(N)LO Simulations of Chargino Production and Decay

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Chargino and Neutralino sector in the MSSM: Reconstruction of SUSY parameters

- Charginos $\tilde{\chi}_i^\pm$ and Neutralinos $\tilde{\chi}_i^0$ in the MSSM: superpositions of gauge and Higgs boson superpartners
- Chargino/ Neutralino sector: SUSY parameters at electroweak scale

  $$\tan\beta,\ \mu\ \text{(Higgs sector)},\ M_1,\ M_2\text{(soft breaking terms)}$$

  can be reconstructed from

  masses of $\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^\pm$, $\tilde{\chi}_1^0$, 2 σ in the $\tilde{\chi}^\pm$ sector

  (Choi ea 98, 00, 01)
- low-scale parameters + evolution to high scales (RGEs):
  $\Rightarrow$ hint at SUSY breaking mechanism (Blair ea, 02)
- requires high precision in ew-scale parameter determination

Experimental accuracy and theoretical next-to-leading-order (NLO) corrections

- experimental errors: obtained from simulation studies (LHC/ ILC study, Weiglein ea, 04)
- generate "experimental data" with known SUSY input parameters
- errors: combination of statistical and systematic errors

combined **LHC + ILC:** ‰

- **Theory**:
  Full NLO SUSY corrections for $\sigma(ee \to \widetilde{\chi}\widetilde{\chi})$ at ILC:
  in the % regime (Fritzsche ea 04, Öller ea 04, 05)
- similar for $\widetilde{\chi}^{\pm}$ decays (Fujimoto ea, Rolbiecki, 07)

$\Rightarrow$ include complete NLO contributions in analyses $\Leftarrow$

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From σ_{tot} to Monte Carlo event generators

MC event generators: Generate event samples
(same form as experimental outcome)

- experiments: see final decay products
- need to compare with simulated event samples
- also: important irreducible background effects

(e.g. Hagiwara ea, 05)

$\Rightarrow$ include NLO results in Monte Carlo Generators $\Leftarrow$

- MC Generator WHIZARD (W. Kilian, T. Ohl, J. Reuter, LC-TOOL-2001-039, arXiv: 0708.4233 [hep-ph])
- so far: LO Monte Carlo Event Generator for $2 \longrightarrow n$ particle processes
- includes various physical models (SM, MSSM, non-commutative geometry, little Higgs models), initial state radiation, parton shower models,...

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MC event generators: How do they work ??

- $m \rightarrow n$ particle process: in phase space Γ_n, $3\,n-4$ independent variables x_i define "event"
- determine random values for x_i (within allowed regions), calculate $d\sigma(x_i)$ ("weight" of event)
- for event simulation, calculate relative probability $p_{\rm evt} = \frac{d\sigma(x_i)}{d\sigma_{\rm max}}$ ("unweigthing")
- hit-and-miss technique: take $r\,\epsilon\,[0,1]$, accept event if $p \geq r$
- in practice: more sophisticated adaption methods for most MC generators

"unweighting" requires $p \geq 0$!

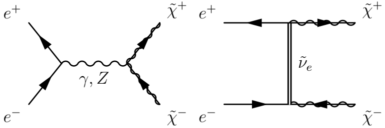
(not always fulfilled for NLO calculations, $\mathcal{O}(\alpha)$ corrections $\propto \ln(k_0)$ can be solved by resumming all orders)

In the following:

- Show results for NLO Event Generation for chargino production
- Show (**preliminary**) results for LO production and decay with SM/ MSSM background

The process: Chargino production at the ILC

- **ILC**: future e^+e^- collider, $\sqrt{s} = 500\,\mathrm{GeV}\,(1\,\mathrm{TeV})$
  "clean" environment, low backgrounds
  $\Rightarrow$ precision-machine, errors $\mathcal{O}(‰)$
- Charginos: (typically) light in the MSSM
  $\Rightarrow$ easily accessible at colliders (ILC/ LHC) $\Leftarrow$
- LO production at the ILC:



- decays: typically long decay chains

$$\text{e.g. } e^+\,e^- \rightarrow \widetilde{\chi}_1^+\,\widetilde{\chi}_1^- \rightarrow \tilde{\tau}_1^+\,\tilde{\tau}_1^-\,\nu_\tau\,\bar{\nu_\tau}\,(\rightarrow \tau^+\,\tau^-\,\nu_\tau\,\bar{\nu_\tau}\,\widetilde{\chi}_1^0\,\widetilde{\chi}_1^0)$$

Part I:
Inclusion of NLO results in WHIZARD

based on hep-ph/0607127 (Kilian ea), hep-ph/0610401 (TR)

NLO cross section contributions

σ_{tot} contributions and dependencies:

- σ_{born}
- virtual $\mathcal{O}(\alpha)$ corrections: $\sigma_{\text{virt}}(\lambda)$
- emission of soft/ hard collinear/ hard non-collinear photons:

  $\sigma_{\text{soft}}(\Delta E_\gamma, \lambda) + \sigma_{\text{hc}}(\Delta E_\gamma,\, \Delta\theta_\gamma) + \sigma_{2\to 3}(\Delta E_\gamma, \Delta\theta_\gamma)$
- higher order initial state radiation: $\sigma_{\text{ISR}} - \sigma_{\text{ISR}}^{\mathcal{O}(\alpha)}(Q)$

  λ: photon mass , ΔE_γ: soft cut , $\Delta\theta_\gamma$: collinear angle

Including FormCalc $\mathcal{O}(\alpha)$ results in WHIZARD (1)

- inclusion in WHIZARD : split photon phase space for real photon into soft/ hard-collinear/ hard non-collinear region:

$$\sigma_{\text{Born}+\gamma} = \sigma_{\text{soft}} + \sigma_{\text{hard, coll}} + \sigma_{\text{hard,noncoll}}$$

- soft photons ($E_\gamma \leq \Delta E_\gamma$): use soft photon approximation, add to virtual contribution ($\Rightarrow$ cancellation of IR divergencies): $\Rightarrow$ integrate over effective matrix element in Γ_2:

$$\sigma_{\text{Born}} + \sigma_{\text{virt}}(\lambda) + \sigma_{\text{soft}}(\Delta E_\gamma, \lambda) = \int d\Gamma_2 \, |\mathcal{M}_{\text{eff}}|^2(\Delta E_\gamma)$$

$$|\mathcal{M}_{\text{eff}}|^2(\Delta E_\gamma) = (1 + f_s(\Delta E_\gamma, \lambda)) \, |\mathcal{M}_{\text{born}}|^2 + 2\, Re(\mathcal{M}_{\text{born}} \, \mathcal{M}^*_{\text{virt}}(\lambda))$$

ΔE_γ: soft photon cut, λ: photon mass

- in practice: create library from FormCalc code, link this to WHIZARD

Including FormCalc $\mathcal{O}(\alpha)$ results in WHIZARD (2)

- hard collinear photons: $E_\gamma > \Delta E_\gamma,\, \theta_\gamma \leq \Delta\theta_\gamma$
  use hard collinear approximation (Dittmaier ea, 1993):

$$\sigma_{\text{hard, coll}} = \int_{\text{hard, coll}} d\Gamma_3 |\mathcal{M}_{2\rightarrow 3}|^2$$
$$\longrightarrow \int d\Gamma_2 \int_0^{x_0} dx_i\, f_\pm(x_i)\, |\mathcal{M}^{(\pm)}_{\text{Born}}|^2 (x_i, s),$$

  x_i: energy fraction of incoming fermion after photon radiation
  integrate in Γ_2
- hard, non-collinear photons: calculated exactly using $\mathcal{M}_{(2\rightarrow 3)}$
  generated by separate WHIZARD run using Γ_3

Fixed order method: Drawback

- Drawback: $|\mathcal{M}_{\text{eff}}|^2 < 0$ for small values of $\frac{\Delta E_\gamma}{\sqrt{s}}$
- well-known problem at LEP
- ad hoc solution: set $|\mathcal{M}_{\text{eff}}|^2 = 0$ for these cases
- too low energy cuts: $\mathcal{O}(\alpha)$ not sufficient, leads to "wrong" σ_{tot}
- remark: event generator specific problem ($\sigma_{\text{tot}} \geq 0$)

Resumming leading logs to all orders

solution to fixed order drawback:
$\Rightarrow$ resumm respective contributions to all orders $\Leftarrow$

- in practice: subtract $\mathcal{O}(\alpha)$ soft + virtual collinear contributions in $\mathcal{M}_{\text{eff}}$:

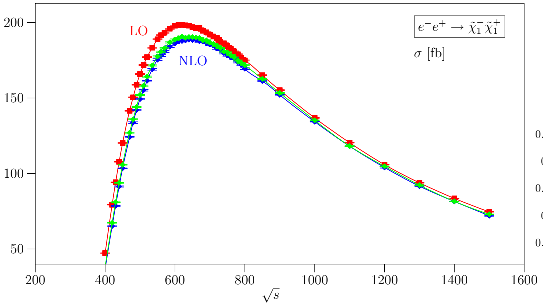
$$\begin{aligned} |\widetilde{\mathcal{M}}_{\text{eff}}|^2 &= (1 + f_s(\Delta E_\gamma))\,|\mathcal{M}_{\text{born}}|^2 + 2\,Re(\mathcal{M}_{\text{born}}\,\mathcal{M}^*_{\text{virt}}) \\ &- 2\,f_s^{ISR,\mathcal{O}(\alpha)}(\Delta E_\gamma)\,|\mathcal{M}_{\text{born}}|^2 \end{aligned}$$

- add the resummed contribution by folding with ISR structure function:

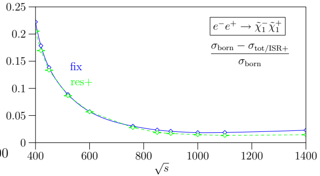
$$\int d\Gamma \int_0^1 dx_1 \int_0^1 dx_2\, f^{\text{ISR}}(x_1)\, f^{\text{ISR}}(x_2)\, |\widetilde{\mathcal{M}}_{\text{eff}}|^2(s, x_i)$$

- $f^{\text{ISR}}(x)$: Initial state radiation (Jadach, Skrzypek, Z.Phys. 1991), describes collinear (real + virtual) photons in leading log accuracy
- $f_s^{\text{ISR},\mathcal{O}(\alpha)}$: soft integrated $\mathcal{O}(\alpha)$ contribution

Results: cross sections



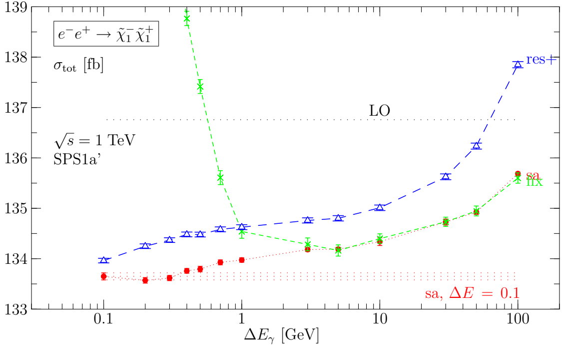
$\sigma(\sqrt{s})$



relative corrections

agrees with results in the literature (Fritzsche ea, Öller ea)

A closer look: ΔE_γ dependence of σ_{tot}



$\sigma_{tot}(\Delta E_\gamma)$:

semianalytic tests soft photon approx shift: 2 - 5 ‰ ($\Delta E_\gamma \leq 10\,\mathrm{GeV}$)

fixed order $|\mathcal{M}|^2 \leq 0$ effects for $\Delta E_\gamma \leq 3\,\mathrm{GeV}$

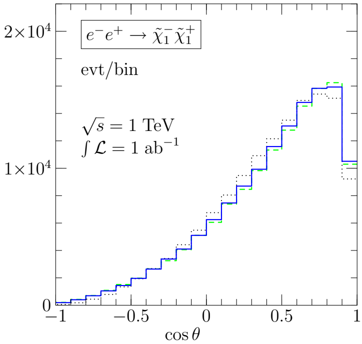
resummation includes higher order effects, 5‰ difference to 'sa' for $\Delta E_\gamma \leq 10\,\mathrm{GeV}$

In summary:

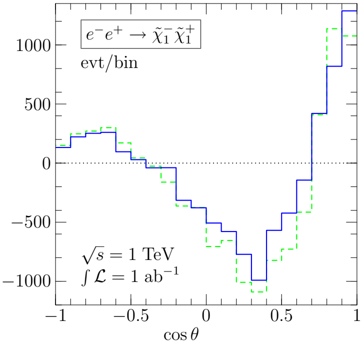
shift in ΔE_γ leads to ‰ effects, match ILC accuracy

$\Rightarrow$ careful choice of ΔE_γ, method important

simulation results: angular distributions



N_{tot}



$N_i - N_{\text{Born}}$

Born, fixed order, resummation

!! more than 1 σ deviation !! $\sqrt{n_{\text{max}}} \approx \mathcal{O}(10^2)$; nbins $= 20$

Part II:
Production and decay at leading order

(Status report)

Signal and (MS)SM backgrounds

- Charginos: unstable particles ⇒ also consider decays
- here: leptonic decay mode

$$e^+ e^- \longrightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \longrightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 e^- \mu^+ \nu_\mu \bar{\nu}_e$$

signal: $e^- \mu^+ + \mathbf{E_{miss}}$

- main SM backgrounds: (W (pair)production, τ pair production)

$$e^+ e^- \longrightarrow \text{anything} \longrightarrow e^- \mu^+ \nu_\mu \bar{\nu}_e (\nu_\tau \bar{\nu}_\tau)$$

- main SUSY backgrounds

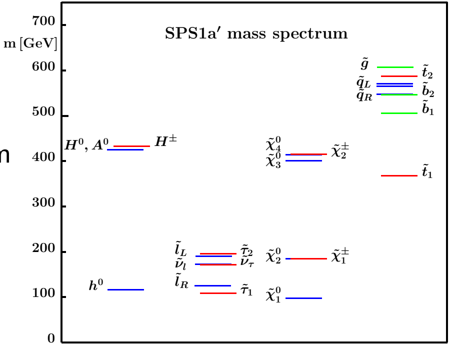
$$e^+ e^- \longrightarrow \text{anything} \longrightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 e^- \mu^+ \nu_\mu \bar{\nu}_e (\nu_\tau \bar{\nu}_\tau)$$

reducible/ irreducible background

Point SPS1a'

- mSUGRA scenario
- according to Snowmass Points (Allanach ea, 02), in agreement with cosmology data/ WMAP ($\tilde{\chi}_1^0$ as DM candidate)

spectrum
$\Longrightarrow$



light sleptons
heavy squarks
some light $\tilde{\chi}$s
all masses < 1 TeV

Signal and background: Total cross sections

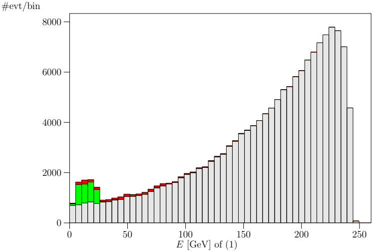
For SPS1a', $\sqrt{s} = 500\,\mathrm{GeV}$

$\sigma_{\text{signal}} = (3.08 \pm 0.08)\,\mathrm{fb}$	$\tilde{\chi}^{\pm}$ production and decay
$\sigma_{\text{SM, 4}} = (150.11 \pm 0.38)\mathrm{fb}$	W pair production
$\sigma_{\text{SM,6}} \sim 14\,\mathrm{fb}$	τ pair production (*)
$\sigma_{\text{SUSY, tot}} = (4.69 \pm 0.01)\,\mathrm{fb}$	signal $+$ irreducible background
$\sigma_{\text{SUSY, 8}} = (4.22 \pm 0.02)\,\mathrm{fb}$	reducible, $\tilde{\tau}$ pair production (*)

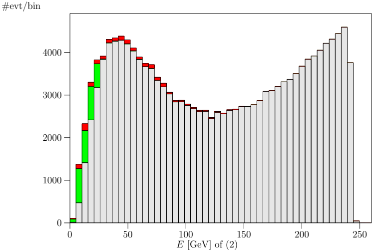
$(*)$: processes not yet included in analysis (remember the "preliminary")

SUSY/ background: 3 %, 70 % of SUSY signal from $\tilde{\chi}^{\pm}$ production

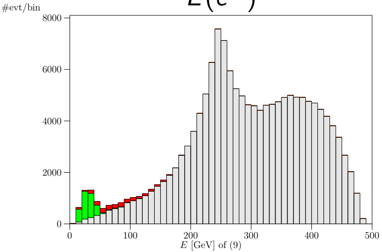
Energy distributions w/o cuts



$E(e^-)$



$E(\mu^+)$



$E(e^-) + E(\mu^+)$

single and combined lepton energies, no cuts
dominated by W production
signal, add background (SUSY)

Cross sections including cuts

Include cuts on lepton energies and emission angles:

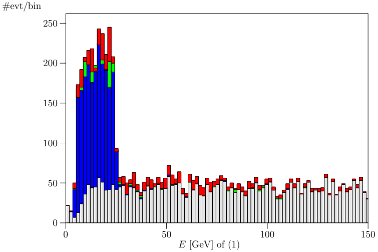
$$\cos\theta_e \stackrel{!}{<} 0.5,\ E(e) \stackrel{!}{<} 150\mathrm{GeV},\ E(\mu) \stackrel{!}{<} 150\,\mathrm{GeV},\ \cos\theta_{e,\mu} \stackrel{!}{\geq} 0.5$$

σ_{signal}	$(3.08 \pm 0.08)\,\mathrm{fb}$	$\longrightarrow$	$(1.83 \pm 0.05)\mathrm{fb}$
$\sigma_{\text{SM, 4}}$	$(150.11 \pm 0.38)\mathrm{fb}$	$\longrightarrow$	$(3.59 \pm 0.66)\mathrm{fb}$,
$\sigma_{\text{SUSY, tot}}$	$(4.69 \pm 0.01)\,\mathrm{fb}$	$\longrightarrow$	$(2.44 \pm 0.05)\mathrm{fb}$

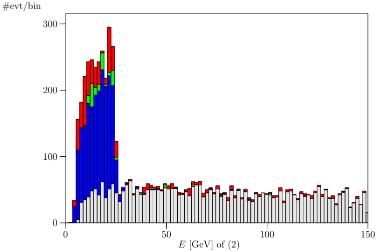
SUSY/ background: 68 % (3 %),
75 % of SUSY signal from $\widetilde{\chi}^{\pm}$ production (70 %)

Cuts significantly reduce SM background
still large number of events ($\int \mathcal{L} = 1\,\mathrm{ab}^{-1}$).

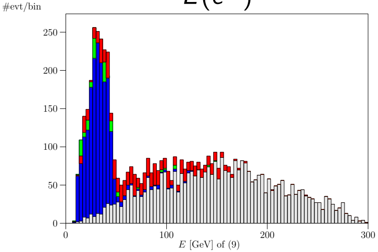
Energy distributions including cuts



$E(e^-)$



$E(\mu^+)$



$E(e^-) + E(\mu^+)$

single and combined lepton energies, with cuts
signal, add background (SUSY)

$\widetilde{\chi}^{\pm}$ and $\tilde{\nu}$ mass determination (1)

- SPS1a': $\widetilde{\chi}_1^{\pm}$ and $\tilde{\nu}_e$ nearly mass degenerate

$$m_{\widetilde{\chi}^{\pm}} = 183.67\,\mathrm{GeV}, \; m_{\tilde{\nu}} = 173.52\,\mathrm{GeV}$$

- $\tilde{\nu}$ decays to $\widetilde{\chi}^0$, ν: can only be observed indirectly
- determination from lepton energy (Freitas ea, 05):

$$m_{\widetilde{\chi}^{\pm}} = \sqrt{s}\,\frac{\sqrt{E_{\text{min}}\,E_{\text{max}}}}{E_{\text{min}} + E_{\text{max}}}, \; m_{\tilde{\nu}} = m_{\widetilde{\chi}^{\pm}}\sqrt{1 - \frac{2\,(E_{\text{min}} + E_{\text{max}})}{\sqrt{s}}},$$

$E_{\text{min, max}}$: edges of lepton energy distributions;
$\widetilde{\chi}_1^{\pm}$, $\tilde{\nu}_e$ are assumed onshell

$\tilde{\chi}^{\pm}$ and $\tilde{\nu}$ mass determination (2)

- read off:

$$E_{\text{min}} = 4.33 \pm 1.73\,\text{GeV}, \; E_{\text{max}} = 25.13 \pm 1.73\,\text{GeV}.$$

-

$$\begin{aligned} m_{\tilde{\chi}^{\pm}} &= 177.04 \pm 2.91\,\text{GeV}\ (183.67), \\ m_{\tilde{\nu}} &= 166.28 \pm 2.74\,\text{GeV}\ (173.52) \end{aligned}$$

- too low !! taking masses as input, obtain:

$$E_{\text{min}} = 4.32\,\text{GeV}, \; E_{\text{max}} = 22.54\,\text{GeV}$$

- read off maximal energy $E_{\text{max}} = 25.13 \pm 1.73\,\text{GeV}$ too large
- offshell effect; still needs further investigation (work in progress)
- take $m_{\tilde{\chi}^{\pm}}$ from threshold scan:

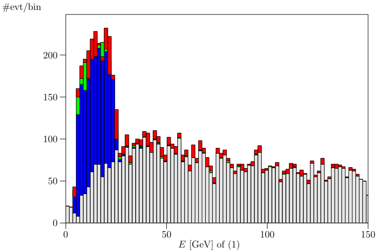
$$m_{\tilde{\nu}} = 172.51 \pm 2.74\,\text{GeV} \quad \checkmark$$

Inclusion of ISR and beamstrahlung

Turn on initial state radiation and beamstrahlung

$$\begin{array}{llll} \sigma_{\text{signal}} & (1.83 \pm 0.05)\text{fb} & \longrightarrow & (1.54 \pm 0.04)\text{fb}, \\ \sigma_{\text{SM, 4}} & (3.59 \pm 0.66)\text{fb} & \longrightarrow & (5.76 \pm 0.08)\text{fb} \\ \sigma_{\text{SUSY, tot}} & (2.44 \pm 0.05)\text{fb} & \longrightarrow & (2.13 \pm 0.05)\text{fb} \end{array}$$

SM background enhanced, SUSY/ background: 37% (68%) $\Rightarrow$ still large



$E(e^-)$

signal, background (SUSY)

mass determination:

$E_{\text{min, max}}$ as before

same conclusions

$\sqrt{s}$ **and parameter point dependent !!**

Summary and Outlook

- Chargino/ neutralino sector of MSSM: high precision in SUSY paramater analysis at EW scale (‰ at ILC)
- same size/ larger NLO corrections

$\Rightarrow$ include NLO results in Monte Carlo Event generators

- successfull inclusion for NLO $e^+e^- \to \widetilde{\chi}^+\widetilde{\chi}^-$ in WHIZARD
- resummation method for photons evades negative weight problem
- NLO as well as higher order contributions significant !!
- started to look at phenomenology of full process at LO including ISR and beamstrahlung
- clear SUSY signal, mass determination for $\tilde{\nu}$ possible
- **however:** some dominant background missing
- offshell effects on mass determination still need investigation

Superpotential and breaking parts

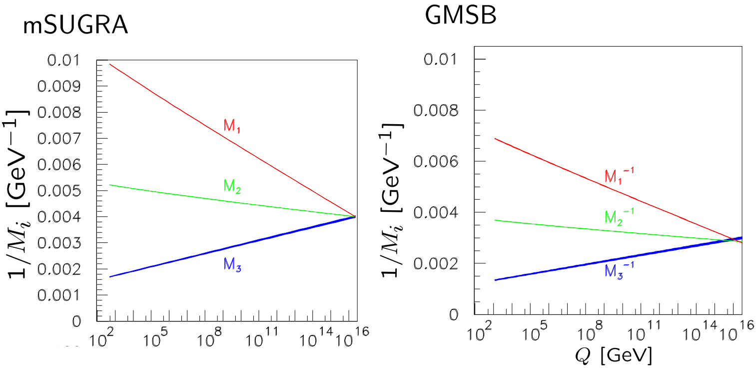
- Superpotential in MSSM

$$W = \bar{u} y_u Q H_u - \bar{d} y_d Q H_d - \bar{e} y_e L H_d + \mu H_u H_d$$

- soft SUSY breaking terms, gauge sector

$$\frac{1}{2}(M_1 \widetilde{B}\widetilde{B} + M_2 \widetilde{W}^a \widetilde{W}^a + M_3 \tilde{g}\tilde{g}) + h.c.$$

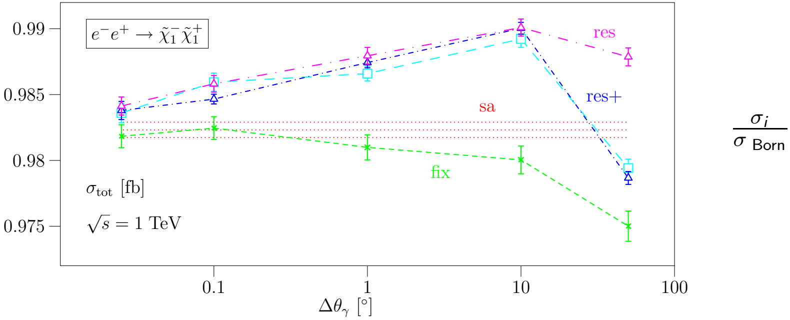
Mass unification in mSUGRA and GMSB



Blair ea, 02

cut dependencies: $\Delta\theta_\gamma$

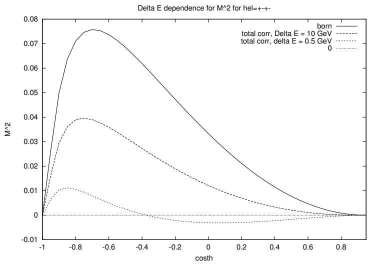
tests: collinear photon approximation



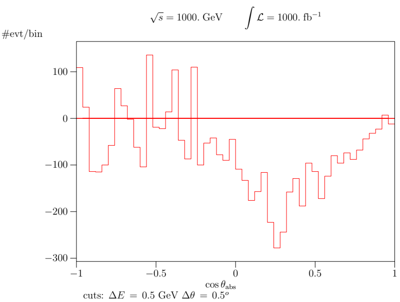
σ_{tot} again larger for resummation method
for higher angles: second order ISR effects between 0.05^o and 0.1^o ($\mathcal{O}$(‰))

Angular distribution: Do we see $|\mathcal{M}|^2 < 0$ effects ?? ($\checkmark$)

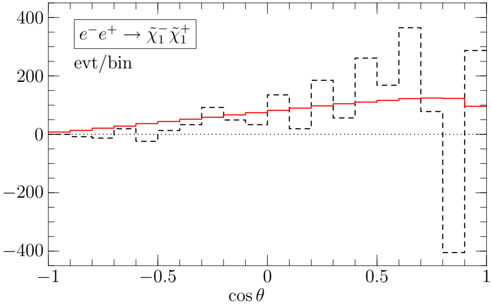
Reminder:
$|\mathcal{M}_{eff}|^2$ behaviour
$(\Delta E_{low} = 0.5 \text{ GeV})$:



angular distribution:



Angular distributions: higher orders



$N_{\mathrm{res},+} - N_{\mathrm{fix}}$
red: 1 standard dev from Born result

N_{res}^{+}: resummation, additionaly $2 \to 3$ folded w ISR; most complete

also higher order contributions statistically significant

η, f_s, hard collinear approximation, $ISR^{\mathcal{O}(\alpha)}$

- $\eta = \frac{2\alpha}{\pi}\left(\log\left(\frac{Q^2}{m_e^2}\right) - 1\right)$ (Q = scale of process)
- $$f_s = -\frac{\alpha}{2\pi} \sum_{i,j = e^\pm} \int_{|\mathbf{k}| \leq \Delta \mathbf{E}} \frac{d^3k}{2\omega_k} \frac{(\pm)\, p_i\, p_j\, Q_i\, Q_j}{p_i\, k\, p_j\, k},$$

  (Denner 1992)

  $\omega_k = \sqrt{\mathbf{k^2} + \lambda^2}$, p_i initial/ final state momenta, k: γ momentum
- hard collinear factor ($\pm$ helicity conserving/ flipping):

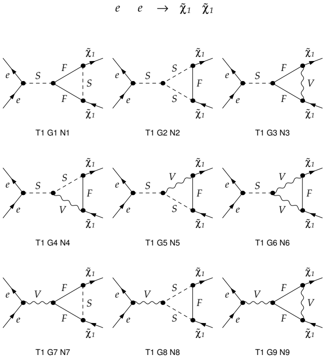
  $$f^+(x) = \frac{\alpha}{2\pi} \frac{1+x^2}{(1-x)} \left(\ln\left(\frac{s\,(\Delta\theta)^2}{4\,m^2}\right) - 1\right), f^-(x) = \frac{\alpha}{2\pi} x.$$

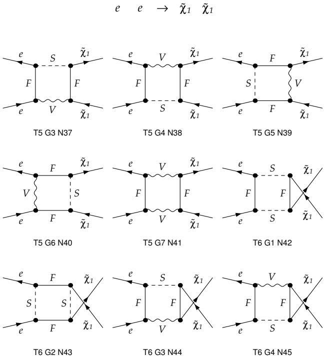
  (Dittmaier 1993)
- $$f_s^{ISR,\mathcal{O}(\alpha)} = \left[\int_{x_0}^{1} f_{ISR}(x)\, dx\right]_{\mathcal{O}(\alpha)} = \frac{\eta}{4}\left(2\ln(1-x_0) + x_0 + \frac{1}{2}x_0^2\right)$$

ISR in its full beauty (Skrzypek ea, 91)

$$
\begin{aligned}
\Gamma_{ee}^{LL}(x, Q^2) \;=\; & \frac{\exp(-\frac{1}{2}\eta\gamma_E + \frac{3}{8}\eta)}{\Gamma(1+\frac{\eta}{2})}\,\frac{\eta}{2}\,(1-x)^{(\frac{\eta}{2}-1)} \\
- \;& \frac{\eta}{4}\,(1+x) + \frac{\eta^2}{16}\left(-2\,(1-x)\log(1-x) - \frac{2\log x}{1-x} + \frac{3}{2}\,(1+x)\log x - \frac{x}{2}\right. \\
- \;& \left.\frac{5}{2}\right) + \left(\frac{\eta}{2}\right)^3\left[-\frac{1}{2}(1+x)\left(\frac{9}{32} - \frac{\pi^2}{12} + \frac{3}{4}\log(1-x) + \frac{1}{2}\log^2(1-x)\right.\right. \\
- \;& \left.\frac{1}{4}\log\,x\log(1-x) + \frac{1}{16}\log^2 x - \frac{1}{4}\mathrm{Li}_2(1-x)\right) \\
+ \;& \frac{1}{2}\,\frac{1+x^2}{1-x}\left(-\frac{3}{8}\log x + \frac{1}{12}\log^2 x - \frac{1}{2}\log x\log(1-x)\right) \\
- \;& \left.\frac{1}{4}\,(1-x)\left(\log(1-x) + \frac{1}{4}\right) + \frac{1}{32}\,(5-3x)\log x\right]\;;\;\eta \;=\; \frac{2\,\alpha}{\pi}\left(\log\left(\frac{Q^2}{m_e^2}\right) - 1\right)
\end{aligned}
$$

Some NLO matrix elements





Some NLO matrix elements