Soft and collinear enhancements to top quark and Higgs cross sections

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- **Soft and collinear corrections**
- **Resummation and NNNLO expansions**
- **Single top production -**

  t **and** s **channels and** tW **production**
- **Higgs production via** $b\bar{b} \to H$

Soft and collinear gluon corrections

Incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (low-energy) gluons

For the process $p_1 + p_2 \to p_3 + p_4$

define $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, $u = (p_2 - p_3)^2$ and $s_4 = s + t + u - m_3^2 - m_4^2$, $z = Q^2/s$

At threshold $s_4 \to 0$ (1PI) or $z \to 1$ (PIM)

Soft corrections $\left[\frac{\ln^k(s_4/M^2)}{s_4}\right]_+$ or $\left[\frac{\ln^k(1-z)}{1-z}\right]_+$

$k \leq 2n - 1$ for the $\mathcal{O}(\alpha_s^n)$ corrections

Resum (exponentiate) these soft corrections

At NLL accuracy requires one-loop calculations in the eikonal approximation

Also purely collinear corrections $\ln^k(s_4/M^2)$ or $\ln^k(1-z)$

Soft and collinear gluon corrections

The n-th order corrections in the partonic cross section

$$\hat{\sigma}^{(n)}(z) = V^{(n)}\,\delta(1-z) + \sum_{k=0}^{2n-1} S_k^{(n)} \left[\frac{\ln^k(1-z)}{1-z}\right]_+ + \sum_{k=0}^{2n-1} C_k^{(n)} \ln^k(1-z)$$

Near threshold soft corrections are dominant and provide excellent approximations to the full cross section

Examples: top pair and single top production

jet, direct photon, or W production at high p_T

In other cases purely collinear corrections also have to be included to get a good approximation (e.g. Higgs production).

The hadronic cross section

$$\sigma = \sum_f \int dx_1 dx_2\, \phi_{f_1/p}(x_1,\mu_F)\, \phi_{f_2/\bar{p}}(x_2,\mu_F)\, \hat{\sigma}(s,t,u,\mu_F,\mu_R,\alpha_s)$$

Resummed cross section

Resummation follows from factorization properties of the cross section

- performed in moment space

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E^{f_i}(N_i)\right] \exp\left[\sum_j E'^{f_j}(N_j)\right] \exp\left[\sum_i 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\, \gamma_{i/i}\left(N_i, \alpha_s(\mu)\right)\right]$$

$$\times \exp\left[\sum_i 2d_{\alpha_s} \int_{\mu_R}^{\sqrt{s}} \frac{d\mu}{\mu}\, \beta\left(\alpha_s(\mu)\right)\right] H^{f_i f_j}\left(\alpha_s(\mu_R)\right)$$

$$\times \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_j} \frac{d\mu}{\mu}\, \Gamma_S^{\dagger\, f_i f_j}\left(\alpha_s(\mu)\right)\right] \tilde{S}^{f_i f_j}\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}_j}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_j} \frac{d\mu}{\mu}\, \Gamma_S^{f_i f_j}\left(\alpha_s(\mu)\right)\right]$$

where

$$\sum_i E^{f_i}(N_i) = -\sum_i C_i \int_0^1 dz \frac{z^{N_i-1}-1}{1-z} \left\{\int_{(1-z)^2}^1 \frac{d\lambda}{\lambda} \frac{\alpha_s(\lambda s)}{\pi} + \frac{\alpha_s((1-z)^2 s)}{\pi}\right\} + \mathcal{O}(\alpha_s^2)$$

$$\sum_j E'^{f_j}(N_j) = \sum_j \int_0^1 dz \frac{z^{N_j-1}-1}{1-z} \left\{C_j \int_{(1-z)^2}^{1-z} \frac{d\lambda}{\lambda} \frac{\alpha_s(\lambda s)}{\pi} - B_j^{(1)} \frac{\alpha_s((1-z)s)}{\pi} - C_j \frac{\alpha_s((1-z)^2 s)}{\pi}\right\} + \mathcal{O}(\alpha_s^2)$$

$C_i = C_F = (N_c^2-1)/(2N_c)$, $B_q^{(1)} = 3C_F/4$ **for quarks;** $C_i = C_A = N_c$, $B_g^{(1)} = \beta_0/4$ **for gluons**

Γ_S **is the soft anomalous dimension - a matrix in color space**

NNNLO expansions of resummed cross section

Invert back to momentum space and expand to arbitrary order

NLO soft and collinear gluon corrections

$$\hat{\sigma}^{(1)} = F^B \frac{\alpha_s(\mu_R^2)}{\pi} \left\{ c_3 \left[\frac{\ln(1-z)}{1-z}\right]_+ + c_2 \left[\frac{1}{1-z}\right]_+ + c_1^\mu \, \delta(1-z) \right.$$
$$\left. + c_3^c \ln(1-z) + c_2^c \right\}$$

NNLO soft gluon corrections

$$\hat{\sigma}^{(2)} = F^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ \frac{1}{2} c_3^2 \left[\frac{\ln^3(1-z)}{1-z}\right]_+ + \cdots + \frac{1}{2} c_3 c_3^c \ln^3(1-z) + \cdots \right\}$$

NNNLO soft gluon corrections

$$\hat{\sigma}^{(3)} = F^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{ \frac{1}{8} c_3^3 \left[\frac{\ln^5(1-z)}{1-z}\right]_+ + \cdots + \frac{1}{8} c_3^2 c_3^c \ln^5(1-z) + \cdots \right\}$$

Top quark production

Dominant process is pair production $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$

Very good agreement of theory (with soft-gluon corrections) with Tevatron data

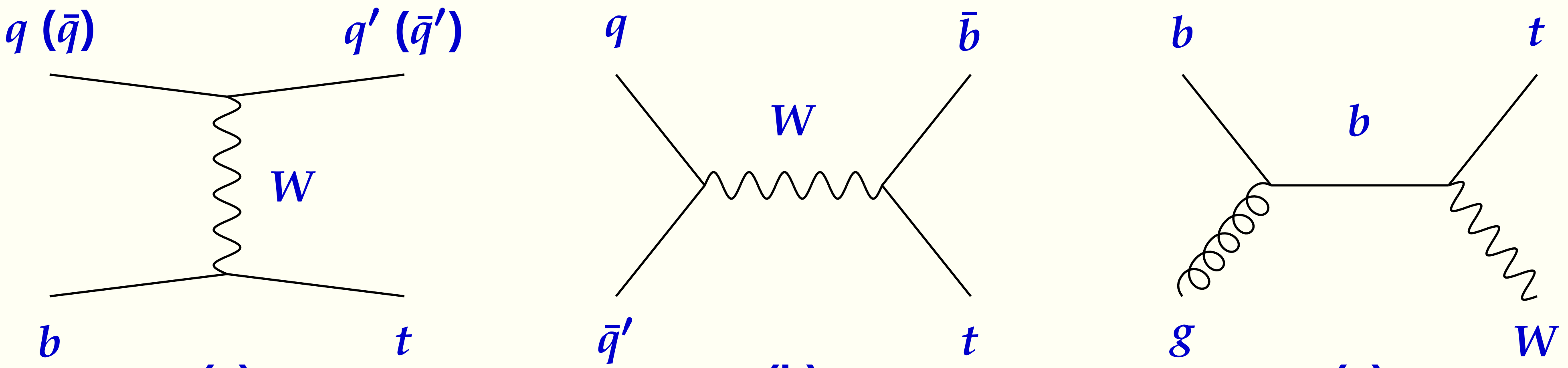
Recent evidence for single top production - cross section consistent with theory

Opportunities for study of electroweak properties of the top

Top quark mass value lowered from $\sim$ 175 GeV to $\sim$ 170 GeV

Single top quark production

Partonic processes at LO



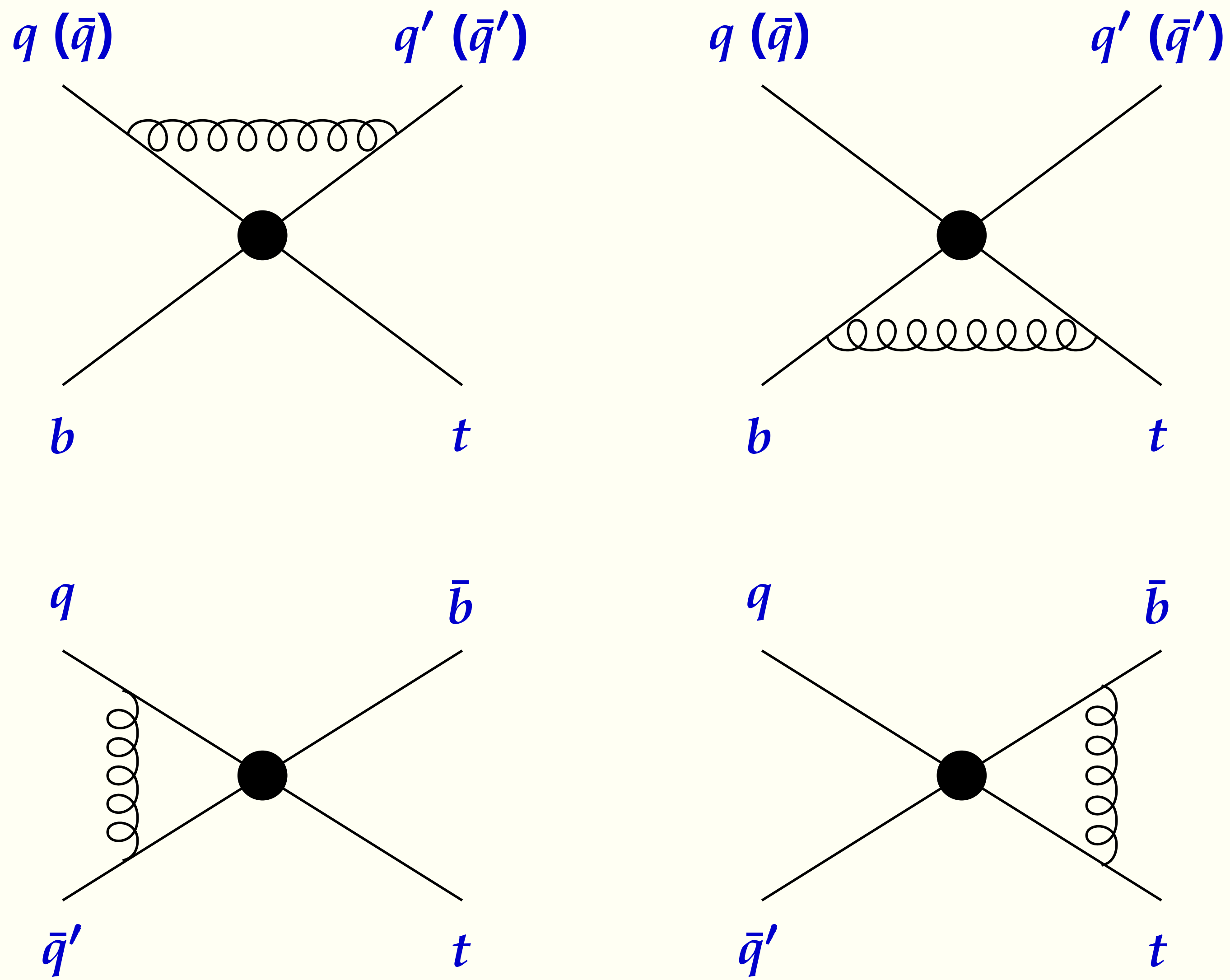
(a) (b) (c)

(a) t channel: $qb \to q't$ and $\bar{q}b \to \bar{q}'t$ ($ub \to dt$ and $\bar{d}b \to \bar{u}t$, etc.)

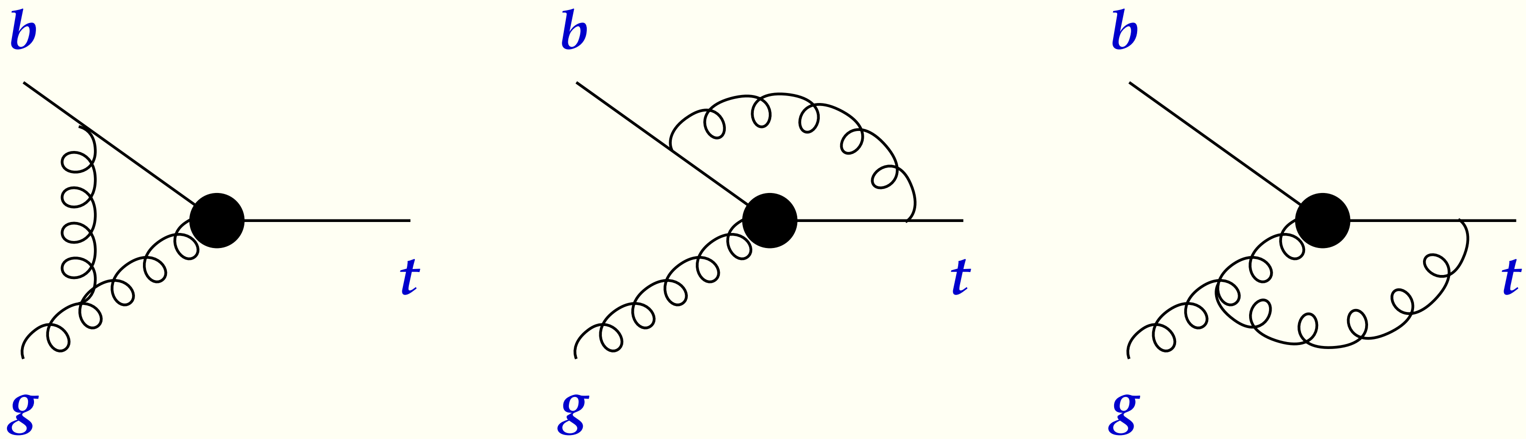
(b) s channel: $q\bar{q}' \to \bar{b}t$ ($u\bar{d} \to \bar{b}t$, etc)

(c) associated tW production: $bg \to tW^-$

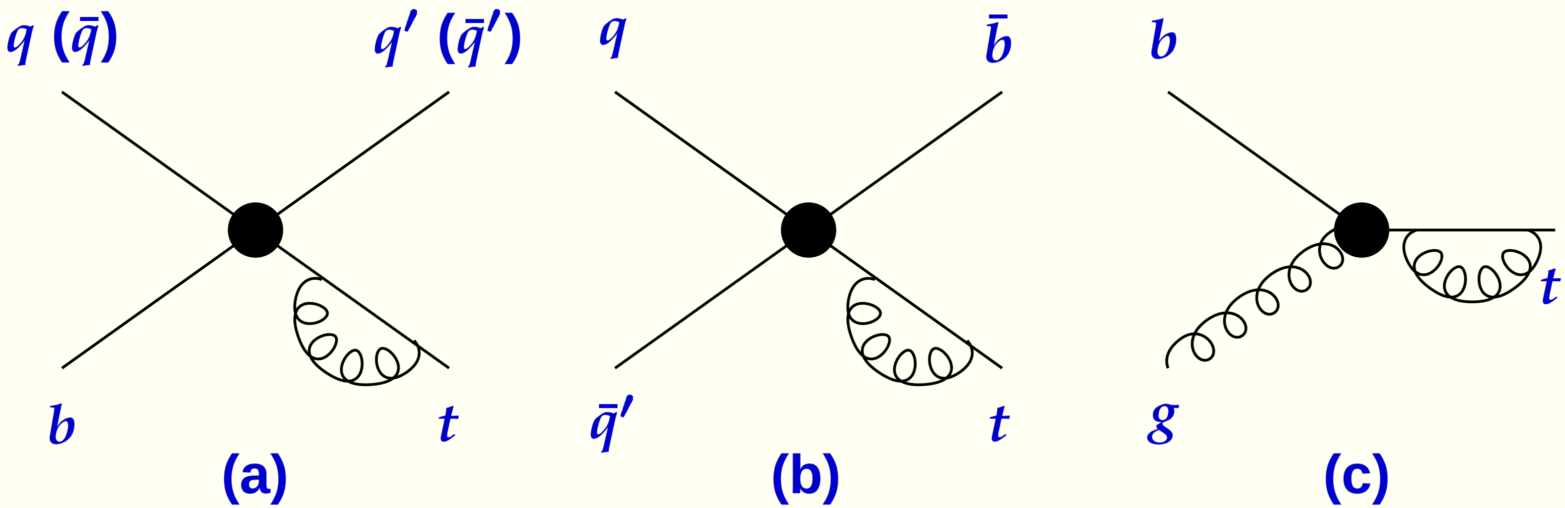
One-loop eikonal vertex corrections to the soft function in the t and s channels



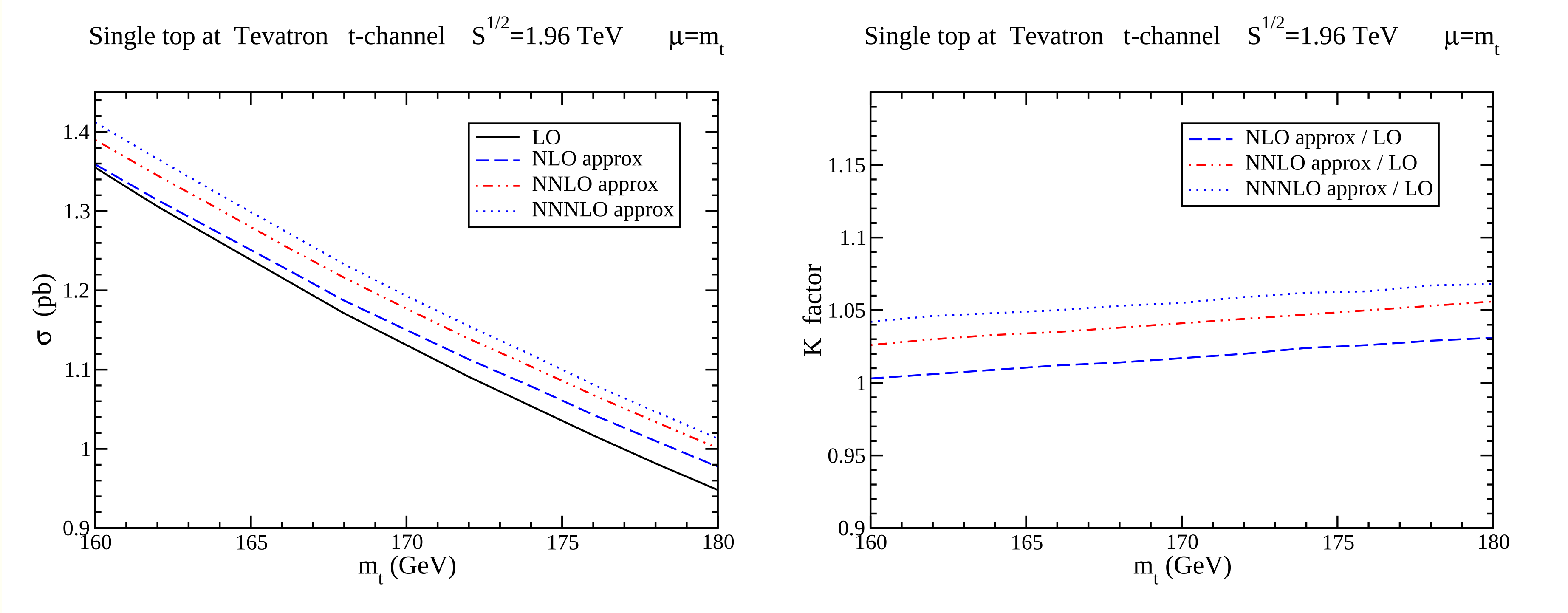
One-loop eikonal vertex corrections to the soft function in the tW channel



Top-quark eikonal self-energy one-loop corrections: (a) t channel; (b) s channel; (c) associated tW production



Single top production at the Tevatron - t channel



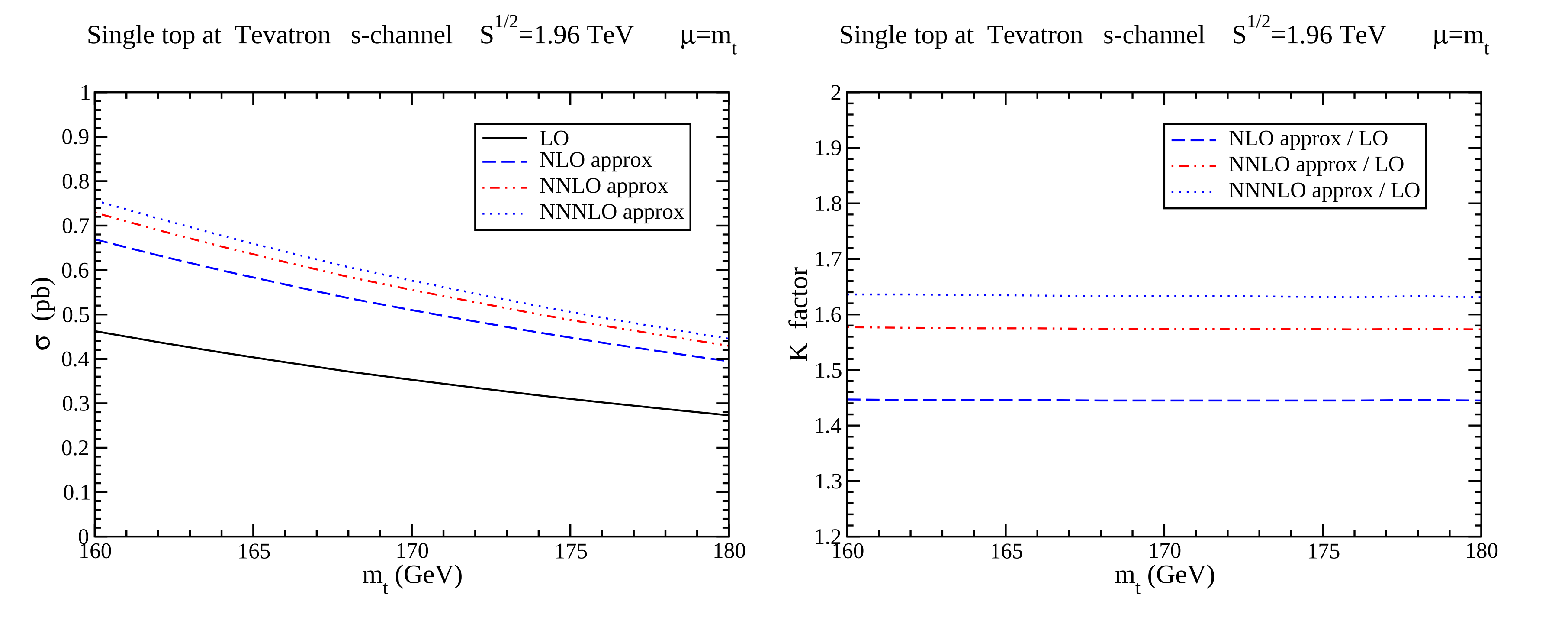
Matched cross section (exact NLO + soft gluon corrections through NNNLO)

$$\sigma^{t-\text{channel}}(m_t = 170\,\text{GeV}) = \underbrace{1.17^{+0.02}_{-0.01}}_{\text{scale}} \underbrace{\pm 0.06}_{\text{pdf}} \text{ pb} = 1.17 \pm 0.06 \text{ pb}$$

$$\sigma^{t-\text{channel}}(m_t = 175\,\text{GeV}) = \underbrace{1.08^{+0.02}_{-0.01}}_{\text{scale}} \underbrace{\pm 0.06}_{\text{pdf}} \text{ pb} = 1.08 \pm 0.06 \text{ pb}$$

Cross section for anti-top production is identical

Single top production at the Tevatron - s channel



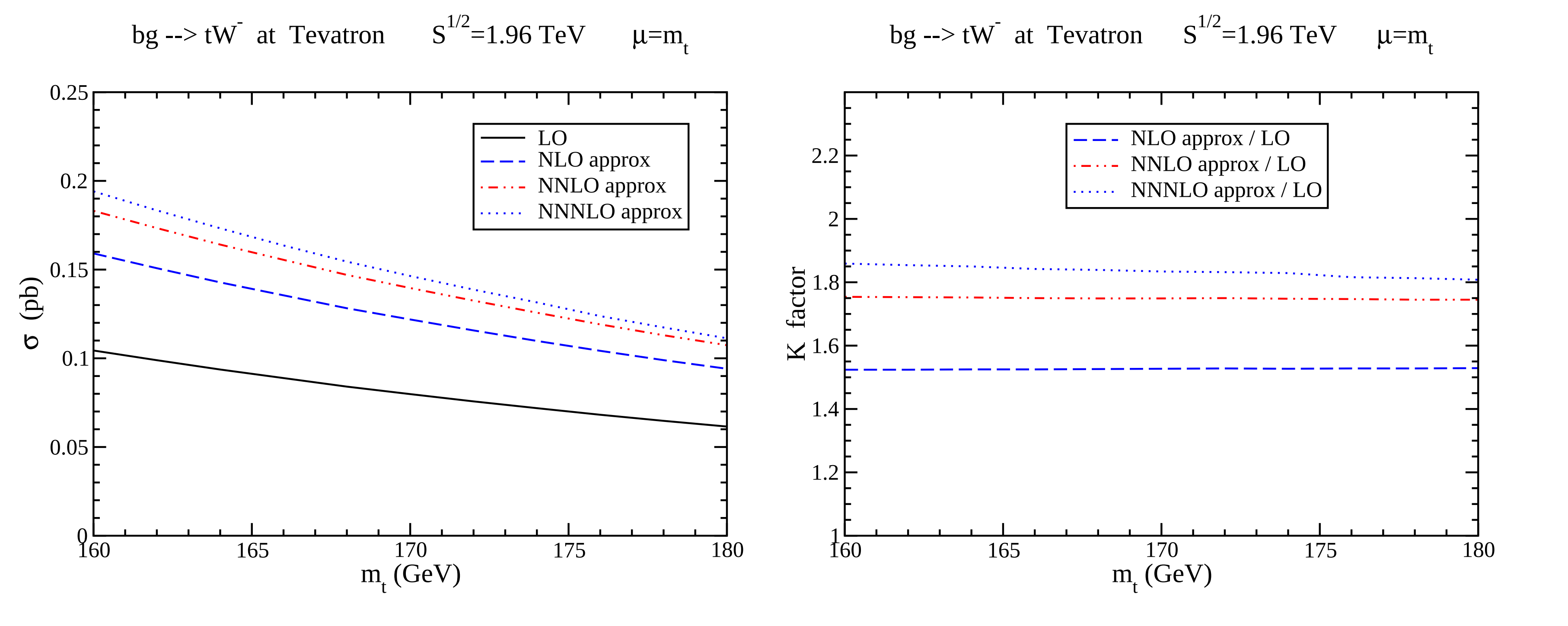
Matched cross section (exact NLO + soft gluon corrections through NNNLO)

$$\sigma^{s-\mathrm{channel}}(m_t = 170\,\mathrm{GeV}) = 0.56 \underset{\text{scale}}{\pm 0.02} \underset{\text{pdf}}{\pm 0.01}\ \mathrm{pb} = 0.56 \pm 0.03\ \mathrm{pb}$$

$$\sigma^{s-\mathrm{channel}}(m_t = 175\,\mathrm{GeV}) = 0.49 \underset{\text{scale}}{\pm 0.02} \underset{\text{pdf}}{\pm 0.01}\ \mathrm{pb} = 0.49 \pm 0.02\ \mathrm{pb}$$

Cross section for anti-top production is identical

Single top production at the Tevatron - tW channel



Approximate NNNLO cross section

$$\sigma^{tW}(m_t = 170\,\mathrm{GeV}) = \mathbf{0.15} \pm \underbrace{\mathbf{0.02}}_{\text{scale}} \pm \underbrace{\mathbf{0.03}}_{\text{pdf}}\ \mathbf{pb} = \mathbf{0.15} \pm \mathbf{0.03}\ \mathbf{pb}$$

$$\sigma^{tW}(m_t = 175\,\mathrm{GeV}) = \mathbf{0.13} \pm \underbrace{\mathbf{0.02}}_{\text{scale}} \pm \underbrace{\mathbf{0.02}}_{\text{pdf}}\ \mathbf{pb} = \mathbf{0.13} \pm \mathbf{0.03}\ \mathbf{pb}$$

Cross section for anti-top production is identical

Single top production at the LHC - t channel

Threshold corrections not a good approximation of full QCD corrections

Exact NLO cross section

$$\sigma_{\rm top}^{t-{\rm channel}}(m_t=170\,{\rm GeV}) = 152 \pm \underset{\text{scale}}{\underset{\uparrow}{5}} \pm \underset{\text{pdf}}{\underset{\uparrow}{3}}\ {\rm pb} = 152 \pm 6\ {\rm pb}$$

$$\sigma_{\rm top}^{t-{\rm channel}}(m_t=175\,{\rm GeV}) = 146 \pm \underset{\text{scale}}{\underset{\uparrow}{4}} \pm \underset{\text{pdf}}{\underset{\uparrow}{3}}\ {\rm pb} = 146 \pm 5\ {\rm pb}$$

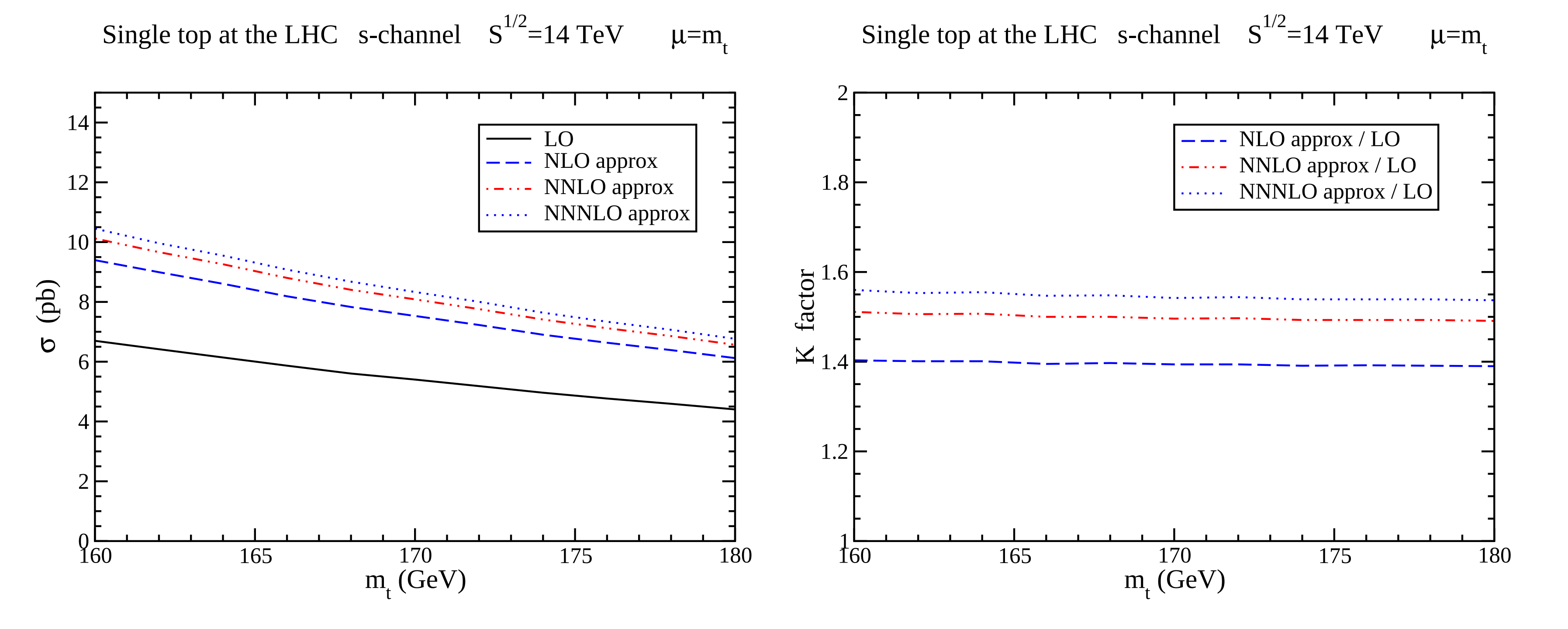
Antitop production at the LHC - t channel

Exact NLO cross section

$$\sigma_{\rm antitop}^{t-{\rm channel}}(m_t=170\,{\rm GeV}) = 93 \pm \underset{\text{scale}}{\underset{\uparrow}{3}} \pm \underset{\text{pdf}}{\underset{\uparrow}{2}}\ {\rm pb} = 93 \pm 4\ {\rm pb}$$

$$\sigma_{\rm antitop}^{t-{\rm channel}}(m_t=175\,{\rm GeV}) = 89 \pm \underset{\text{scale}}{\underset{\uparrow}{3}} \pm \underset{\text{pdf}}{\underset{\uparrow}{2}}\ {\rm pb} = 89 \pm 4\ {\rm pb}$$

Single top production at the LHC - s channel

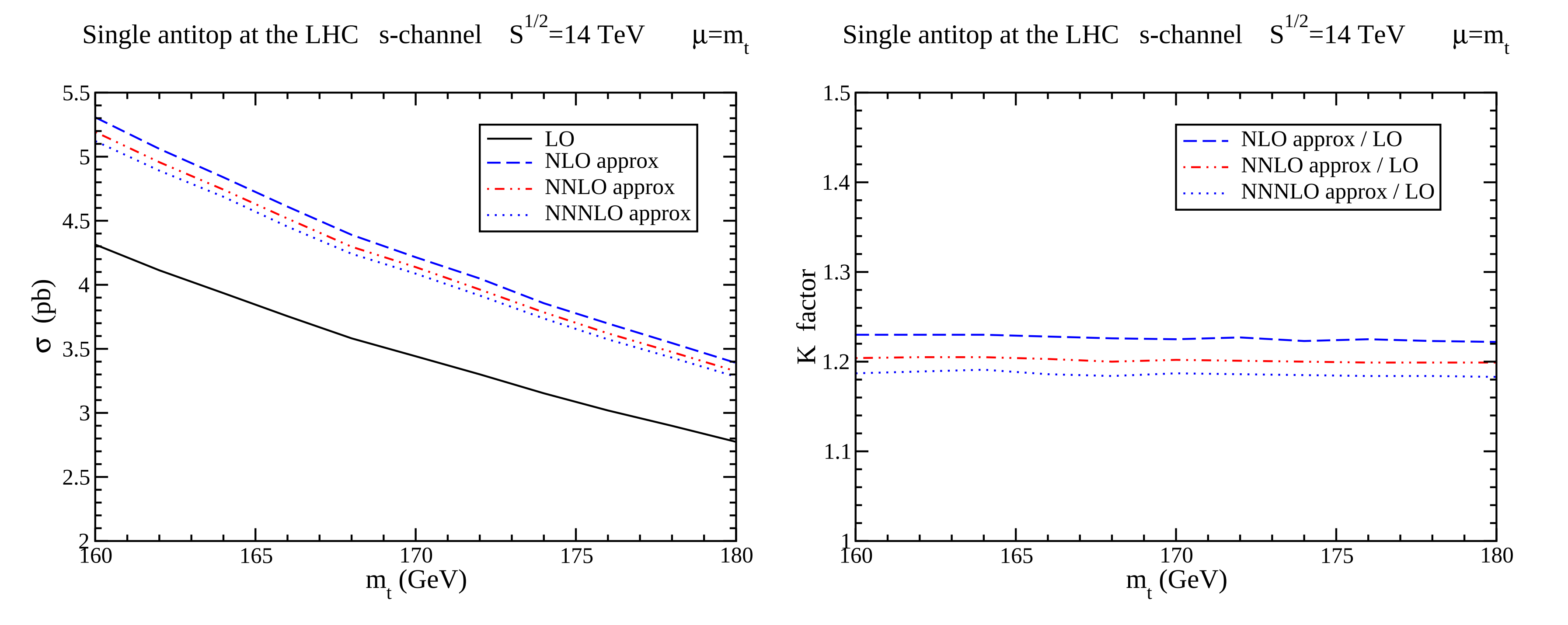


Matched cross section (exact NLO + soft gluon corrections through NNLO)

$$\sigma_{\rm top}^{s-\rm channel}(m_t = 170\,{\rm GeV}) = \underset{\substack{\uparrow \\ \text{scale}}}{8.0^{+0.6}_{-0.5}} \underset{\substack{\uparrow \\ \text{pdf}}}{\pm 0.1}\ {\rm pb} = 8.0^{+0.6}_{-0.5}\ {\rm pb}$$

$$\sigma_{\rm top}^{s-\rm channel}(m_t = 175\,{\rm GeV}) = \underset{\substack{\uparrow \\ \text{scale}}}{7.2^{+0.6}_{-0.5}} \underset{\substack{\uparrow \\ \text{pdf}}}{\pm 0.1}\ {\rm pb} = 7.2^{+0.6}_{-0.5}\ {\rm pb}$$

Single antitop production at the LHC - s channel

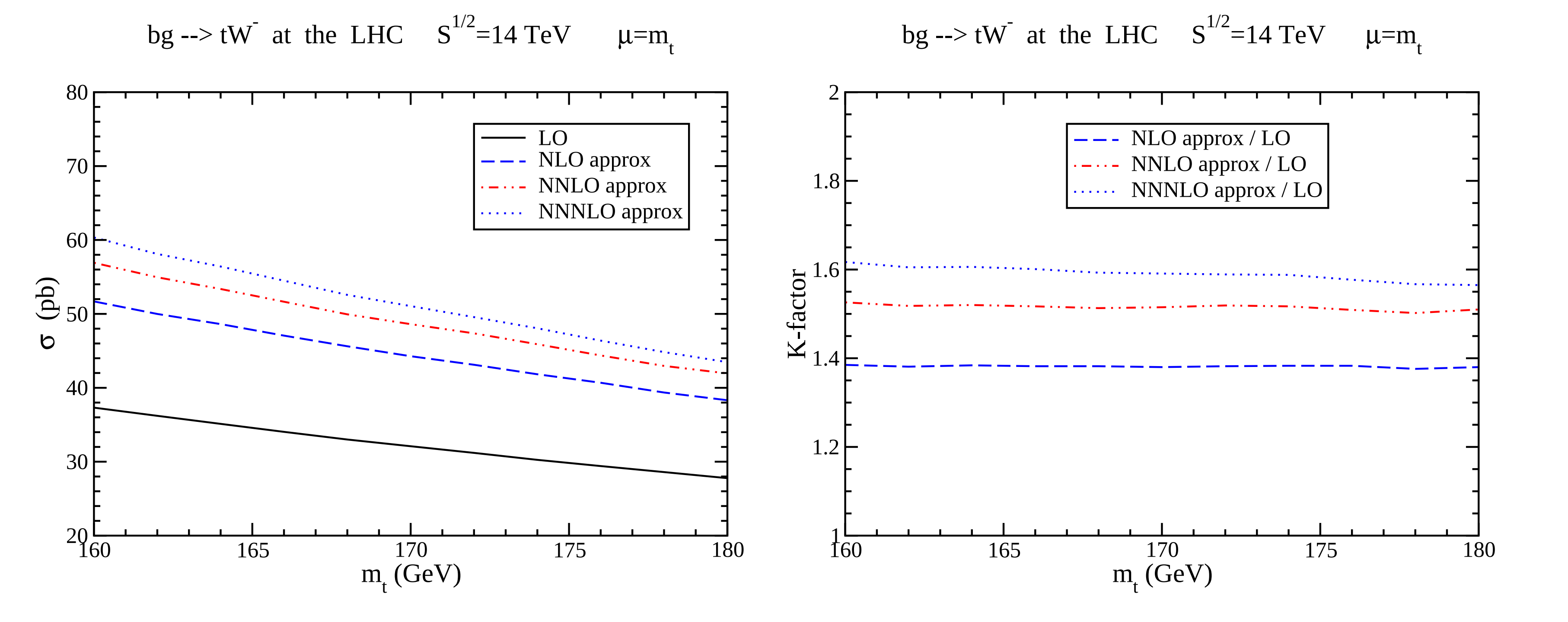


Matched cross section (exact NLO + soft gluon corrections through NNLO)

$$\sigma^{s-\text{channel}}_{\text{antitop}}(m_t = 170\,\text{GeV}) = 4.5 \pm \underset{\text{scale}}{\underset{\uparrow}{0.1}} \pm \underset{\text{pdf}}{\underset{\uparrow}{0.1}}\ \text{pb} = 4.5 \pm 0.2\ \text{pb}$$

$$\sigma^{s-\text{channel}}_{\text{antitop}}(m_t = 175\,\text{GeV}) = 4.0 \pm \underset{\text{scale}}{\underset{\uparrow}{0.1}} \pm \underset{\text{pdf}}{\underset{\uparrow}{0.1}}\ \text{pb} = 4.0 \pm 0.2\ \text{pb}$$

Single top production at the LHC - tW channel



Matched cross section (exact NLO + soft gluon corrections through NNNLO)

$$\sigma^{tW}(m_t = 170\,\text{GeV}) = 44 \underset{\text{scale}}{\underset{\uparrow}{\pm 5}} \underset{\text{pdf}}{\underset{\uparrow}{\pm 1}}\ \text{pb} = 44 \pm 5\ \text{pb}$$

$$\sigma^{tW}(m_t = 175\,\text{GeV}) = 41 \underset{\text{scale}}{\underset{\uparrow}{\pm 4}} \underset{\text{pdf}}{\underset{\uparrow}{\pm 1}}\ \text{pb} = 41 \pm 4\ \text{pb}$$

Cross section for anti-top production is identical

Higgs production at the Tevatron and the LHC

Search for Higgs one of most important goals at both colliders

Main Standard Model production channel $gg \to H$

$b\bar{b} \to H$ **important in MSSM at high** $\tan\beta$

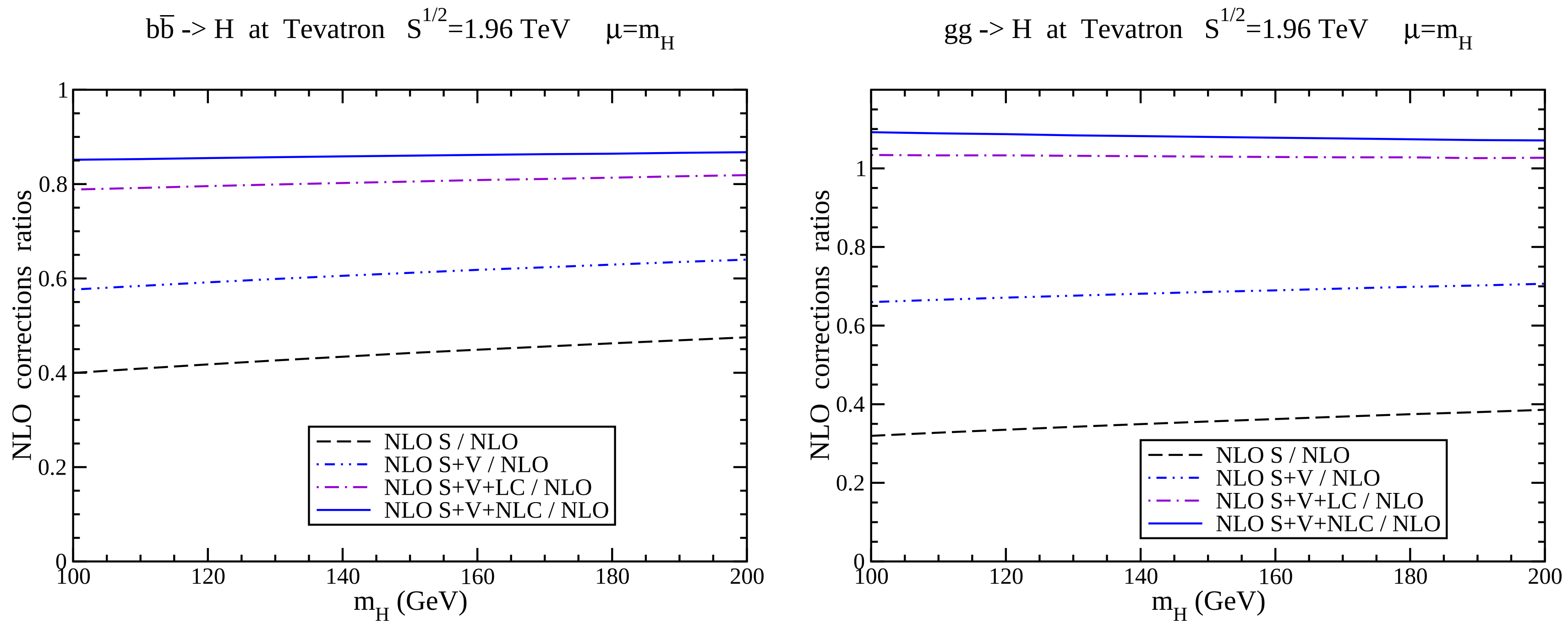
Very simple color structure and kinematics (like Drell-Yan)

Full corrections known to NNLO

Soft-gluon approximation inadequate - purely collinear terms must be added

Can calculate all soft corrections fully to NNNLO

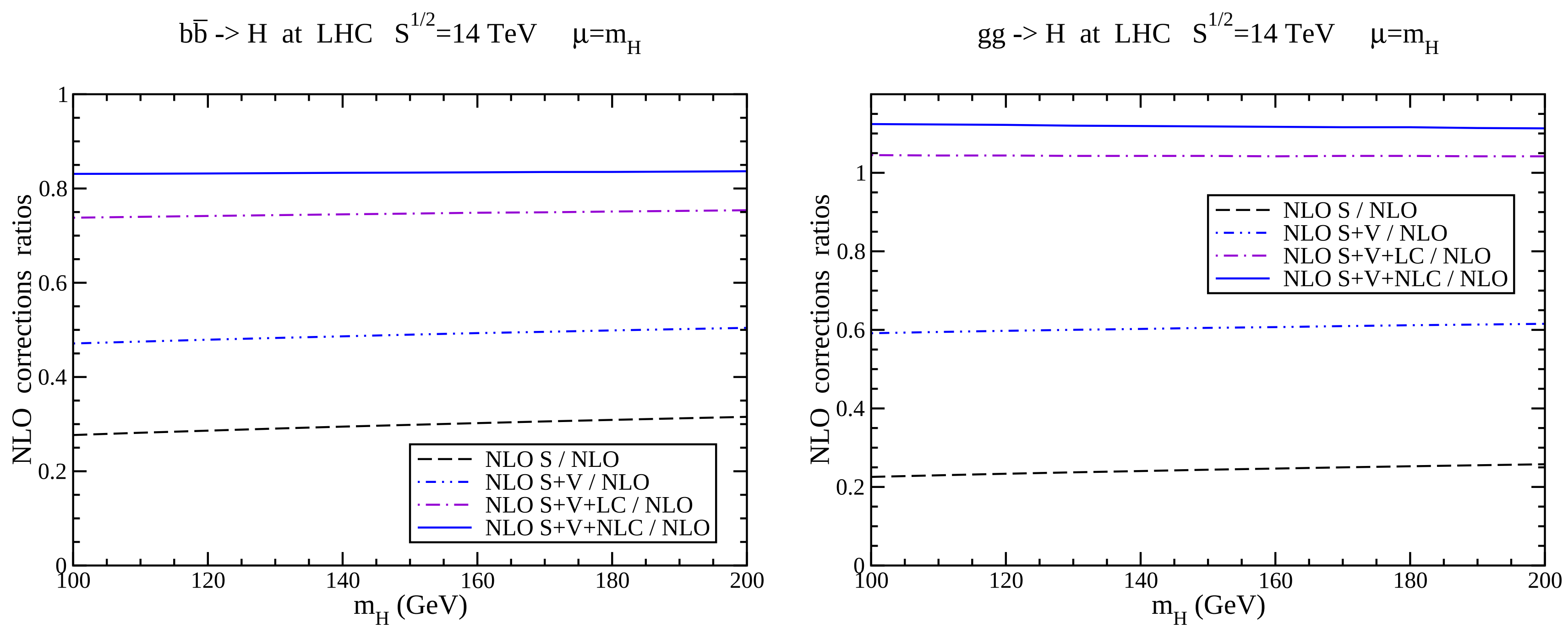
Higgs production at NLO at the Tevatron



soft (or soft+virtual) corrections inadequate

inclusion of collinear terms provides very good approximation

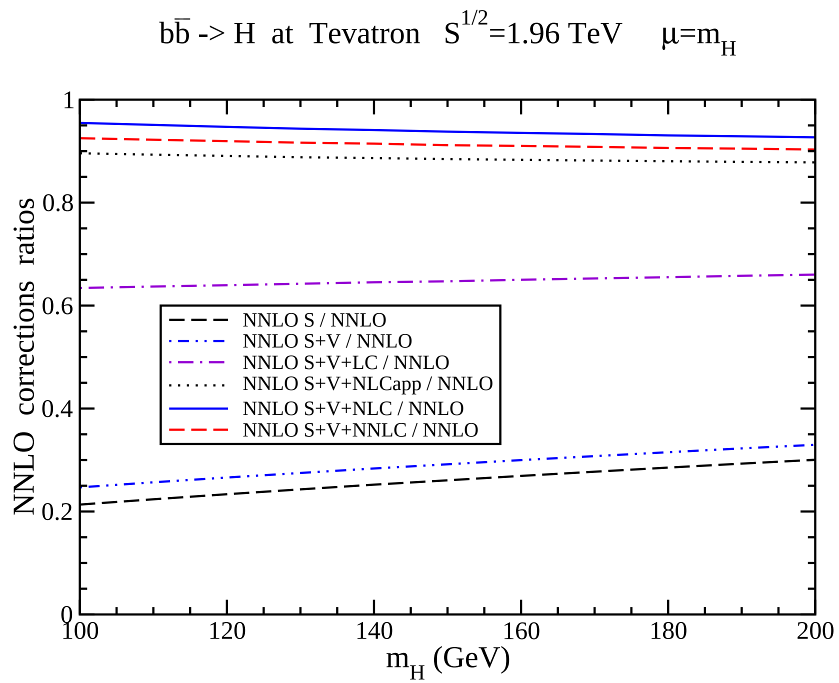
Higgs production at NLO at the LHC

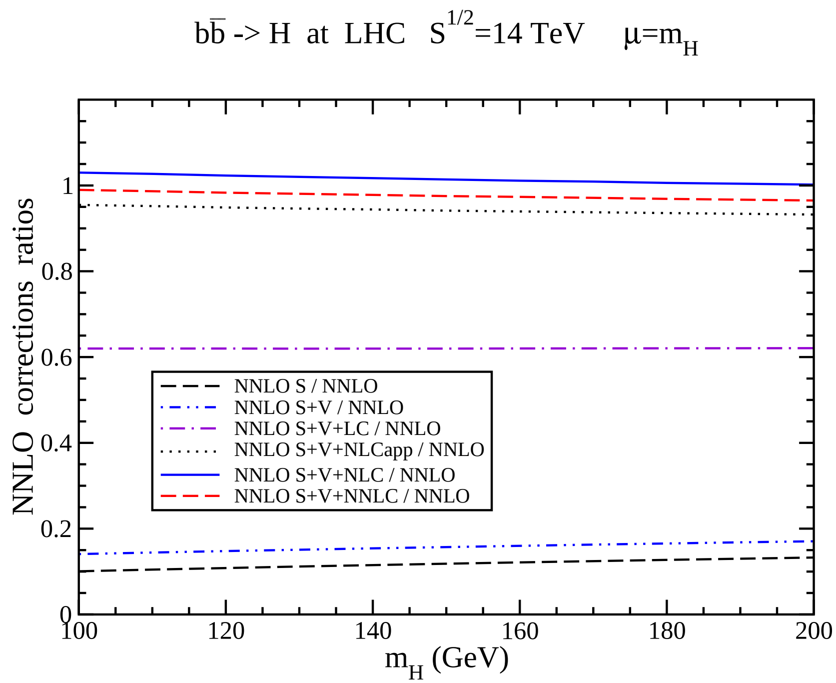


again soft (or soft+virtual) corrections (even more) inadequate

inclusion of collinear terms provides very good approximation

Higgs production at NNLO via $b\bar{b} \to H$

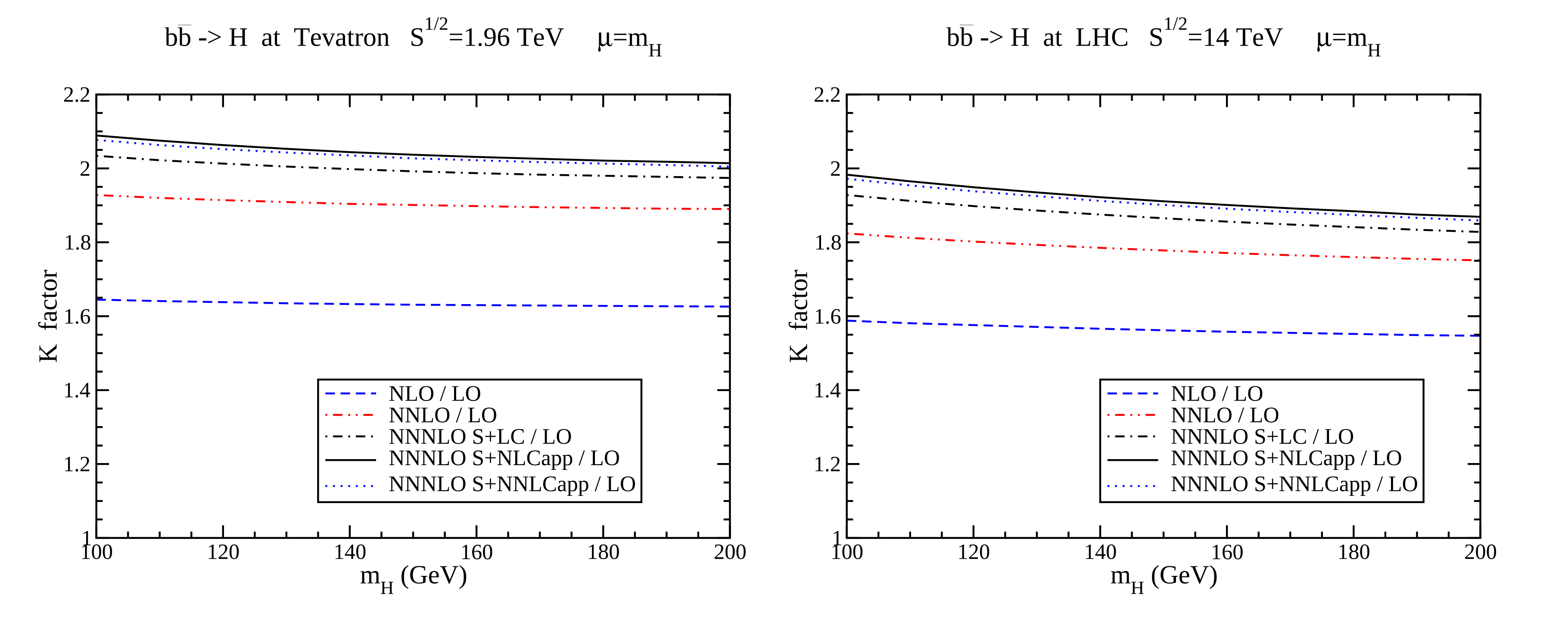




soft (or soft+virtual) corrections very inadequate

inclusion of collinear terms (especially at NL+ accuracy) provides excellent approximation

Higgs production via $b\bar{b} \to H$: K factors

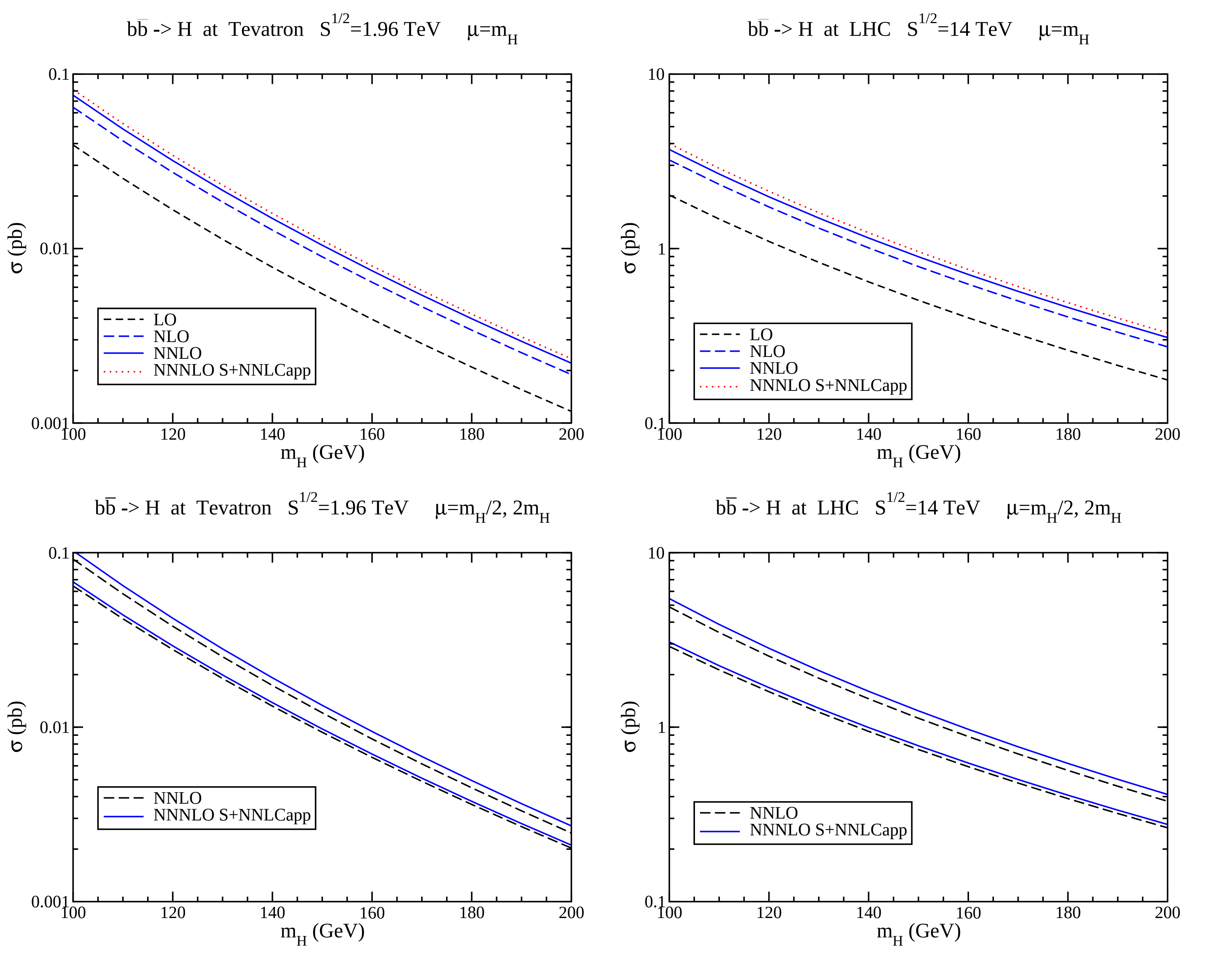


Large K factors even at NLO

NNLO and NNNLO corrections are important

Quite similar at Tevatron and LHC

Higgs production via $b\bar{b} \to H$: SM cross sections



Summary

- Soft and collinear corrections important in cross sections
- Resummation and NNNLO expansions
- Single top production - soft approximation works well
- Higgs production - collinear+soft approximation is excellent