Fully Unintegrated Parton Correlation Functions and Factorization in Lowest Order Hard Scattering

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(Work in collaboration with
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In This Talk:

- Very careful review of standard parton model kinematics in DIS (parton model).
- Need for a more general treatment of factorization.
- Overview of a treatment that maintains exact kinematics for initial and final states.
- Main complications.
- Treatment of higher orders.
- Unsolved problems and future work.

Factorization

- Statement of a factorization theorem.

$$\sigma = C \otimes f + \mathcal{O}((\Lambda/Q)^a \, |\sigma|)$$

Hard scattering coefficient

Universal parton distribution function (PDF)

Power suppressed corrections

Importance for Phenomenology

- Factorization in DIS, etc…

  +

- Matrix element calculations

  +

- Universal PDFs

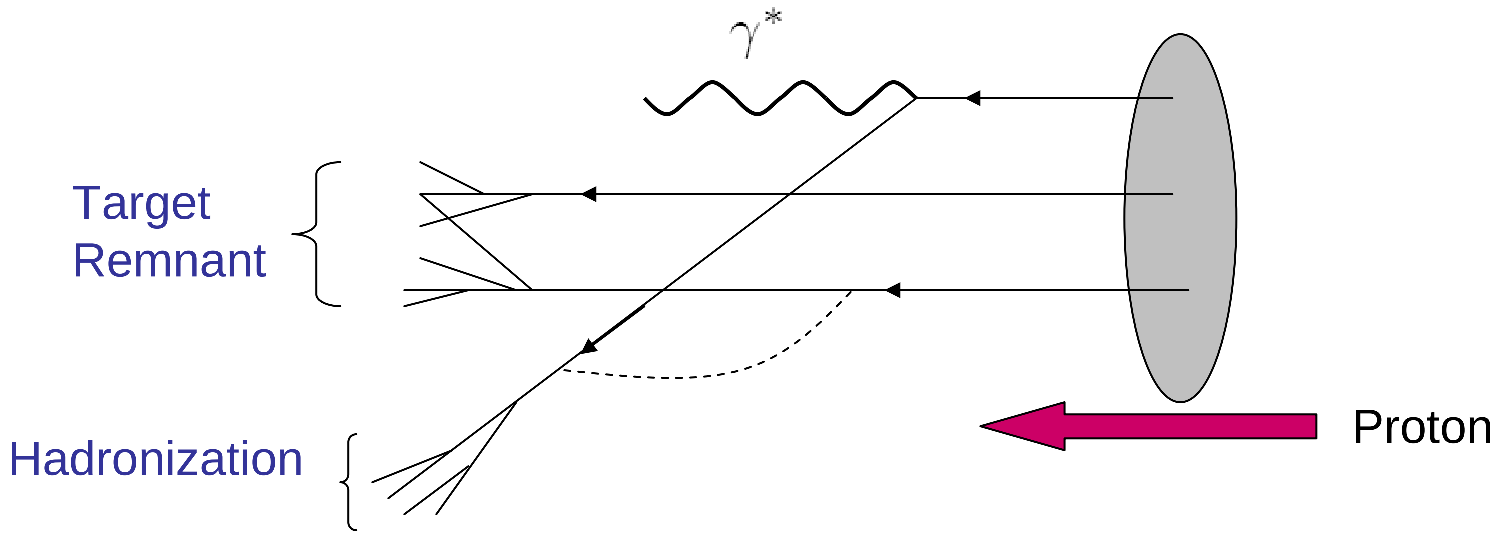
  +

- Evolutions equations

Factorization → Predictions for "new" physics, LHC, etc…

Example: Lowest Order DIS:

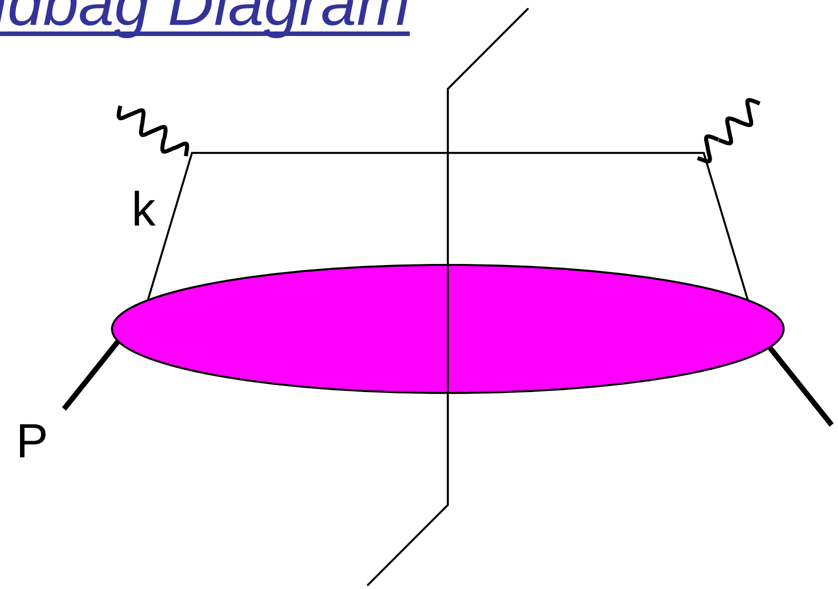
Conventional Parton Model Intuition



How to transition to field theory?

Types of PDFs

DIS – Handbag Diagram



- In general, what should be used for k?

Types of PDFs

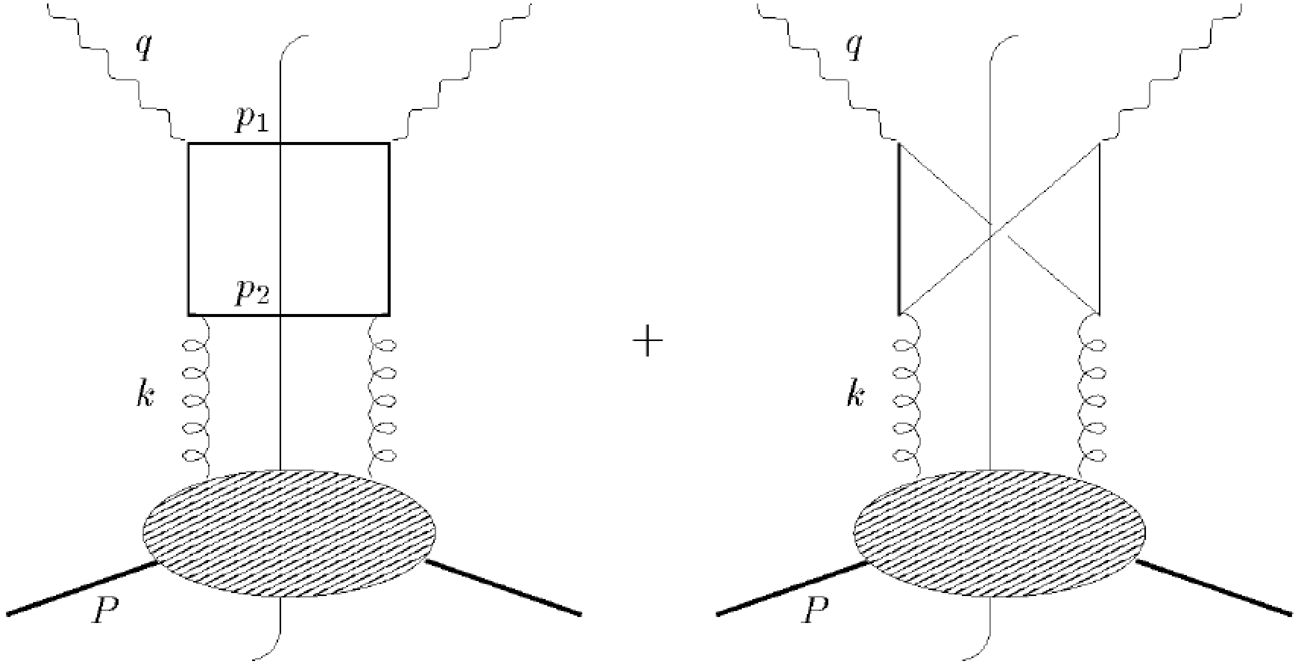
- Integrated PDFs:
  - Standard x-dependent PDFs - k_T and k^2 are integrated over.
  - Well-known operator definitions of classic factorization theorems.

- Unintegrated PDFs:
  - Depend on k_T, *but still integrated over invariant energy*.
  - Some consistent operator definitions proposed.

    (e.g., Hautmann and Soper, Phys.Rev.D 75, 074020 2007)

- Parton Correlation Functions (Including *Fully Unintegrated* PDFs):
  - Differential in all components of four-momentum.
  - Refers to *fully* unintegrated PDFs as well as jet-factors, and soft factors. *(e.g., Watt, Martin, and Ryskin Eur.Phys.J. C31,73 (2003))*

Example: $c\bar{c}$ photoproduction

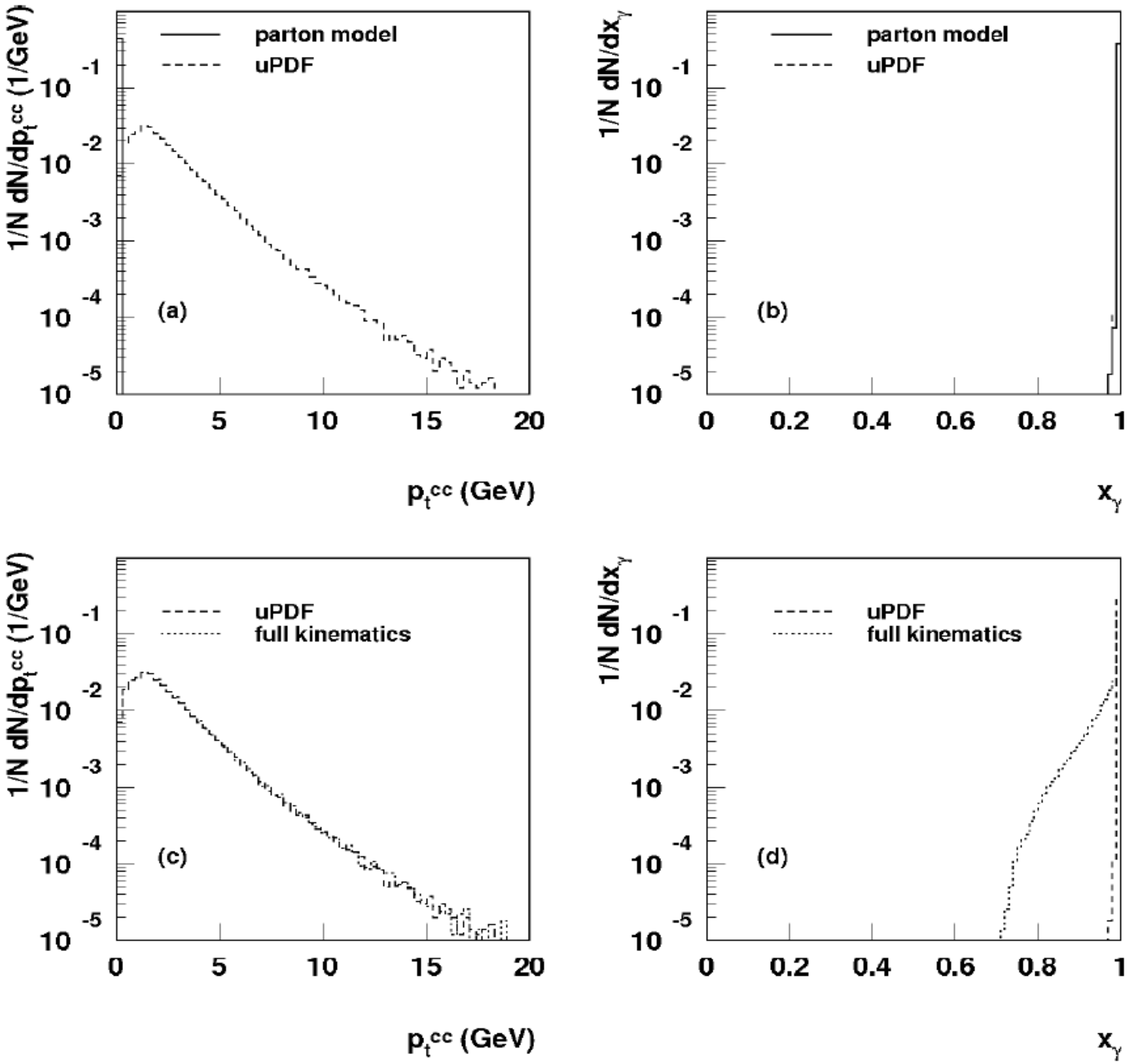


- Try:
  - Parton model kinematics: k_T and k^2 are small and integrated over in gluon PDF.
  - k_T- dependent PDF, but k^2 still integrated over.
  - Exact kinematics – all components of parton four momentum are kept.

(Collins and Jung: hep-ph/0508280)

Errors in final state kinematics

$c\bar{c}$
Pair-production
In DIS.

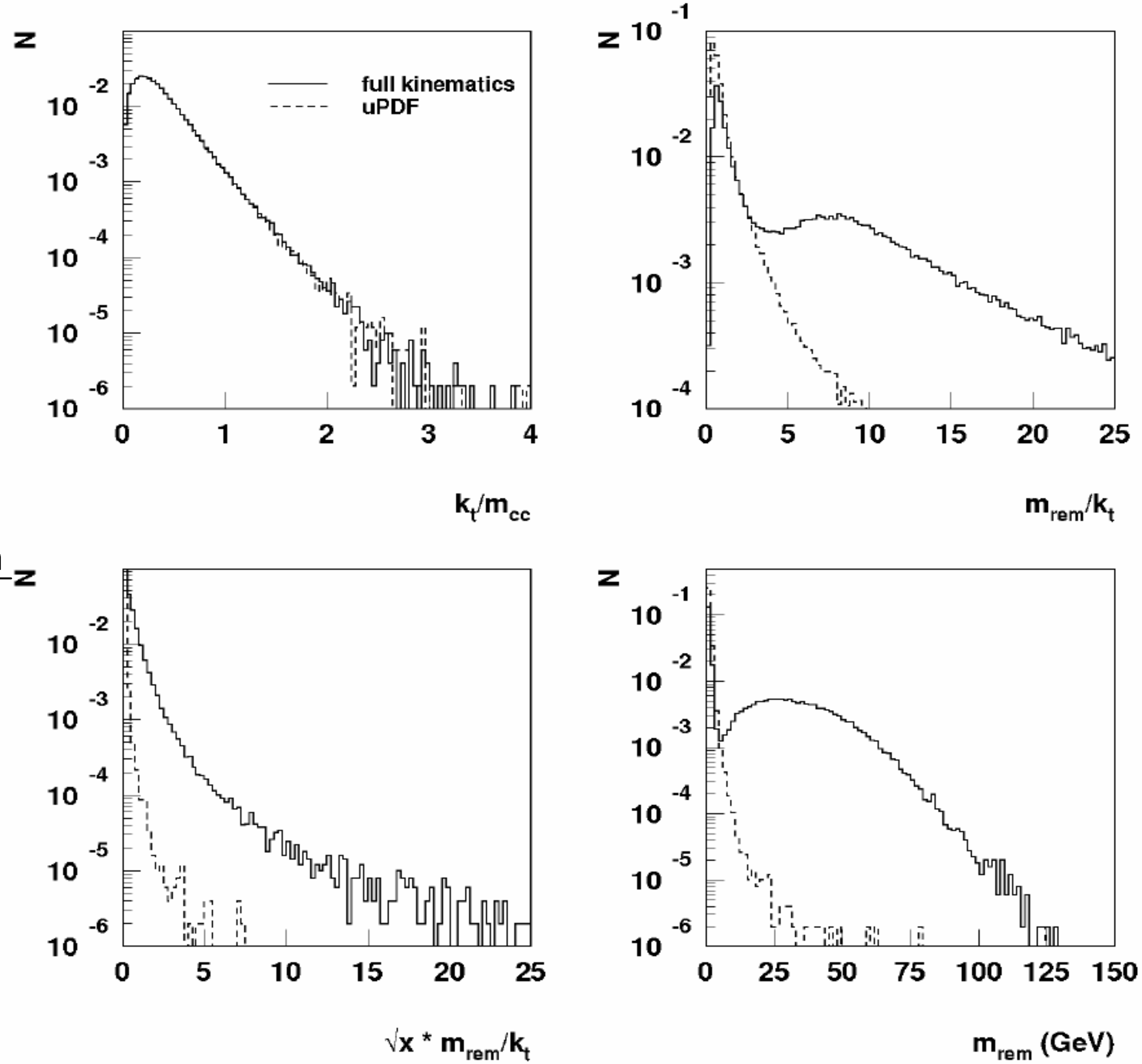


(Collins and Jung: hep-ph/0508280)

Errors in final state kinematics

$c\bar{c}$
Pair-production

$$k^2 = -\frac{k_T^2 + x m_{rem}^2}{1 - x}$$

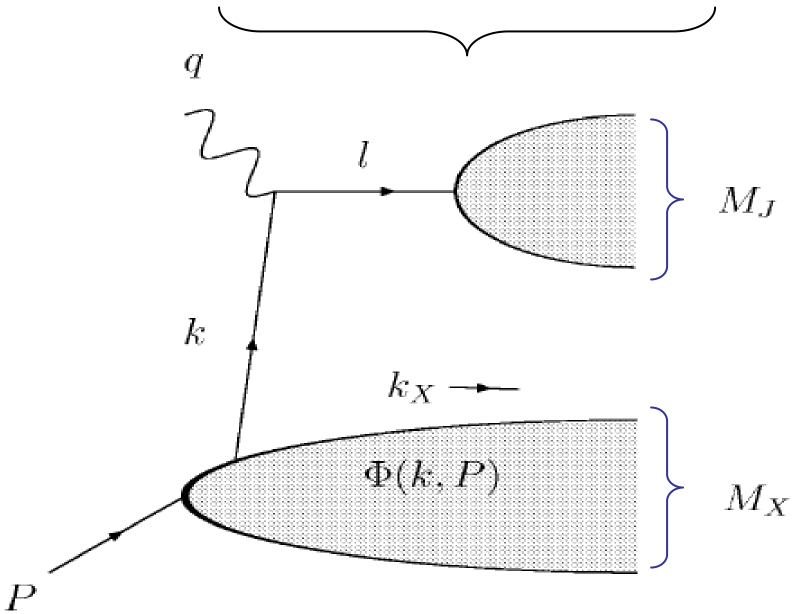


(Calculation from H. Jung using Cascade Monte Carlo)

Kinematics and Final States

Large-x

Two jets with fixed invariant energy in final state:



$$s = (1 - x)M_p^2 + \frac{Q^2}{x}(1 - x)$$

$$\Rightarrow \quad k_T^2 < \frac{(1 - x)}{4}M_p^2 + \frac{Q^2}{4x}(1 - x)$$

But k_T runs to order Q^2 in the def. of the PDF!

Standard Kinematical Approximations

- Re-assignment of final state kinematics – mismatch between true kinematics and parton model approximation.

- Good approximation for integrated quantities.

- Not good enough if details of final states are considered.

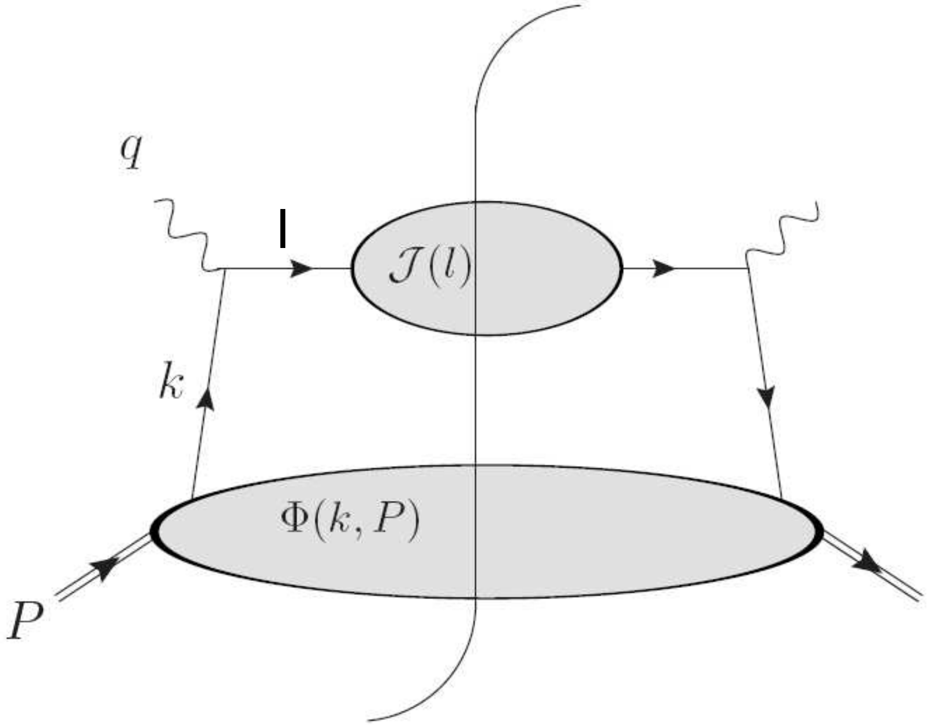
- In some cases, *exact* kinematics are needed.

- What about factorization?

Unapproximated LO graph

(Additional soft/collinear gluons to be considered later.)

$$W^{\mu\nu}(q,P) = \sum_j \frac{e_j^2}{4\pi} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\gamma^\mu \mathcal{J}(k+q)\gamma^\nu \Phi(k,P)]$$



large component

small component

$$P = \left(P^+, \frac{M^2}{2P^+}, \mathbf{0}_T\right)$$

$$q = \left(-xP^+, \frac{Q^2}{2xP^+}, \mathbf{0}_T\right)$$

Reproducing the basic parton model:

- Utilize parton model kinematics:
  - $k = (k^+, k^-, \mathbf{k}_T)$
  - $l = k + q = \left(k^+ - xP^+, \frac{Q^2}{2xP^+} + k^-, \mathbf{k}_T\right)$
  - $\text{small } l^2 \text{ means } k^+ \approx xP^+$
- Inside target bubble write: $k \to (xP^+, k^-, \mathbf{k}_T)$
- Inside the jet bubble write: $l \to \left(l^+, \frac{Q^2}{2xP^+}, \mathbf{0}_T\right)$
- Use parton model values <u>in hard vertex</u>:

$$k \to \hat{k} = (xP^+, 0, \mathbf{0}_T)$$
$$l \to \hat{l} = \left(0, \frac{Q^2}{2xP^+}, \mathbf{0}_T\right)$$

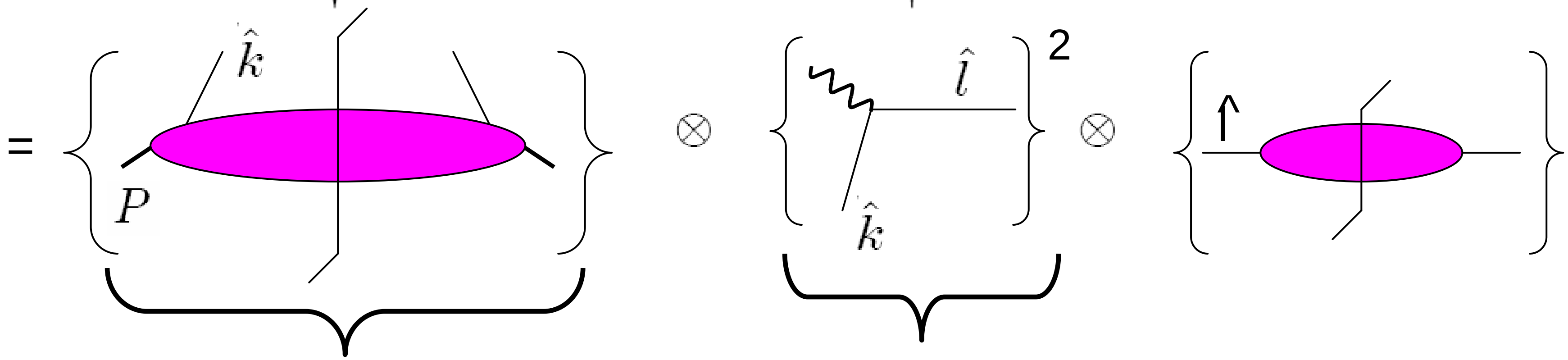
- Keep only large Dirac matrices.

Reproducing the basic parton model:

- Hadronic Tensor Becomes:

Set to one by
unitarity argument

$$W^{\mu\nu}(q,P) \simeq \frac{e_j^2}{4\pi}\left\{\int \frac{dk^- d^2\mathbf{k}_T}{(2\pi)^4}\Phi_j^+(x_{Bj}P^+,k^-,\mathbf{k}_T)\right\} \mathrm{Tr}\left[\gamma^\mu\gamma^+\gamma^\nu\gamma^-\right] \left\{\int dl^+ \mathcal{J}_j^-(l^+,q^-,\mathbf{0}_T)\right\}$$



Parton Distribution???

- UV divergent – requires renormalization.

- Not gauge invariant.

LO partonic
structure functions.

Note shift in final state kinematics !!

The Standard PDF

Operator definition:

(Reproduces integral form up to c.t.)

$$f_j(x_{\rm Bj},\mu) \;=\; \int \frac{dw^-}{4\pi} e^{-ix_{\rm Bj}p^+w^-} \langle p|\bar\psi(0,w^-,\mathbf{0}_T) V_w^\dagger(u_{\rm J}) \gamma^+ V_0(u_{\rm J}) \psi(0)|p\rangle_R$$

$$u_{\rm J} = (0,1,\mathbf{0}_T)$$

Light-like Wilson lines for gauge invariance:

$$V_w(n) = P\exp\left(-ig\int_0^\infty d\lambda\, n\cdot A(w+\lambda n)\right)$$

$$V_w^\dagger(u_{\rm J})V_0(u_{\rm J}) = P\exp\left(-ig\int_0^{w^-} d\lambda\, u_{\rm J}\cdot A(\lambda u_{\rm J})\right)$$

Complications With Unintegrated PDFs

- Generalization:

$$P(x, \mathbf{k}_t, \mu) \stackrel{??}{=} \int \frac{dw^- d\mathbf{w}_T}{16\pi^3} e^{-ixp^+ w^- + i\mathbf{k_T}\cdot\mathbf{w}_T} \times \langle p \left| \bar{\psi}(0, w^-, \mathbf{w}_T) V_w^\dagger(u_J) I_{u_J;w,0} \gamma^+ V_0(u_J) \psi(0) \right| p \rangle$$

- Wilson line needed to link points at infinity.

(Belitsky et. al Nucl. Phys.B 656, 165 2003)

- Light-cone divergence in outgoing quark direction!

- Divergence persists even when gluon mass and UV cutoff are included.

Proposal:

(Collins, Rogers, Stasto arXiv:0708:2833)

- Maintain exact over-all kinematics of initial and final states.

- Maintain explicit factors for final states.

- Non-perturbative factors should depend on all components of four-momentum. *(PDFs, jet factors, soft factors)*

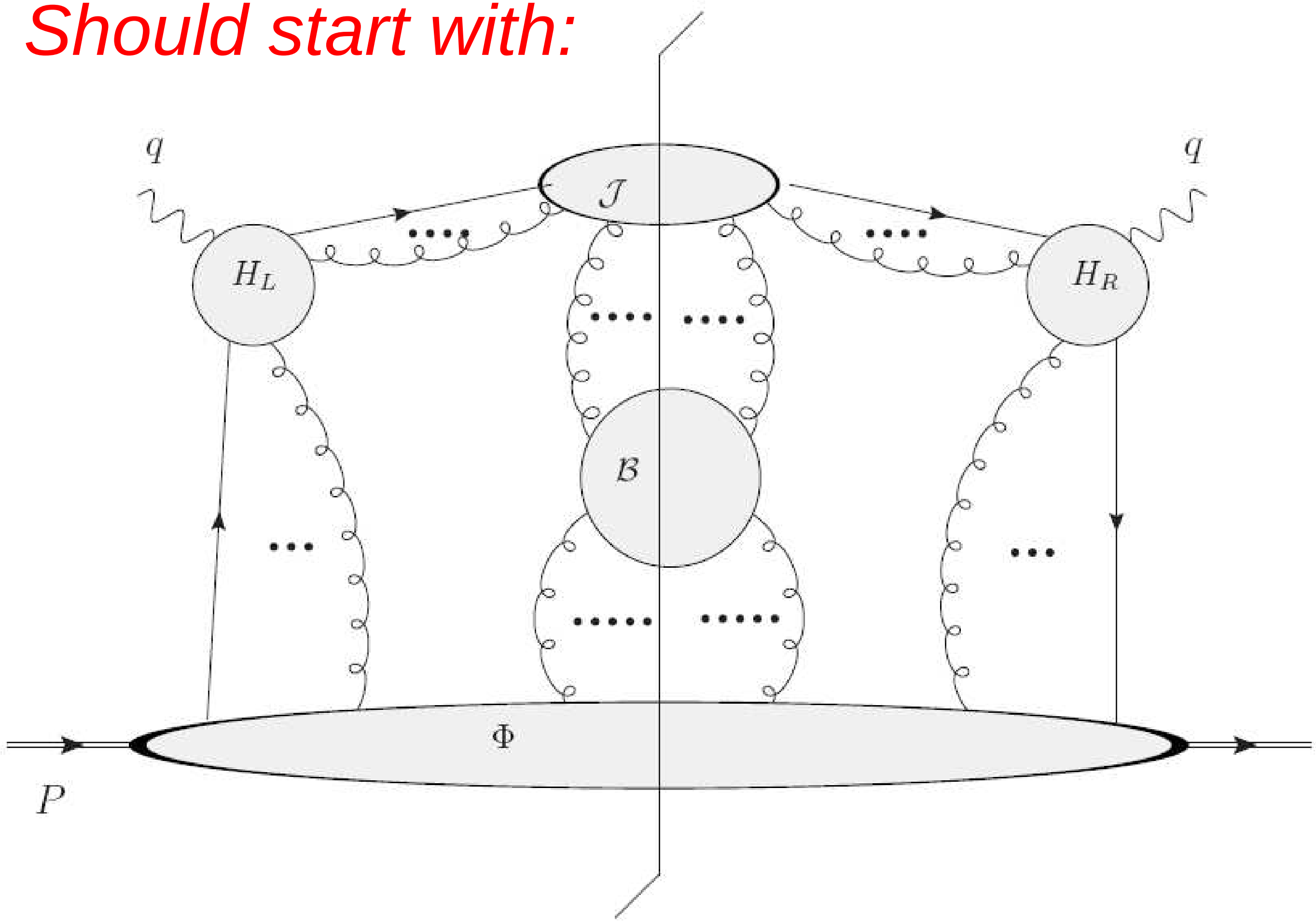
- What about factorization?

What is needed?

- General factorization formula with power suppressed corrections.
- Well-defined operator definitions for PCFs.
- Hard scattering matrix calculated with on-shell Feynman graphs.
- Higher orders rely on subtractive formalism – *detailed treatment of lowest order hard scattering needed as a first step.*

General Graphical Structure

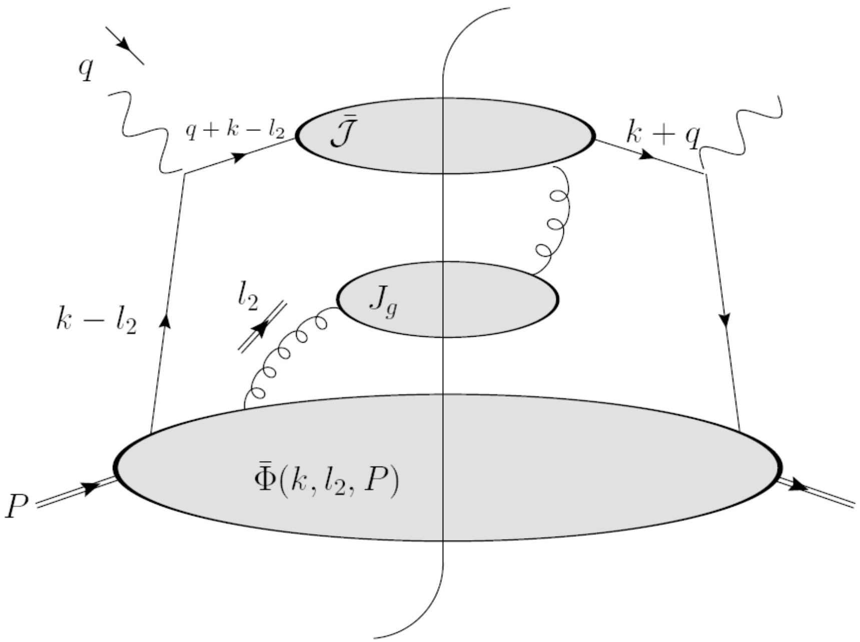
Should start with:

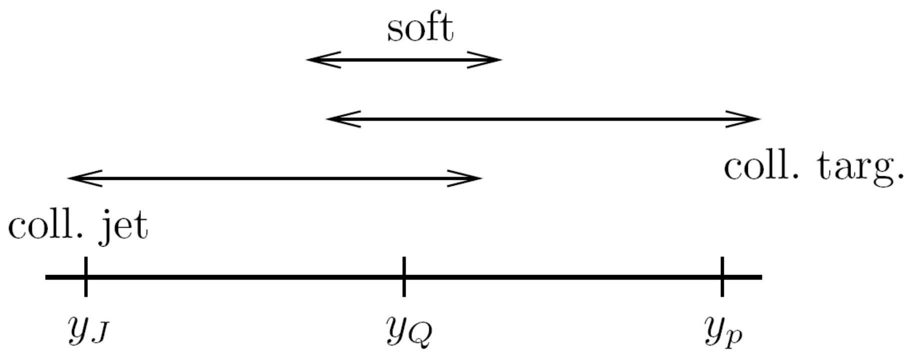


Must disentangle soft and collinear gluons to get topological factorization…

Graphical example:

Single extra gluon





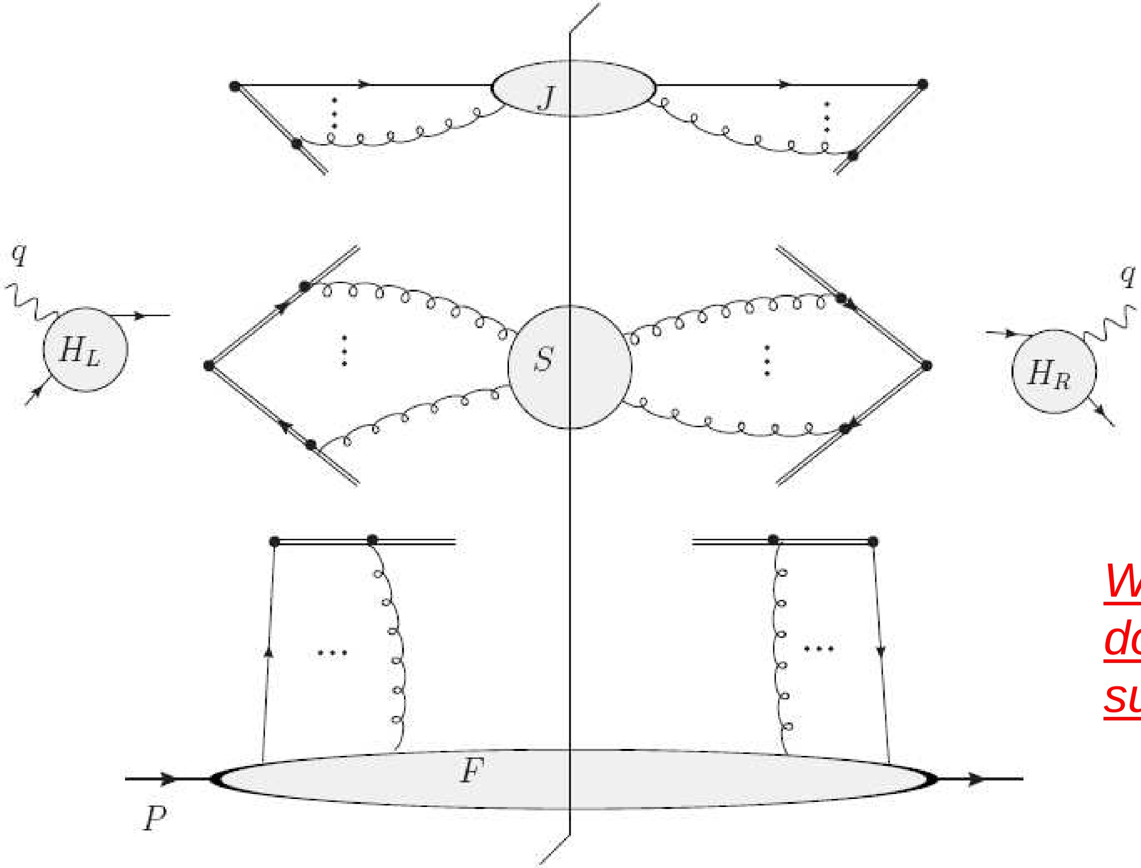
Range of gluon rapidity

Strategy Overview

- Propose gauge invariant operator definitions of PCFs.
- Consider extra soft/collinear gluon attachments that contribute to the leading region.
  - Characterize regions, R, of gluon momentum.
- Apply *consistent* soft, target collinear, and jet-collinear approximations.
  - No approximations in any the initial or final state bubbles!
- Sum over graphs, Γ, and apply Ward identities to obtain topological factorization.
- Apply double counting subtractions to larger regions.
- Identify PCFs and obtain factorization formula:

$$\sigma = \sum_R \sum_\Gamma C_R \Gamma + \textit{power suppressed corrections}$$

Topological Factorization:



We also need double counting subtractions.

A formula of this type is the goal.

$$\sigma = C \otimes F \otimes J \otimes S + \mathcal{O}((\Lambda/Q)^a|\sigma|), \qquad a > 0$$

Defining PCFs

❑ What are the required features?

- Depend on all components of momentum. (No approximations on kinematics.)
- Wilson lines to enforce gauge invariance should be non-light-like:

$$n_{\rm T} = (1, -e^{-2y_{\rm T}}, \mathbf{0}_T) \qquad n_{\rm J} = (e^{-|2y_J|}, 1, \mathbf{0}_T) \qquad n_s = (-e^{y_s}, e^{-y_s}, \mathbf{0}_T)$$

$$y_{\rm T} >> 0 \qquad y_{\rm J} << 0$$

- Rapidity variables, y_T and y_J, effectively cut-off rapidity divergences.
- Soft rapidity y_s characterizes boundary between left and right moving partons.

Defining PCFs

Example: Soft factor (coordinate space)

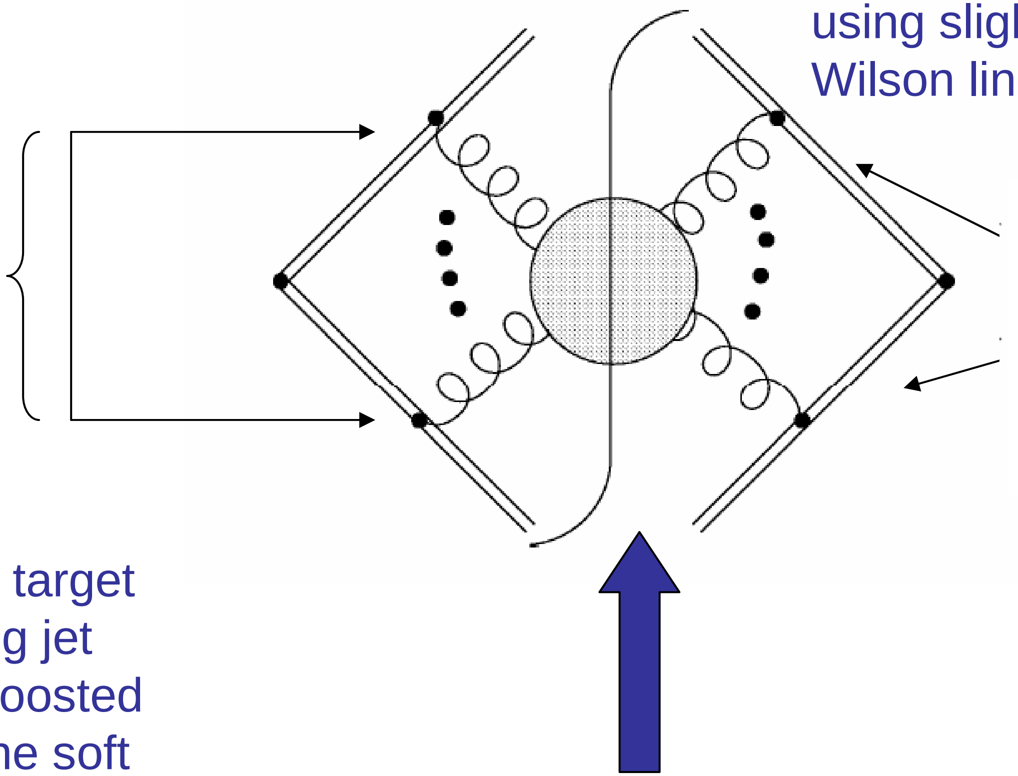
$$\tilde{S}(w, y_{\mathrm{T}}, y_{\mathrm{J}}, \mu) = \langle 0 | I^{\dagger}_{n_{\mathrm{T}};w,0} V_w(n_{\mathrm{T}}) V^{\dagger}_w(n_{\mathrm{J}}) I_{n_{\mathrm{J}};w,0} V_0(n_{\mathrm{J}}) V^{\dagger}_0(n_{\mathrm{T}}) | 0 \rangle_R$$

- Scale μ for renormalization of standard UV divergences.
- Non-light like directions n_T and n_J Wilson lines – y_T and y_J are effectively regulators of the light-cone divergences.
- Fourier transform to momentum space.
- Evolution with y_T and y_J?
- Similar issues involved target PCF and jet PCF. (More complications due to double counting subtractions.)

Example: graphical structure of the soft PCF:

- Cutoff high rapidity gluons by using slightly non-light-like Wilson lines

Eikonal lines near the target and jet directions



$$n_{\rm T} = (1, -e^{-2y_{\rm T}}, \mathbf{0}_T)$$

$$n_{\rm J} = (e^{-|2y_J|}, 1, \mathbf{0}_T)$$

$$y_{\rm T} >> 0,\ y_{\rm J} << 0$$

- Both the target and outgoing jet are highly boosted relative to the soft gluons.

Soft gluons and final state bubble

Summary

- Looking at details of final states requires precise treatment of kinematics.
- Exact kinematics requires the use of fully unintegrated parton correlation functions (PCFs) in both initial and final states. *(fully unintegrated PDFs, soft factor, jet factors)*
- Without usual kinematic approximations, factorization needs to be reconsidered.
- Exact operator definitions needed.
- Detailed lowest order treatment needed first – higher orders follow from subtractive formalism.

Fully Unintegrated Approach

Advantages

- Generality.
- Needed for complicated events and for details of final states.
- Hard scattering is an ordinary function – not generalized function (e.g. delta-functions/plus-distribution)

Disadvantages

- Very complicated – much theoretical work is still needed.
- PCFs depend on multiple parameters.

What has been done?

- Factorization for scalar theory.

  (Collins, Zu, JHEP 03 (2005) 059)

- Detailed treatment of factorization for the case of a single outgoing jet.

- Derived formula using candidate operator definitions for the PCFs.

- Account for multiple gluon exchanges (in the abelian case).

- Method extendable to other processes.

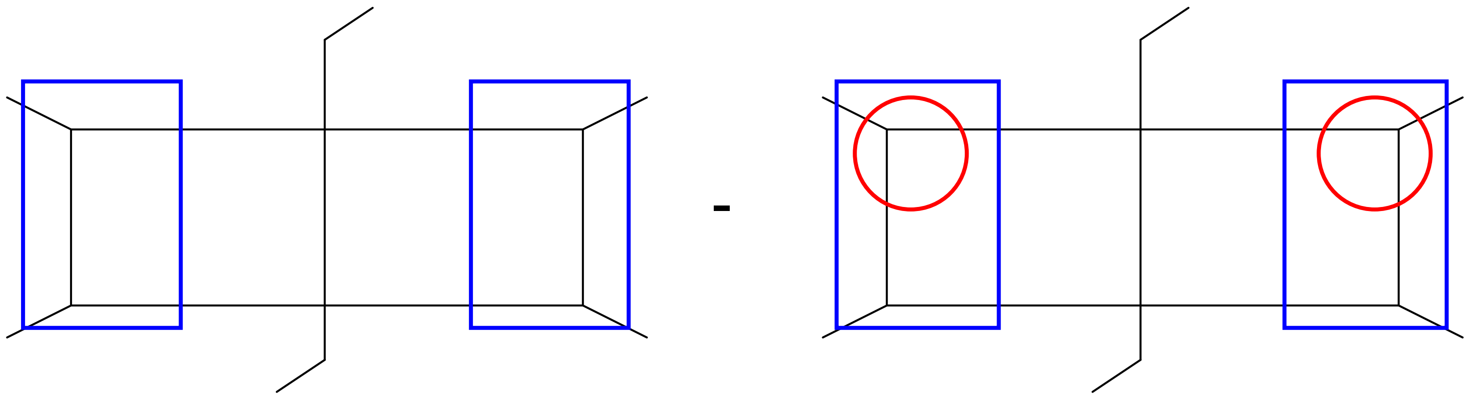
Outlook

- Factorization for higher order hard scattering?
- Extension to other (more exclusive) processes needed.
- Ward identity arguments in non-Abelian case?
- Evolution equations?
  - Relation to CSS formalism?
- Recovery of other approaches in appropriate limits? (e.g. BFKL, CCFM, etc…)

Current Work - Higher Order Hard Scattering

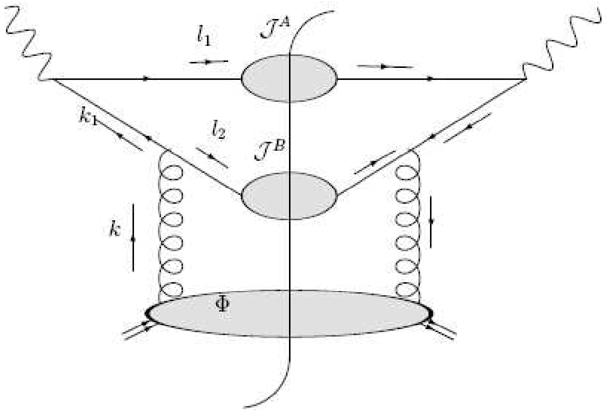
- Explicit implementation of subtractive formalism:

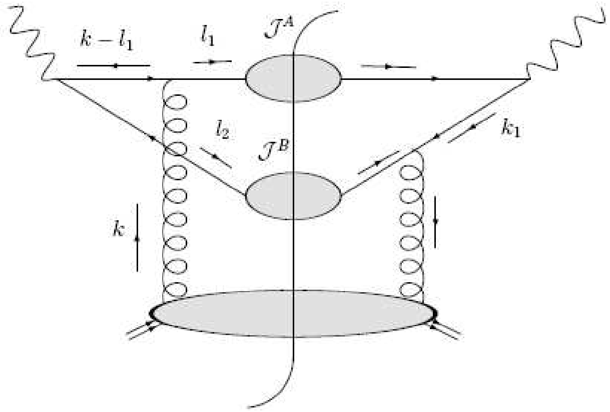
(Collins, Zu, JHEP 03 (2005) 059)



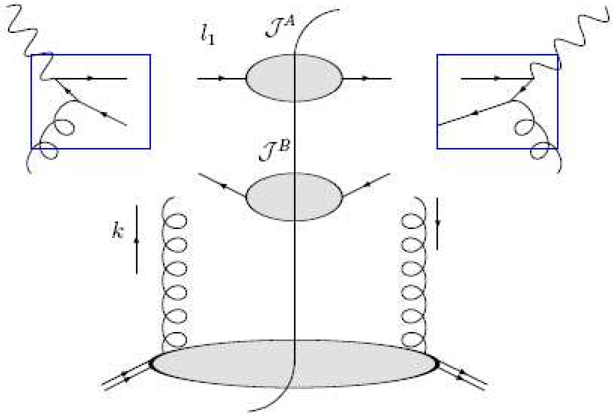
- Multiple levels of approximation.

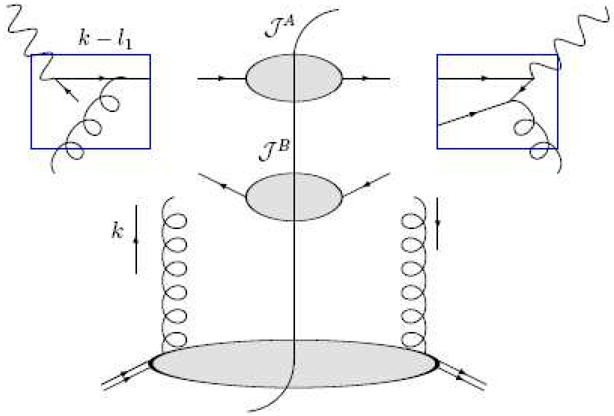
Current Work- Higher Orders Hard Scattering





(Neglecting extra soft and collinear lines for now.)

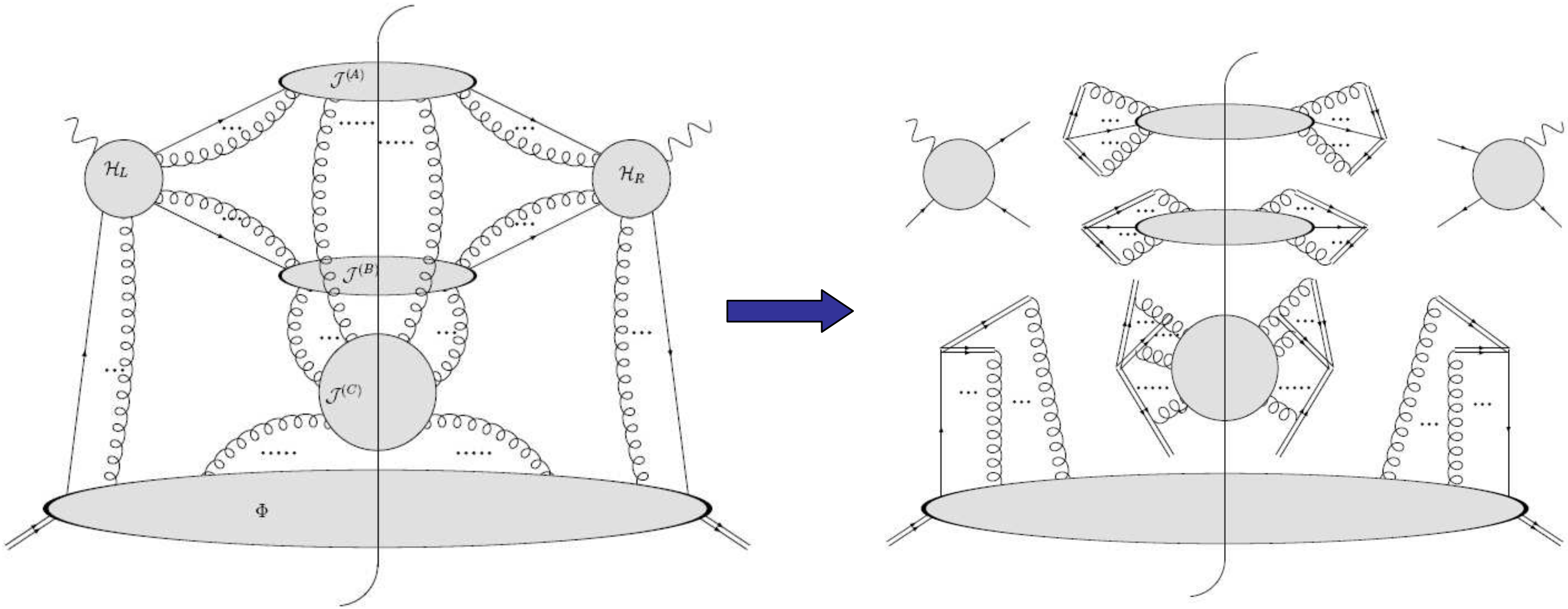




+ subtraction terms.

Extract Hard Scattering…

Factorization Beyond LO



Conclusions:

- Exact kinematics needed. (Unintegrated PDFs not enough.)
  (Basic program outlined for scalar theory by Collins and Zu (2005))
- Requires exact definitions for parton correlation functions.
- We have defined parton correlation functions and derived a factorization formula for the case of an abelian gauge theory.
  (Strongly suggestive of a structure for the non-abelian case.)
- Much work to be done.