Parton Shower as QCD Prediction

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CRACOW EPIPHANY CONFERENCE

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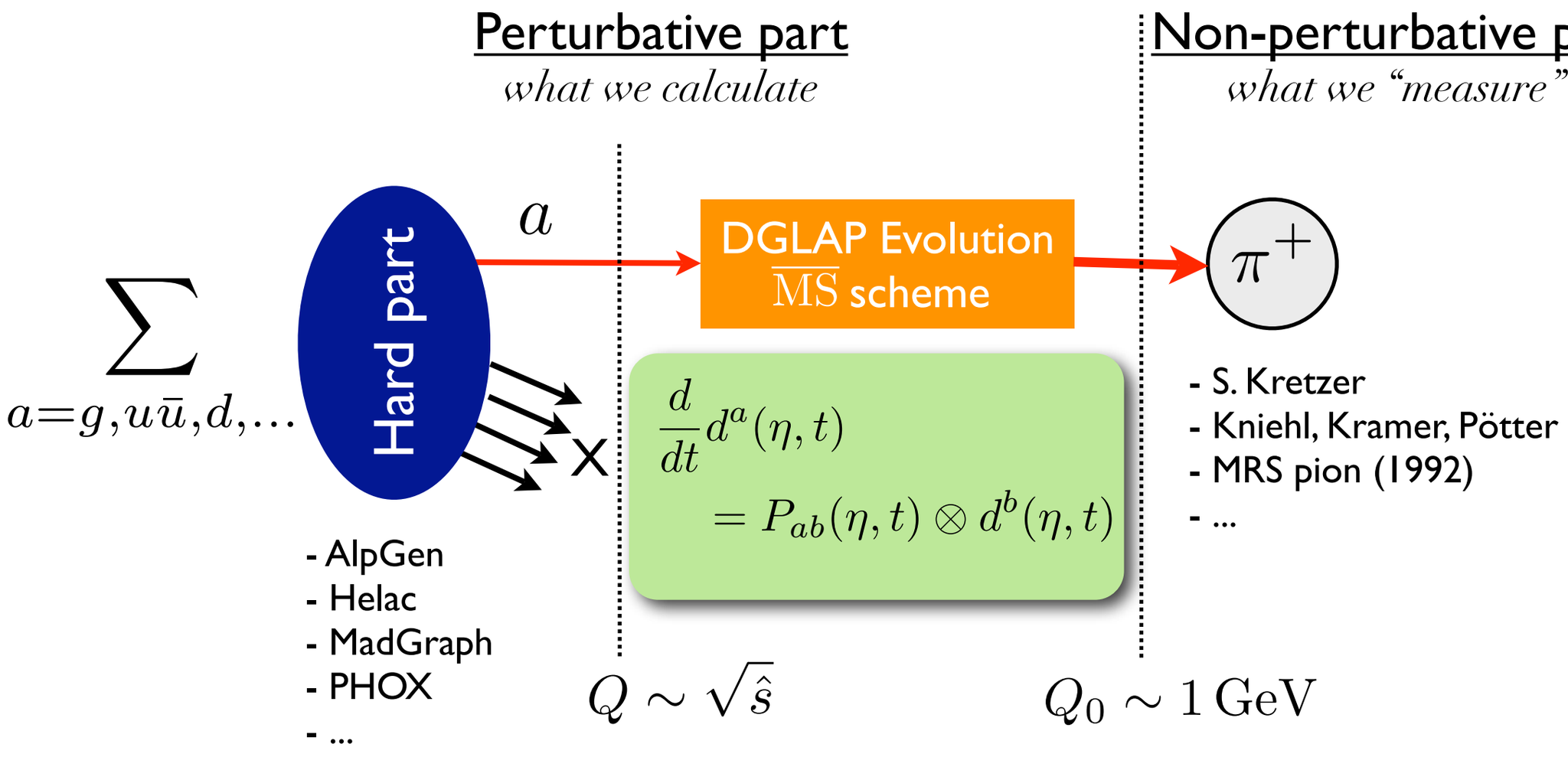
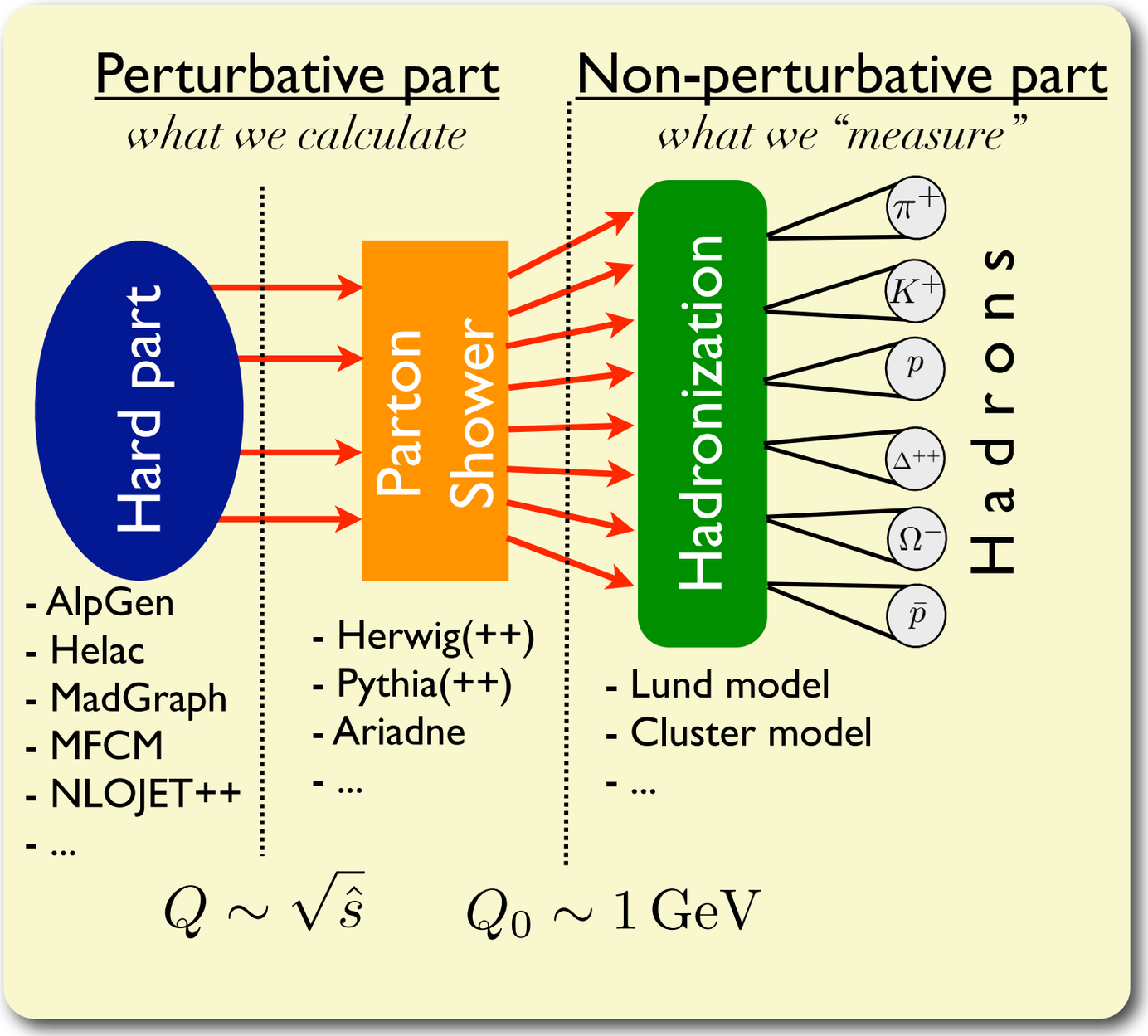
Introduction

Structure of the one-hadron inclusive cross section



Introduction

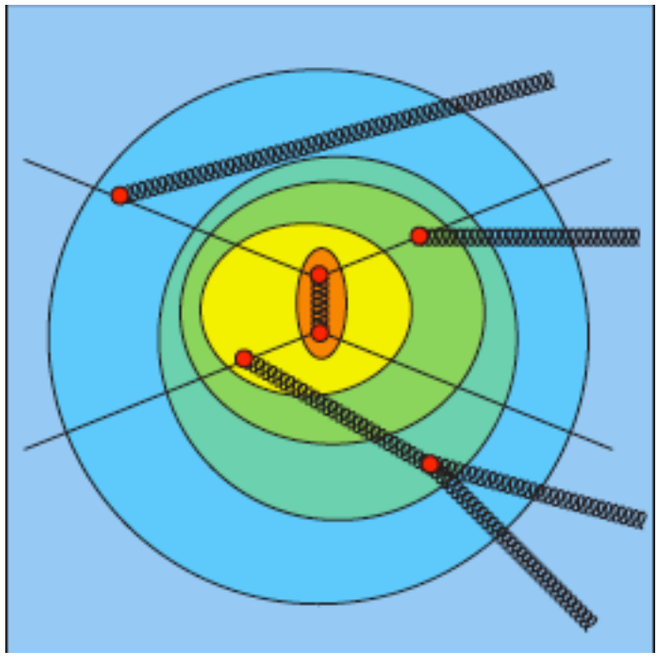
Structure of the Monte Carlo algorithm



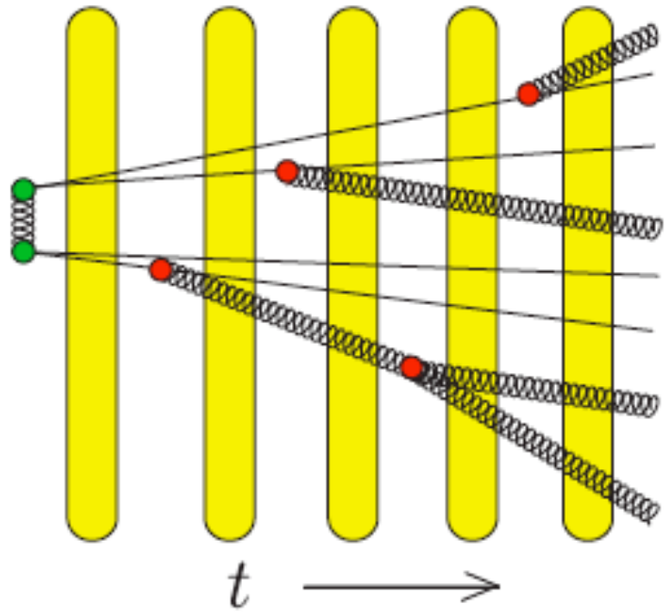
- Define the shower very general level
- Adding higher order
- Go beyond the leading color approximation
- Spin correlation
- Matching to fix order calculations

Introduction

Think of shower branching as developing in a “time” that goes from most virtual to least virtual.



Real time picture



Shower time picture

Thus shower time proceeds backward in physical time for initial state radiation.

Density Matrix

The physical cross section is

$$\sigma[F] = \sum_m \int [d\{p,f\}_m] \overbrace{f_{a/A}(\eta_\mathrm{a},\mu_F^2)\, f_{b/B}(\eta_\mathrm{b},\mu_F^2)}^{\text{parton distributions}} \frac{1}{2\eta_\mathrm{a}\eta_\mathrm{b} p_A\cdot p_B}$$
$$\times \langle \mathcal{M}(\{p,f\}_m)| \underbrace{F(\{p,f\}_m)}_{\text{observable}} \underbrace{|\mathcal{M}(\{p,f\}_m)\rangle}_{\text{matrix element}}$$

It is useful to write this as trace in the color and spin space

$$\sigma[F] = \sum_m \int [d\{p,f\}_m] \,\mathrm{Tr}\{\underbrace{\rho(\{p,f\}_m)}_{\text{density operator in color} \otimes \text{ spin space}} F(\{p,f\}_m)\}$$

Density Matrix

The physical cross section is

$$\sigma[F] = \sum_m \int [d\{p,f\}_m] \overbrace{f_{a/A}(\eta_\mathrm{a},\mu_F^2)\, f_{b/B}(\eta_\mathrm{b},\mu_F^2)}^{\text{parton distributions}} \frac{1}{2\eta_\mathrm{a}\eta_\mathrm{b} p_A\cdot p_B}$$

$$\int [d\{p,f\}_m] \equiv \frac{1}{m!}\prod_{i=1}^{m}\left\{\sum_{f_i}\int \frac{d^4p_i}{(2\pi)^4}\,2\pi\delta_+(p_i^2)\right\}\sum_a\int_0^1 d\eta_\mathrm{a}\sum_b\int_0^1 d\eta_\mathrm{b}$$
$$\times (2\pi)^4\delta\left(\eta_\mathrm{a}p_A+\eta_\mathrm{b}p_B-\sum_{i=1}^{m}p_i\right)$$

It

and spin space

$$\sigma[F] = \sum_m \int [d\{p,f\}_m]\,\mathrm{Tr}\{\underbrace{\rho(\{p,f\}_m)}F(\{p,f\}_m)\}$$

density operator in color $\otimes$ spin space

Density Matrix

The density operator is

$$\rho(\{p,f\}_m) = \left|\mathcal{M}(\{p,f\}_m)\right\rangle \frac{f_{a/A}(\eta_\mathrm{a},\mu_F^2) f_{b/B}(\eta_\mathrm{b},\mu_F^2)}{2\eta_\mathrm{a}\eta_\mathrm{b} p_A\cdot p_B} \left\langle\mathcal{M}(\{p,f\}_m)\right|$$

or expanding it on a color and spin basis

$$\rho(\{p,f\}_m) = \sum_{s,c}\sum_{s',c'} \left|\{s,c\}_m\right\rangle A(\{p,f,s',c',s,c\}_m) \left\langle\{s',c'\}_m\right|$$

We use conventional treatment of spin, thus we have orthogonal basis:

$$\left\langle\{s'\}_m\middle|\{s\}_m\right\rangle = \prod_{i=\mathrm{a},\mathrm{b},1,\ldots,m} \delta_{s_i}^{s'_i}$$

Density Matrix

In the color space we use a basis which is normalized but not orthogonal:

$$\langle \{c\}_m | \{c\}_m \rangle = 1 \quad \text{but} \quad \langle \{c'\}_m | \{c\}_m \rangle = \mathcal{O}(1/N_c^2) \quad \text{for } \{c'\}_m \neq \{c\}_m$$

It is useful to introduce a dual basis $|\{c\}_m\rangle_D$ that is defined by

$${}_D\langle \{c'\}_m | \{c\}_m \rangle = \delta^{\{c'\}_m}_{\{c\}_m}$$

and the completeness relations are

$$1 = \sum_{\{c\}_m} |\{c\}_m\rangle \; {}_D\langle \{c\}_m| \qquad \text{and} \qquad 1 = \sum_{\{c\}_m} |\{c\}_m\rangle_D \; \langle \{c\}_m|$$

Classical States

The set of functions $A(\{p, f, s', c', s, c\}_m)$ forms a vector space.

Basis: $\left|\{p, f, s', c', s, c\}_m\right)$

Completeness relation :

$$1 = \sum_m \int \left[d\{p, f, s', c', s, c\}_m\right] \left|\{p, f, s', c', s, c\}_m\right) \left(\{p, f, s', c', s, c\}_m\right|$$

where

$$\int \left[d\{p, f, s', c', s, c\}_m\right] \equiv \int \left[d\{p, f\}_m\right] \sum_{s_{\rm a}, s'_{\rm a}, c_{\rm a}, c'_{\rm a}} \sum_{s_{\rm b}, s'_{\rm b}, c_{\rm b}, c'_{\rm b}} \prod_{i=1}^{m} \left\{ \sum_{s_i, s'_i, c_i, c'_i} \right\}$$

Inner product of the basis states:

$$\left(\{p, f, s', c', s, c\}_m \middle| \{\tilde{p}, \tilde{f}, \tilde{s}', \tilde{c}', \tilde{s}, \tilde{c}\}_{\tilde{m}}\right) = \delta_{m,\tilde{m}}\ \delta(\{p, f, s', c', s, c\}_m; \{\tilde{p}, \tilde{f}, \tilde{s}', \tilde{c}', \tilde{s}, \tilde{c}\}_{\tilde{m}})$$

Classical State

A physical state which is related to the density matrix:

$$|A) = \int \left[d\{p, f, s', c', s, c\}_m\right] A(\{p, f, s', c', s, c\}_m) \left|\{p, f, s', c', s, c\}_m\right)$$

or the vector corresponding to the measurement function

$$\left(F\middle|\{p, f, s', c', s, c\}_m\right) = \left\langle\{s', c'\}_m\middle|F(\{p, f\}_m)\middle|\{s, c\}_m\right\rangle$$

Then the cross section that corresponding to this measurement function is

$$\sigma[F] = \left(F\middle|A\right)$$

Parton Shower Evolution

We use an evolution variable e.g.:

$$\log \frac{Q^2}{q^2} = t \in [0, \infty]$$

The evolution is given by
a linear operator

$$|A(t)) = U(t, t_0)|A(t_0))$$

Group decomposation

$$U(t_3, t_2)\, U(t_2, t_1) = U(t_3, t_1)$$

Preserves the normalization

$$(1|A(t_0)) = 1 \quad \Rightarrow \quad (1|U(t, t_0)|A(t_0)) = 1$$

Parton Shower Evolution

$$U(t_3,t_1) = \underbrace{N(t_3,t_1)}_{\text{No-change operator}} + \overbrace{\int_{t_1}^{t_3} dt_2\, U(t_3,t_2)\,\mathcal{H}(t_2)\,N(t_2,t_1)}^{\text{Splitting part}}$$

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Parton Shower Evolution

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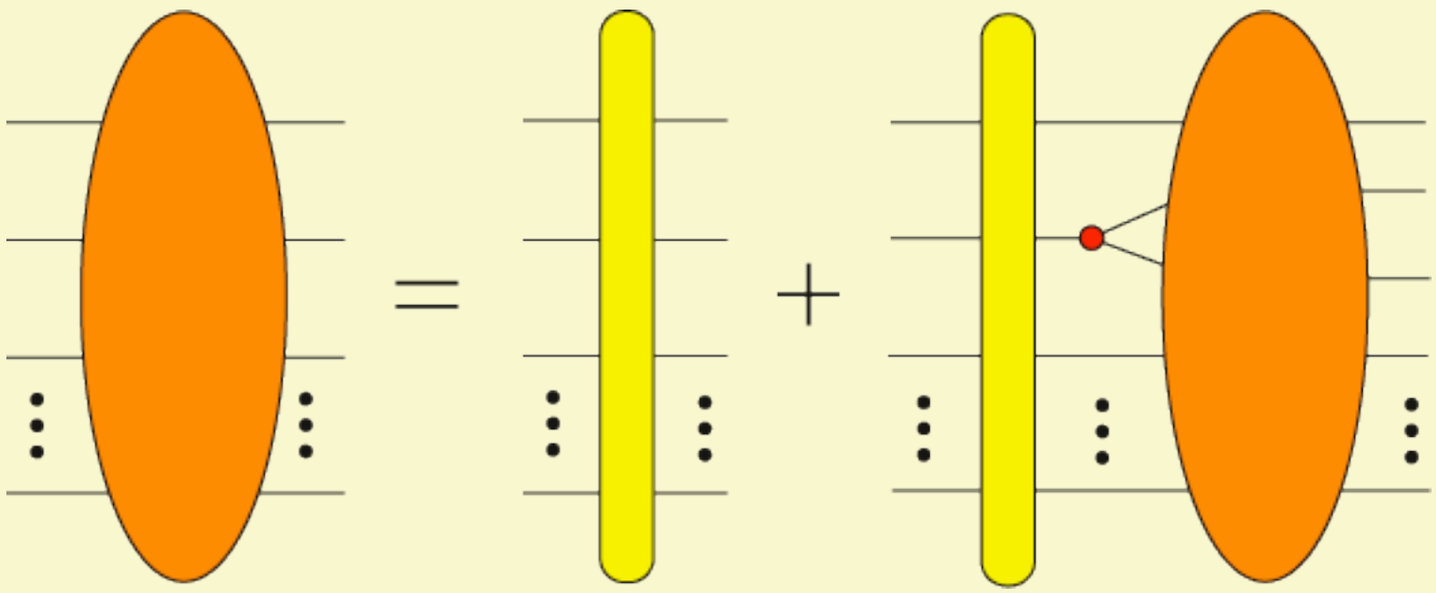
Group d $U(t_3,t_1)$

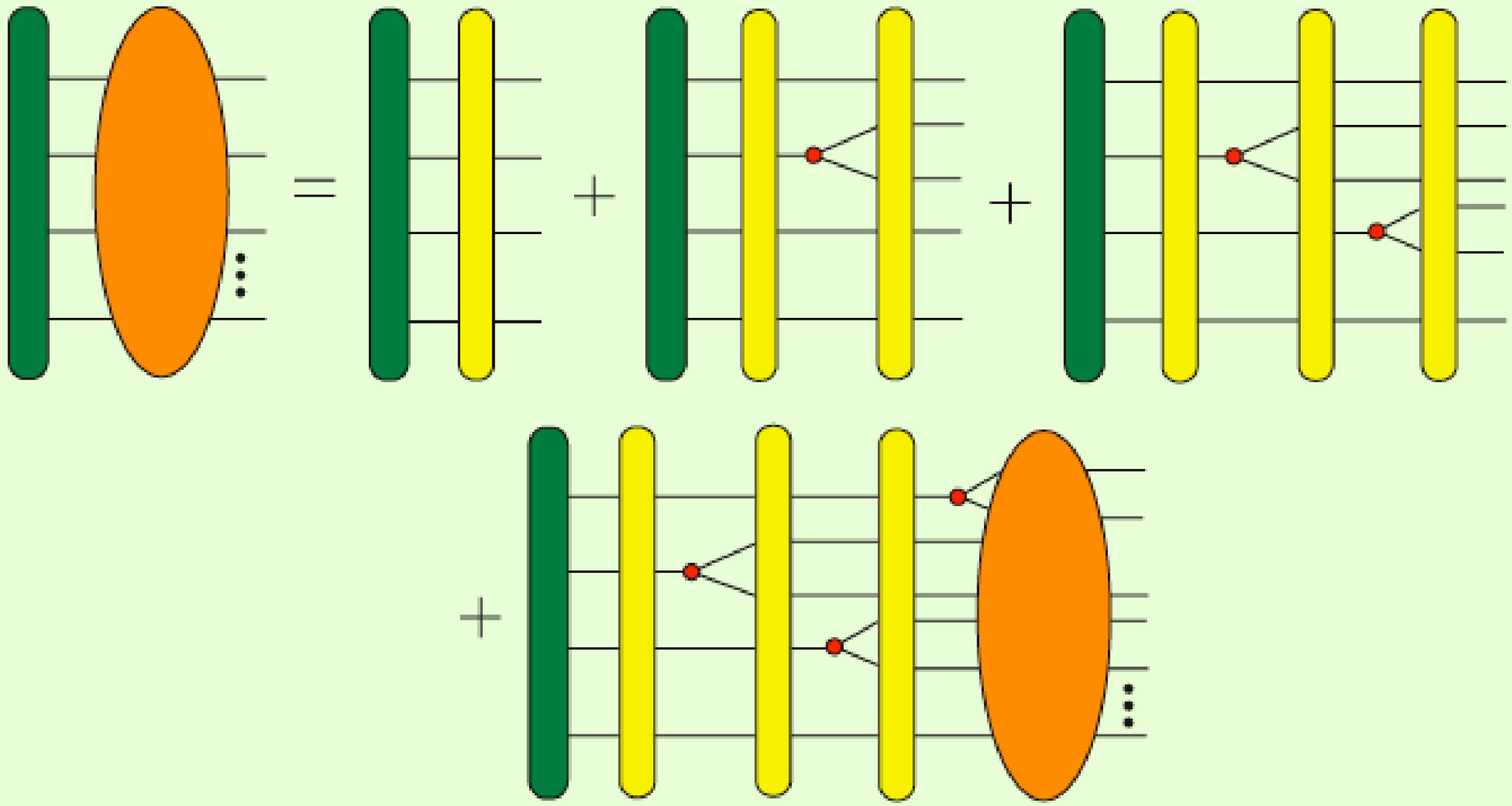
Preserve

$(1|A$ $= 1$



Parton Shower Evolution

Start with $2 \to 2$ cross section and iterate the evolution equation, say, twice:



No-change Operator

The operator $N(t',t)$ leaves the basis states $|\{p,f,s',c',s,c\}_m)$ unchanged

$$N(t',t)|\{p,f,s'c',s,c\}_m) = \underbrace{\Delta(\{p,f,c',c\}_m;t',t)}_{\text{Sudakov factor}}|\{p,f,s'c',s,c\}_m)$$

Consistently with the group decomposition property and with the initial condition $N(t,t)=1$ for Δ we have

$$\Delta(\{p,f,c',c\}_m;t_2,t_1) = \exp\left(-\int_{t_1}^{t_2} d\tau \underbrace{\omega(\tau;\{p,f,c',c\}_m)}_{\text{related to the parton splitting}}\right)$$

No-change Operator

From the normalization conditions

$$(1|U(t,t')|\{p,f,...\}_m) = (1|\{p,f,...\}_m)$$

and

$$(1|\{p,f,s',c',s,c\}_m) = \langle\{s'\}_m|\{s\}_m\rangle\langle\{c'\}_m|\{c\}_m\rangle$$

the relation between the splitting operator and $\omega(\tau;\{p,f,c',c\}_m)$ is

$$\langle\{s'\}_m|\{s\}_m\rangle\,\langle\{c'\}_m|\{c\}_m\rangle\,\omega(t,\{p,f,c',c\}_m) = \big(1\big|\mathcal{H}(t)\big|\{p,f,s',c',s,c\}_m\big)$$

At this point we need to discuss the structure of $\mathcal{H}(t)$!

Splitting Operator

The splitting operator consists of two terms

$$\mathcal{H}(t) = \mathcal{H}_I(t) + \mathcal{V}(t)$$

$\mathcal{H}_I(t)$ describes the parton splitting and based on the factorization property of the matrix element. It increases the number of partons and changes spins and colors.

$$\left(1\middle|\mathcal{H}_I(t)\middle|\{p,f,s',c',s,c\}_m\right) = 2\left\langle\{s'\}_m\middle|\{s\}_m\right\rangle \underbrace{\left\langle\{c'\}_m\middle|h(t,\{p,f\}_m)\middle|\{c\}_m\right\rangle}_{\text{non-trivial color structure}}$$

At leading order level:

$$h^{(0)}(t,\{p,f\}_m) = \underbrace{\mathbf{1}P(t,\{p,f\}_m)}_{\text{collinear}} + \sum_{\substack{i,k\\ i\neq k}} \underbrace{\boldsymbol{T}_i\cdot\boldsymbol{T}_k S_{ik}(t;\{p\}_m)}_{\text{pure soft}}$$

Splitting Operator

The operator $\mathcal{V}(t)$ represents the contributions of the virtual graphs. It keeps the number of partons, flavors, spins unchanged but changes the color.

$$\begin{aligned}
&\mathcal{V}(t)\Big|\{p,f,s',c',s,c\}_m\Big) \\
&\quad= \sum_{\{\tilde{c}\}_m} \Big|\{p,f,s',c',s,\tilde{c}\}_m\Big)\ {}_D\Big\langle\{\tilde{c}\}_m\Big|v(t,\{p,f\}_m)\Big|\{c\}_m\Big\rangle \left[1-\delta^{\{\tilde{c}\}_m}_{\{c\}_m}\right] \\
&\quad+ \sum_{\{\tilde{c}'\}_m} \Big|\{p,f,s',\tilde{c}',s,c\}_m\Big)\ \Big\langle\{c'\}_m\Big|v(t,\{p,f\}_m)\Big|\{\tilde{c}'\}_m\Big\rangle_D \left[1-\delta^{\{\tilde{c}'\}_m}_{\{c'\}_m}\right]
\end{aligned}$$

The singularities of the virtual graphs cancel those of the real emission graphs, thus the obvious choice is

$$v(\{p,f\}_m) = -h(\{p,f\}_m)$$

Sudakov Exponent

The relation between the splitting operator and the Sudakov exponent is

$$\left\langle \{s'\}_m \middle| \{s\}_m \right\rangle \left\langle \{c'\}_m \middle| \{c\}_m \right\rangle \omega(t, \{p, f, c', c\}_m) = \left(1 \middle| \mathcal{H}(t) \middle| \{p, f, s', c', s, c\}_m\right)$$

From this we have

$$\omega(t, \{p, f, c', c\}_m) = {}_D\left\langle \{c\}_m \middle| h(\{p, f\}_m) \middle| \{c\}_m \right\rangle + \left\langle \{c'\}_m \middle| h(\{p, f\}_m) \middle| \{c'\}_m \right\rangle_D$$

where

$$h^{(0)}(t, \{p, f\}_m) = \underbrace{\mathbf{1} P(t, \{p, f\}_m)}_{\text{collinear}} + \sum_{\substack{i,k \\ i \neq k}} \underbrace{\boldsymbol{T}_i \cdot \boldsymbol{T}_k S_{ik}(t; \{p\}_m)}_{\text{pure soft}}$$

Comments

- The Sudakov exponent exponentiates the color conserving part of the inclusive splitting operator $h(t, \{p, f\}_m)$. Fortunately the leading color part always conserves the color.

- The subleading color part is not exponentiated but it is subtracted. These contributions are treated perturbatively like the splitting terms in $\mathcal{H}_I(t)$.

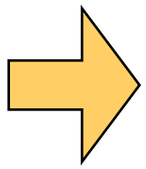
- Thus we have two perturbative parameters:

$$\alpha_s \qquad \text{and} \qquad \frac{1}{N_c^2}$$

Leading Color Approx.

Neglecting all the $1/N_c^2$ contributions the color basis becomes orthogonal and the dual basis is identical to the color basis:

$$\left|\{c\}_m\right\rangle_D = \left|\{c\}_m\right\rangle + \mathcal{O}(1/N_c^2)$$



No subtraction: $\mathcal{V}(t) = \mathcal{O}(1/N_c^2)$

The Sudakov factor exponentiates the whole inclusive splitting function

$$\omega(t, \{p, f, c', c\}_m) = 2\left\langle\{c\}_m\middle|h(\{p, f\}_m)\middle|\{c\}_m\right\rangle + \mathcal{O}(1/N_c^2)$$

Splitting Operator

The splitting operator is based on the soft and collinear factorization formulas. At LO level we have

$$\mathcal{H}_I(t) = \mathcal{D}_{\rm a}(t) + \mathcal{D}_{\rm b}(t) + \sum_{i=1}^{m} \mathcal{D}_i(t) + \mathcal{O}(\alpha_s^2)$$

- The splitting kernel is organized according to the collinear splittings (even the soft radiations)
- This decomposition can be also done at higher order level.
- We need to define phase space mapping to get *m+1* parton configurations from *m* parton configurations. This can be also done systematically even for higher order.
- It is important that the phase space mapping must be exact since the classical states are defined on phase space surface in the momentum space.
- The mapping must be consistent with the higher order; *e.g.* we cannot use the Catani-Seymour dipole factorization and phase space mapping.

Shower Cross Section

The evolution starts from the kinematically simplest configuration and the shower cross section is

$$\sigma[F] = \left(F\middle|D(t_{\mathrm{f}})U(t_{\mathrm{f}},t_2)\middle|\mathcal{M}_2\right)$$

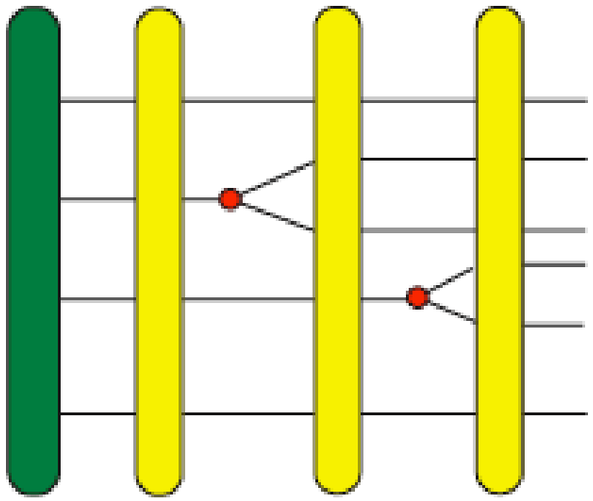
$D(t_{\mathrm{f}})$ represents the hadronization. Tuning is allowed only here.

If we have an leading order shower than the corresponding hard configuration should based on the tree level matrix elements.

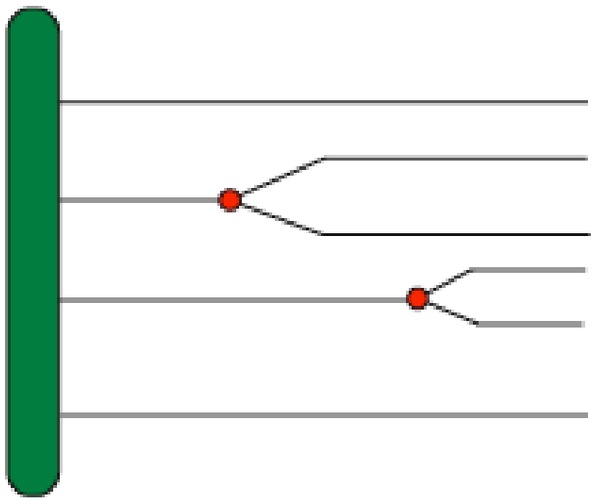
Adding Exact Matrix Elements

- Having the LO shower defined, we can calculate any cross section, 2-jet, 3-jet,...
- Only the $2 \to 2$ Born matrix elements are considered.
- For 3-jet we should consider at least the $2 \to 3$ Born matrix elements.
- Since α_s is large it would be useful to consider the contributions of $2 \to 4$ Born and $2 \to 3$ 1-loop matrix elements in the 3-jet calculation.

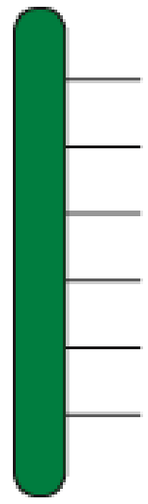
Matching at Born Level



Standard shower



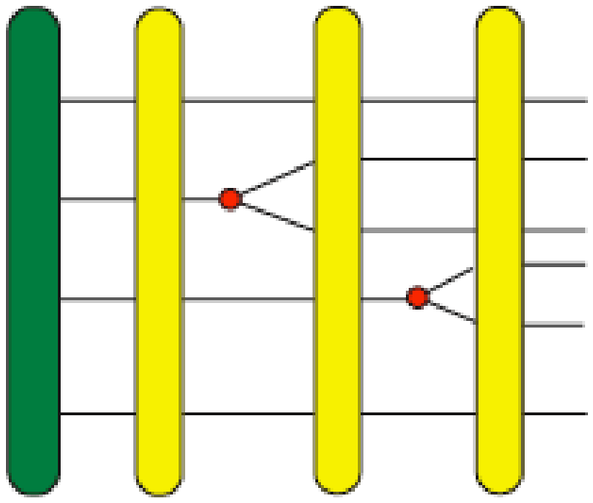
Small p_T approximation



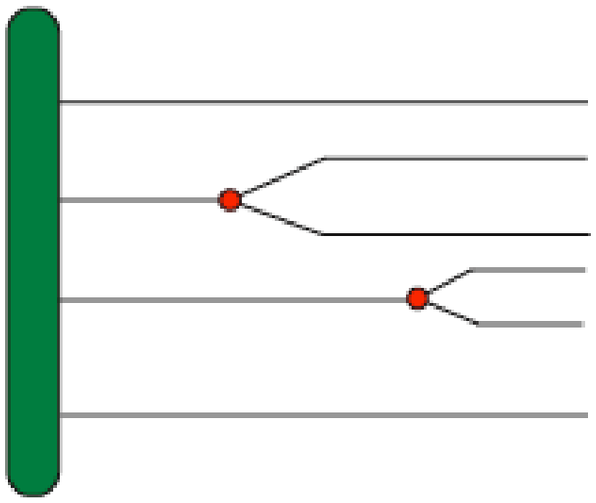
$|\mathcal{M}|^2$

- The shower approximation relies on the small p_T splittings.
- May be the exact matrix element would be better.
- But that lacks the Sudakov exponents.

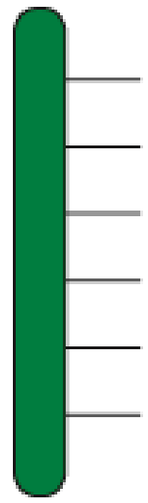
Matching at Born Level



Standard shower



Small p_T approximation



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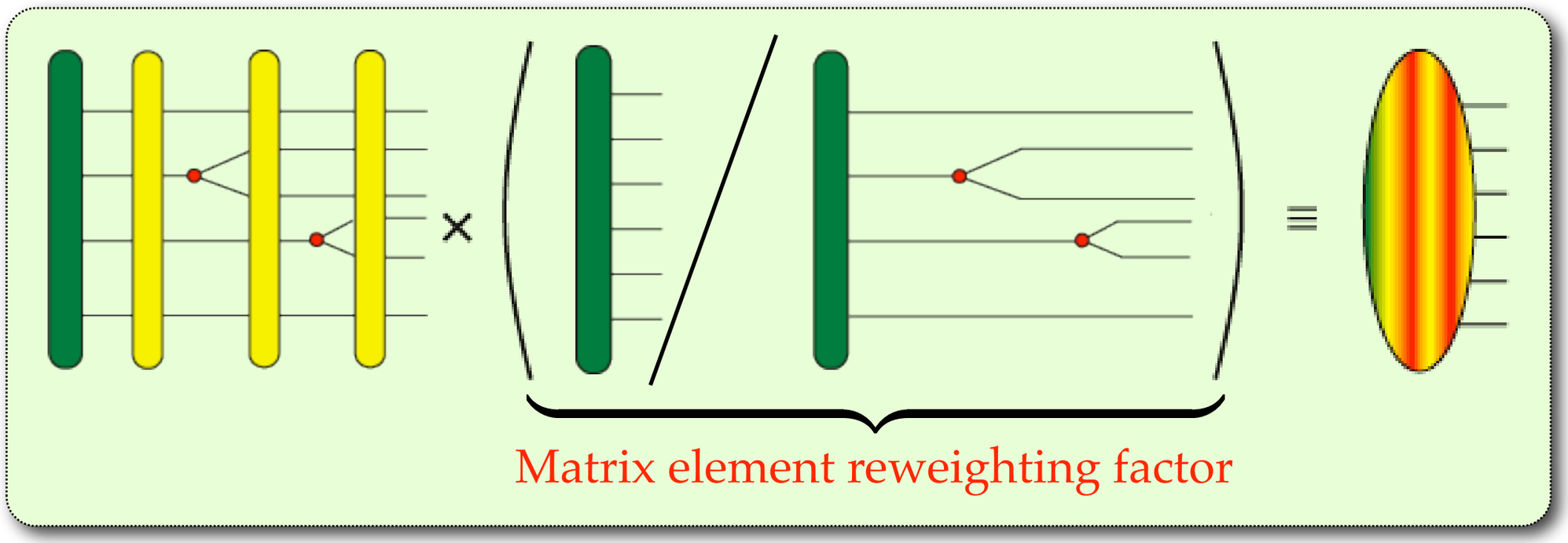


Adjoint Splitting Operator

Let us define the operator $\mathcal{H}^\dagger(t)$ according to

$$\left(F\middle|\mathcal{H}(t)\middle|A\right) = \left(A\middle|\mathcal{H}^\dagger(t)\middle|F\right)$$

$$\mathcal{H}^\dagger(t) = \mathcal{H}_I^\dagger(t) + \mathcal{V}^\dagger(t)$$

Since $\mathcal{H}_I(t)$ always increases the number of partons $\mathcal{H}_I^\dagger(t)$ always decreases it. This operator is the fully exclusive version of the fix order calculation's subtraction terms.

For multiple emission:

$$\left(F\middle|\mathcal{H}(t_m)\mathcal{H}(t_{m-1})\cdots\mathcal{H}(t_1)\middle|A\right) = \left(A\middle|\mathcal{H}^\dagger(t_1)\cdots\mathcal{H}^\dagger(t_{m-1})\mathcal{H}^\dagger(t_m)\middle|F\right)$$

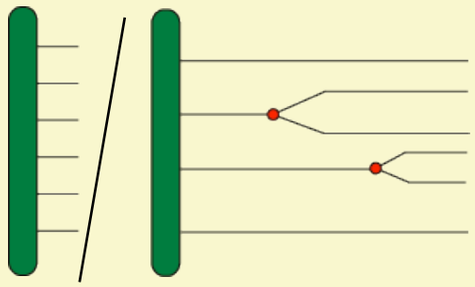
Approximated Matrix Element

For a given m-parton configuration the Born level approximated matrix element is

$$(\mathcal{A}_m|\{p,f,...\}_m) = \int_{t_2}^{t_f} dt_3 \cdots \int_{t_{m-1}}^{t_f} dt_m \, (\mathcal{M}_2|\mathcal{H}_I^\dagger(t_3)\mathcal{H}_I^\dagger(t_4)\cdots\mathcal{H}_I^\dagger(t_m)|\{p,f,...\}_m)$$

The matrix element reweighting factor is

$$w_M = \frac{}{} = \begin{cases} \dfrac{(\mathcal{M}_m|\{p,f,..\}_m)}{(\mathcal{A}_m|\{p,f,...\}_m)} & \text{if } \mathcal{M}_m \text{ is known} \\ 1 & \text{otherwise} \end{cases}$$



and the reweighting operator is

$$W_M(t_f,t_2) = \sum_m \int \left[d\{p,f,...\}_m\right] |\{p,f,..\}_m)(\{p,f,..\}_m|$$
$$\times\, w_M(\{p,f,s',c',s,c\}_m)$$

Approximated Matrix Element

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$$(\mathcal{A}_m|\{p,f,...\}_m) = \int_{t_2}^{t_f} dt_3 \cdots \int_{t_{m-1}}^{t_f} dt_m \, (\mathcal{M}_2|\mathcal{H}_I^\dagger(t_3)\mathcal{H}_I^\dagger(t_4)\cdots\mathcal{H}_I^\dagger(t_m)|\{p,f,...\}_m)$$

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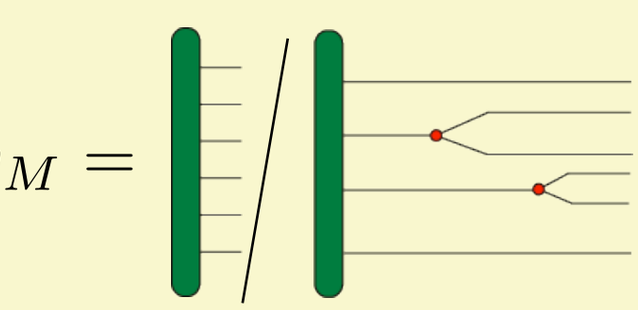
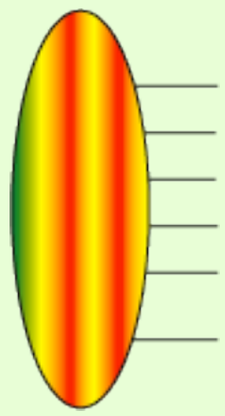


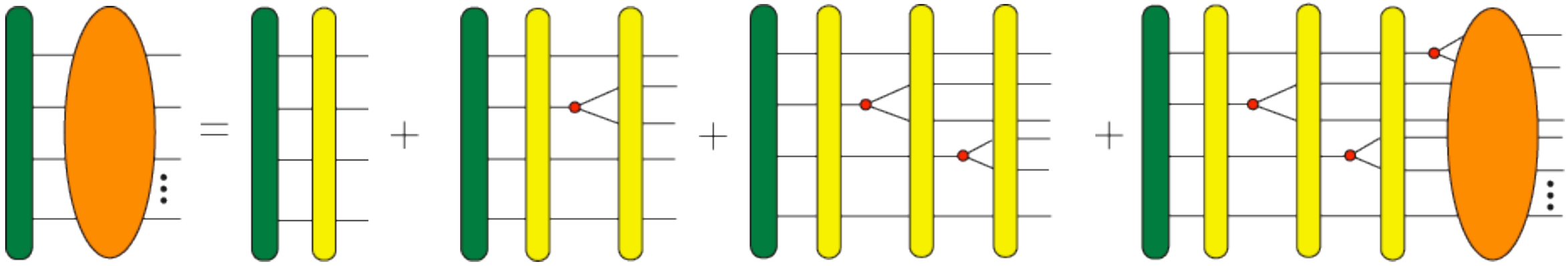
and th

$$V \quad = \int_{t_2}^{t_f} dt_3 \int_{t_3}^{t_f} dt_4 N(t_f,t_4) W_M(t_f,t_2)$$
$$\times \mathcal{H}_I(t_4) N(t_4,t_3) \mathcal{H}_I(t_3)\, N(t_3,t_2)|\mathcal{M}_2)$$



Matching at Born level

At leading color level when $\mathcal{V}(t) = 0$ we have



Expanding one step of the shower

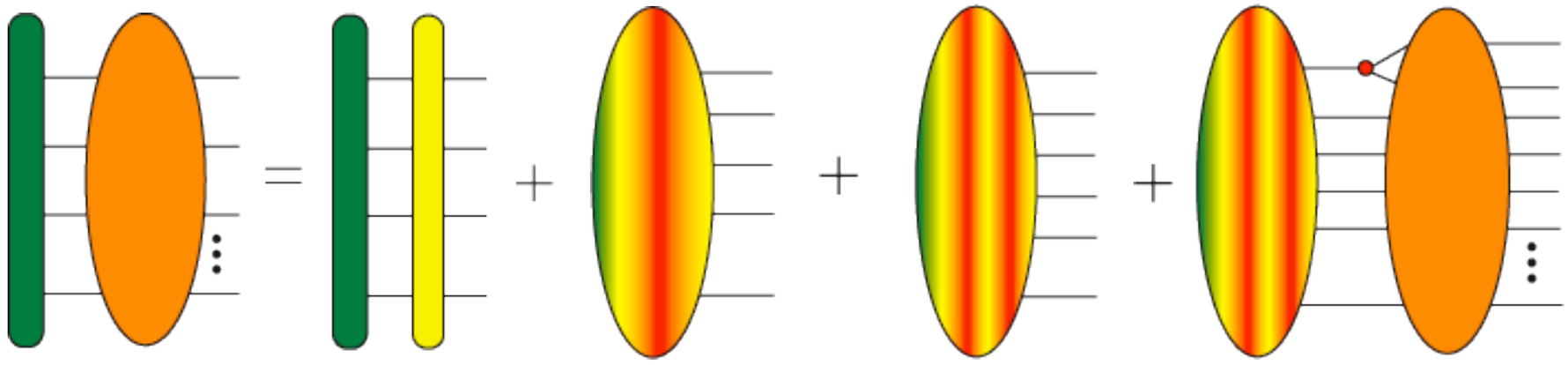
$$\big|A(t_{\rm f})\big) = N(t_{\rm f}, t_2)\big|\mathcal{M}_2\big) + \int_{t_2}^{t_{\rm f}} dt_3\, U(t_{\rm f}, t_3)(\mathcal{H}_I(t_3) + \mathcal{V}(t_t))N(t_3, t_2)\big|\mathcal{M}_2\big)$$

Assuming that we know the $2 \to 3$ Born matrix elements

$$\big|A_M(t_{\rm f})\big) = N(t_{\rm f}, t_2)\big|\mathcal{M}_2\big) + \int_{t_2}^{t_{\rm f}} dt_3\, U(t_{\rm f}, t_3)(W_M(t_{\rm f}, t_2)\mathcal{H}_I(t_3) + \mathcal{V}(t_t))N(t_3, t_2)\big|\mathcal{M}_2\big)$$

Matching at Born level

At leading color level when $\mathcal{V}(t) = 0$ we have



Expanding one step of the shower

$$\big|A(t_{\rm f})\big) = N(t_{\rm f}, t_2)\big|\mathcal{M}_2\big) + \int_{t_2}^{t_{\rm f}} dt_3\, U(t_{\rm f}, t_3)(\mathcal{H}_I(t_3) + \mathcal{V}(t_t))N(t_3, t_2)\big|\mathcal{M}_2\big)$$

Assuming that we know the $2 \to 3$ Born matrix elements

$$\big|A_M(t_{\rm f})\big) = N(t_{\rm f}, t_2)\big|\mathcal{M}_2\big) + \int_{t_2}^{t_{\rm f}} dt_3\, U(t_{\rm f}, t_3)(W_M(t_{\rm f}, t_2)\mathcal{H}_I(t_3) + \mathcal{V}(t_t))N(t_3, t_2)\big|\mathcal{M}_2\big)$$

Matching at Born Level

After some algebra

$$\big|A_M(t_{\rm f})\big) = \underbrace{U(t_{\rm f},t_2)\big|\mathcal{M}_2\big)}_{\text{Standard shower}} + \int_{t_2}^{t_{\rm f}} dt_3\, U(t_{\rm f},t_3)\, \underbrace{\big[W_M(t_{\rm f},t_2),\mathcal{H}_I(t_3)\big]}_{W_M(t_{\rm f},t_2)\mathcal{H}_I(t_3)-\mathcal{H}_I(t_3)W_M(t_{\rm f},t_2)}\, N(t_3,t_2)\big|\mathcal{M}_2\big)$$

The second term doesn't change the LL and NLL structure

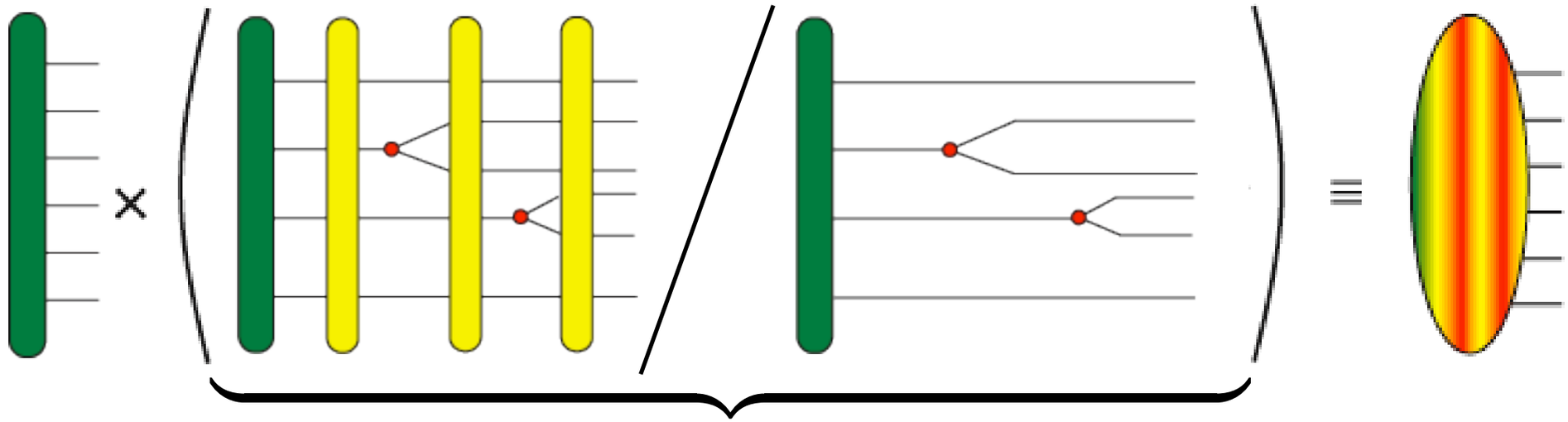
$$\big[W_M(t_{\rm f},t_2),\mathcal{H}_I(t_3)\big]\big|\mathcal{M}_2\big) \sim \big|\mathcal{M}_3\big) - \mathcal{H}_I(t_3)\big|\mathcal{M}_2\big)$$

Assuming we know $\mathcal{M}_3$, $\mathcal{M}_4$, ..., $\mathcal{M}_N$ then

$$\begin{aligned}\big|A_M(t_{\rm f})\big) &= U(t_{\rm f},t_2)\big|\mathcal{M}_2\big) \\ &+ \sum_{m=3}^{N} \int_{t_2}^{t_{\rm f}} dt_3 \int_{t_3}^{t_{\rm f}} dt_4 \cdots \int_{t_{m-1}}^{t_{\rm f}} dt_m\, U(t_{\rm f},t_m)\big[W_M(t_{\rm f},t_2),\mathcal{H}_I(t_m)\big] \\ &\qquad \times N(t_m,t_{m-1})\mathcal{H}_I(t_{m-1})N(t_{m-1},t_{m-2})\cdots\mathcal{H}_I(t_3)\,N(t_3,t_2)\big|\mathcal{M}_2\big)\end{aligned}$$

Matching at Born Level

There is another way to do the reweighting



Sudakov reweighting factor

$$W_\Delta(t_{\rm f},t_2;t) = \sum_m \int \Big[d\{p,f,s',c',s,c\}_m\Big] \Big|\{p,f,..\}_m\Big)\Big(\{p,f,..\}_m\Big|$$

$$\times \lim_{\delta\to 0} \int_{t_2}^{t} dt_{m-1} \int_{t_2}^{t_{m-1}} dt_{m-2} \cdots \int_{t_2}^{t_4} dt_3$$

$$\times \frac{\Big(\mathcal{M}_2\Big|N(t_3,t_2)\mathcal{H}_I^\dagger(t_3)\cdots N(t,t_{m-1})\mathcal{H}_I^\dagger(t)\Big|\{p,f,..\}_m\Big)}{\Big(\mathcal{A}_m(t_{\rm f},t_2)\Big|\{p,f,s',c',s,c\}_m\Big)+\delta}$$

Matching at Born Level

The improved shower formulae is

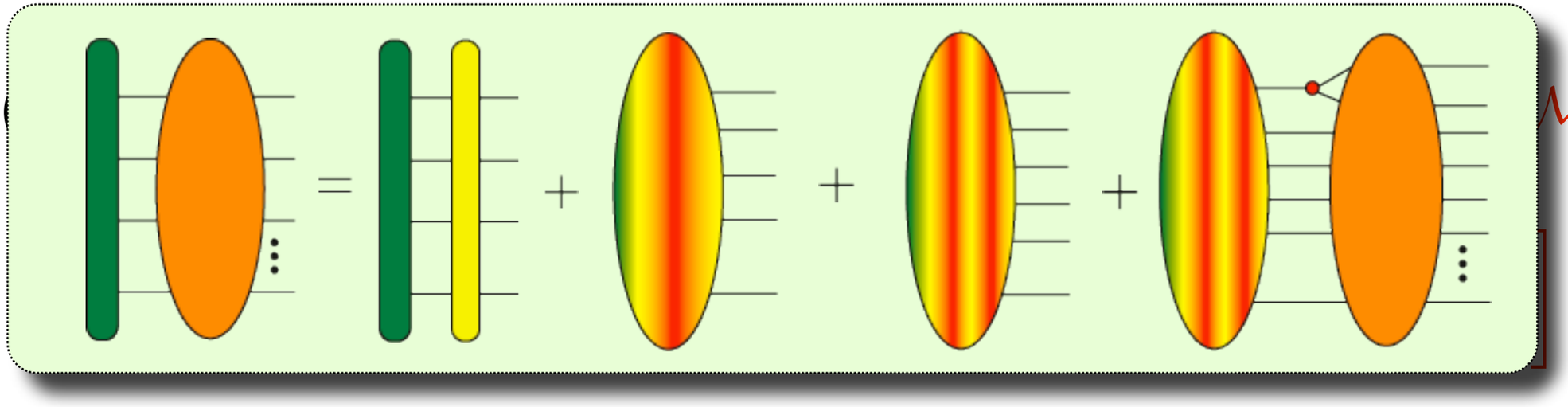
$$\big|A_\Delta(t_\mathrm{f})\big) = U(t_\mathrm{f},t_2)\big|\mathcal{M}_2\big) + \sum_{m=3}^{N}\int_{t_2}^{t_\mathrm{f}} dt_m U(t_\mathrm{f},t_m)\Big[W_\Delta(t_\mathrm{f},t_2;t_m)\big|\mathcal{M}_m\big) - \mathcal{H}_I(t_m)\int_{t_2}^{t_m} dt' N(t_m,t')W_\Delta(t_\mathrm{f},t_2;t')\big|\mathcal{M}_{m-1}\big)\Big]$$

and at leading color level this is even simpler

$$\big|A_\Delta(t_\mathrm{f})\big) = N(t_\mathrm{f},t_2)\big|\mathcal{M}_2\big) + \sum_{m=3}^{n-1}\int_{t_2}^{t_\mathrm{f}} dt_m\, N(t_\mathrm{f},t_m)W_\Delta(t_\mathrm{f},t_2;t_m)\big|\mathcal{M}_m\big) + \int_{t_2}^{t_\mathrm{f}} dt_n\, U(t_\mathrm{f},t_n)W_\Delta(t_\mathrm{f},t_2;t_n)\big|\mathcal{M}_n\big)$$

Matching at Born Level

The improved shower formulae is

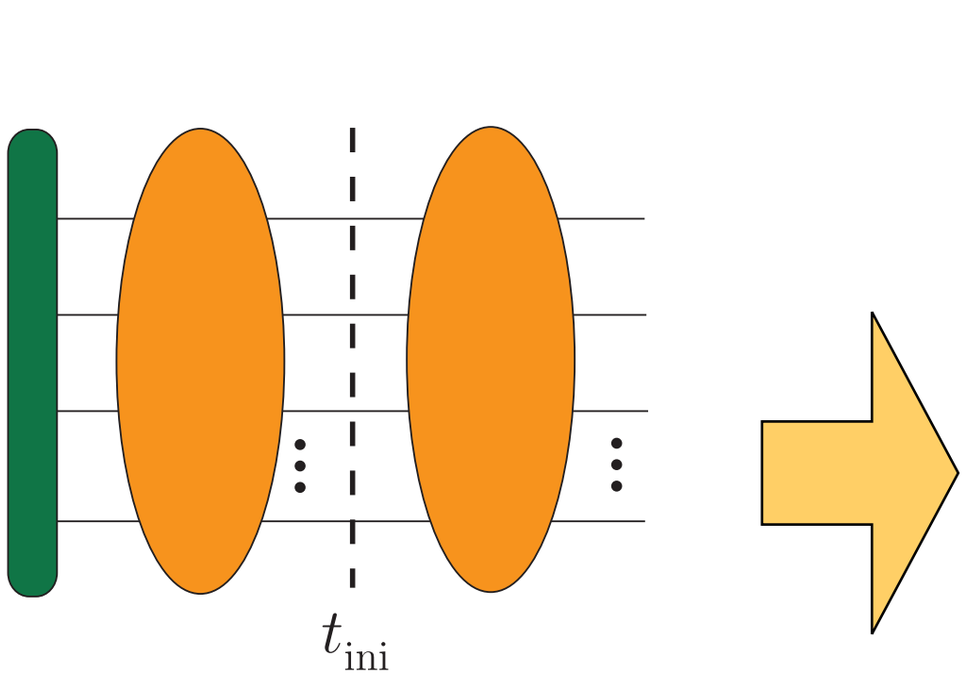
$|A_\Delta($  $\mathcal{M}_m)$

and at leading color level this is even simpler

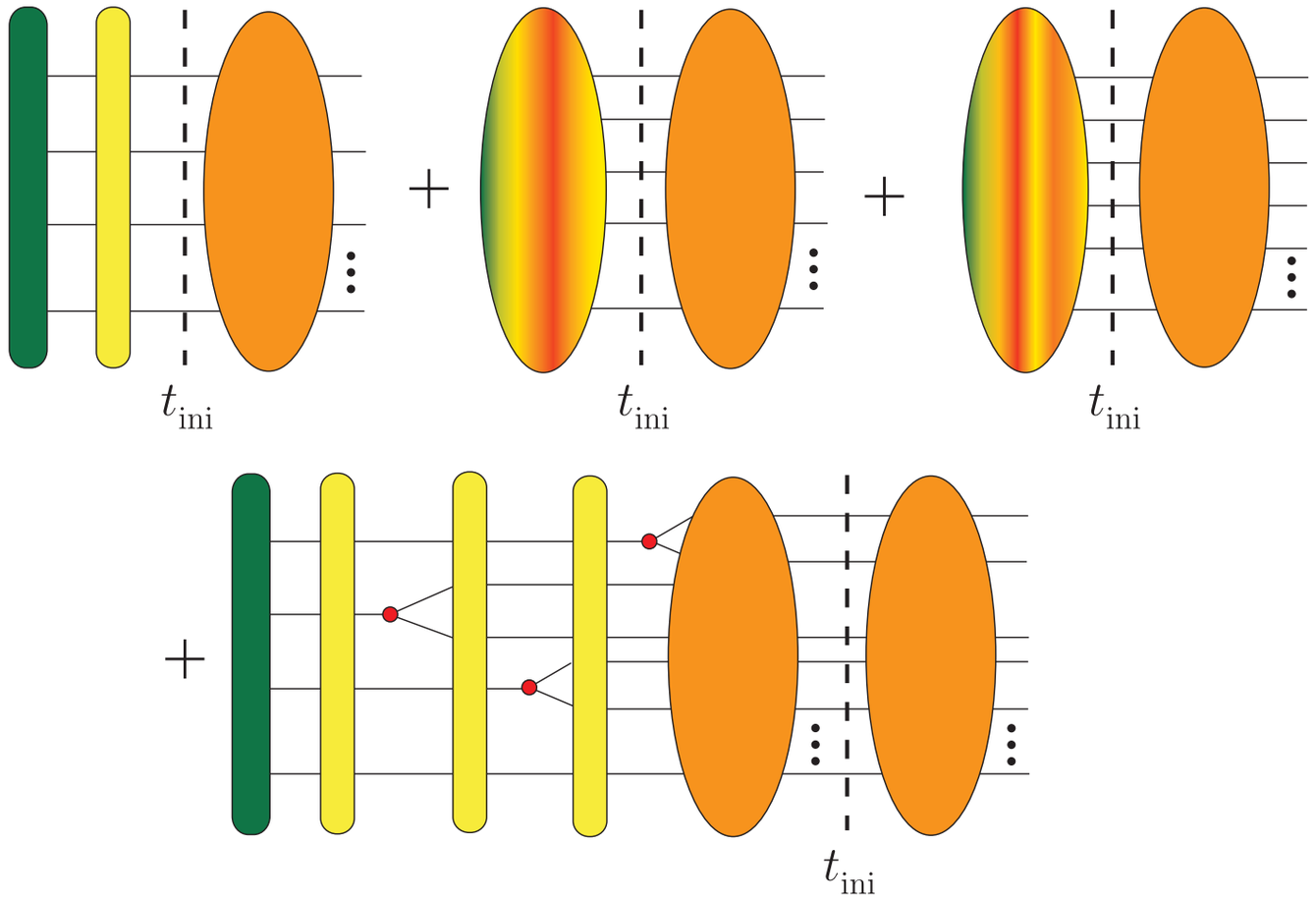
$$\big|A_\Delta(t_\mathrm{f})\big) = N(t_\mathrm{f},t_2)\big|\mathcal{M}_2\big) + \sum_{m=3}^{n-1}\int_{t_2}^{t_\mathrm{f}} dt_m\, N(t_\mathrm{f},t_m) W_\Delta(t_\mathrm{f},t_2;t_m)\big|\mathcal{M}_m\big)$$
$$+ \int_{t_2}^{t_\mathrm{f}} dt_n\, U(t_\mathrm{f},t_n) W_\Delta(t_\mathrm{f},t_2;t_n)\big|\mathcal{M}_n\big)$$

CKKW without Equations

Two steps calculation



CKKW break the evolution
into $0 < t < t_{\rm ini}$ and $t_{\rm ini} < t < t_{\rm f}$



- CKKW use improve weighting for $0 < t < t_{\rm ini}$
- For $t_{\rm ini} < t < t_{\rm f}$ they have standard shower with transverse momentum veto
- They use the kT algorithm and NLL Sudakov factors to do the reweighting.

CKKW@NLO
ZN and D. Soper,
hep-ph/0503053

NLO Calculations

The NLO fix order calculations can be organized by the following way

$$\sigma_{\rm NLO} = \int_N d\sigma^B + \int_{N+1} \left[d\sigma^R - d\sigma^A \right]$$
$$+ \int_N \left[d\sigma^B \otimes \boldsymbol{I}(\epsilon) + d\sigma^V \right]_{\epsilon=0} + \int_N d\sigma^B \otimes \left[\boldsymbol{K} + \boldsymbol{P}(\mu_F) \right]$$

The Born ($d\sigma^B$) and the real ($d\sigma^R$) are based on the N and N+1 parton matrix elements, respectively and $d\sigma^V$ is the contribution of the virtual graphs. The operators $\boldsymbol{I}(\epsilon), \boldsymbol{K}, \boldsymbol{P}$ are universal.

It is useful to define the subtraction term based on the shower splitting operator

$$d\sigma^A \sim \int_0^\infty dt \left(\mathcal{M}_N \left| \mathcal{H}_I^\dagger(t) \right| \{p, f, ...\}_{N+1} \right)$$

Matching at NLO level

Let us calculate the N-jet cross section. The matrix element improved cross section is

$$\begin{aligned}\left(F_N\middle|A_\Delta(t_{\rm f})\right) = &\int_{t_2}^{t_{\rm f}} dt_N \left(F_N\middle|N(t_{\rm f},t_N)W_\Delta(t_{\rm f},t_2;t_N)\middle|\mathcal{M}_N\right)\\ &+\int_{t_2}^{t_{\rm f}} dt_{N+1}\left(F_N\middle|U(t_{\rm f},t_{N+1})W_\Delta(t_{\rm f},t_2;t_{N+1})\middle|\mathcal{M}_{N+1}\right)\\ &+\underbrace{\left[\cdots\mathcal{V}(t)\cdots\right]}+\mathcal{O}\left[\mathcal{V}(t)^2\right]\end{aligned}$$

[-- It is too long to display. --]

Expanding it in α_s then we have

$$\begin{aligned}\left(F_N\middle|A_\Delta\right) = &\int_N d\sigma^B + \int_{N+1}\left[d\sigma^R - d\sigma^A\right]\\ &+\int_N\left[d\sigma^S[\mathcal{V}(t)] + d\sigma^B\, W_\Delta^{(1)}\right]+\mathcal{O}(\alpha_s^2)\end{aligned}$$

Matching at NLO level

Fixing the shower formulae by subtracting the approximated and adding correct 1-loop contributions, thus we have

$$\left(F_N\middle|A_\Delta^{\rm NLO}(t_{\rm f})\right) = \left(F_N\middle|A_\Delta(t_{\rm f})\right) - \left[\cdots\mathcal{V}(t)\cdots\right] + \int_{t_2}^{t_{\rm f}} dt_N \left(F_N\middle|U(t_{\rm f},t_N)W_\Delta(t_{\rm f},t_2;t_N)\middle|\mathcal{M}_N^{(1)}\right)$$

where

$$\int [d\{s',c',s,c\}_N]\left(M_N^{(1)}\middle|\{p,f,...\}_N\right) \sim -\frac{\alpha_s}{2\pi}W_\Delta^{(1)}\,|\mathcal{M}_N|^2 + |\mathcal{M}_N|^2 \otimes \left(\boldsymbol{K}+\boldsymbol{P}(\mu_F)\right) + \Big[|\mathcal{M}_N|^2_{1-loop} + |\mathcal{M}_N|^2 \otimes \boldsymbol{I}(\epsilon)\Big]_{\epsilon=0} \quad ,$$

Conclusions

The things I talked about:

- We defined a very general formalism to define and implement parton shower algorithm.
- We can consider spin correlation and color correlation beyond the leading color approximation.
- The treatment of the subleading color part is still perturbative (not exponentiated).
- The algorithm is capable to deal with the higher order corrections in α_s once the splitting kernels are known.
- We have general method to match the LO shower to Born and NLO matrix elements.

Conclusion

The things I didn't talk about:

- The algorithm is Lorentz covariant/invariant.
- Based on exact phase space.
- No technical cuts, parameters. Only the infrared cutoff parameter.
- Color coherence (angular ordering) is naturally provided without forced angular ordering, vetoing, or other tricks. No azimuthal averaging.
- The evolution parameter doesn't have to be special. Any infrared sensitive parameter is good. Use the simplest, say virtuality.
- Since we defined $\mathcal{H}_I^\dagger(t)$ operator, we defined a new NLO subtraction scheme for fix order NLO calculation.

Conclusions

About this new NLO scheme

- The algorithm is Lorentz covariant/invariant.
- Based on exact phase space factorization.
- Splitting operator is defined fully exclusive way. One can do MC helicity and color sum in the NLO calculation.
- For N-jet calculation the number of the subtraction terms is $(N+1)(N+4)/2$. Compared to the Catani-Seymour dipole subtraction method $(N+1)^2(N+4)/2$.
- Since we have less counter-terms we expect better numerical behavior.