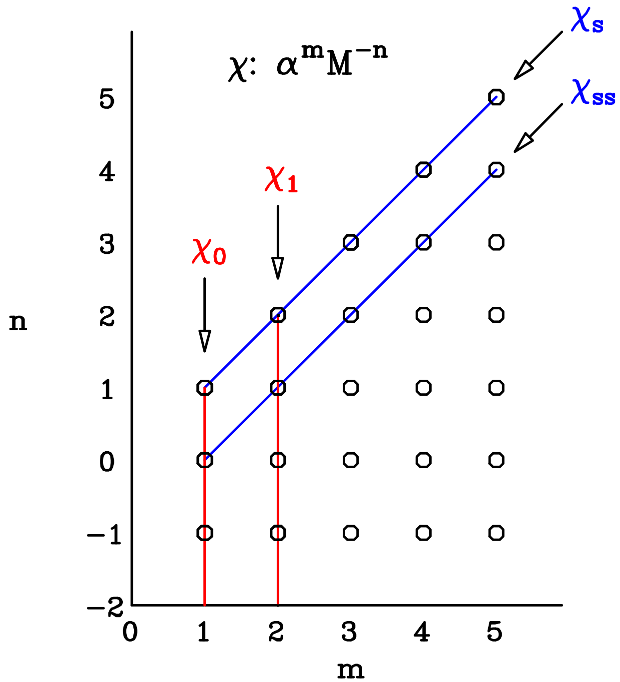
DUAL PERTURBATIVE EXPANSIONS





$\ln Q^2$ EVOLUTION

$$\gamma(N) = \alpha\left(\frac{c^{(1)}_{-1}}{N} + c^{(1)}_0 + \ldots\right) + \alpha^2\left(\frac{c^{(2)}_{-2}}{N^2} + \frac{c^{(2)}_{-1}}{N} + \ldots\right)$$

$$\gamma_s(N) = c^{(1)}_{-1}\frac{\alpha}{N} + c^{(2)}_{-2}\frac{\alpha^2}{N^2} + \ldots$$

$1/N$ POLES $\Leftrightarrow$ $\ln 1/x$

$\ln 1/x$ EVOLUTION

$$\chi(M) = \alpha\left(\frac{\tilde{c}^{(1)}_{-1}}{M} + \tilde{c}^{(1)}_0 + \ldots\right) + \alpha^2\left(\frac{\tilde{c}^{(2)}_{-2}}{M^2} + \frac{\tilde{c}^{(2)}_{-1}}{M} + \ldots\right)$$

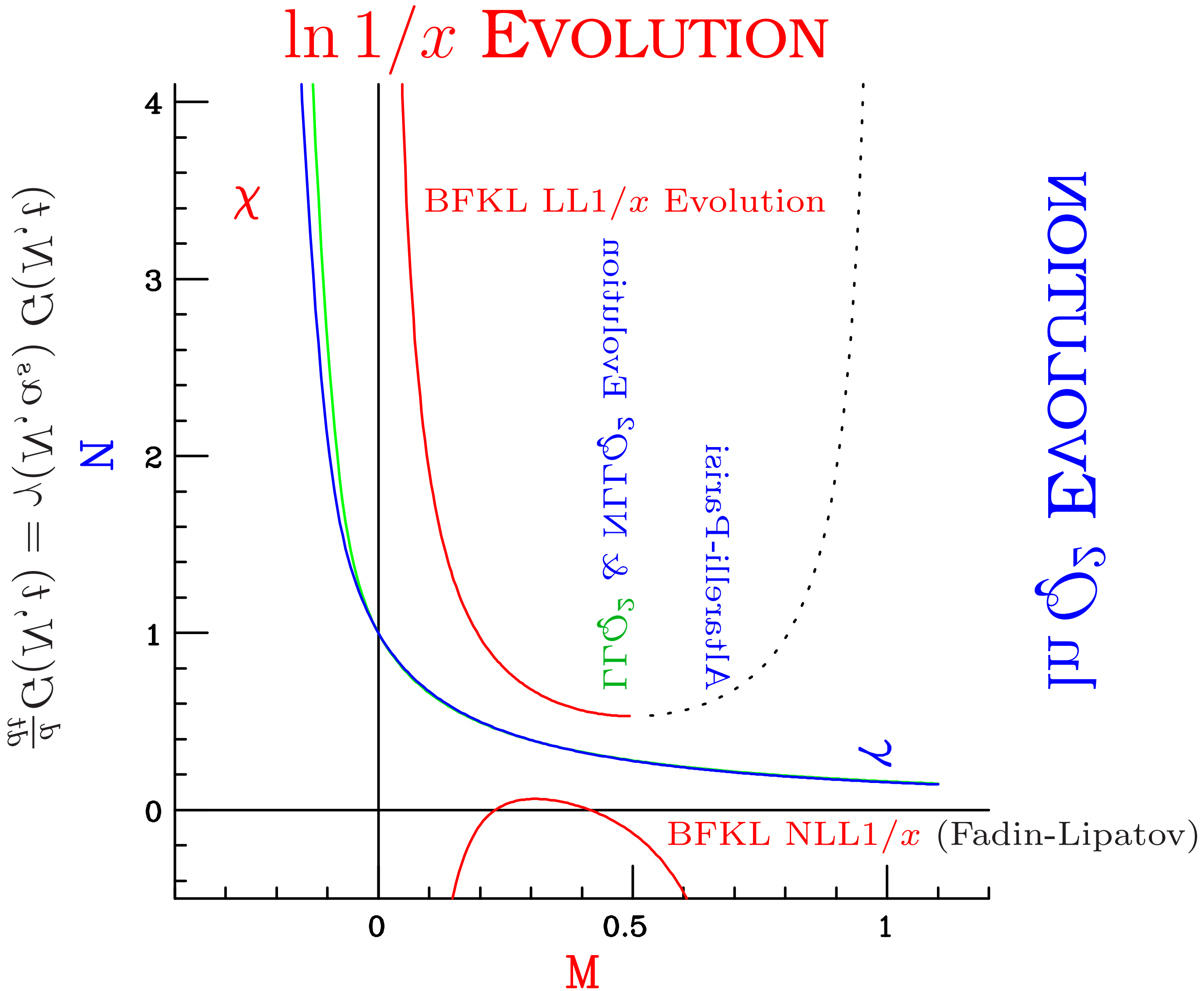
$$\chi_s(M) = \tilde{c}^{(1)}_{-1}\frac{\alpha}{M} + \tilde{c}^{(2)}_{-2}\frac{\alpha^2}{M^2} + \ldots$$

$1/M$ POLES $\Leftrightarrow$ $\ln Q^2$

$$\gamma_0(N) \Longleftarrow\!\!\Longrightarrow \chi_s(\alpha_s/M)$$

$$\gamma_s(\alpha_s/N) \Longleftarrow\!\!\Longrightarrow \chi_0(M)$$

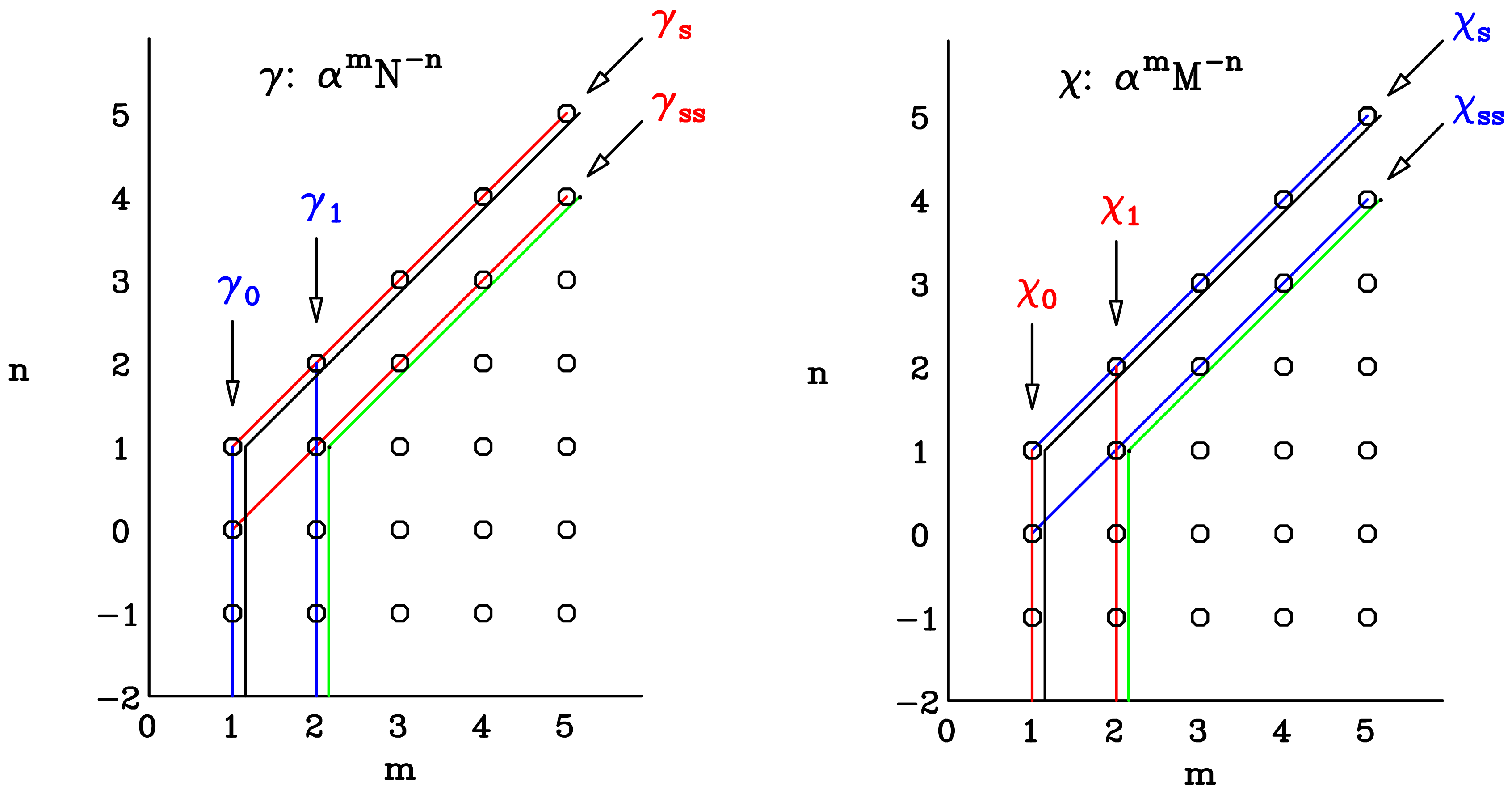
THE PROBLEM WITH LL$1/x$ EVOLUTION: NL CORRECTIONS!



$$\frac{d}{d\xi}G(M,t) = \chi(M,\alpha_s)\ G(M,t)$$

- THE LLQ^2 AND LL$1/x$ KERNELS GREATLY DIFFER FROM EACH OTHER
- THE EXPANSION OF THE LL$1/x$ KERNEL LOOKS VERY UNSTABLE

The Double–Leading Expansion



$$\gamma(N,\alpha_s) = \left[\alpha_s\gamma_0(N) + \gamma_s\left(\frac{\alpha_s}{N}\right) - \frac{n_c\alpha_s}{\pi N}\right] \quad \Leftarrow\ \Rightarrow \chi(M,\alpha_s) = \left[\alpha_s\chi_0(M) + \chi_s\left(\frac{\alpha_s}{M}\right) - \frac{n_c\alpha_s}{\pi M}\right]$$

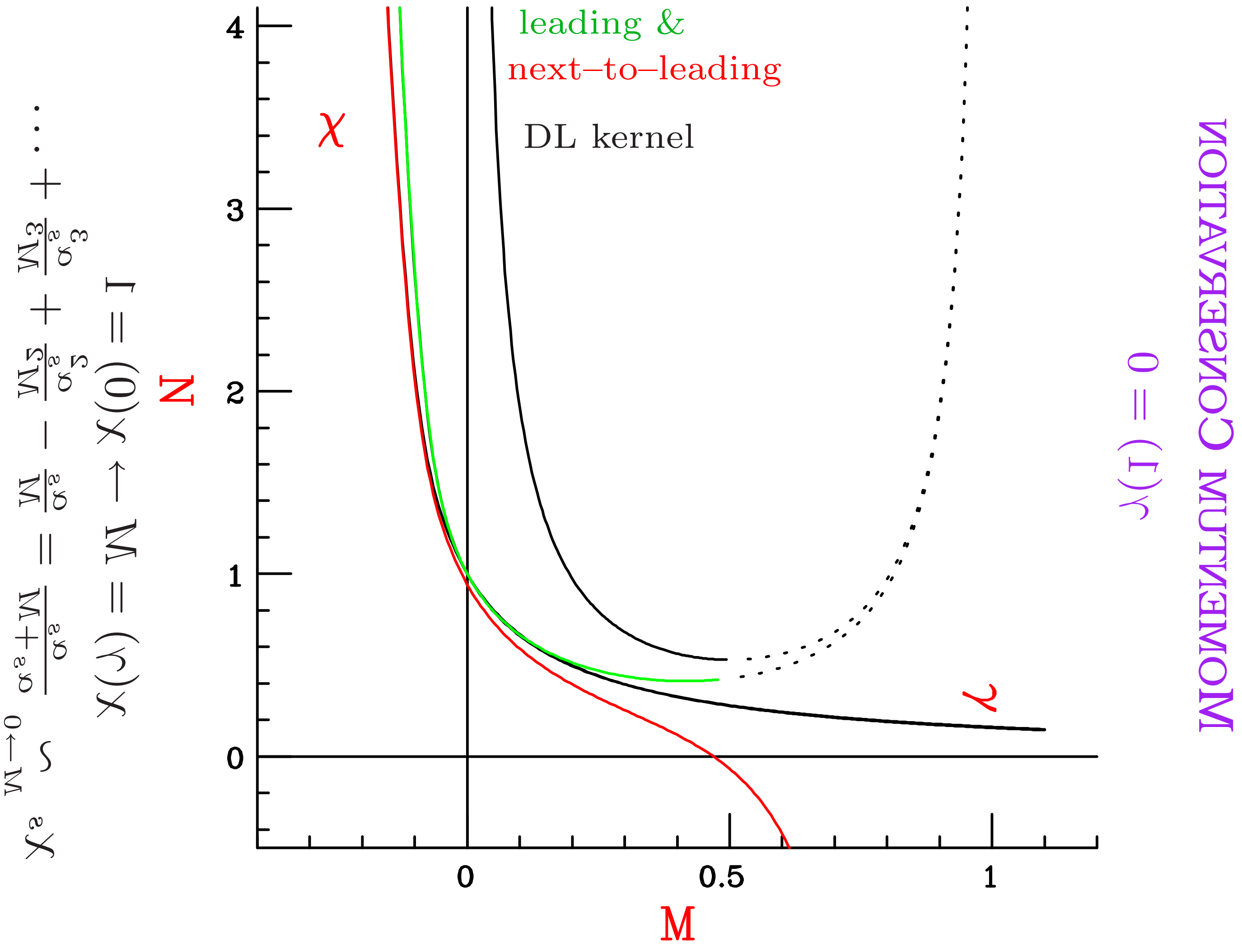
$$+\alpha_s\left[\alpha_s\gamma_1(N) + \gamma_{ss}\left(\frac{\alpha_s}{N}\right) - \alpha_s\left(\frac{e_2}{N^2} + \frac{e_1}{N}\right) - e_0\right] \quad +\alpha_s\left[\alpha_s\chi_1(M) + \chi_{ss}\left(\frac{\alpha_s}{M}\right) - \alpha_s\left(\frac{f_2}{M^2} + \frac{f_1}{M}\right) - f_0\right]$$

$$+\cdots \qquad +\cdots$$

Duality holds order–by–order in the double–leading expansion:

the dual of $\chi_{\rm DL}^{\rm LO}$ is $\gamma_{\rm DL}^{\rm LO}$ up to terms of order $\gamma_{\rm DL}^{\rm NLO}$, and conversely

Double–Leading Evolution

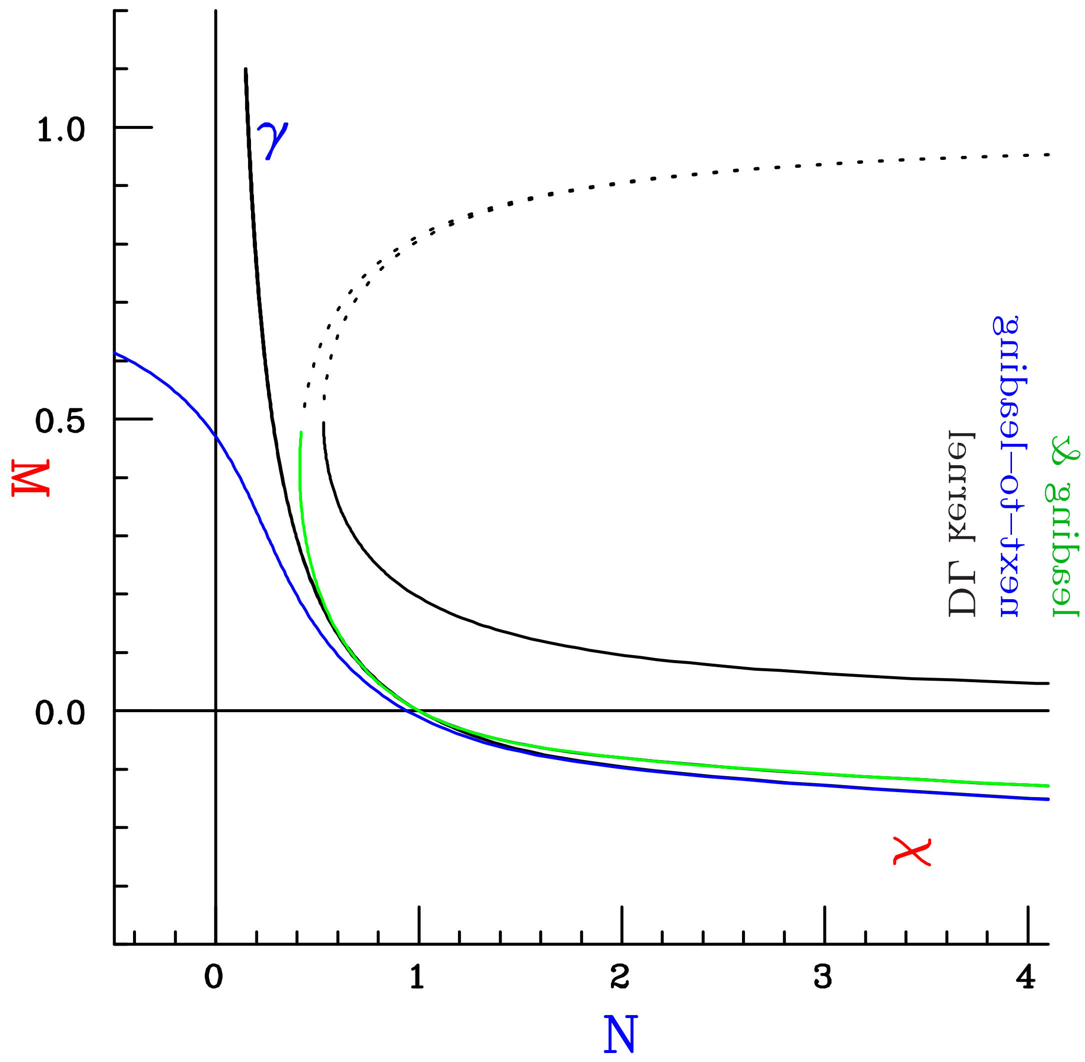


- The DL kernel has a well–behaved perturbative expansion
- DL is close to the LLQ^2 result for $N \gtrsim 0.3 \leftrightarrow M \lesssim 0.2$, close to LL$1/x$ for $M \sim 1/2$

Double–Leading Evolution

Momentum Conservation!

$$\gamma(1) = 0$$



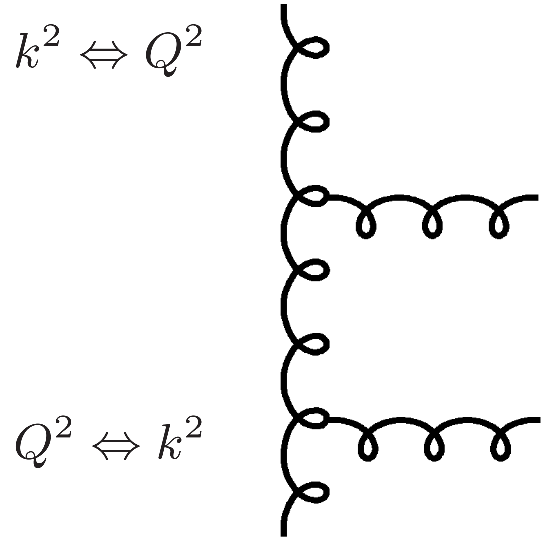
$$\chi(\gamma) = N \to \chi(0) = 1$$

$$\chi_s(M) \underset{M\to 0}{\sim} \frac{\alpha}{\alpha+M} = \frac{\alpha}{M} - \frac{\alpha^2}{M^2} + \frac{\alpha^3}{M^3} + \ldots$$

THE SECOND INGREDIENT: EXCHANGE SYMMETRY

(CIAFALONI, SALAM, 1999)

DIAGRAMS FOR $\ln 1/x$ EVOLUTION KERNEL



$$\frac{d}{d\xi} G(\xi, M) = \chi(M, \alpha_s)\; G(\xi, M)$$

$$\chi(\xi, M) = \int_{-\infty}^{\infty} \frac{dQ^2}{Q^2} \left(\frac{Q^2}{k^2}\right)^{-M} \chi(\xi, \tfrac{Q^2}{k^2})$$

SYMMETRIC UPON INTERCHANGE

OF INITIAL AND FINAL PARTON VIRTUALITIES

$$Q^2 \leftrightarrow k^2 \Leftrightarrow M \leftrightarrow 1 - M$$

COLLINEAR RES. OF $\frac{1}{M}$ POLES $\leftrightarrow$ ANTICOLLINEAR RES. OF $\frac{1}{1-M}$ POLES

SYMMETRY BREAKING

- DIS KINEMATIC VARIABLES $s = \frac{Q^2}{x}$ (small x)
- RUNNING OF THE COUPLING $\alpha_s(Q^2)$

BOTH CAN BE DETERMINED EXACTLY

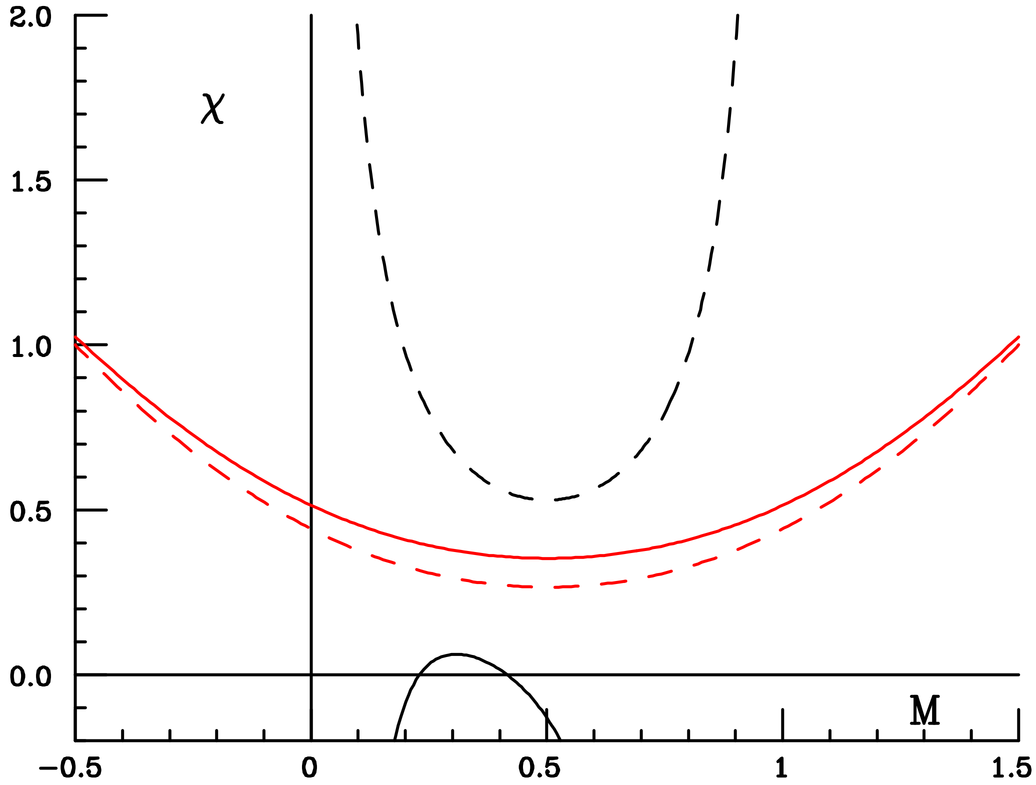
SYMMETRIZED EXPANSION

THE χ KERNEL

MOMENTUM CONSERVATION + SYMMETRY$\Rightarrow \chi$ ALWAYS HAS A MINIMUM

SYMMETRIC VARIABLES

- LO, NLO SYMMETRIC RESUMMED CLOSE TO EACH OTHER
- χ IS AN ENTIRE FCTN (QUADRATIC APPROX. IS EXCELLENT!)
- RESUMMED NLO HIGHER THAN LO



SYMMETRIZED EXPANSION

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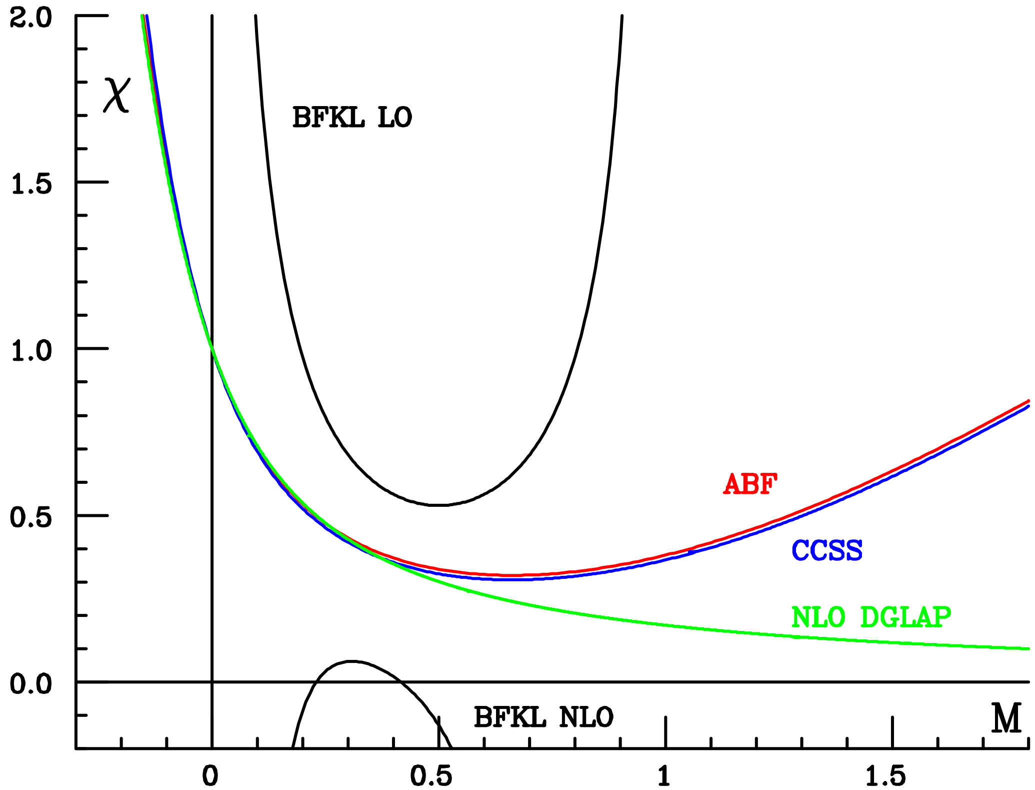
ASYMMETRIC VARIABLES

- LO, NLO SYM. CLOSE TO EACH OTHER
- LO, NLO SYM. CLOSE TO AP
- CURVATURE & INTERCEPT SAME IN SYM. & ASYM. VARIABLES



- RESULT DETERMINED BY MOM. CONS. + SYM.
- AMBIGUITIES MINIMAL, (CFR. ABF VS. CCSS) BUT MATCHING TO GLAP CRUCIAL

THE THIRD INGREDIENT: RUNNING COUPLING

(COLLINS, KWIECINSKI, 1989; ABF, 2001)

- THE RUNNING OF THE COUPLING $\alpha(t) = \alpha_\mu[1 - \beta_0\alpha_\mu t + \ldots]$
  ($t \equiv \ln \frac{Q^2}{\mu^2}$) IS LEADING LOG Q^2, BUT NEXT–TO–LEADING LOG $\frac{1}{x}$
- UPON M-MELLIN TRANSFORMATION ($\ln x$ EVOLUTION)
  $\alpha_s(t)$ BECOMES AN OPERATOR:

$$\alpha_s(M) = \alpha_{\mu^2}[1 + \beta_0\alpha_{\mu^2}\frac{d}{dM} + \ldots]$$

$\Rightarrow$ EVOLUTION EQUATION for $G(N, M)$ with b.c. $H_0(M)$

$$\left(1 - \frac{\alpha_\mu}{N}\right)\chi(M)G(N,M) - H_0(M) = \beta_0\alpha_\mu \frac{d}{dM}G(N,M)$$

- GOOD NEWS: DUALITY STILL HOLDS AT NLO& BEYOND
  –

$$\gamma(\alpha_s(t), \alpha_s(t)/N) = \gamma_s(\alpha_s(t)/N) + \alpha_s(t)\beta_0\Delta\gamma_{ss}(\alpha_s(t)/N) +$$
$$+(\alpha_s(t)\beta_0)^2\Delta\gamma_{sss}(\alpha_s(t)/N) + O(\alpha_s(t)\beta_0)^3$$

  – TERMS $\Delta\gamma_{s^n}$ CAN BE CALCULATED TO ALL ORDERS THROUGH AN OPERATOR APPROACH (BALL & S.F., 05)

THE THIRD INGREDIENT: RUNNING COUPLING

(Collins, Kwiecinski, 1989; ABF, 2001)

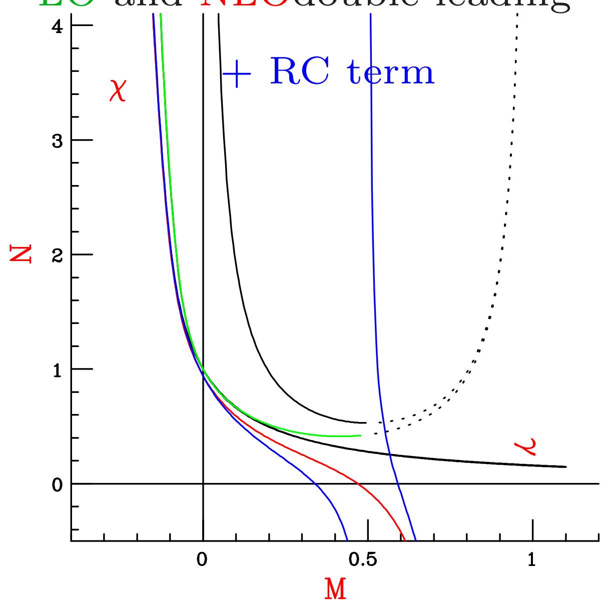
- The running of the coupling $\alpha(t) = \alpha_\mu[1 - \beta_0 \alpha_\mu t + \ldots]$ $(t \equiv \ln \frac{Q^2}{\mu^2})$ is Leading Log Q^2, but Next–to–Leading Log $\frac{1}{x}$
- Upon M-Mellin transformation ($\ln x$ evolution) $\alpha_s(t)$ becomes an operator:

$$\alpha_s(M) = \alpha_{\mu^2}[1 + \beta_0 \alpha_{\mu^2} \frac{d}{dM} + \ldots]$$

$\Rightarrow$ Evolution equation for $G(N, M)$ with b.c. $H_0(M)$

$$\left(1 - \frac{\alpha_\mu}{N}\right) \chi(M) G(N, M) - H_0(M) = \beta_0 \alpha_\mu \frac{d}{dM} G(N, M)$$

- BAD NEWS: PERTURBATIVE INSTABILITY



- NLO r.c. correction not uniformly small as $x \to 0$:

$$\frac{\Delta P_{ss}(\alpha_s, \xi)}{P_s(\alpha_s, \xi)} \underset{\xi \to \infty}{\sim} (\alpha_s \xi)^2$$

- But series of corrections can be computed and summed to all orders

ASYMPTOTIC SOLUTION & LEADING SINGULARITY

ASYMPTOTIC BEHAVIOUR CONTROLLED BY

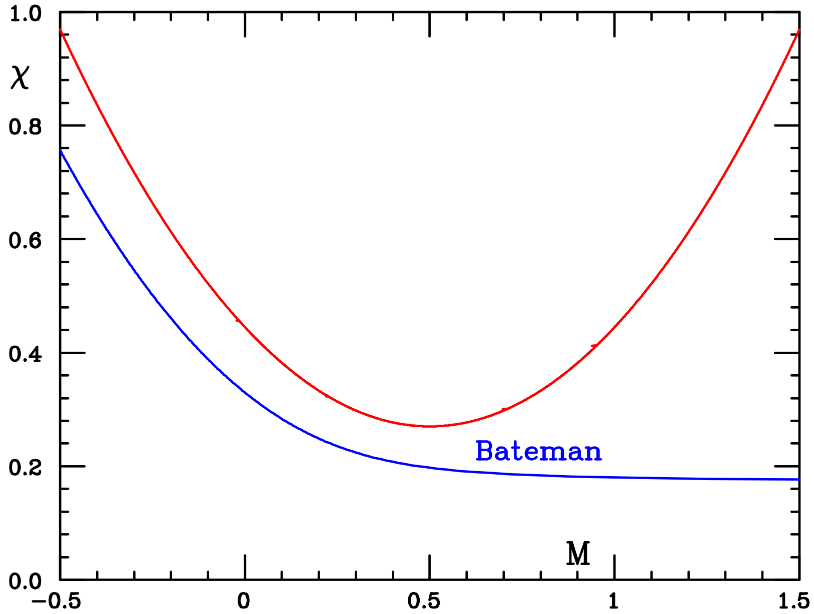
MINIMUM OF $\chi(M) \Leftrightarrow$ RIGHTMOST SING. OF $\gamma(N)$

QUADRATIC KERNEL $\chi_q(\hat{\alpha}_s, M) = [c(\hat{\alpha}_s) + \frac{1}{2}\kappa(\hat{\alpha}_s)(M - M_s)^2]$

CAN SOLVE EXACTLY WITH LINEARIZED $c(\hat{\alpha}_s)$, $\kappa(\hat{\alpha}_s)$

IN TERMS OF BATEMAN FUNCTION $K_\nu(x)$:

THE EFFECTIVE RESUMMED KERNEL



- $G(N,t) \propto K_{2B(\alpha_s,N)}\left[\frac{1}{\beta_0 \bar{\alpha}_s(t) A(\alpha_s,N)}\right]$
  A, B DEPEND ON α_s, N TRHOUGH c, κ
- ASYMPTOTIC LEADING LOG SMALL x SERIES RESUMMED
- BRANCH CUT IN γ REPLACED BY SIMPLE POLE

PUTTING EVERYTHING TOGETHER

THE RESUMMED ANOMALOUS DIMENSION:

$$\gamma^{rc}_{\Sigma\,NLO}(\alpha_s(t),N) = \gamma^{rc,\,pert}_{\Sigma\,NLO}(\alpha_s(t),N) + \gamma^{B}(\alpha_s(t),N) - \gamma^{B}_{s}(\alpha_s(t),N) - \gamma^{B}_{ss}(\alpha_s(t),N)$$
$$-\gamma^{B}_{ss,0}(\alpha_s(t),N) + \gamma_{\rm match}(\alpha_s(t),N) + \gamma_{\rm mom}(\alpha_s(t),N)$$

- $\gamma^{rc,\,pert}_{\Sigma\,NLO}(\alpha_s(t),N)$ CONTAINS ALL TERMS WHICH ARE UP TO NLO IN THE DOUBLE-LEADING EXPANSION, SYMMETRIZED (SO ITS DUAL χ HAS A MINIMUM)

- $\gamma^{B}(\alpha_s(t),N)$ RESUMS THE SERIES OF SINGULAR RUNNING COUPLIG CORRECTIONS

- $\gamma^{B}_{s}(\alpha_s(t),N)$, $\gamma^{B}_{ss}(\alpha_s(t),N)$ $\gamma^{B}_{ss,0}(\alpha_s(t),N)$ ARE DOUBLE COUNTING SUBTRACTIONS BETWEEN THE PREVIOUS TWO

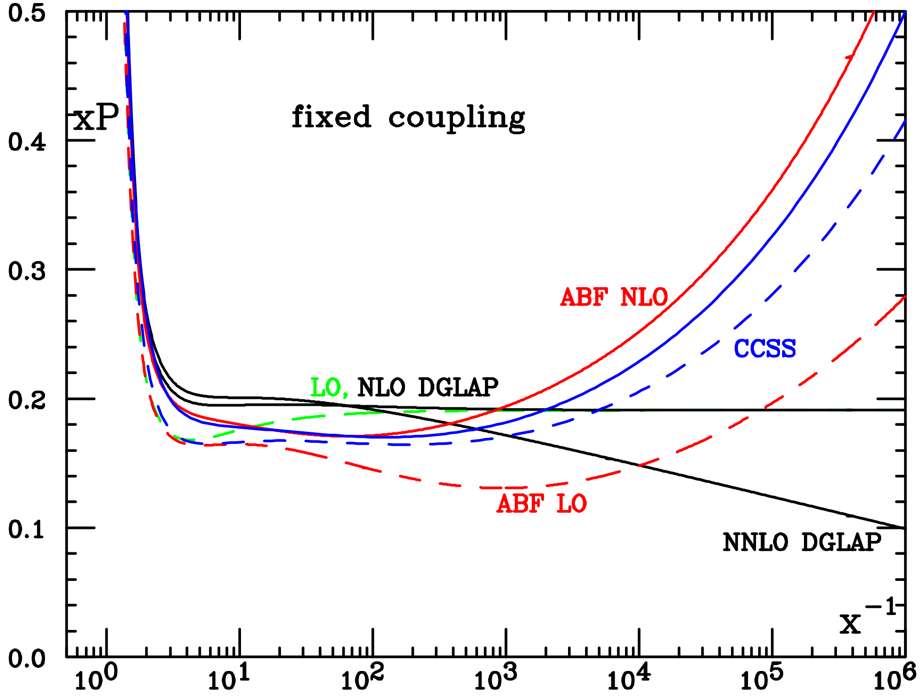
- $\gamma_{\rm mom}$ SUBTRACTS SUBLEADING TERMS WHICH RUIN MOMENTUM CONSERVATION

- $\gamma_{\rm match}$ SUBTRACTS ANY CONTRIBUTION WHICH DEVIATES FROM NLO GLAP AND AT LARGE N DOESN'T DROP AT LEAST AS $\frac{1}{N}$

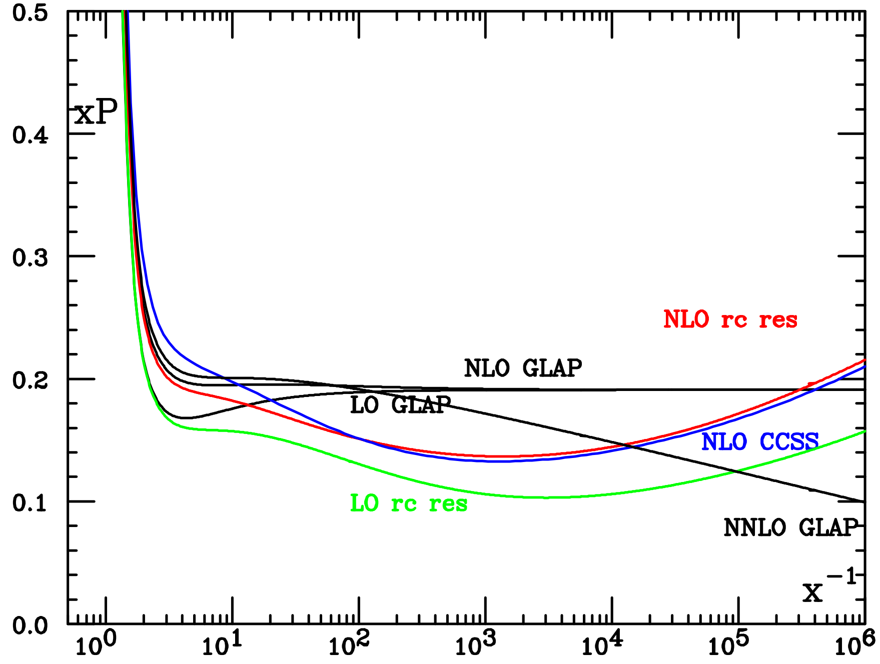
RESUMMATION: GENERAL FEATURES

THE SPLITTING FUNCTION

FIXED COUPLING



RUNNING COUPLING



- RESUMMED EXPANSION CONVERGES RAPIDLY
- BEHAVIOUR FOR $x < 10^{-2}$ VERY STABLE
- CAREFUL MATCHING OF SMALL x RUNNING COUPLING TERMS REQUIRED

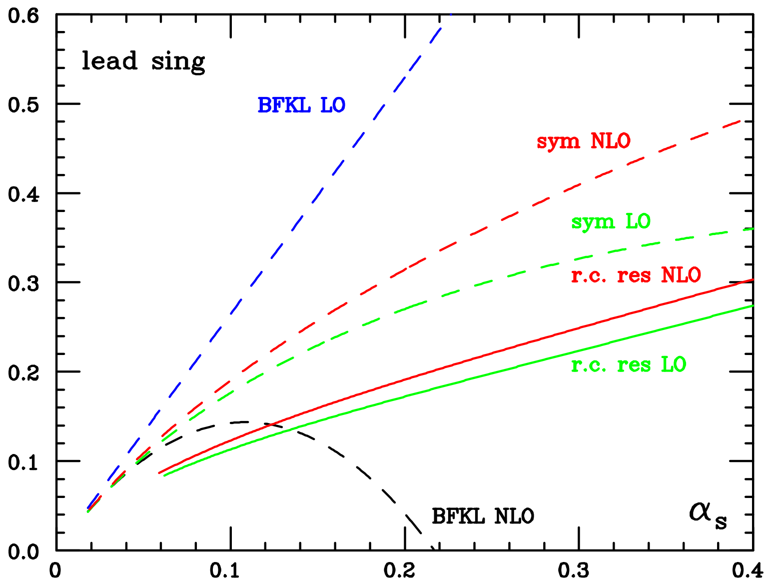
  compare with CCSS $x \sim 0.2$

RESUMMATION: GENERAL FEATURES

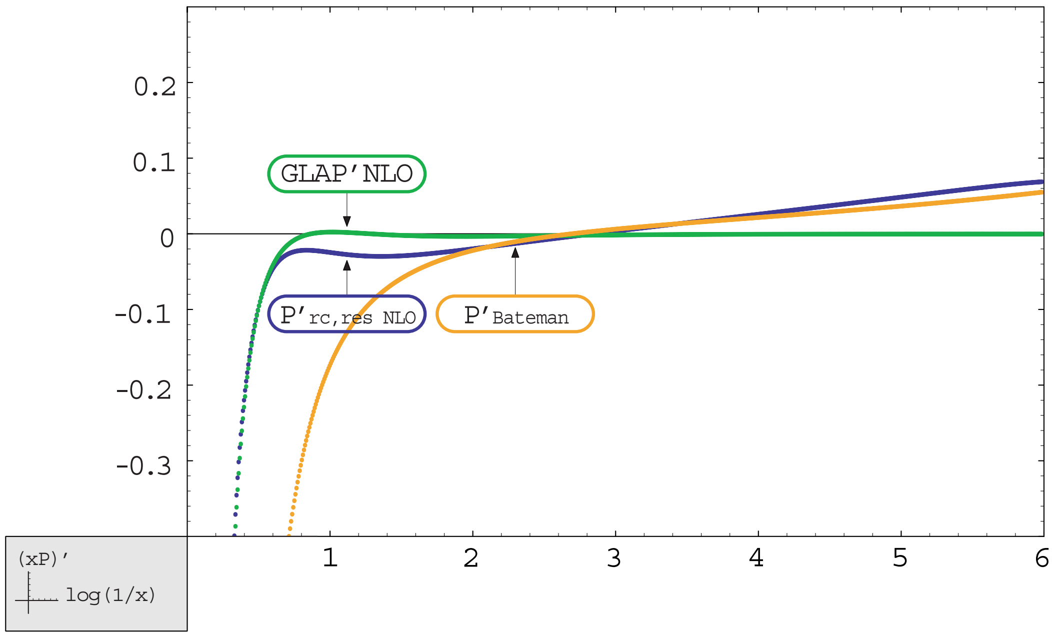
SMALL x BEHAVIOUR

SINGULARITY IN ANOM. DIM. AT $N = \alpha \Rightarrow$ ASYMPT. SMALL-x POWER $G \sim x^{-\alpha}$

SMALL-x POWER VS. α_s



SLOPE OF THE SPLITTING FUNCTION



- ABOVE $x \gtrsim 0.2$ SPLITTING FUNCTION COINCIDES NLO GLAP
- BELOW $x \lesssim 10^{-2}$ SPLITTING FUNCTION COINCIDES WITH SMALL x ASYMPTOTIC SOLUTION (C. Frugiuele, 2007)
- SMALL x INTERCEPT & CURVATURE DETERMINE RESUMMED BEHAVIOUR

MATCHING AND PHENOMENOLOGY

RESUMMATION: FROM EVOLUTION TO PHYSICAL OBSERVABLES

SCHEME CHOICE

2×2 ANOM. DIM. MATRIX
$\longrightarrow$ 2 EIGENVECTORS,
ONLY ONE AFFECTED
BY RESUMMATION
(GLUON AT LO)

$\Rightarrow$ THE RELATION BETWEEN (Q,G) AND EIGENVECTORS IS A SCHEME CHOICE

COMPLICATIONS

- UNPHYSICAL SINGULARITIES DUE TO EIGENVALUE CROSSING
- MUST TRANSF. FROM $Q_0\overline{\mathrm{MS}}$ USED IN RESUM TO $\overline{\mathrm{MS}}$ USED AT FIXED ORDER

ALTERNATIVE APPROACH (CCSS): MATRIX BFKL EQUATION (IN PROGRESS)

COEFFICIENT FUNCTION RESUMMATION

RESUMMED COEFFICIENT FUNCTION AFFECTED BY UNPHYSICAL SINGULARITIES

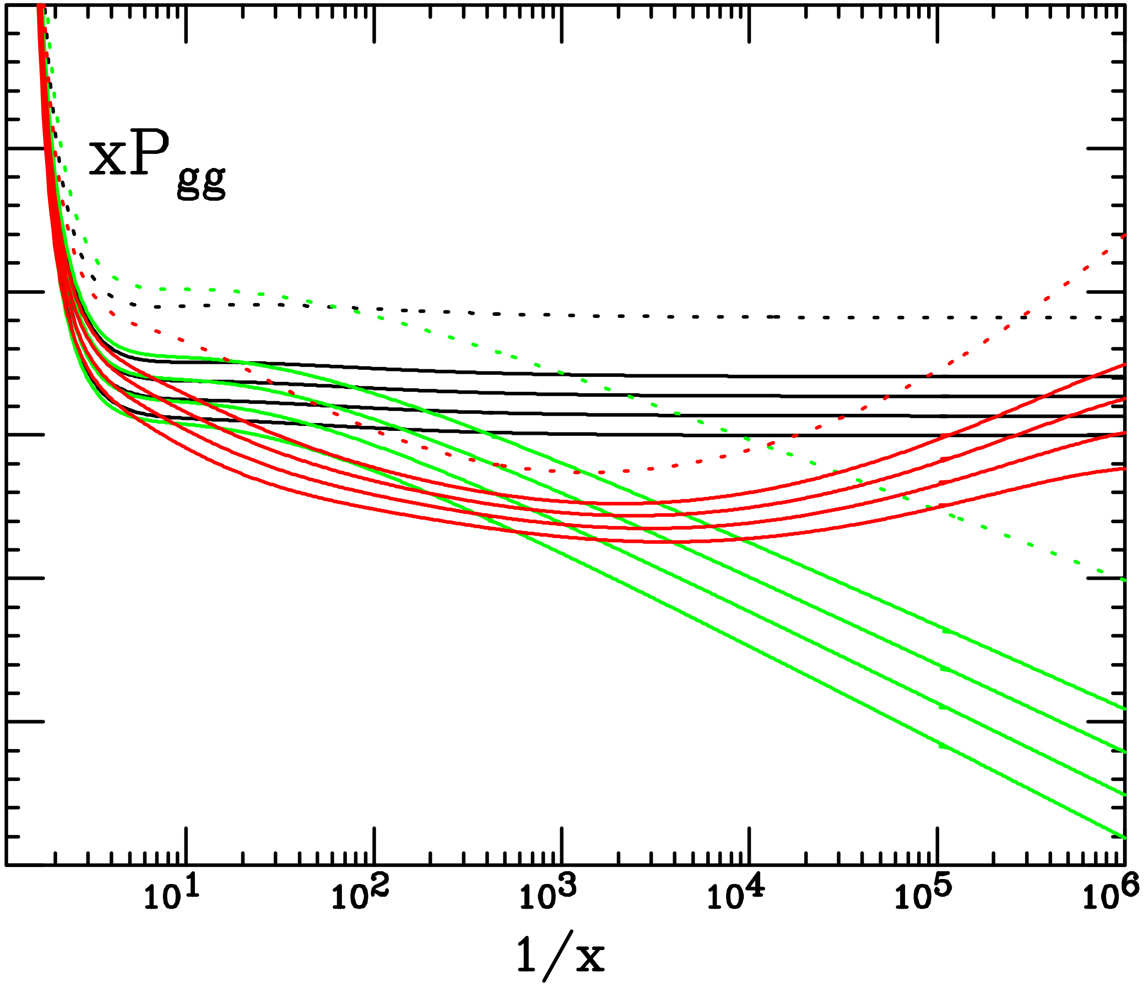
$\Rightarrow$ REMOVED BY RUNNING COUPLING RESUMMATION

$n_f \neq 0$: THE GLUON SECTOR

- MUST REMOVE CUT FROM AP DIAGONALIZATION (P_+ NOT WELL DEFINED)
- n_f DEPENDENCE NOT NEGLIGIBLE
- SMALL x RISE SOFTENED BY COUPLING TO QUARKS

P_{gg}, $n_f = 0, 3, 4, 5$ (top to bottom)

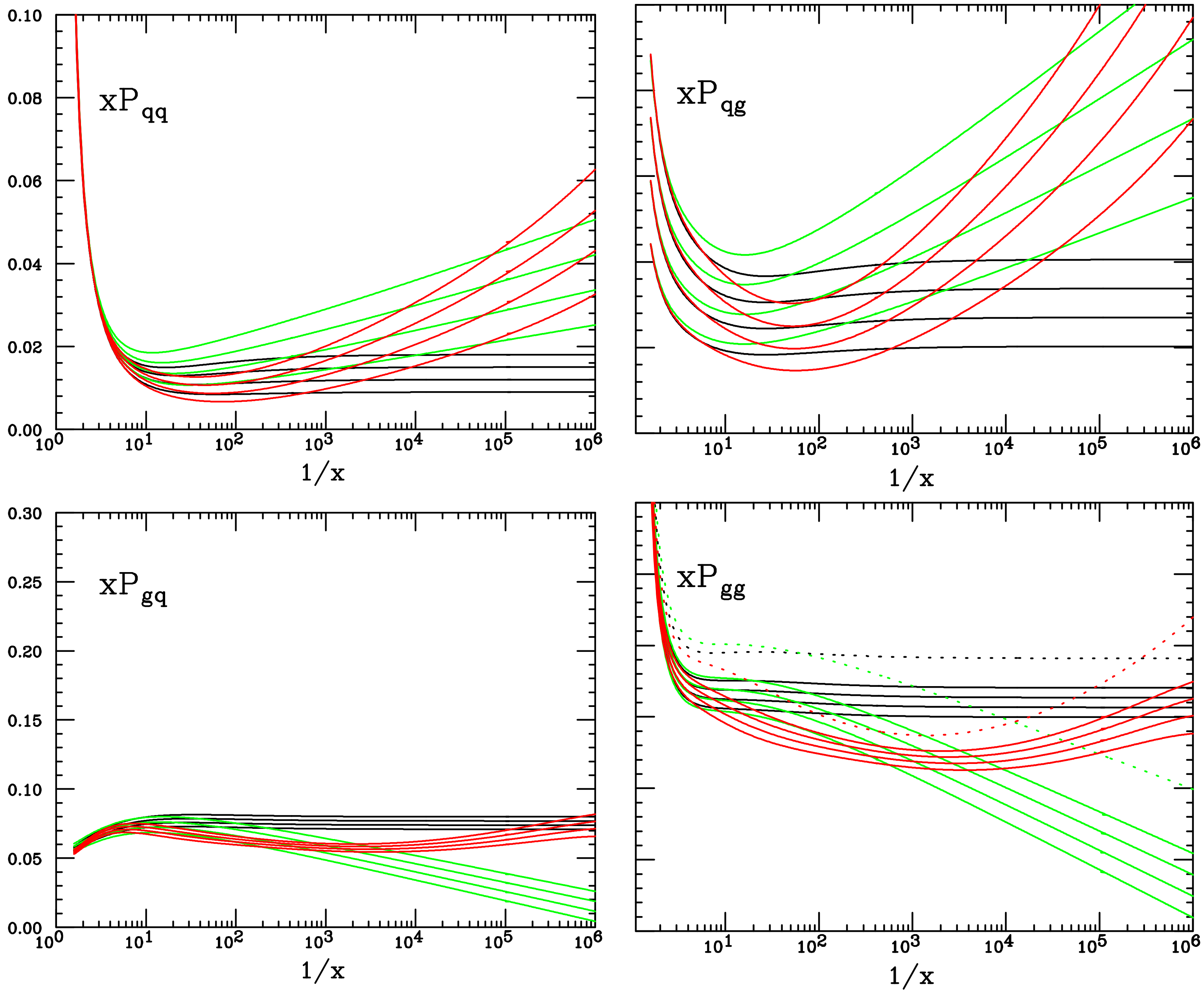
NLO, NNLO, RESUMMED



THE SPLITTING FUNCTION MATRIX

n_f dependence: $n_f = [0,]3,4,5$

NLO, NNLO, RESUMMED



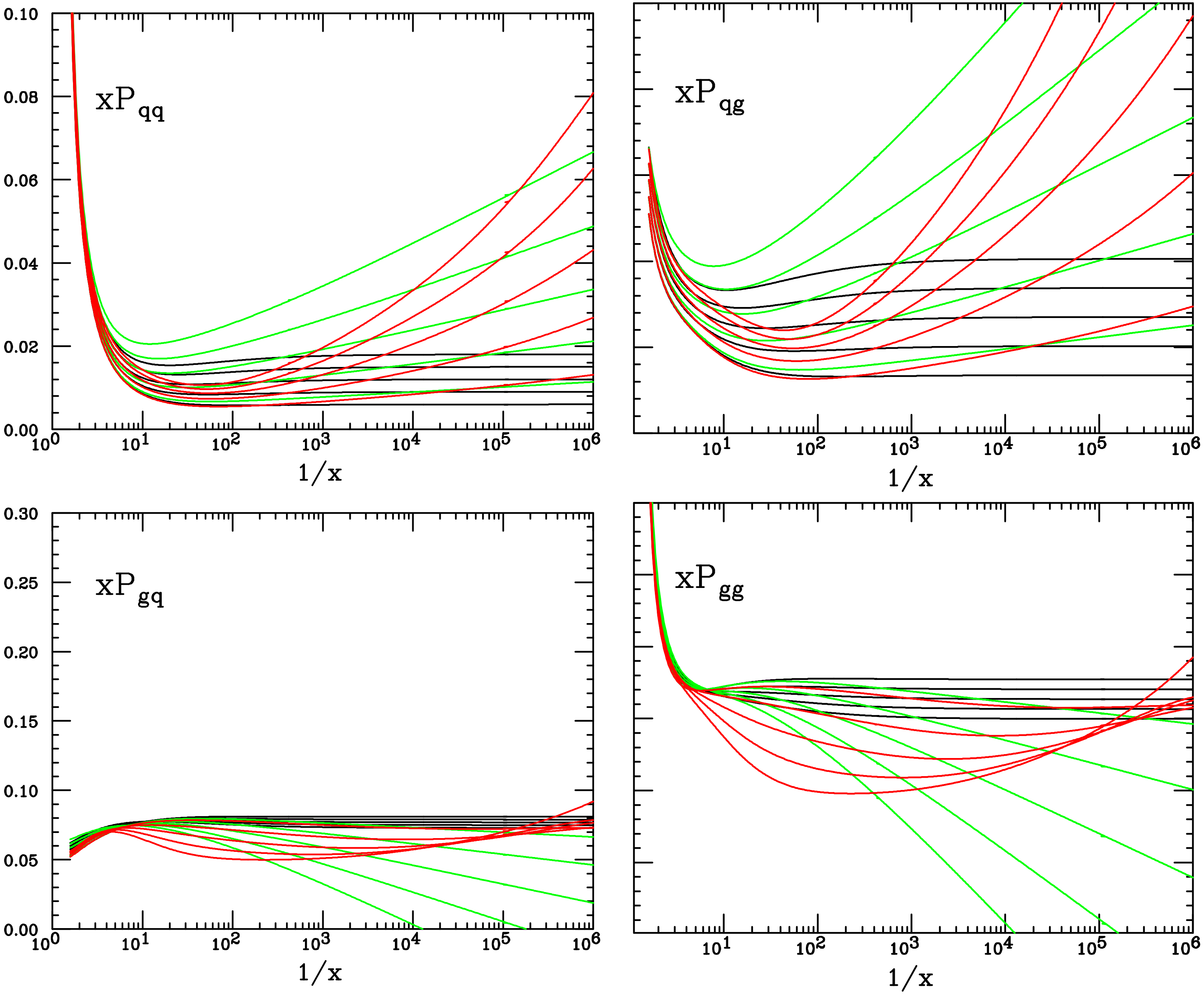
$\Uparrow n_f$ GROWS

$\Downarrow n_f$ GROWS

THE SPLITTING FUNCTION MATRIX

α_s dependence: $\alpha_s = 0.1,\ 0.15,\ 0.2,\ 0.25,\ 0.3$

NLO, NNLO, RESUMMED



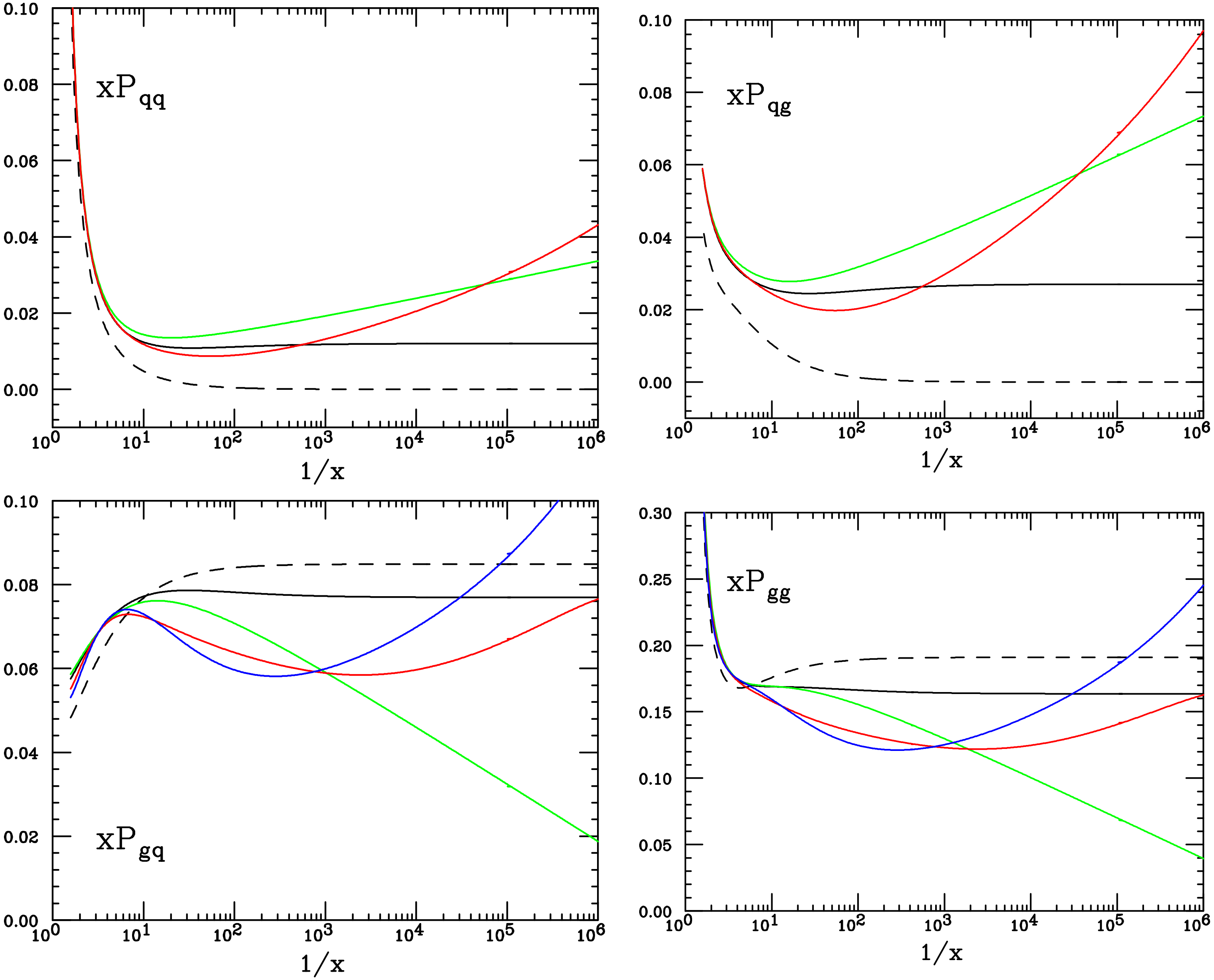
⇑ α_s GROWS

⇓ α_s GROWS

THE SPLITTING FUNCTION MATRIX

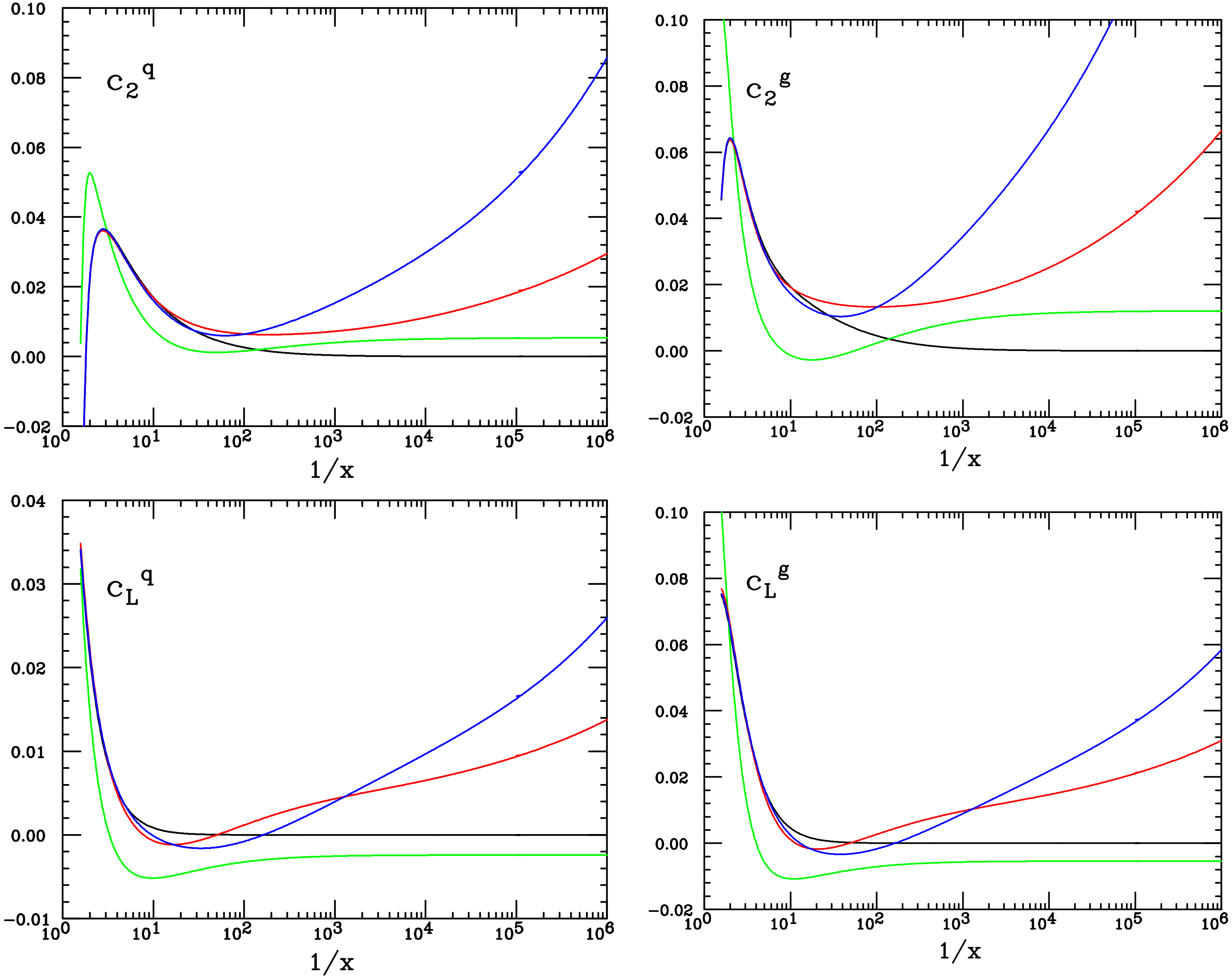
SMALL x SCHEME DEPENDENCE (ONLY AFFECTS GLUON SECTOR):

LO (DASH), NLO, NNLO, RESUMMED ($Q_0\overline{\mathrm{MS}}$) RESUMMED ($\overline{\mathrm{MS}}$) $n_f = 4$, $\alpha_s = 0.2$



THE COEFFICIENT FUNCTION MATRIX

NLO, NNLO, RESUMMED ($Q_0\overline{MS}$) RESUMMED ($\overline{MS}$) $n_f = 4$, $\alpha_s = 0.2$

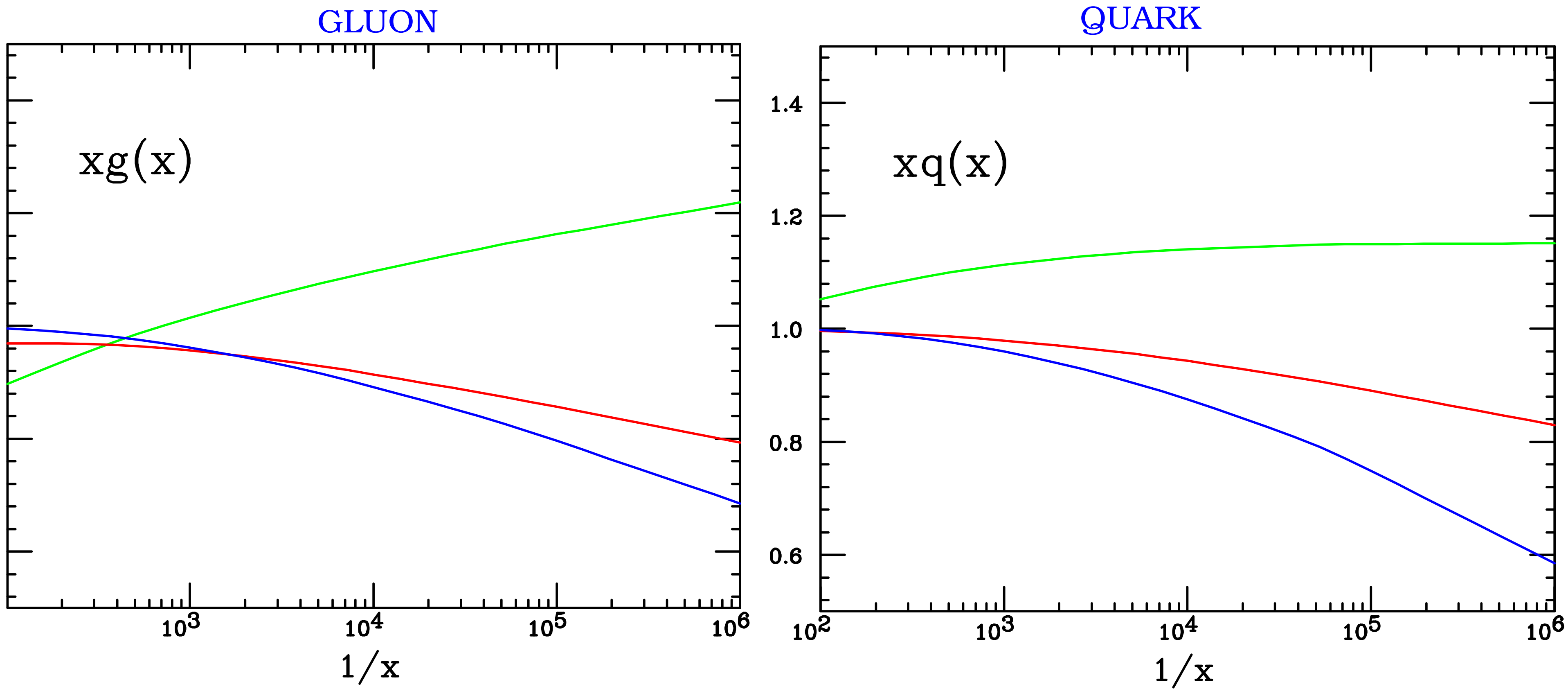


HOW DO THE INITIAL PDFS CHANGE?

KEEP F_2 & F_L FIXED AT $Q_0 = 5$ GeV

COMPUTE $K(x) \equiv q^{\rm new}(x,Q_0^2)/q^{\rm NLO}(x,Q_0^2);\; g^{\rm new}(x,Q_0^2)/g^{\rm NLO}(x,Q_0^2)$

NNLO, RESUMMED $Q_0\overline{\rm MS}$, RESUMMED $\overline{\rm MS}$



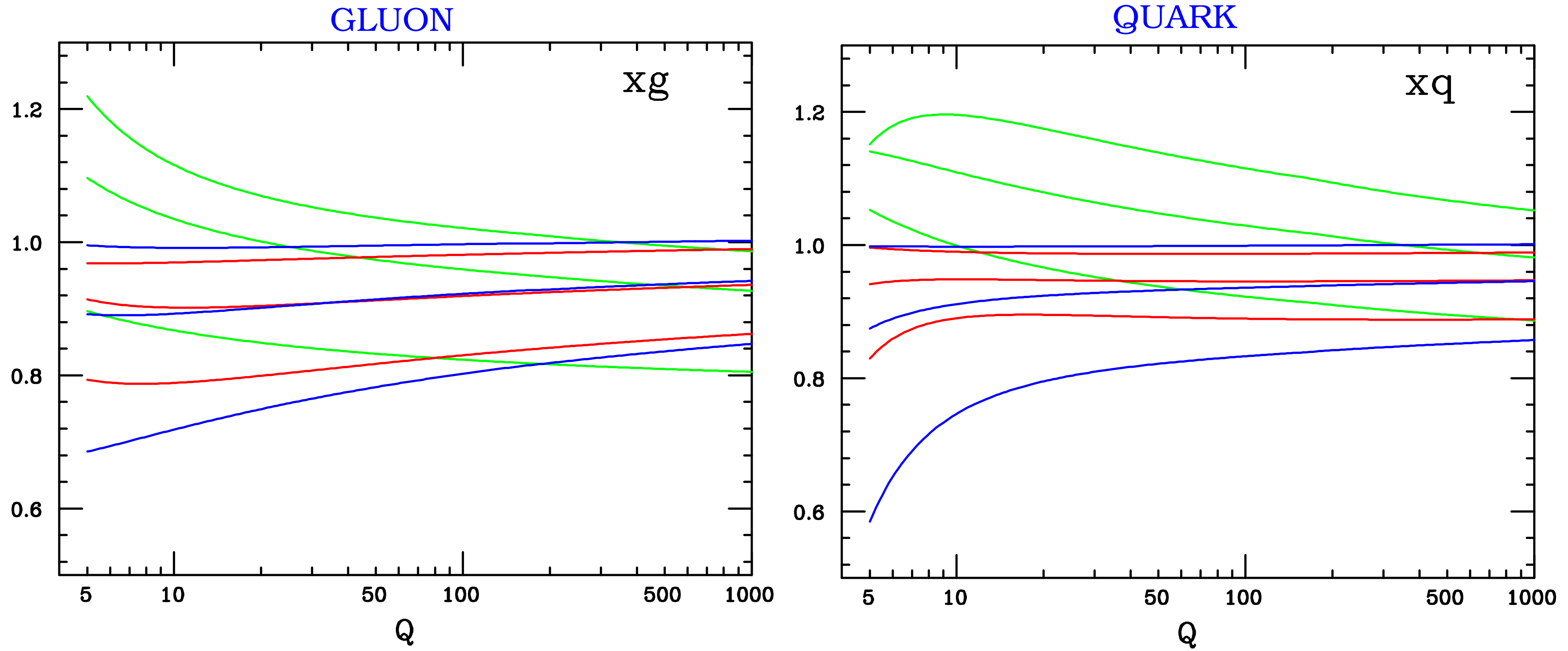
- EFFECT OF RESUMMATION COMPARABLE TO NNLO BUT STABLE!
- RESUMMED SUPPRESSION DUE TO LARGER COEFFICIENT FUNCTIONS

HOW DO PDFS CHANGE WITH SCALE?

KEEP F_2 & F_L FIXED AT $Q_0 = 5$ GEV

COMPUTE $K(Q) \equiv q^{\rm new}(x,Q^2)/q^{\rm NLO}(x,Q^2);\ g^{\rm new}(x,Q^2)/g^{\rm NLO}(x,Q^2)$

NNLO, RESUMMED $Q_0\overline{\rm MS}$, RESUMMED $\overline{\rm MS}$; $x = 10^{-2},\ 10^{-4},\ 10^{-6}$

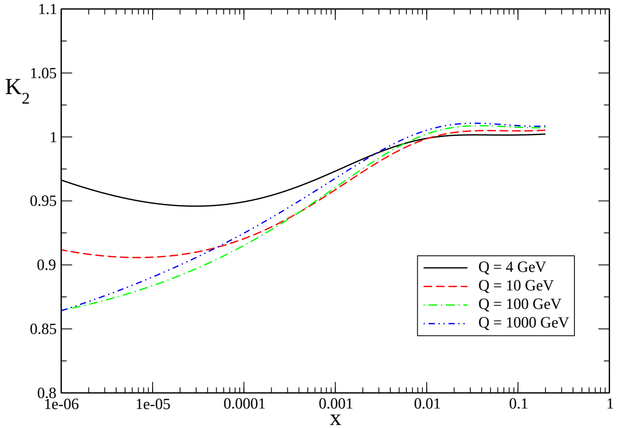


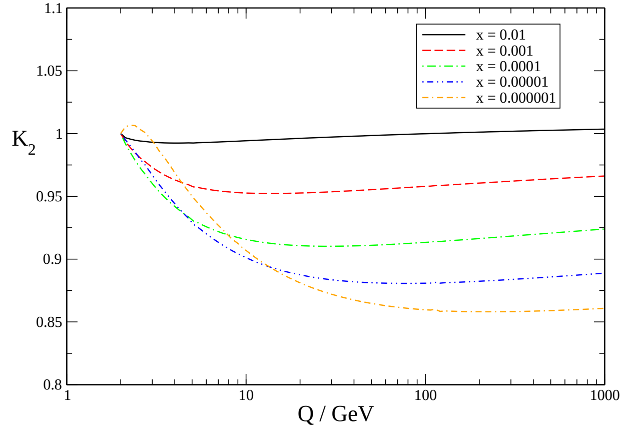
- EVOLUTION WASHES OUT THE DIFFERENCES

PHYSICAL OBSERVABLES: FIXED INPUT

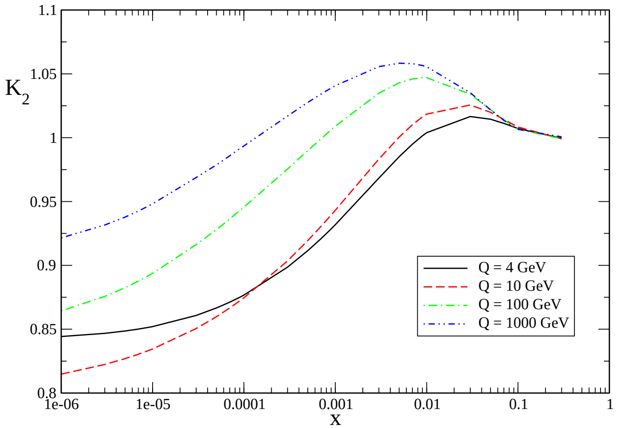
evolve pdf fixed in the DIS scheme (fixed starting F_2)

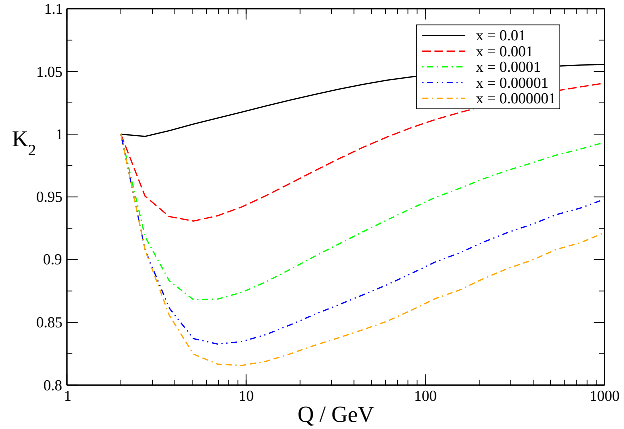
compute $K(x) \equiv F_2^{\text{resum}}(x, Q^2)/F_2^{\text{NLO}}(x, Q^2)$





COMPARISON WITH THE THORNE-WHITE APPROACH:





TW VS. ABF:

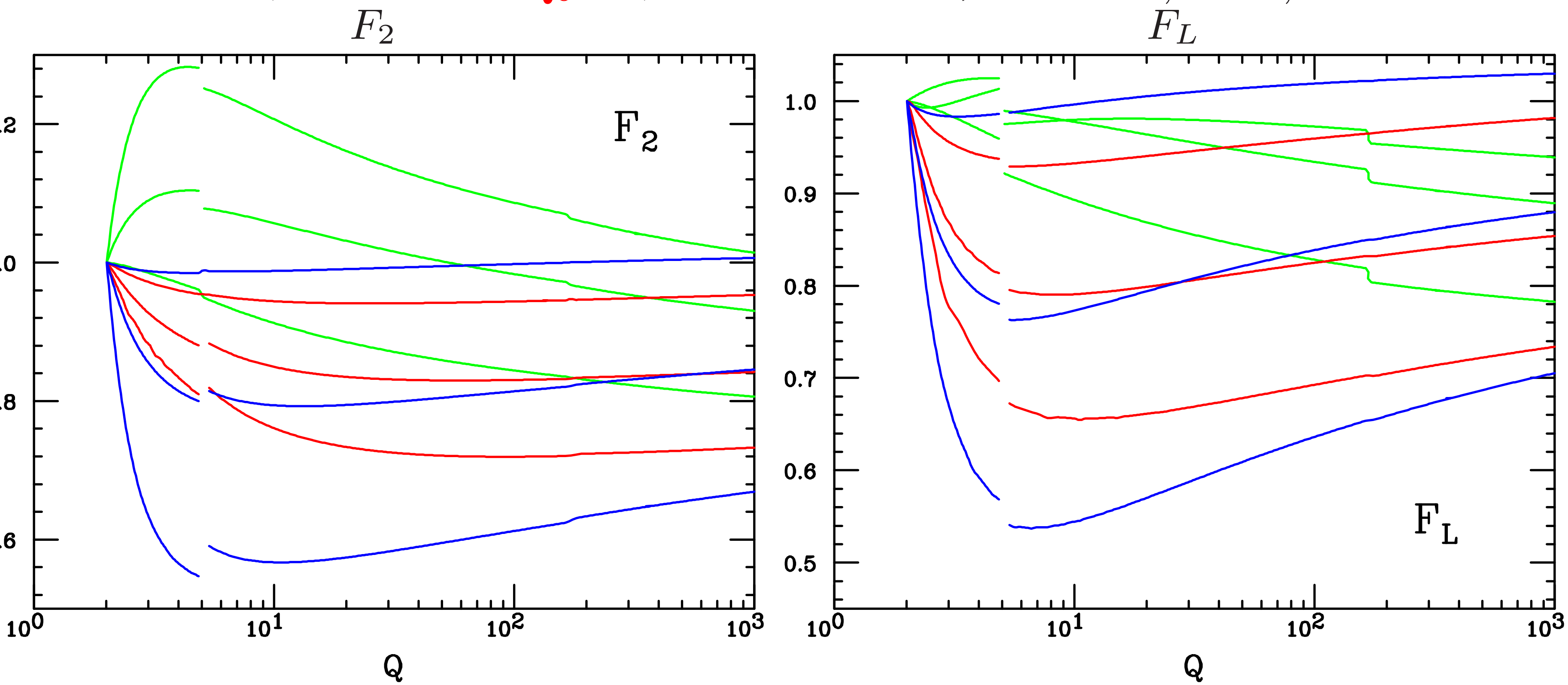
- QUALITATIVELY SIMILAR, TW LESS STABLE (K LARGER & OSCILLATORY): NO COLLINEAR-ANTICOLL. RESUMMATION?
- TW Q^2 DEP. DOES NOT FLATTEN AT LARGE SCALE (TW): SCHEME NOT FULLY CONSISTENT?
- TW $K \neq 1$ AT $x \gtrsim 0.01$: LARGE x MATCHING?

EFFECT ON PHYSICAL OBSERVABLES

KEEP F_2 & F_L FIXED AT $Q_0 = 2$ GeV

COMPUTE $K(Q) \equiv F_2^{\rm new}(x,Q^2)/F_2^{\rm NLO}(x,Q^2);\ F_L^{\rm new}(x,Q^2)/F_L^{\rm NLO}(x,Q^2)$

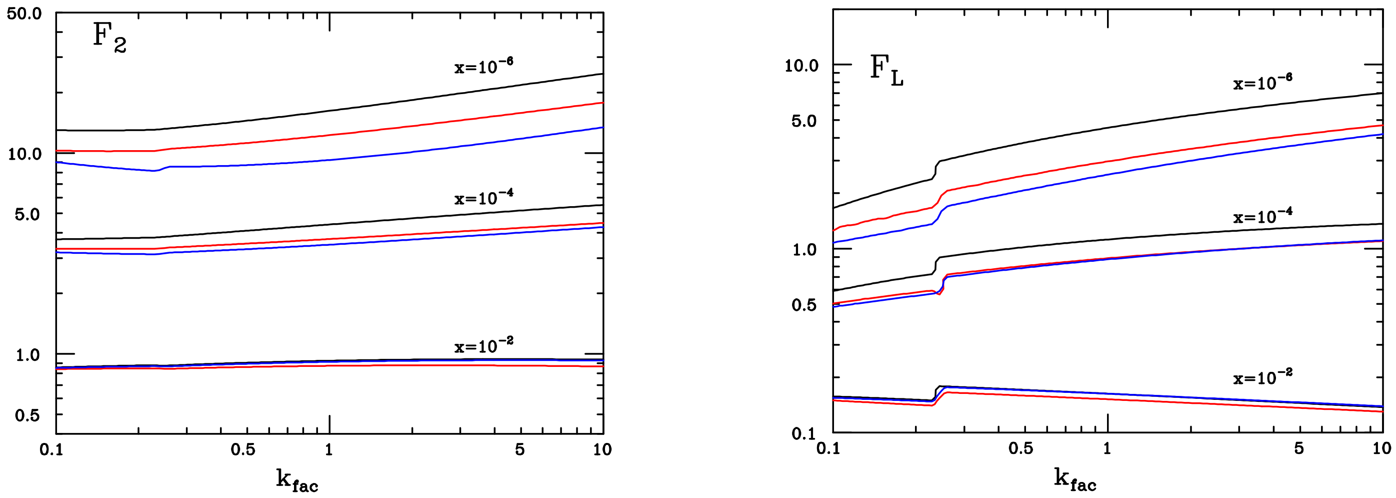
NNLO, RESUMMED $Q_0\overline{\rm MS}$, RESUMMED $\overline{\rm MS}$; $x = 10^{-2},\ 10^{-4},\ 10^{-6}$



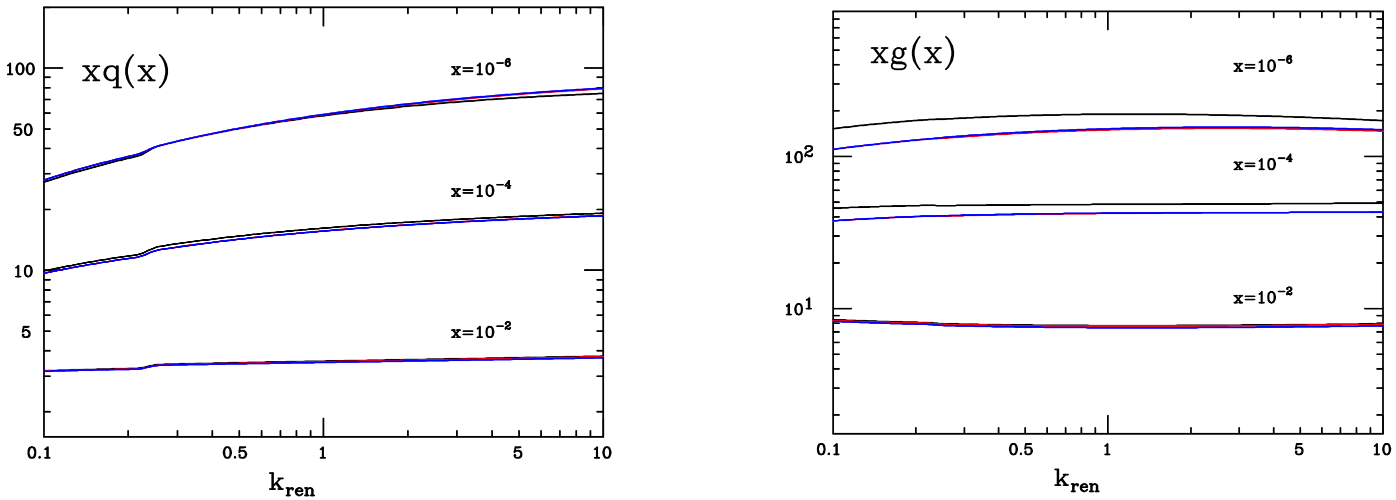
- EFFECT OF RESUMMATION COMPARABLE TO NNLO
- RESUMMED SUPPRESSION DUE TO DIP IN EVOLUTION & PDF SUPPR. LOW SCALE
- SCHEME DEPENDENCE SMALLER THAN FOR PDFs
- EVOLUTION WASHES OUT THE DIFFERENCES

STABILITY OF PHYSICAL OBSERVABLES

FACTORIZATION SCALE VARIATION: NLO, RESUMMED $Q_0\overline{\rm MS}$, RESUMMED $\overline{\rm MS}$



REN. SCALE VARIATION OF PDFS



- SCALE DEPENDENCE SIMILAR AT RESUMMED AND FIXED ORDER
  $\Rightarrow$ RESUMMED PERT. EXPANSION AS GOOD AS STANDARD

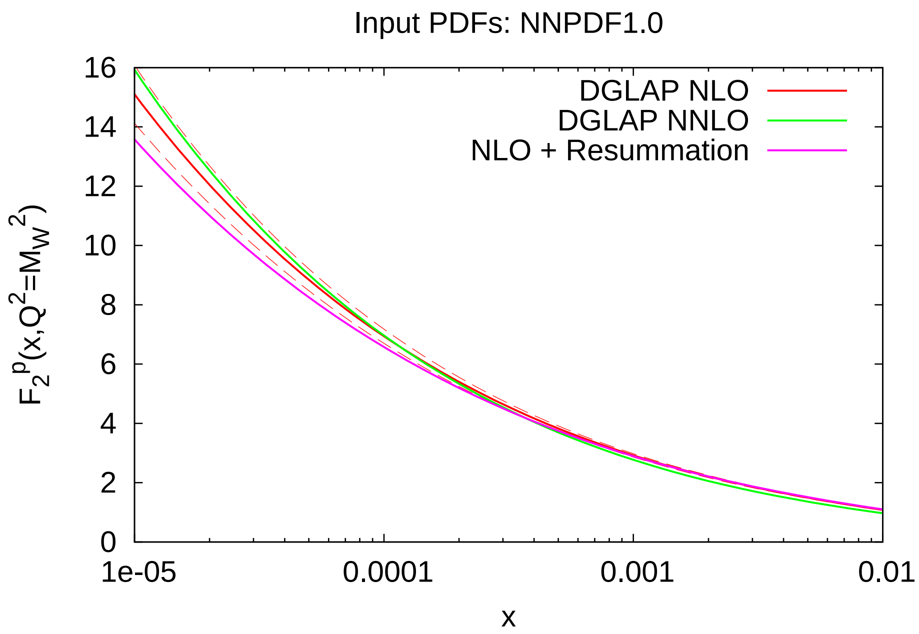
- SCALE DEP OF F_2 SMALLER THAN SCALE DEP OF q (F_L less stable: starts at NLO)

STRUCTURE FUNCTIONS: LHeC PHENOMENOLOGY

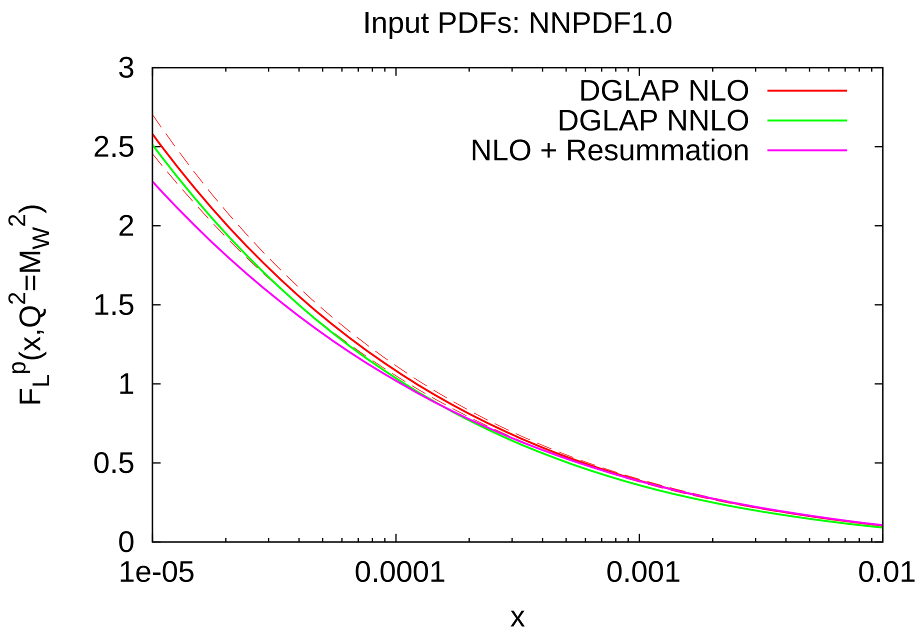
USING K-FACTORS, BASED ON NNPDF1.0 PARTONS

$$Q^2 = M_W^2;\ \text{NLO},\ \text{NNLO},\ \text{RES.}$$

F_2



F_L

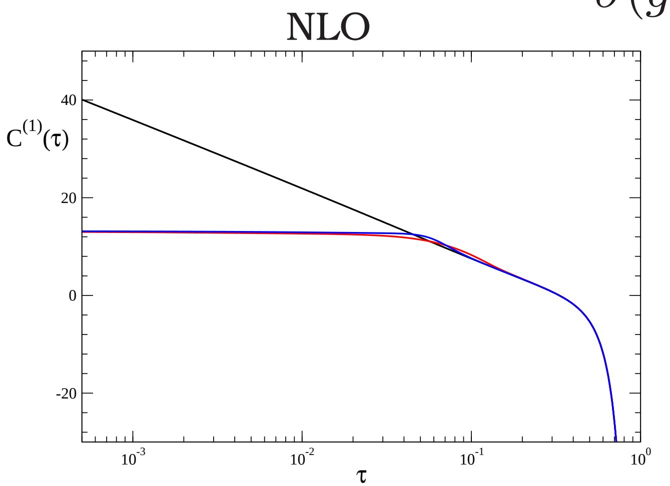


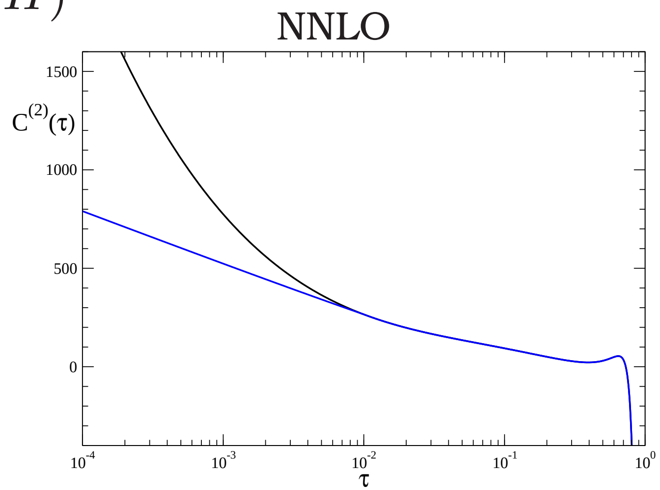
- RESUMMATION SIZABLE IN LHC/LHeC REGION
- EFFECT OUTSIDE NLO PDF ERROR BAND
- NNLO CORRNS SMALLER

RESUMMED HARD CROSS SECTIONS: RECENT PROGRESS

- HIGH ENERGY FACTORIZATION ONLY AVAILABLE AT LEADING $\ln x$ (Catani, Ciafaloni, Hautmann 93)
- LEADING NONTRIVIAL CORRECTIONS KNOWN FOR DIS (Catani, Ciafaloni, Hautmann 92-94), HQ PHOTO–, ELECTRO– (Catani, Ciafaloni, Hautmann 90-92) AND HADRO–PRODUCTION (Ball, K. Ellis 01)
- RECENTLY COMPUTED ALSO FOR DRELL-YAN (Ball, Marzani 08)
- RECENT PROGRESS IN THE COMPUTATION OF HIGGS PRODUCTION IN gg FUSION:
  - LEADING SINGULARITIES COMPUTED IN $m_\top \to \infty$ LIMIT (UNPHYSICAL DOUBLE LOGS) (Hautmann 02)
  - LEADING SINGULARITIES COMPUTED NUMERICALLY IN PHYSICAL CASE, RESULT UP NNLO USED TO IMPROVED FIXED ORDER (Marzani, Ball, del Duca, S.F., Vicini, 08)

$$\sigma(gg \to H)$$





$m_{\rm top} \to \infty$, EXACT SMALL x, EXACT

CONCLUSIONS

- WE KNOW HOW INCLUDE ALL LOG ENHANCED TERMS UP TO NLO WITH CONTROL OF FACTORIZATION SCHEME
- RESULTS STABLE, AMBIGUITIES SMALL
- DIS @ HERA: EFFECTS AS LARGE AS NNLO, OPPOSITE SIGN
- PHENOMENOLOGY FOR HADRONIC PROCESSES AT LHC BEHIND THE CORNER!

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LHC WILL PROBE PHYSICS UNDER EXTREME CONSITIONS:
WE BETTER USE THE BEST THEORY WE HAVE

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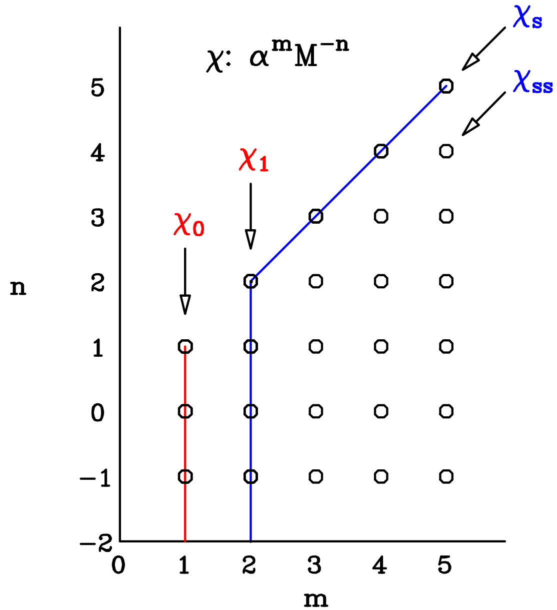
FORTUNATELY WE HAVE GOT A FEW EXTRA MONTHS TO WORK ON IT....

EXTRAS

RESUMMATION: THE CSS APPROACH

- SOLVE NUMERICALLY (ξ, Q^2) SPACE INTEGRAL EQUATION
$$G(\xi,Q^2,Q_0^2) = G_0(Q^2,Q_0^2)\Theta(\xi) + \int_0^\xi d\xi' \int_{Q_0^2}^{Q^2} d\bar{Q}^2 K(\xi-\xi';Q^2,Q_0^2)G(\xi',Q^2,Q_0^2)$$

KERNEL $K(\xi-\xi';Q^2,Q_0^2)$ OBTAINED BY INVERSE MELLIN FROM COLLINEAR–RESUMMED



- NLO BFKL $\alpha_s(Q^2,Q_0^2)\chi(M)$
(EFFECTIVELY, BFKL + MOMENTUM + LO GLAP)

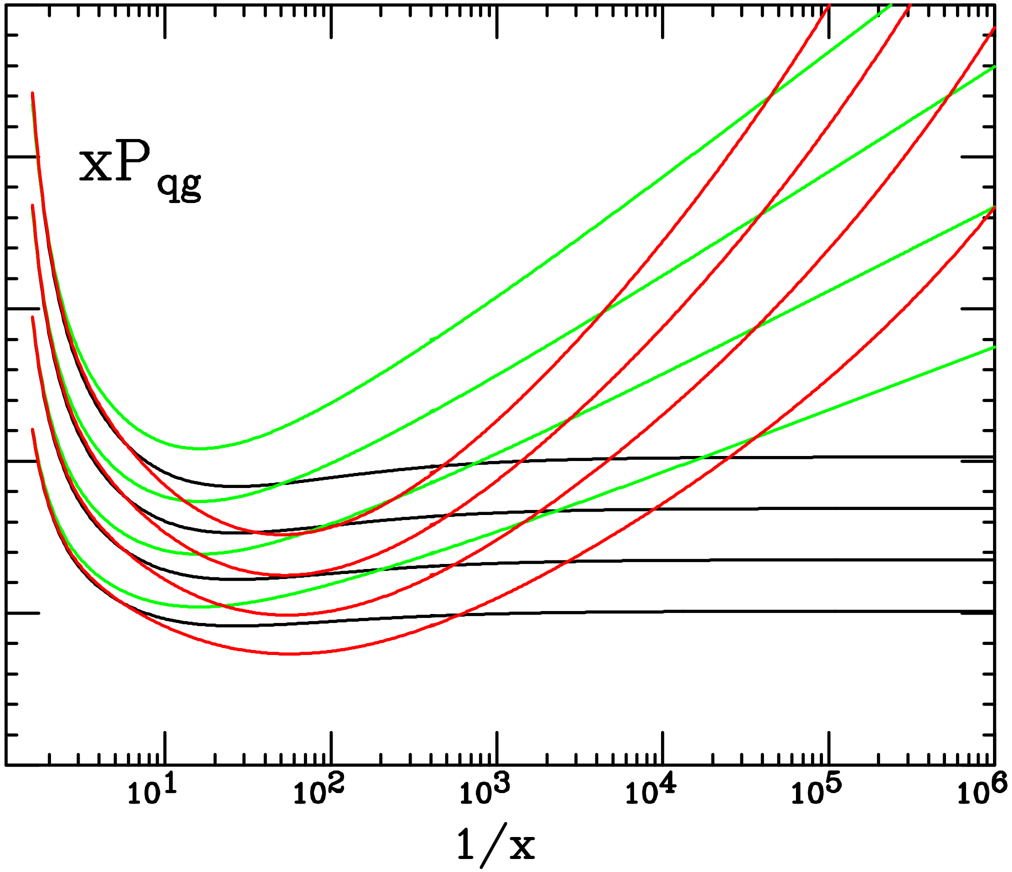
- DETERMINE NUMERICALLY $P_{\rm eff}$ SUCH THAT
$$\frac{d}{dt}G(x,Q^2,Q_0^2) = \int_x^1 \frac{dz}{z} P\left(\frac{x}{z},Q^2\right) G(x,Q^2,Q_0^2)$$
- ADVANTAGE: CAN TREAT RUNNING COUPLING EXACTLY
- DISADVANTAGE: RESULT IS ONLY NUMERICAL $\Rightarrow$ MATCHING TO GLAP HARD

$n_f \neq 0$: THE QUARK SECTOR

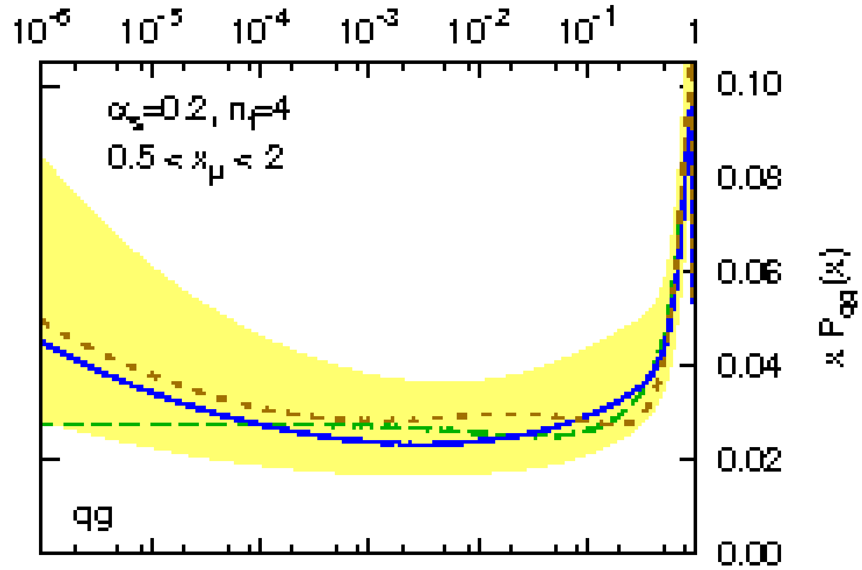
COMPARISON WITH CCSS: QUALITATIVELY SIMILAR (BUT CCSS RISE MILDER)

P_{qg}, $n_f = 0, 3, 4, 5$ (top to bottom)

NLO, NNLO, RESUMMED



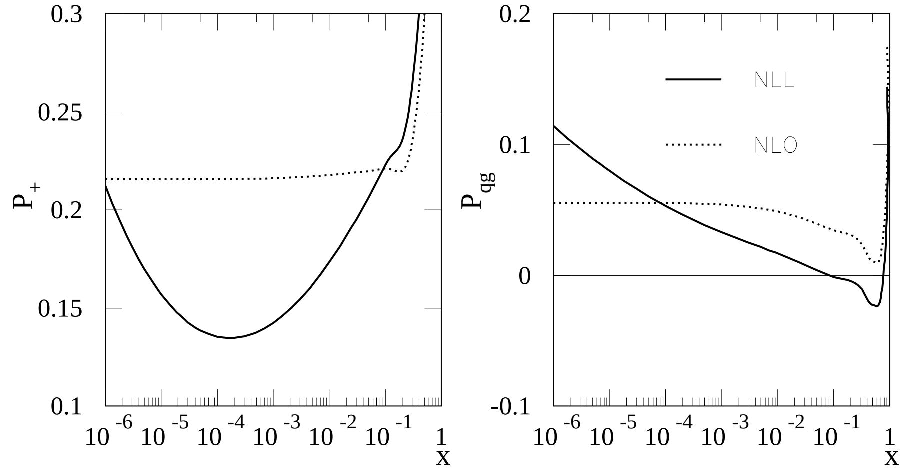
CCSS



RESUMMATION: THE TW APPROACH

- BFKL NLO TERMS INCLUDED, BUT NO SYMMETRIZATION
- DOUBLE COUNTING SUBTRACTED BUT NO PERTURBATIVE MATCHING TO NLO
- INCONSISTENT FACTORIZATION SCHEME IN QUARK SECTOR (Q_0DIS EVOLUTION, DIS COEFF FCTN.)
- RUNNING COUPLING CORRECTIONS FACTORIZED AS IN ABF, BUT THROUGH TREATED BY (DIVERGENT, ASYMPTOTIC) PERTURBATIVE EXPANSION
- NL RESUMMATION OF COEFFICIENT FUNCTIONS (NL IMPACT FACTORS APPROXIMATELY INCLUDED)
- HEAVY QUARK THRESHOLDS INCLUDED

NLO VS. RESUMMED SPLITTING

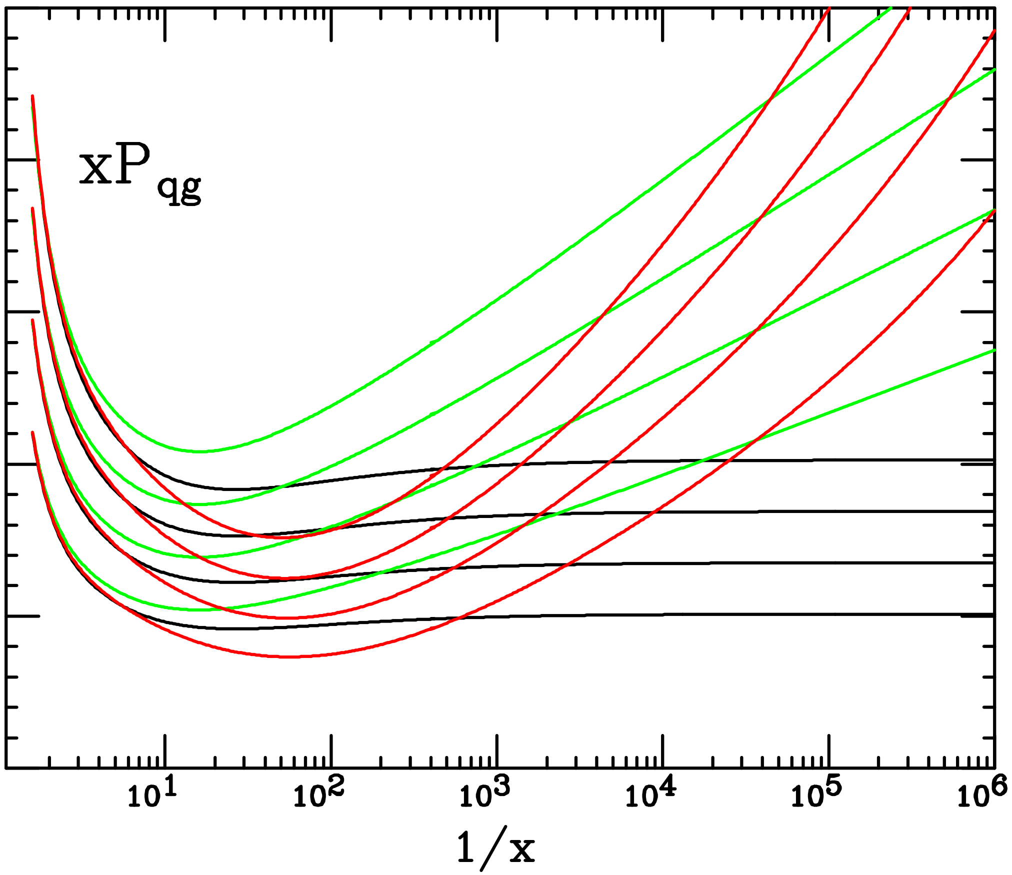


$n_f \neq 0$: THE QUARK SECTOR

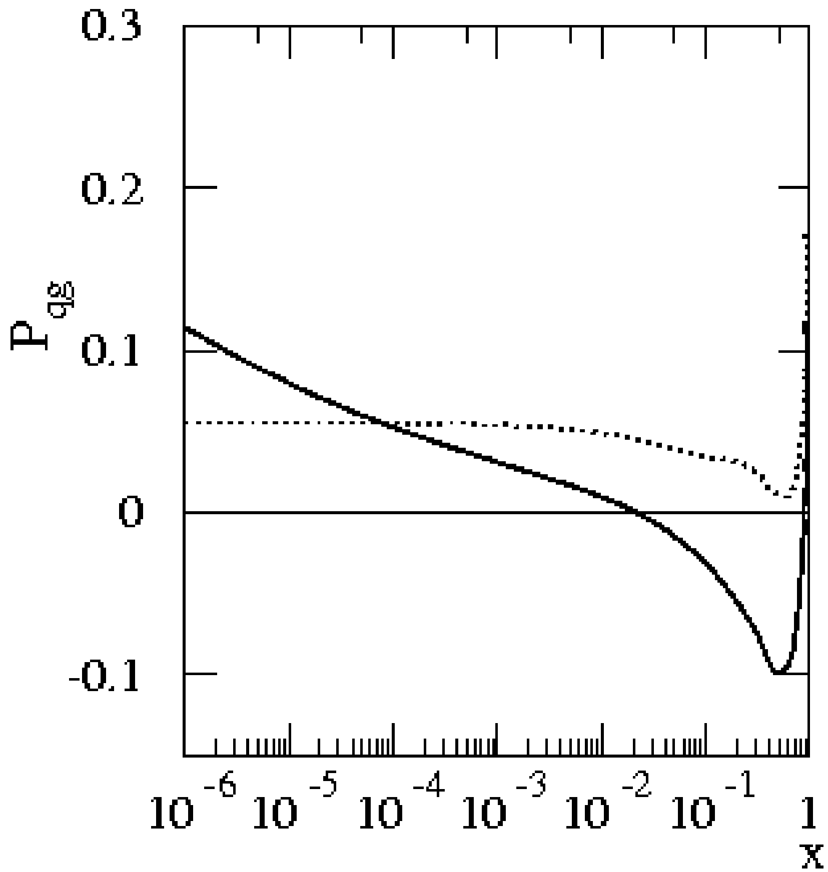
COMPARISON WITH THORNE & WHITE: QUALITATIVELY NOT SO SIMILAR (TW DIP)

P_{qg}, $n_f = 0, 3, 4, 5$ (top to bottom)

NLO, NNLO, RESUMMED



THORNE & WHITE

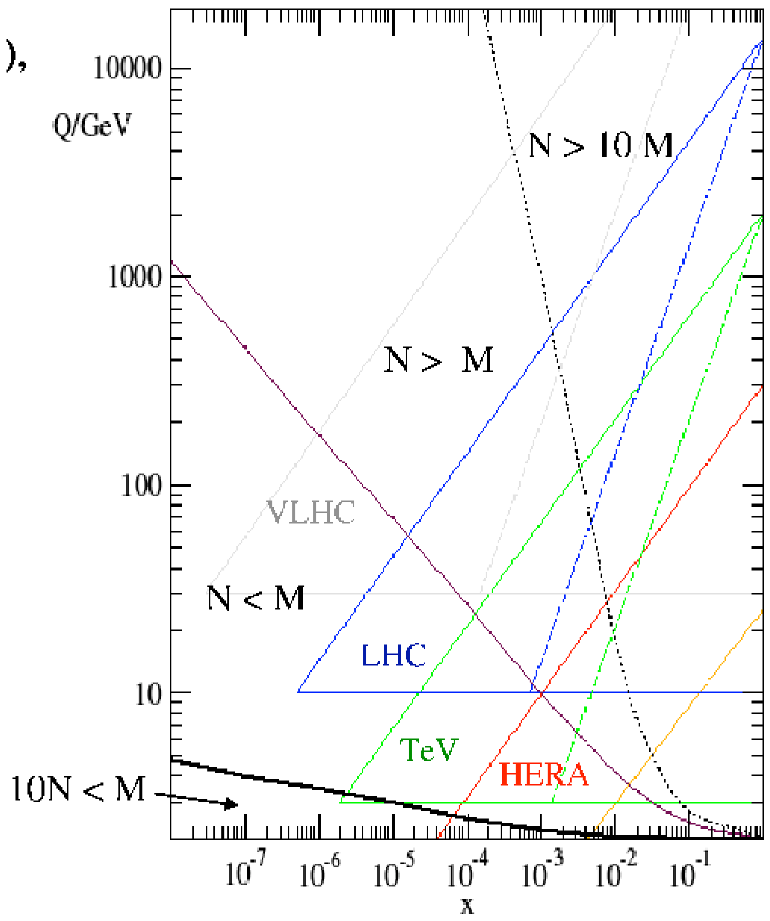


RESUMMED HARD CROSS SECTIONS:

WHEN DO WE NEED THEM?

HARD CROSS SECTIONS $\otimes$ PARTON LUMINOSITY $\Rightarrow$ PHYSICAL OBSERVABLES

- ENERGY FACTORIZATION $\Rightarrow$ HARD CROSS SECTION DEPENDS ON x AND Q^2
- SMALL x RESUMMATION $\Leftrightarrow$ Q^2 DEPENDENCE
- IN MELLIN SPACE $\frac{1}{N} \leftrightarrow \ln\frac{1}{x}$; $\frac{1}{M} \leftrightarrow \ln Q^2$
- IF M>>N $\Rightarrow$ SMALL x (HIGH ENERGY) RESUMMATION
  IF N>>M $\Rightarrow$ LARGE Q^2 (COLLINEAR) RESUMMATION
- UPON CONVOLUTION $N \leftrightarrow \frac{\partial}{\partial \ln x}$; $\quad M \leftrightarrow \frac{\partial}{\partial \ln Q^2}$
  ACTING ON PARTON LUMINOSITY
- ESTIMATE FROM x, Q^2 DEP. OF PARTON LUMI



SMALL x RESUMMATION SMALLER THAN COLLINEAR RESUMMATION REGION

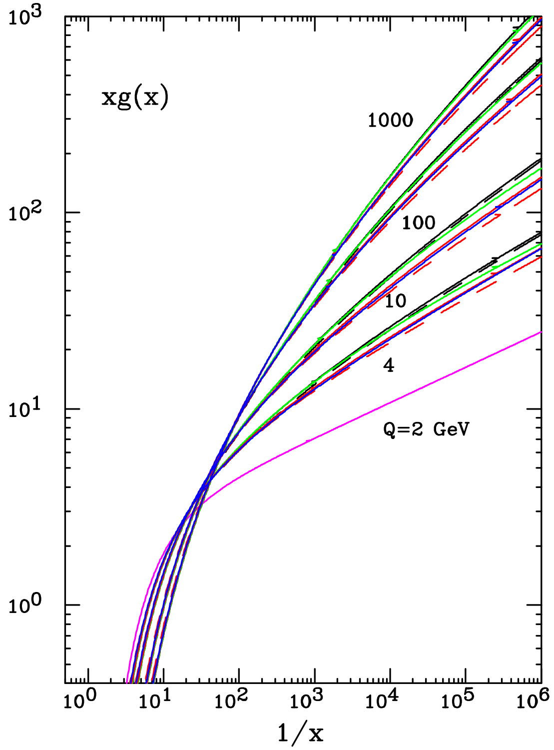
$\Rightarrow$ FOLLOWS FROM ASYMPTOTIC FREEDOM

QUARK AND GLUON EVOLUTION

evolve toy $G = (x, Q_0) = x^{-0.18}(1-x)^5$, $Q(x, Q_0) = \frac{1}{3} G(x, Q_0)$, $Q_0 = 2$ GeV

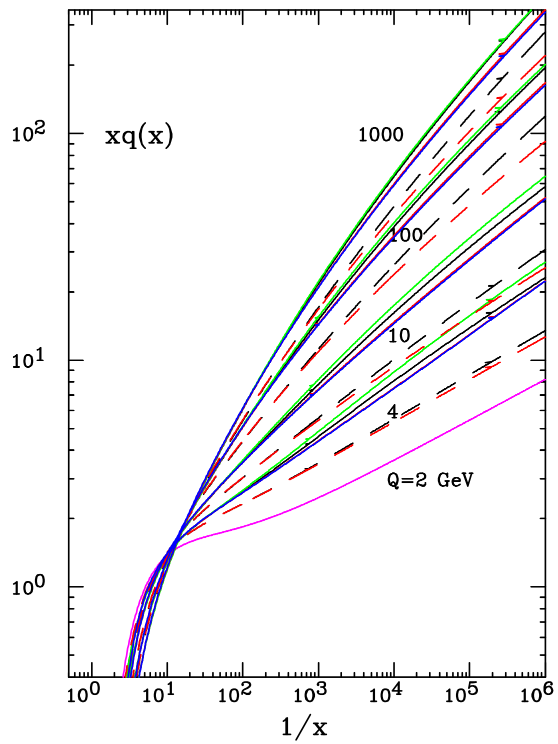
GLUON

$Q = 2,\ 10,\ 100,\ 1000$ GeV (bottom to top)



QUARK

NLO, NNLO, RESUMMED



- LO VS NLO DIFFERENCE LARGER THAN FIXED VS RESUMMED
- RESUMMED LIES BETWEEN NLO & NNLO
- RESUMMATION EFFECT SIZABLE AT MEDIUM-LARGE Q^2
- RESUMMED GLUON BELOW UNRESUMMED, QUARK JUST BELOW