From chiral quark models to high-energy processes

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Dedicated to the memory of Jan Kwieciński

- *Gravitational and higher-order form factors of the pion in chiral quark models*, WB, ERA, Phys. Rev. D78 (2008) 094011
- *Generalized parton distributions of the pion in chiral quark models and their QCD evolution*, WB , ERA, **Krzysztof Golec-Biernat**, Phys. Rev. D77 (2008) 034023
- *Pion-photon Transition Distribution Amplitudes in the Spectral Quark Model*, WB, ERA, Phys. Lett. B649 (2007) 49
- *Photon distribution amplitudes and light-cone wave functions in chiral quark models*, **Alexander E. Dorokhov**, WB, ERA, Phys. Rev. D74 (2006) 054023

Others:

- Praszałowicz, Rostworowski, Bzdak, Kotko
- Noguera, Vento, Theussl, Courtoy
- Tiburzi, Miller
- Bochum, Tübingen groups (nucleon)

"Low energy meets high energy"

- Goal: to explore the soft structure of hadrons
- Chiral quark models (NJL . . .) predict the non-perturbative soft hadronic matrix elements
- Inclusive and exclusive high-energy processes and lattice calculations provide the relevant data
- **Matching the model to QCD** at a low quark-model scale Q_0, **QCD evolution** to experimental scales
- In other words, chiral quark models provide dynamically the non-perturbative initial conditions for the QCD evolution
- Numerous predictions . . .

Definition of Generalized Parton Distributions

Twist-2 even-parity GPDs of the pion

non-singlet:

$$\epsilon_{3ab}\,\mathcal{H}^{q,I=1}(x,\zeta,t)=\int \frac{dz^-}{4\pi}e^{ixp^+z^-}\langle\pi^b(p+q)|\bar{\psi}(0)[0,z]\gamma^+\psi(z)\,\tau_3|\pi^a(p)\rangle\Big|_{z^+=0,z^\perp=0}$$

singlet:

$$\delta_{ab}\,\mathcal{H}^{q,I=0}(x,\zeta,t)=\int \frac{dz^-}{4\pi}e^{ixp^+z^-}\langle\pi^b(p+q)|\bar{\psi}(0)[0,z]\gamma^+\psi(z)|\pi^a(p)\rangle\Big|_{z^+=0,z^\perp=0}$$

$$\delta_{ab}\,\mathcal{H}^{g}(x,\zeta,t)=\int \frac{dz^-}{4\pi p^+}e^{ixp^+z^-}\langle\pi^b(p+q)|F_c^{+\mu}(0)[0,z]^{cc'}g_{\mu\nu}F_{c'}^{+\nu}(z)|\pi^a(p)\rangle\Big|_{z^+=0,z^\perp=0}$$

where $p^2 = m_\pi^2$, $q^2 = -2p\cdot q = t$, $\zeta = q^+/p^+$

ζ - momentum transfer along the light cone

Reviews:

- K. Goeke, M. V. Polyakov, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401, hep-ph/0106012
- M. Diehl, Phys. Rept. 388 (2003) 41, hep-ph/0307382
- A. V. Belitsky, A. V. Radushkin, Phys.Rept.418(2005)1, hep-ph/0504030
- ...

GPDs provide rich information of the structure of hadrons which may come from such processes as $ep \to ep\gamma$, $\gamma p \to pl^+l^-$, $ep \to epl^+l^-$, or from lattices. Small cross sections of exclusive processes require very high accuracy experiments. First results for the nucleon are coming from HERMES and CLAS, also COMPASS, H1, ZEUS

Formal features

Symmetric notation: $\xi = \frac{\zeta}{2-\zeta}$, $X = \frac{x-\zeta/2}{1-\zeta/2}$, with $0 \leq \xi \leq 1$, $-1 \leq X \leq 1$

$H^{I=0}(X,\xi,t) = -H^{I=0}(-X,\xi,t)$, $H^{I=1}(X,\xi,t) = H^{I=1}(-X,\xi,t)$.

For $X \geq 0$ we have $\mathcal{H}^{I=0,1}(X,0,0) = q(X)$ - the usual PDF

The following **sum rules** hold:

$$\forall\xi: \quad \int_{-1}^{1} dX\, H^{I=1}(X,\xi,t) = 2F_V(t),$$
$$\int_{-1}^{1} dX\, X\, H^{I=0}(X,\xi,t) = 2\theta_2(t) - 2\xi^2\theta_1(t),$$

where $F_V(t)$ is the electromagnetic form factor, while $\theta_1(t)$ and $\theta_2(t)$ are the gravitational form factors (related to the charge conservation and the momentum sum rule in DIS)

The **polynomiality** conditions (Lorentz invariance, time reversal, and hermiticity):

$$\int_{-1}^{1} dX\, X^{2j}\, H^{I=1}(X,\xi,t) = 2\sum_{i=0}^{j} A_{2j+1,2i}(t)\xi^{2i},$$

$$\int_{-1}^{1} dX\, X^{2j+1}\, H^{I=0}(X,\xi,t) = 2\sum_{i=0}^{j+1} A_{2j+2,2i}(t)\xi^{2i},$$

where A's are the generalized form factors (GFFs)

Another way to look at GFFs:

$$\left\langle \pi^+(p') \right| \overline{u}(0)\, \gamma^{\{\mu}\, i\overleftrightarrow{D}^{\mu_1} i\overleftrightarrow{D}^{\mu_2} \dots i\overleftrightarrow{D}^{\mu_{n-1}\}}\, u(0) \left| \pi^+(p) \right\rangle =$$

$$2P^{\{\mu}P^{\mu_1}\dots P^{\mu_{n-1}\}}A_{n0}(t) + 2\sum_{\substack{k=2\\ \text{even}}}^{n} q^{\{\mu}q^{\mu_1}\dots q^{\mu_{k-1}}P^{\mu_k}\dots P^{\mu_{n-1}\}}\, 2^{-k} A_{nk}(t)$$

GPDs may be viewed as an infinite collection of GFFs

The **positivity bound**:

$$|H_q(X,\xi,t)| \le \sqrt{q(x_{\rm in})q(x_{\rm out})}, \quad \xi \le X \le 1.$$

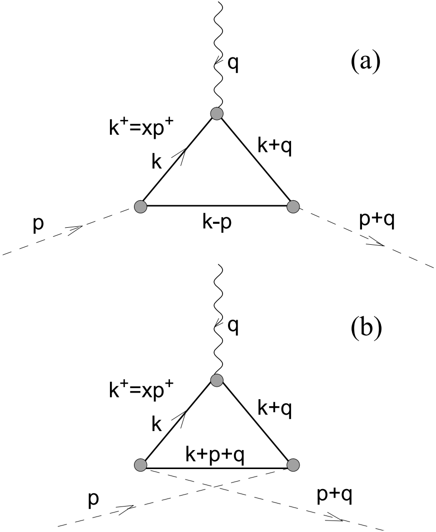
where $x_{\rm in} = (x+\xi)/(1+\xi)$, $x_{\rm out} = (x-\xi)/(1-\xi)$.

Finally, a **low-energy theorem** $H_{I=1}(2z-1,1,0) = \phi(z)$ holds, where ϕ is the pion distribution amplitude (DA)

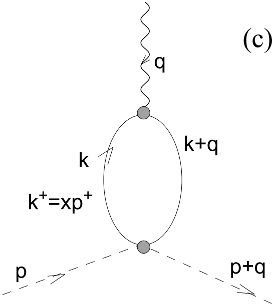
Above relations and bounds impose severe constraints on the form of the GPDs

All are satisfied in our quark-model calculation

QM evaluation of the GPDs



Large-N_c = one loop



Direct (a), crossed (b), and contact (c) contribution (D-term) to the GPD of the pion (wavy line: γ^+)

PDF, QM

With $\zeta = t = 0$, the GPD becomes the PDF. The Nambu–Jona-Lasinio model (Davidson, Arriola, 1995) gives

$$q(x) = 1$$

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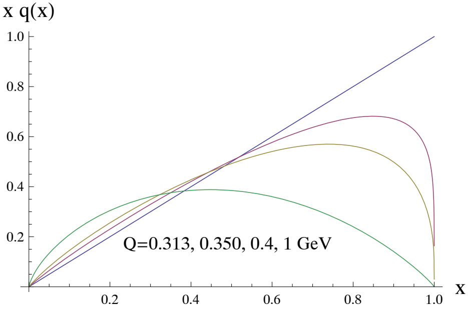
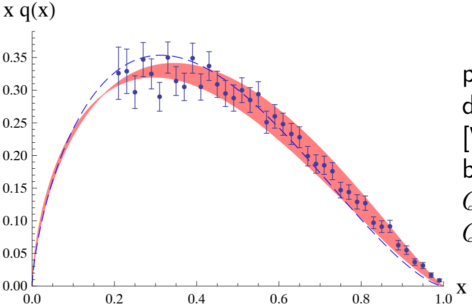
$$q(x) = 1$$

LO DGLAP QCD evolution (good at intermediate x) of the non-singlet part to growing scales



PDF, QM vs. E615

LO DGLAP QCD evolution of the non-singlet part to the scale $Q^2 = (4\text{ GeV})^2$ of the E615 Fermilab experiment:



points: Drell-Yan from E615
dashed: reanalysis of data [Wijesooriya et al., 2005]
band: valence QM PDF evolved to $Q = 4$ GeV from the QM scale $Q_0 = 313^{+20}_{-10}$ MeV

The quark-model scale Q_0

Various ways to fix: PDF, DA, moments

From experiment, the momentum fraction carried by the valence quarks is [SMRS 1992, GRS 1999]

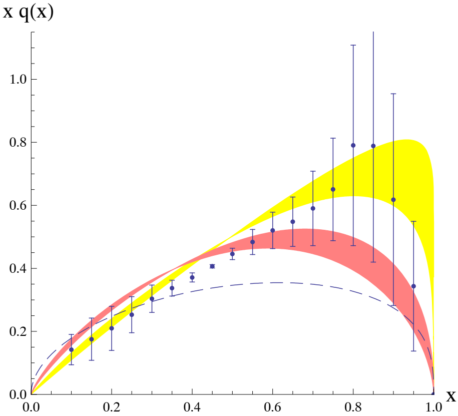
$$\langle x \rangle_v = 0.47(2) \quad \text{at} \quad Q^2 = 4 \text{ GeV}^2$$

QM scale = no gluons, may evolve backwards until $\langle x \rangle_v = 1$
$\rightarrow$ quark-model scale for NJL

$$Q_0 = 313^{+20}_{-10} \text{ MeV}$$

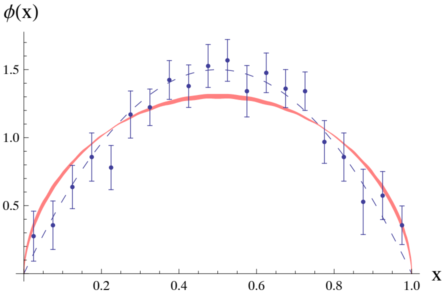
(here for the so called local model, for other QM Q_0 may vary)
At this scale $\alpha(Q_0^2)/(2\pi) = 0.34$, which makes the evolution very fast for the scales close to the initial value – calls for improvement!

PDF, QM vs. lattice



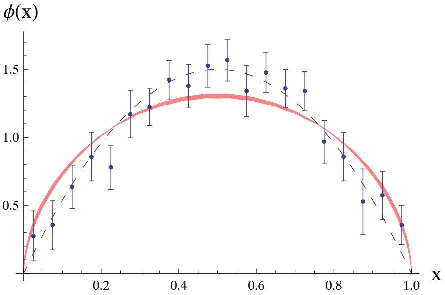
points: transverse lattice [Dalley, van de Sande, 2003]
yellow: QM evolved to 0.35 GeV
pink: QM evolved to 0.5 GeV
dashed: GRS parameterization at 0.5 GeV

PDA, QM vs. E791 and lattice data



points: E791 data from di-jet production in $\pi + A$
band: QM at $Q = 2$ GeV
dashed line: asymptotic form ($Q \to \infty$)

PDA, QM vs. E791 and lattice data



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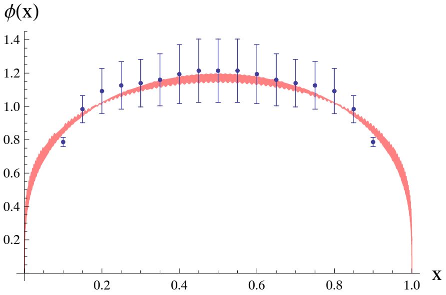


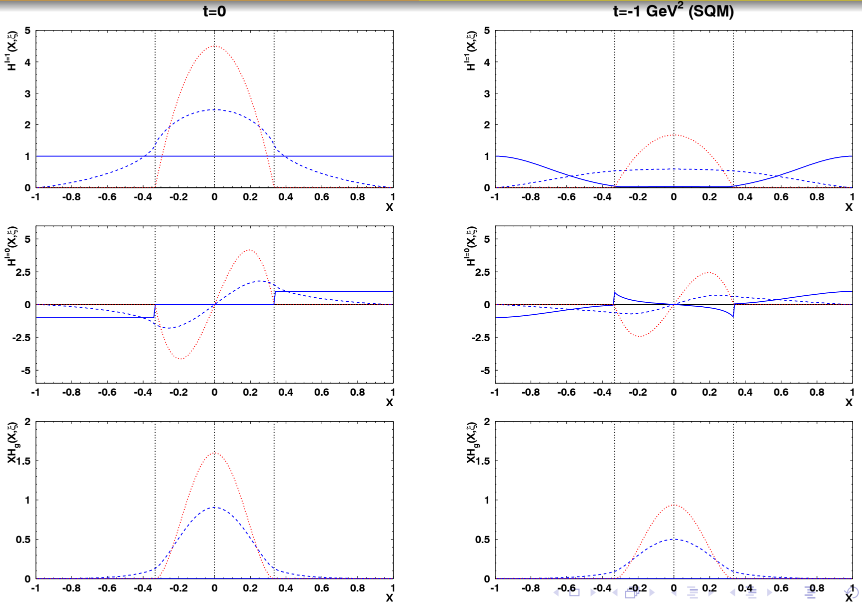
points: transverse lattice data [Dalley, van de Sande, 2003]
band: QM at $Q = 0.5$ GeV

GPD in chiral quark models

Analytic formulas derived for GPD in two models: NJL and SQM (Spectral Quark Model), all formal properties satisfied, non-trivial formulas fit in two long lines, **no factorization of the t-dependence** - sheds light on possible parameterizations

Similar studies in [Praszałowicz, Rostworowski, 2003]

Next slide:
LO DGLAP-ERBL evolution for SQM with $\xi = 1/3$. Solid - initial condition, dashed - evolved to $Q^2 = (4\mathrm{GeV})^2$, dotted - asymptotic form. Code: [Golec-Biernat, Martin, 1999]


t=0
t=-1 GeV^2 (SQM)
$H^{I=1}(X,\xi)$
$H^{I=0}(X,\xi)$
$XH_g(X,\xi)$
X

Gravitational form factors

Electromagnetic current:

$J_V^\mu = \sum_{q=u,d,\dots} \bar{q}(x) \frac{\tau_a}{2} \gamma^\mu q(x)$

Energy-momentum tensor:

$\Theta^{\mu\nu} = \sum_{q=u,d,\dots} \bar{q}(x) \frac{\mathrm{i}}{2} \left(\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu\right) q(x) + \mathrm{gluons}$

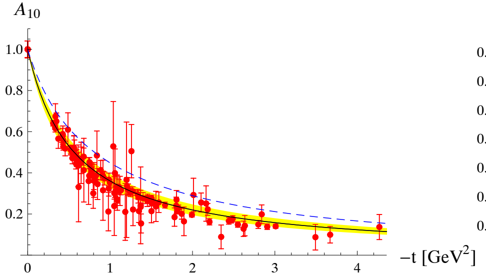
Two structures (form factors):

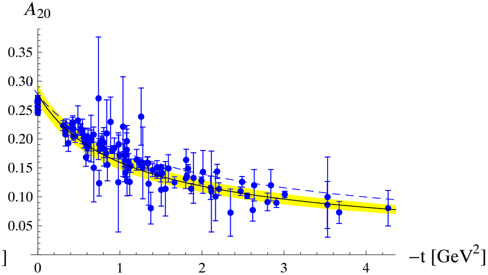
$$\langle \pi^b(p') \mid \Theta^{\mu\nu}(0) \mid \pi^a(p) \rangle = \frac{1}{2} \delta^{ab} \left[(g^{\mu\nu} q^2 - q^\mu q^\nu) \Theta_1(q^2) + 4 P^\mu P^\nu \Theta_2(q^2) \right]$$

traceless tensor – Θ_1 and scalar – Θ_2?

Lattice, exclusive processes

Full-QCD Euclidean lattice results



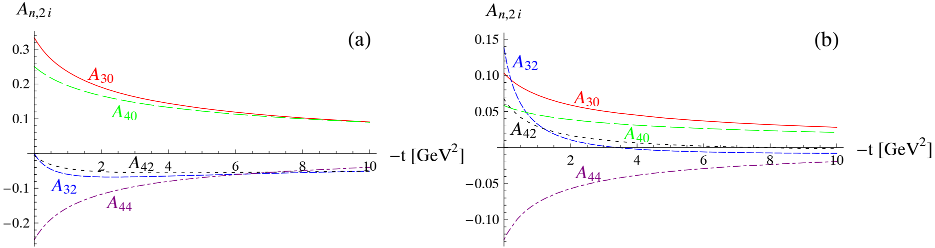


The EM FF (left) and the quark part of the gravitational form factor Θ_1 (right) in SQM (solid line) and NJL (dashed line), compared to data from [Brömmel et al., 2005-7]

Quark-model relation: $\langle r^2 \rangle_\Theta = \frac{1}{2} \langle r^2 \rangle_V$

Matter more concentrated than charge!

Higher-order form factors - predictions



The quark GFFs $A_{3,2i}$ and $A_{4,2i}$ at the quark-model scale $Q_0 \sim 320$ MeV (a) and at the lattice scale $Q = 2$ GeV (b)

Quark moments at $t = \xi = 0$

With the notation $\langle x^n \rangle = A_{n+1,0}(0)$, one finds at the lattice scale of $Q = 2$ GeV [Brömmel et al., 2007]

$$\langle x \rangle = 0.271 \pm 0.016$$
$$\langle x^2 \rangle = 0.128 \pm 0.018$$
$$\langle x^3 \rangle = 0.074 \pm 0.027$$
$$\text{(lattice)}$$

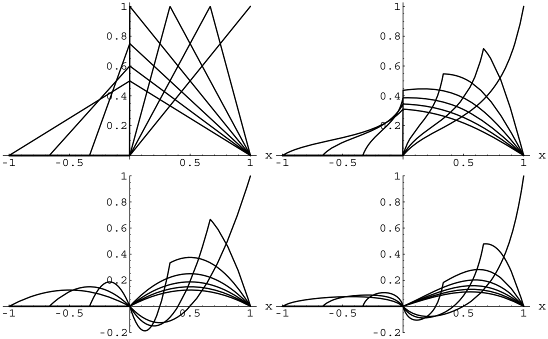
while in QM after the LO DGLAP evolution to the lattice scale

$$\langle x \rangle = 0.28 \pm 0.02$$
$$\langle x^2 \rangle = 0.10 \pm 0.02$$
$$\langle x^3 \rangle = 0.06 \pm 0.01$$
$$\text{(chiral quark models)}$$

Agreement within uncertainties

Pion-photon Transition Distribution Amplitudes

[Pire, Szymanowski, 2005] (as the GPD, but between the π and γ states)



Top: vector TDA for $t = 0$ (left) and $t = -0.4$ GeV (right), ζ: -1, $-2/3$, $-1/3$, 0, $1/3$, $2/3$, and 1. Bottom: the same for the axial TDA, SQM at the scale Q_0 [WB+ERA, 2007]

1. Link between high- and low-energy analyses
2. Quark models provide the initial conditions for the QCD evolution
3. Analytic formulas – useful for general properties, (*e.g.*, no factorization of the t-dependence
4. Q_0 very low – need for improvement of the evolution
5. With naive evolution the overall agreement with the data and lattice simulations very reasonable (PDF, DA, GFFs, GPD, photon DA, TDA, . . .)
6. The electromagnetic and gravitational form factors do not evolve with the scale (they correspond to conserved currents), while the higher-order GFFs do
7. Predictions can be further tested with future lattice simulations for higher-order form factors. The behavior is non-trivial, with form factors having different signs, magnitude, and asymptotic fall-off.
8. GPDs of the **nucleon**: more challenging (Bochum, Tübingen) but experimental data exist

[12 June 2003]