

# Precise predictions from the MSSM

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# Outline

*specific class of SUSY models:*

## MINIMAL SUPERSYMMETRIC STANDARD MODEL (MSSM)

- Introduction
- Standard particles
- Higgs bosons
- SUSY particles
- Conclusions

# The Minimal Supersymmetric Standard Model (MSSM)

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Superpartners for Standard Model particles:

$$[u, d, c, s, t, b]_{L,R} \quad [e, \mu, \tau]_{L,R} \quad [\nu_{e,\mu,\tau}]_L \quad \text{Spin } \frac{1}{2}$$

$$[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}]_{L,R} \quad [\tilde{e}, \tilde{\mu}, \tilde{\tau}]_{L,R} \quad [\tilde{\nu}_{e,\mu,\tau}]_L \quad \text{Spin } 0$$

$$g \quad \underbrace{W^\pm, H^\pm} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \quad \text{Spin } 1 / \text{Spin } 0$$

$$\tilde{g} \quad \tilde{\chi}_{1,2}^\pm \quad \tilde{\chi}_{1,2,3,4}^0 \quad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector: two Higgs doublets, physical states:  
 $h^0, H^0, A^0, H^\pm$

# masses and mixing of SUSY particles through soft-breaking

## model parameters

- gaugino masses:  $M_1, M_2, M_3$
- sfermion masses:  $M_L, M_{\tilde{u}_R}, M_{\tilde{d}_R}$   
for each doublet of squarks and sleptons
- trilinear coupling:  $A_{\tilde{f}}$  for each  $\tilde{f}$   
→  $L$ - $R$  sfermion mixing
- supersymmetric Higgsino mass parameter:  $\mu$
- Higgs sector parameters:  $M_A, \tan \beta = v_2/v_1$

● **chargino masses:**  $m_{\tilde{\chi}_{1,2}^{\pm}}$  from  $M_2, \mu$

$$\begin{pmatrix} M_2 & \sqrt{2}M_W \sin \beta \\ \sqrt{2}M_W \cos \beta & \mu \end{pmatrix}$$

● **neutralino masses:**  $m_{\tilde{\chi}_{1,2,3,4}^0}$  from  $M_1, M_2, \mu$

$$\begin{pmatrix} M_1 & 0 & -M_Z s_W \cos \beta & M_Z s_W \sin \beta \\ 0 & M_2 & M_Z c_W \cos \beta & -M_Z c_W \sin \beta \\ -M_Z s_W \cos \beta & M_Z c_W \cos \beta & 0 & -\mu \\ M_Z s_W \sin \beta & -M_Z c_W \sin \beta & -\mu & 0 \end{pmatrix}$$

● sfermion masses:  $m_{\tilde{f}_{1,2}}$  from  $M_L, M_{\tilde{f}_R}, A_f$

$$\begin{pmatrix} m_f^2 + M_L^2 + M_Z^2 c_{2\beta} (I_f^3 - Q_f s_W^2) & m_f (A_f - \mu \kappa) \\ m_f (A_f - \mu \kappa) & m_f^2 + M_{\tilde{f}_R}^2 + M_Z^2 c_{2\beta} Q_f s_W^2 \end{pmatrix}$$

with

$$\kappa = \{\cot \beta; \tan \beta\} \quad \text{for} \quad f = \{u, d\}$$

## Precision analysis required for

- indirect tests of SUSY through
  - virtual SUSY effects in precision observables
- precision studies for SUSY particles
  - determination of masses & couplings
  - reconstruction of model parameters
- direct *versus* indirect tests
  - precision observables for precisely measured SUSY parameters
  - consistency check

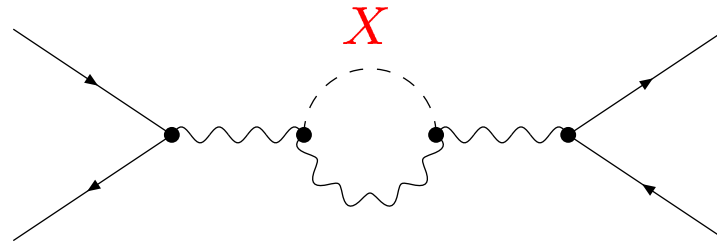
## Processes with external

- (i) standard particles
- (ii) Higgs bosons, especially light Higgs  $h^0$
- (iii) SUSY particles

# Standard particles

Test of theory at quantum level:

Sensitivity to loop corrections



- $\mu$  lifetime:  $M_W, \Delta r, G_F$
- $Z$  observables:  $g_V, g_A, \sin^2 \theta_{\text{eff}}, \Gamma_Z, \dots$

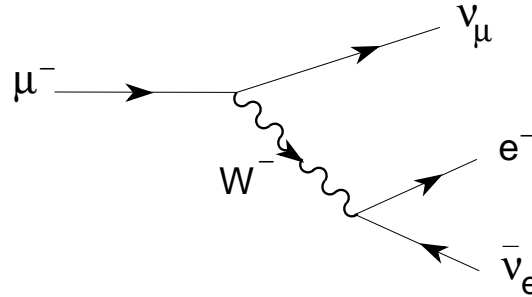
*[Heinemeyer, WH, Weiglein, Phys. Rep. 425 (2006) 265]*

**new:**  $M_W$  with 2-loop improvements  $\mathcal{O}(\alpha\alpha_s, \alpha_t^2, \alpha_b^2, \alpha_t\alpha_b)$   
and complex parameters

*[Heinemeyer, WH, Stöckinger, A. Weber, Weiglein, hep-ph/0604147]*



# $M_W - M_Z$ correlation

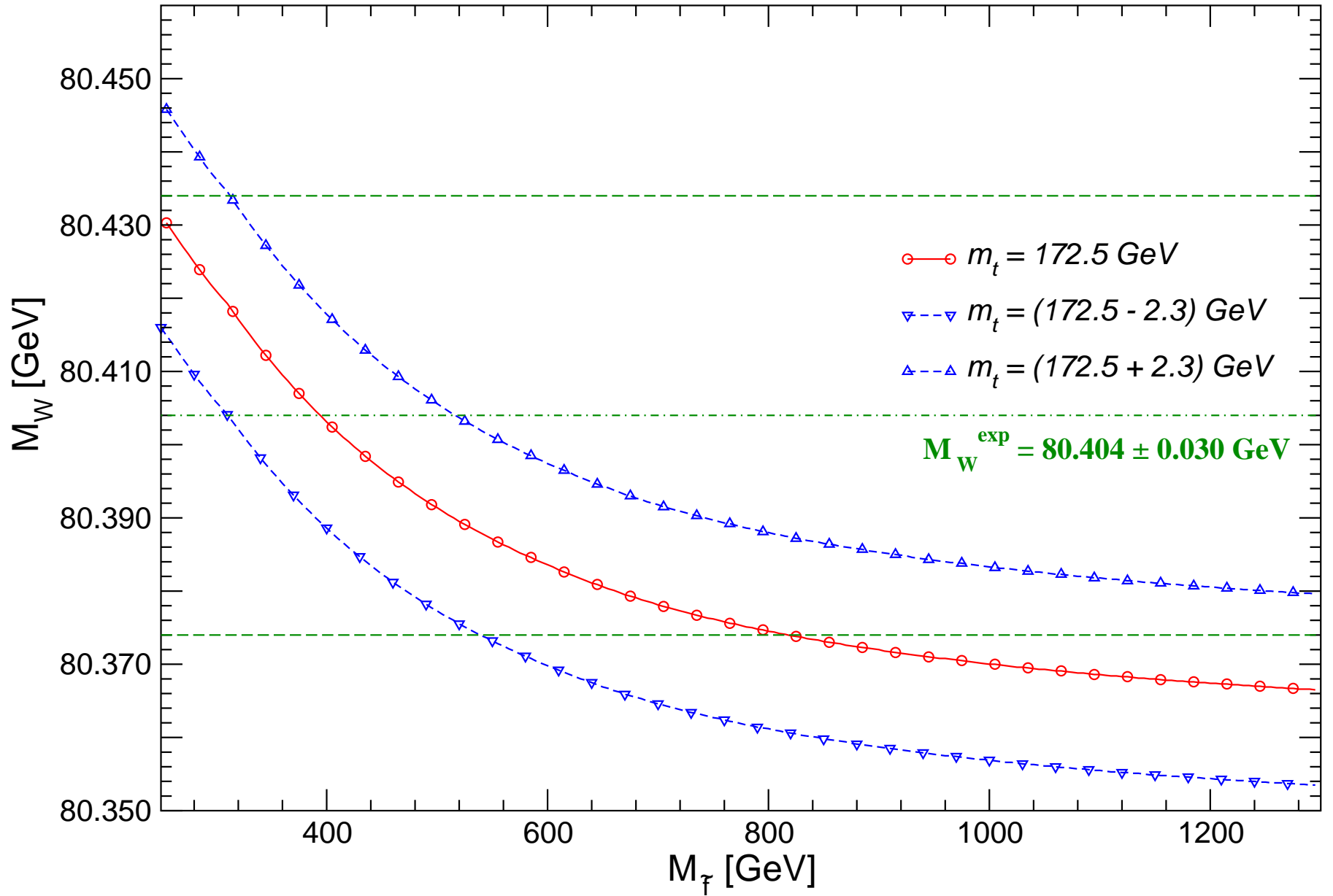


$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2 (1 - M_W^2/M_Z^2)} (1 + \Delta r)$$

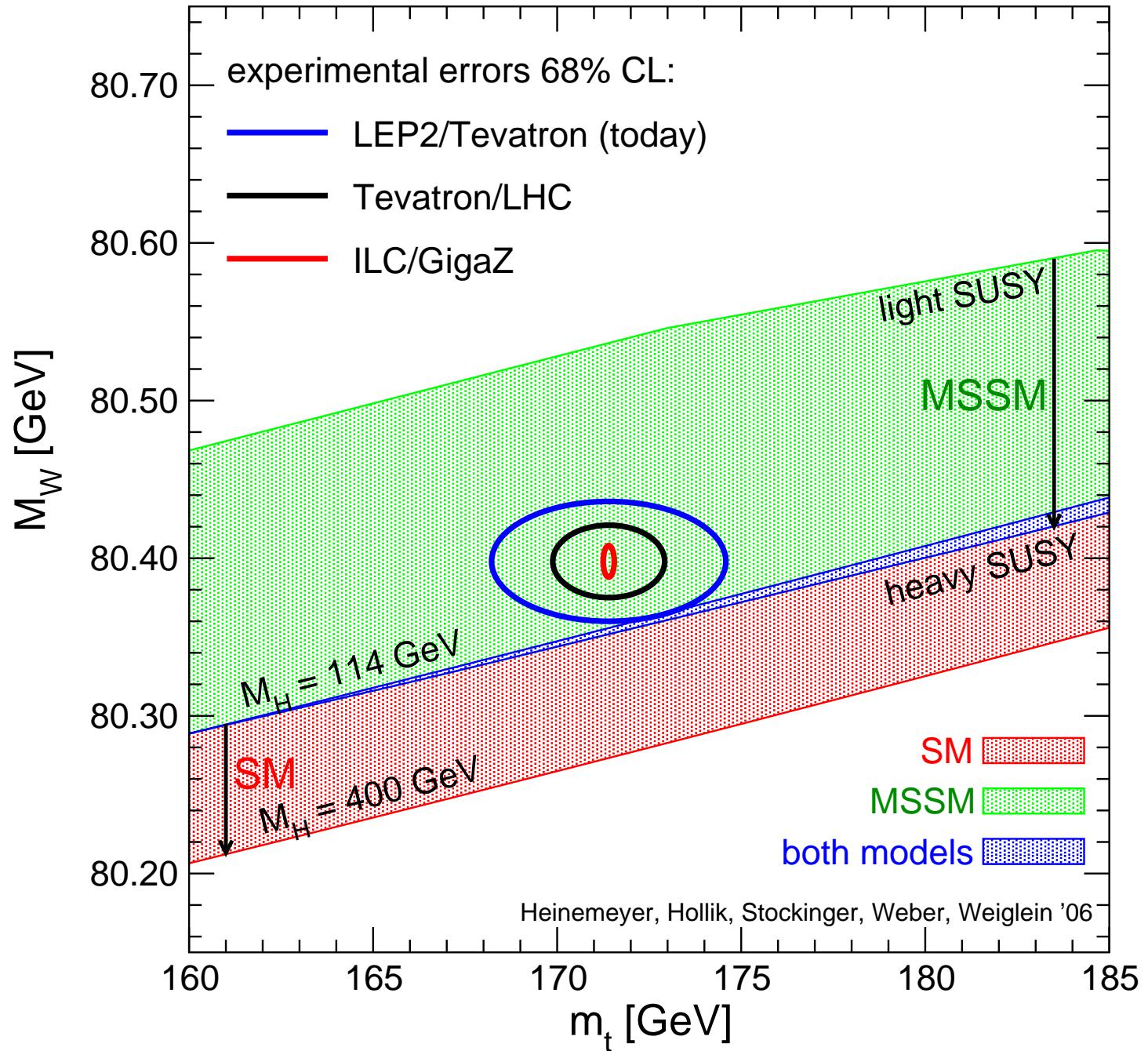
$\Delta r$  : quantum correction,  $\Delta r = \Delta r(m_t, X_{\text{SUSY}})$

$\rightarrow M_W = M_W(\alpha, G_F, M_Z, m_t, X_{\text{SUSY}})$

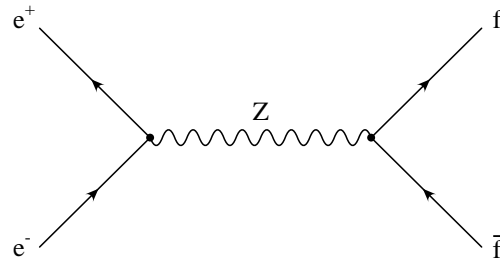
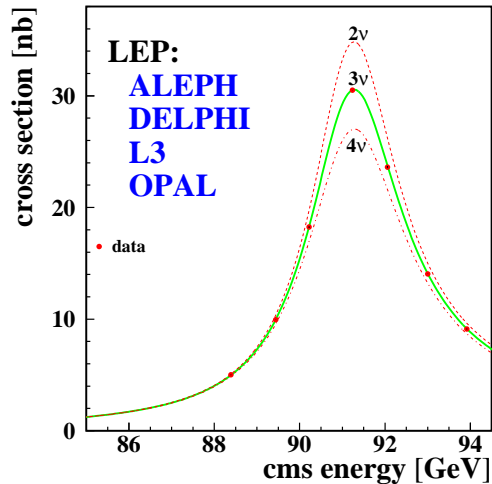
$X_{\text{SUSY}}$  = set of non-standard model parameters



$$\tan \beta = 10, \quad M_A = \mu = M_2 = M_{\tilde{g}} = 300 \text{ GeV}$$



# Z resonance

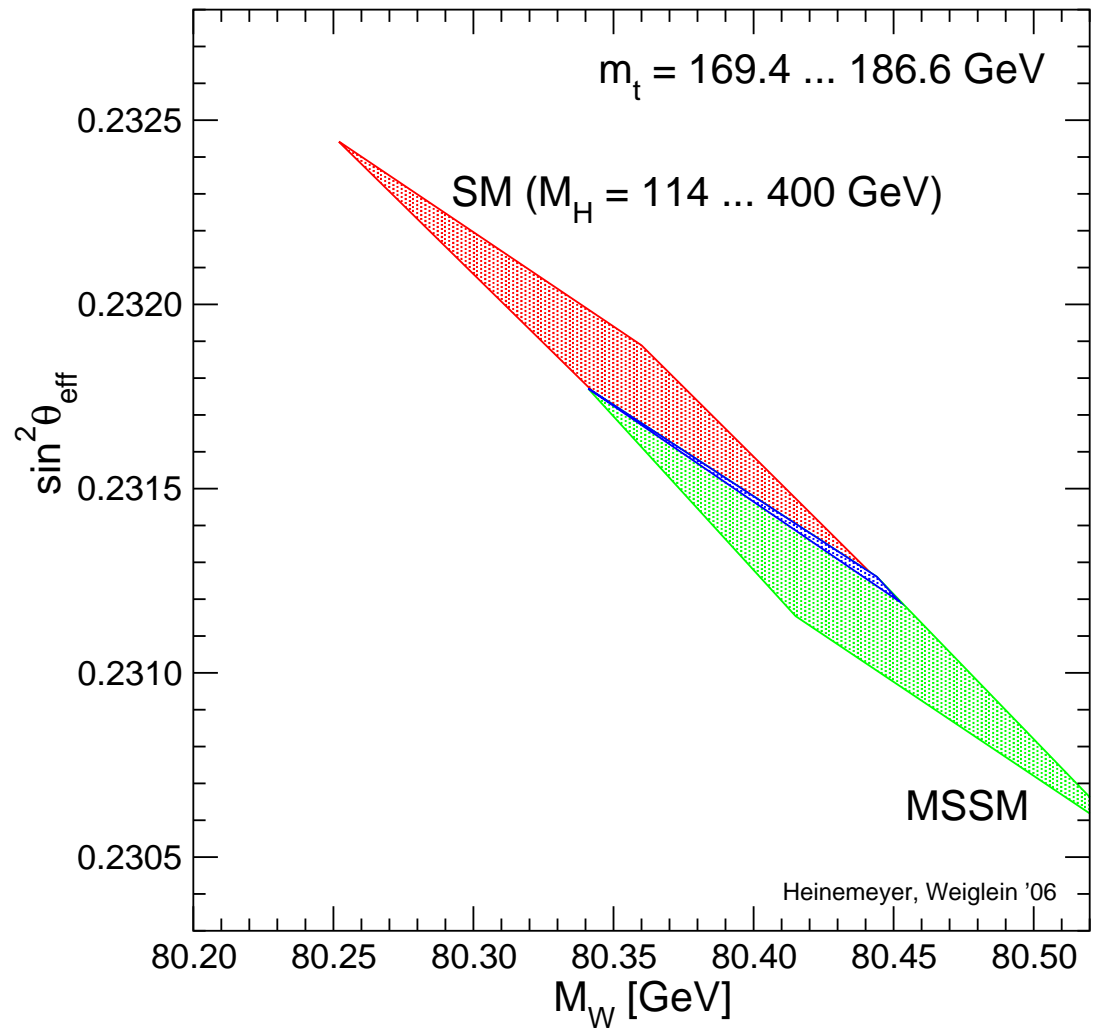


- effective Z boson couplings with higher-order  $\Delta g_{V,A}$

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

- effective ew mixing angle (for  $f = e$ ):

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4} \left( 1 - \text{Re} \frac{g_V^e}{g_A^e} \right)$$



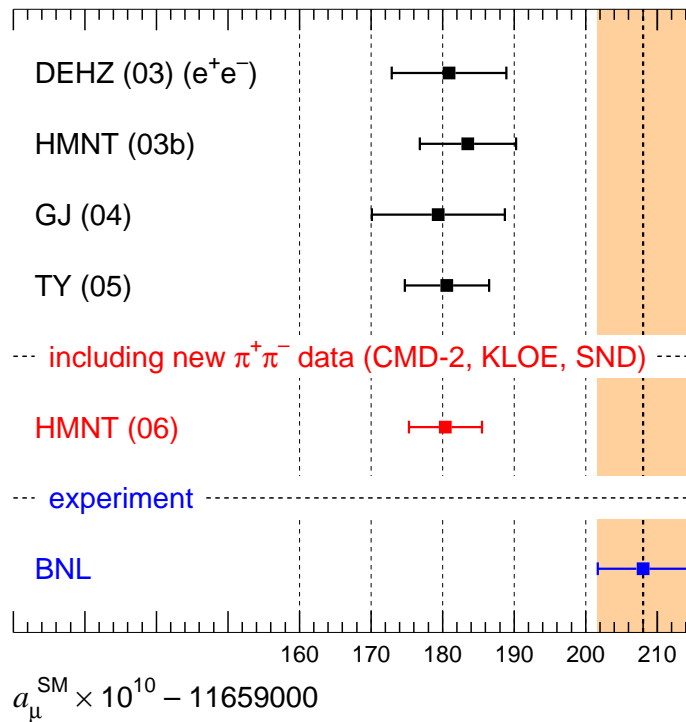
# Anomalous g-factor of the muon

- Dirac theory:  $g = 2$
- QED, 1-loop order:  $g = 2 + \frac{\alpha}{\pi}$
- Standard Model prediction  
QED part: 4-loop (5-loop estimate)  
Electroweak part: 2-loop
- **Experiment 2004: Brookhaven E821**

$$a_{\mu} = \frac{g - 2}{2} = 11659208(6) \cdot 10^{-10}$$

above the SM prediction

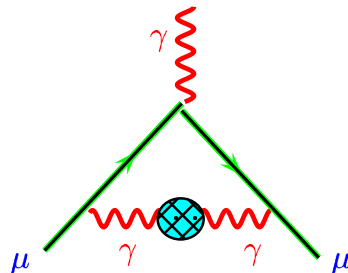
# Theory versus experiment



*Hagiwara, Martin, Nomura, Teubner*

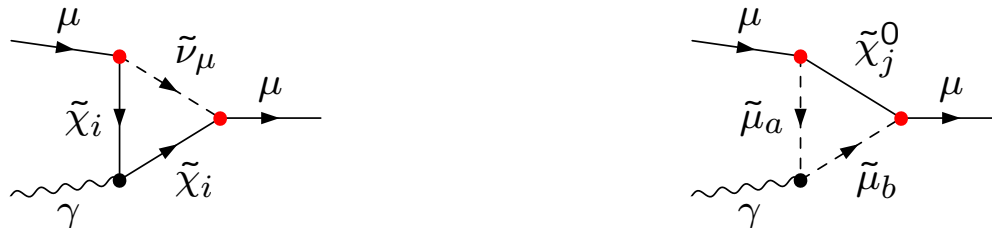
$e^+e^-$  data based SM prediction:  $3.4 \sigma$  below exp. value

theory uncertainty from hadronic vacuum polarization



# $g - 2$ with supersymmetry

new contributions from virtual SUSY partners of  $\mu$ ,  $\nu_\mu$  and of  $W^\pm$ ,  $Z$



extra terms

$$+ \frac{\alpha}{\pi} \frac{m_\mu^2}{M_{\text{SUSY}}^2} \cdot \frac{v_2}{v_1}$$

can provide missing contribution for

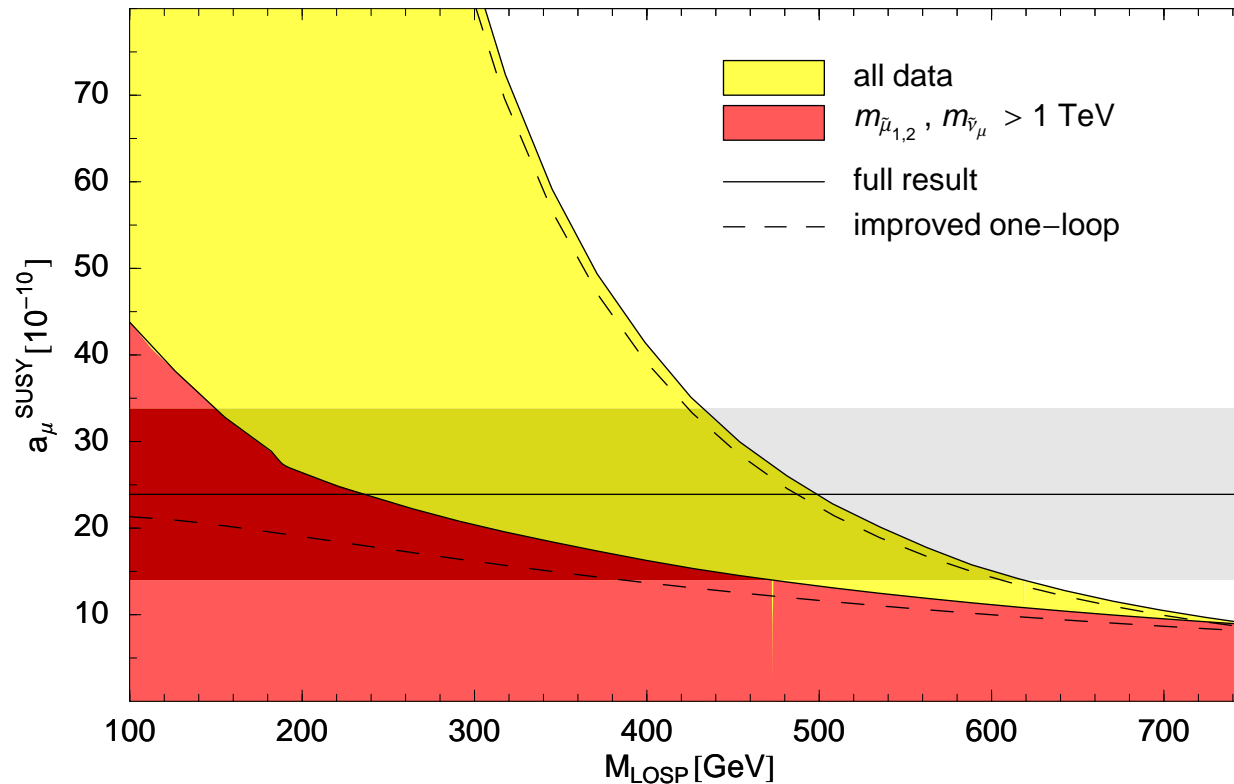
$$M_{\text{SUSY}} = 200 - 600 \text{ GeV}$$

2-loop calculation [*Heinemeyer, Stöckinger, ...*]



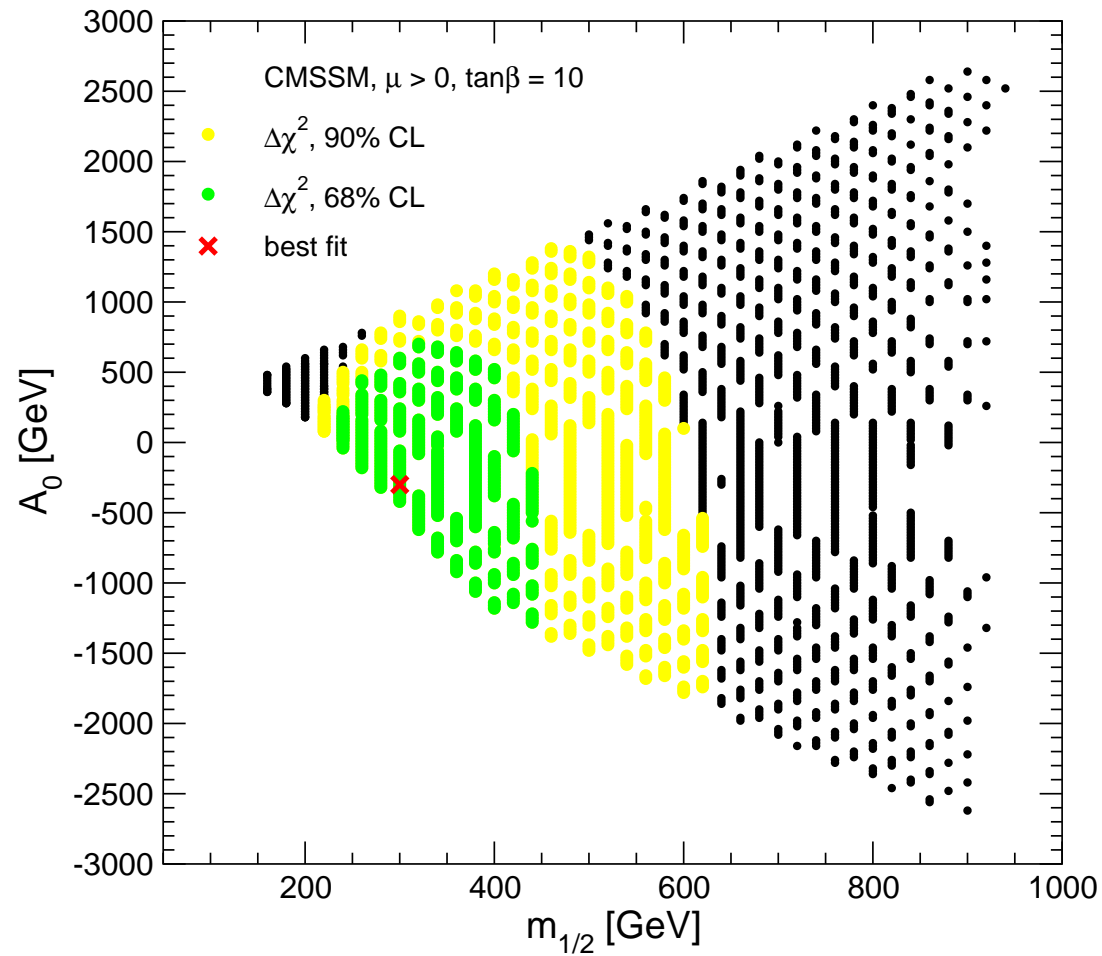
# scan over SUSY parameters compatible with EW and $b \rightarrow s\gamma$ constraints $(\tan \beta = 50)$

[Stöckinger]



LOSP = lightest observable SUSY particle ( $\chi_1^{\pm}, \chi_2^0, \dots$ )

# fits in SUGRA model:



[Ellis, Heinemeyer, Olive, Weiglein]

# Higgs bosons in the MSSM

MSSM Higgs potential contains two Higgs doublets:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ + \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

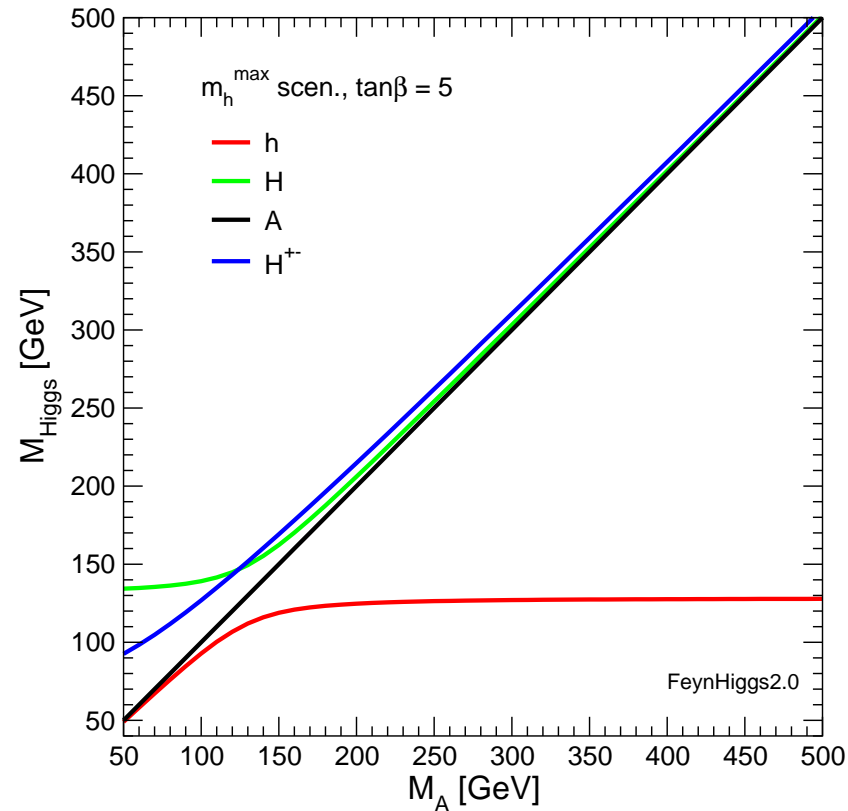
gauge couplings, in contrast to SM

Five physical states:  $h^0, H^0, A^0, H^\pm$

Input parameters:  $\tan \beta = \frac{v_2}{v_1}, M_A$

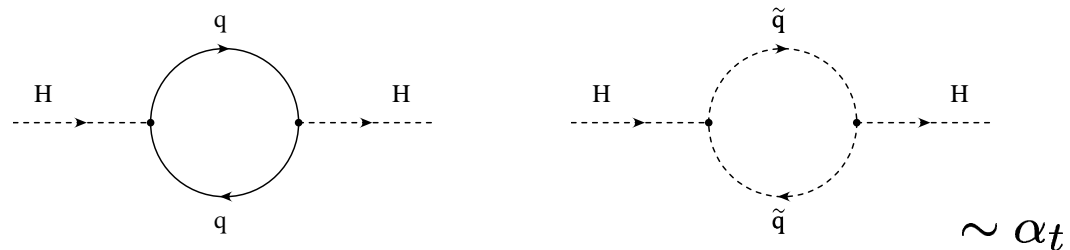
$\Rightarrow m_h, m_H, \text{mixing angle } \alpha, m_{H^\pm}$ : no free parameters

# Spectrum of Higgs bosons in the MSSM (example)



large  $M_A$ :  $h^0$  like SM Higgs boson  $\sim$  decoupling regime

$m_h^0$  strongly influenced by quantum effects, e.g.



# determination of masses and couplings at higher order

- physical states  $h, H, A, H^\pm$
- conventional input:  $M_A, \tan \beta = v_2/v_1$

dressed  $h, H$  propagators, renormalized self-energies  $\hat{\Sigma}$

$$(\Delta_{\text{Higgs}})^{-1} = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_H(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{Hh}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_h(q^2) \end{pmatrix}$$

- $\det = 0 \rightarrow m_{h,H}^{\text{pole}}$
- diagonalization  $\rightarrow$  effective couplings ( $\alpha_{\text{eff}}$ )

1-loop: complete

2-loop:

- QCD corrections  $\sim \alpha_s \alpha_t, \alpha_s \alpha_b$
- Yukawa corrections  $\sim \alpha_t^2$

present theoretical uncertainty:

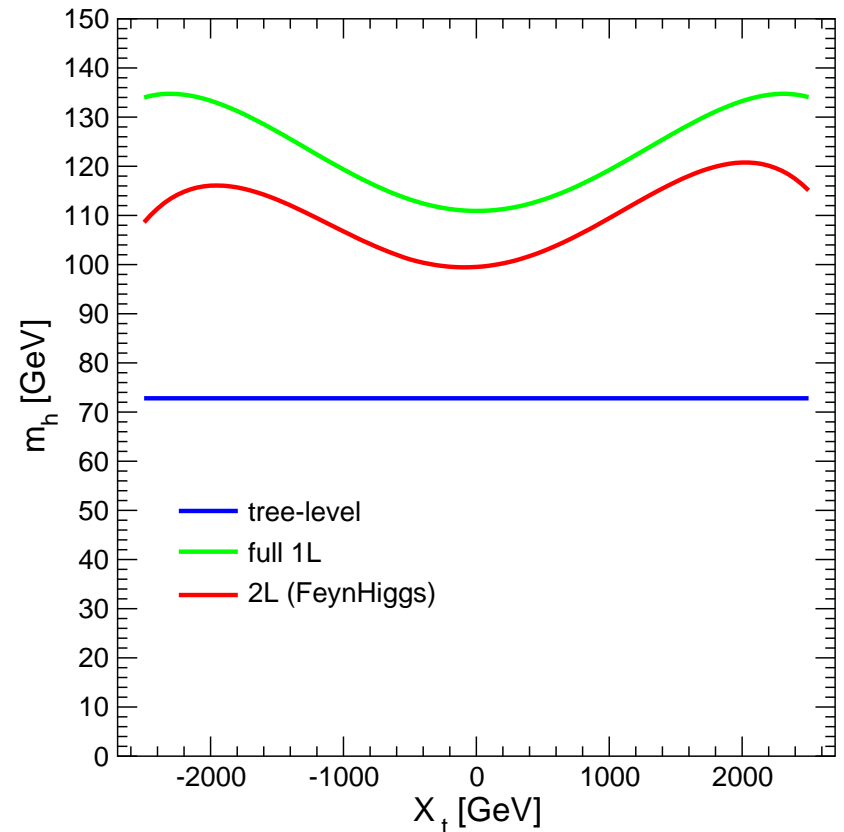
$$\delta m_h \simeq 3\text{-}4 \text{ GeV}$$

[Degrassi, Heinemeyer, WH, Slavich,  
Weiglein]

new version

FeynHiggs2.5

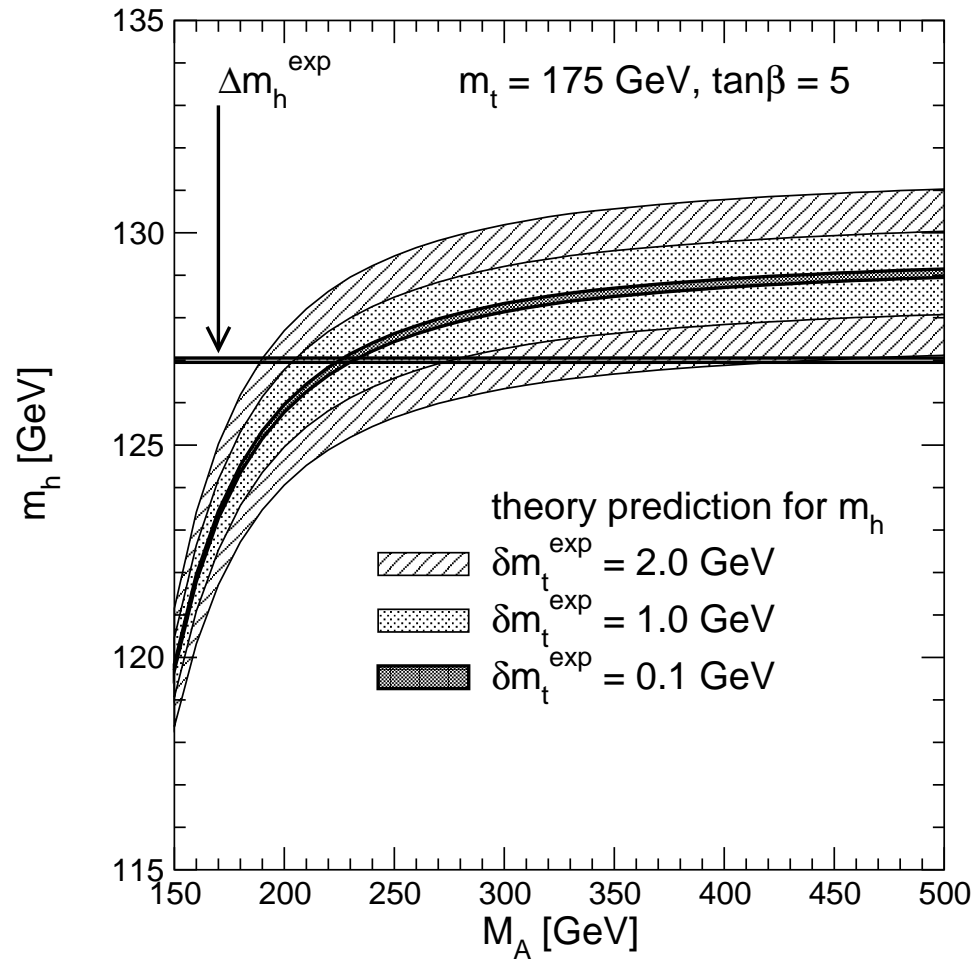
$m_{h^0}$  prediction at different levels of accuracy:



$\tan \beta = 3, \quad M_{\tilde{Q}} = M_A = 1 \text{ TeV}, \quad m_{\tilde{g}} = 800 \text{ GeV}$

$X_t$  : top-squark mixing parameter

$$X_t = A_t - \mu \cot \beta$$



dependent on all SUSY particles and masses/mixings through Higgs self-energies

## Recent developments:

### 1. Counterterms at two-loop order

ST identities valid in dimensional reduction (DR)

DR scheme consistent with symmetric counterterms

*[WH, Stöckinger]*

### 2. $\mathcal{O}(\alpha_s \alpha_b)$ beyond $m_b^{\text{eff}}$ approximation

$m_b^{\text{eff}} = \frac{m_b}{1 + \Delta m_b}$  in  $\alpha_b$  Yukawa coupling

$\Delta m_b = \text{non-decoupling SUSY contribution} \sim \alpha_s \mu \tan \beta$

*[Heinemeyer, WH, Rzehak, Weiglein]*

small shifts  $\sim$  few GeV, but stabilizes prediction

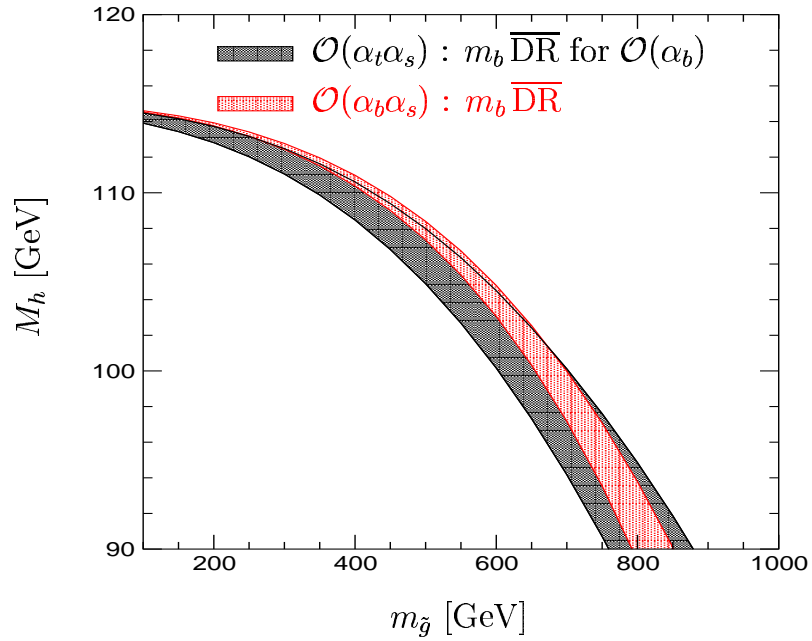
### 3. MSSM with complex parameters

tree level: CP conserving Higgs sector

loop level: CP violation  $\leftarrow$  **other sectors**

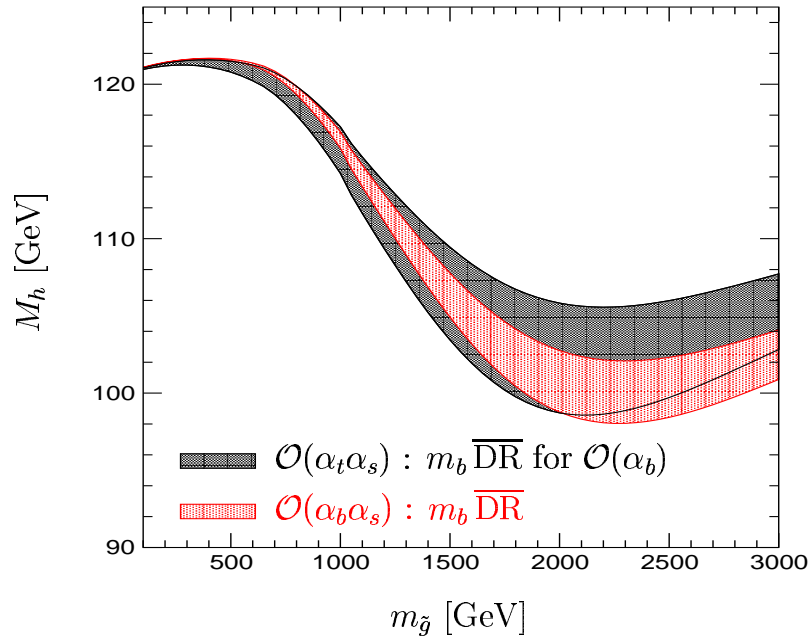
*[Frank, Heinemeyer, WH, Rzehak, Weiglein]*





$$m_t/2 < \mu^{\overline{\text{DR}}} < 2m_t$$

$$M_A = \begin{cases} 120 \text{ GeV} \\ 700 \text{ GeV} \end{cases}$$



## The Higgs sector of the cMSSM at the loop-level:

Complex parameters enter via loop corrections:

- $\mu$  : Higgsino mass parameter
- $A_{t,b,\tau}$  : trilinear couplings  $\Rightarrow X_{t,b,\tau} = A_{t,b,\tau} - \mu^* \{\cot \beta, \tan \beta\}$  complex
- $M_{1,2}$  : gaugino mass parameter (one phase can be eliminated)
- $M_3$  : gluino mass parameter

$\Rightarrow$  can induce  $\mathcal{CP}$ -violating effects

Result:

$$(A, H, h) \rightarrow (h_3, h_2, h_1)$$

with

$$m_{h_3} > m_{h_2} > m_{h_1}$$

$\Rightarrow$  strong changes in Higgs couplings to SM gauge bosons and fermions

## Inclusion of higher-order corrections:

→ Feynman-diagrammatic approach

Propagator / mass matrix with higher-order corrections:

$$\begin{pmatrix} q^2 - M_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$  ( $i, j = h, H, A$ ) : renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0 \Rightarrow \mathcal{CPV}$ ,  $\mathcal{CP}$ -even and  $\mathcal{CP}$ -odd fields can mix

propagator matrix  $\Delta$ :

$$\Delta^{-1}(q^2) = q^2 \mathbf{1} - \mathbf{m}_{\text{tree}}^2 + \hat{\Sigma}(q^2)$$

renormalized self energies:

$$\hat{\Sigma} = \Sigma + \text{counter terms}$$

- renormalization of tadpoles:

$$T_h + \delta T_h = 0, \quad T_H + \delta T_H = 0, \quad T_A + \delta T_A = 0$$

- renormalization of  $M_{H^\pm}$ :  $\delta M_{H^\pm}^2 = \Sigma_{H^\pm}(M_{H^\pm}^2)$   
on-shell condition for pole mass

- renormalization of  $\tan \beta$ :

$$\begin{aligned} \tan \beta = \frac{v_2}{v_1} &\rightarrow \sqrt{\frac{Z_{H_2}}{Z_{H_1}}} \cdot \frac{v_2 + \delta v_2}{v_1 + \delta v_1} \\ &= \frac{v_2}{v_1} \left( 1 + \overbrace{\delta Z_{H_2}}^{\overline{DR}} - \delta Z_{H_1} + \frac{\delta v_2}{v_2} - \frac{\delta v_1}{v_1} \right) \\ &= 0 \end{aligned}$$

propagator matrix  $\Delta$ :

$$\Delta^{-1}(p^2) = p^2 \mathbf{1} - \mathbf{m}_{\text{tree}}^2 + \hat{\Sigma}(p^2)$$

- $\hat{\Sigma}(p^2)$  contain imaginary parts
- Higgs boson masses are complex poles:  
 $s_0 = M^2 - iM\Gamma$
- zeros of determinant:  $\det[\Delta^{-1}(s_0)] = 0$

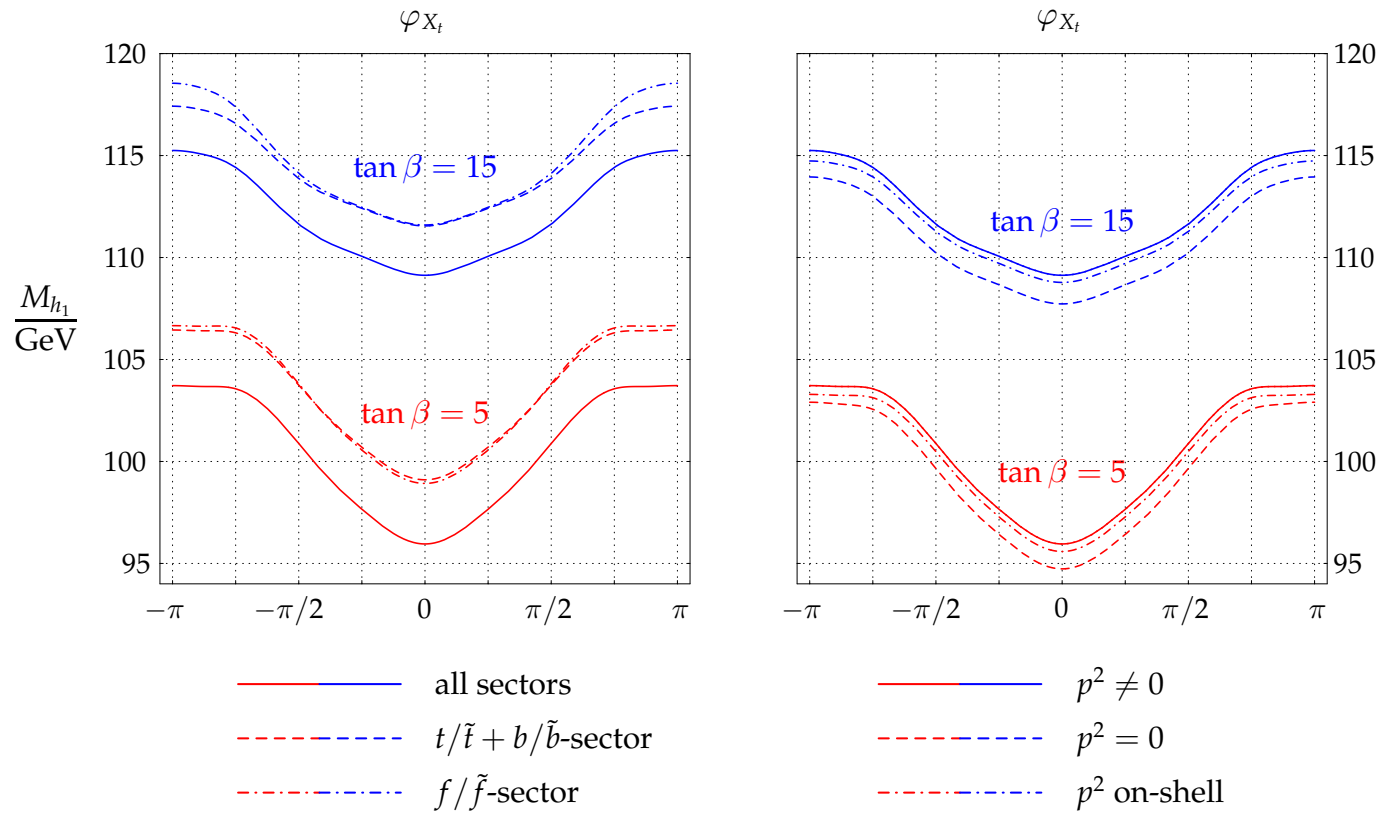
$p^2 = 0$  approximation:

$$\mathbf{M}^2 \simeq \mathbf{m}_{\text{tree}}^2 - \hat{\Sigma}(0)$$

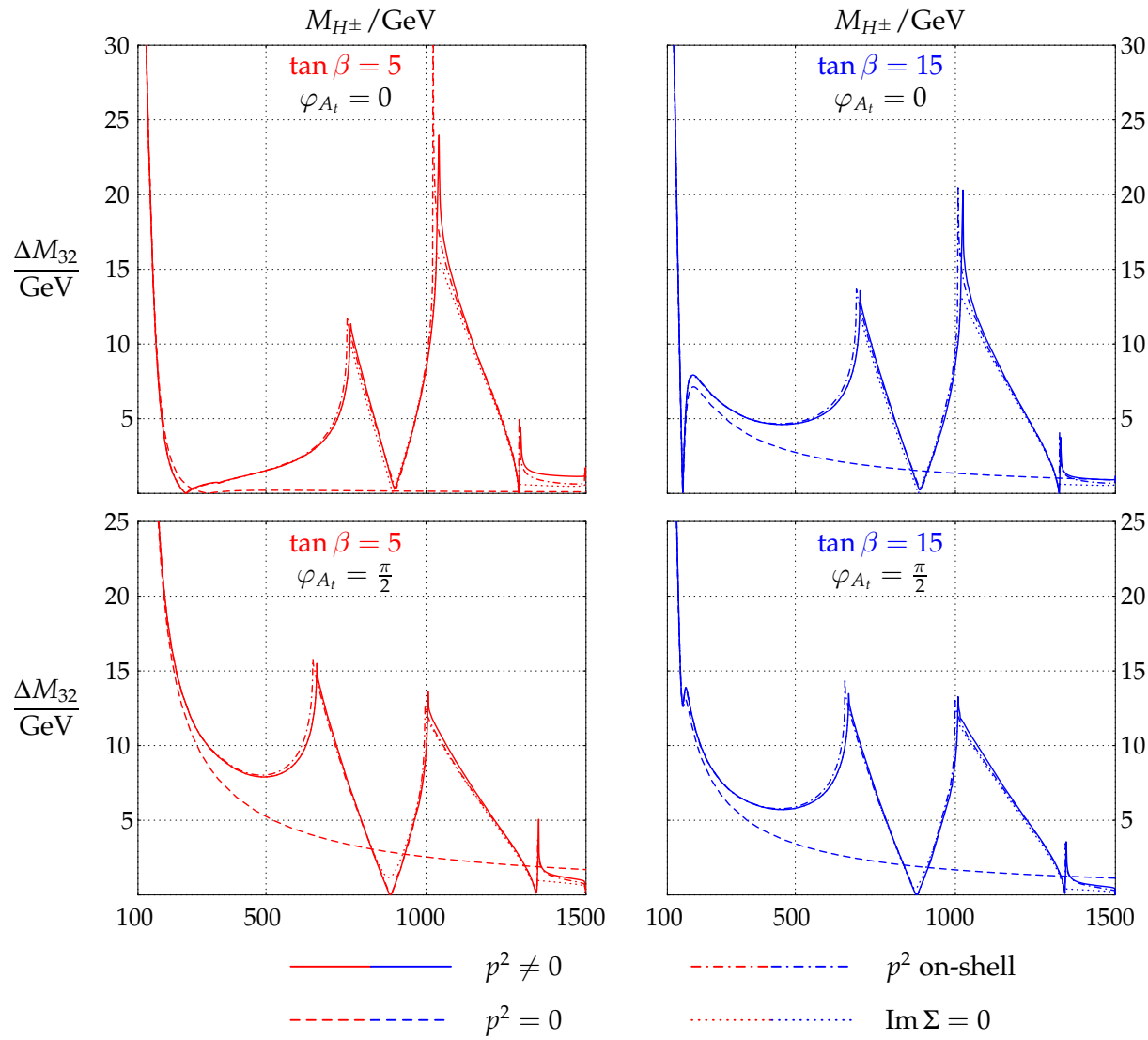
diagonalized by orthogonal matrix **R**

on-shell approximation:

$$\hat{\Sigma}_{ii}(m_i^2), \hat{\Sigma}_{ij}\left(\frac{m_i^2 + m_j^2}{2}\right)$$

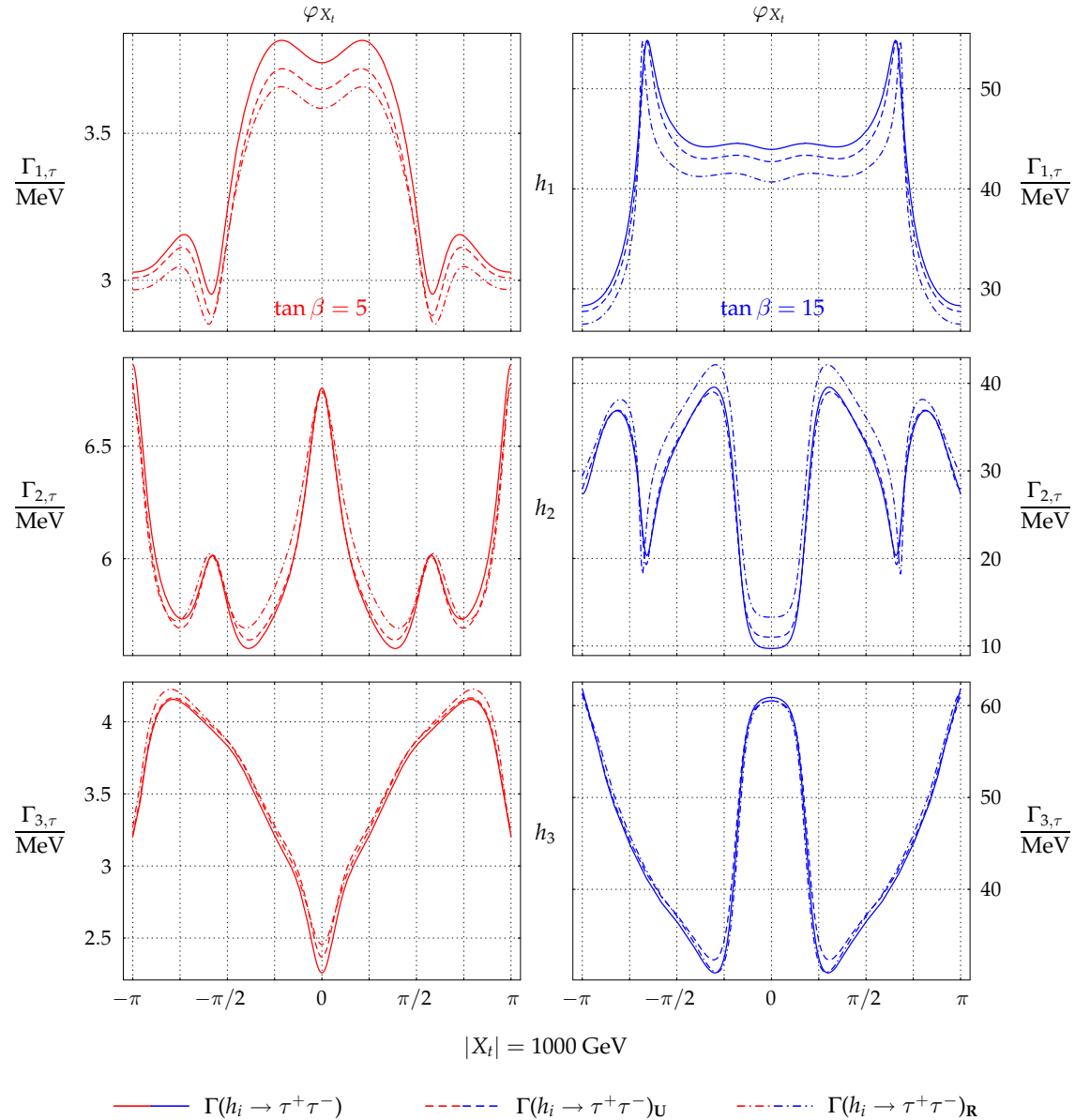


[Frank, Hahn, Heinemeyer, WH, Rzehak, Weiglein]



# implications for couplings

example:  $h_i \rightarrow \tau^+ \tau^-$





present status:

effective potential approximation + RGE

*[Carena, Ellis, Pilaftsis, Wagner]*

complete at one-loop order

*[Frank, Hahn, Heinemeyer, WH, Rzehak, Weiglein]*

leading two-loop contributions of  $\mathcal{O}(\alpha_s\alpha_t)$

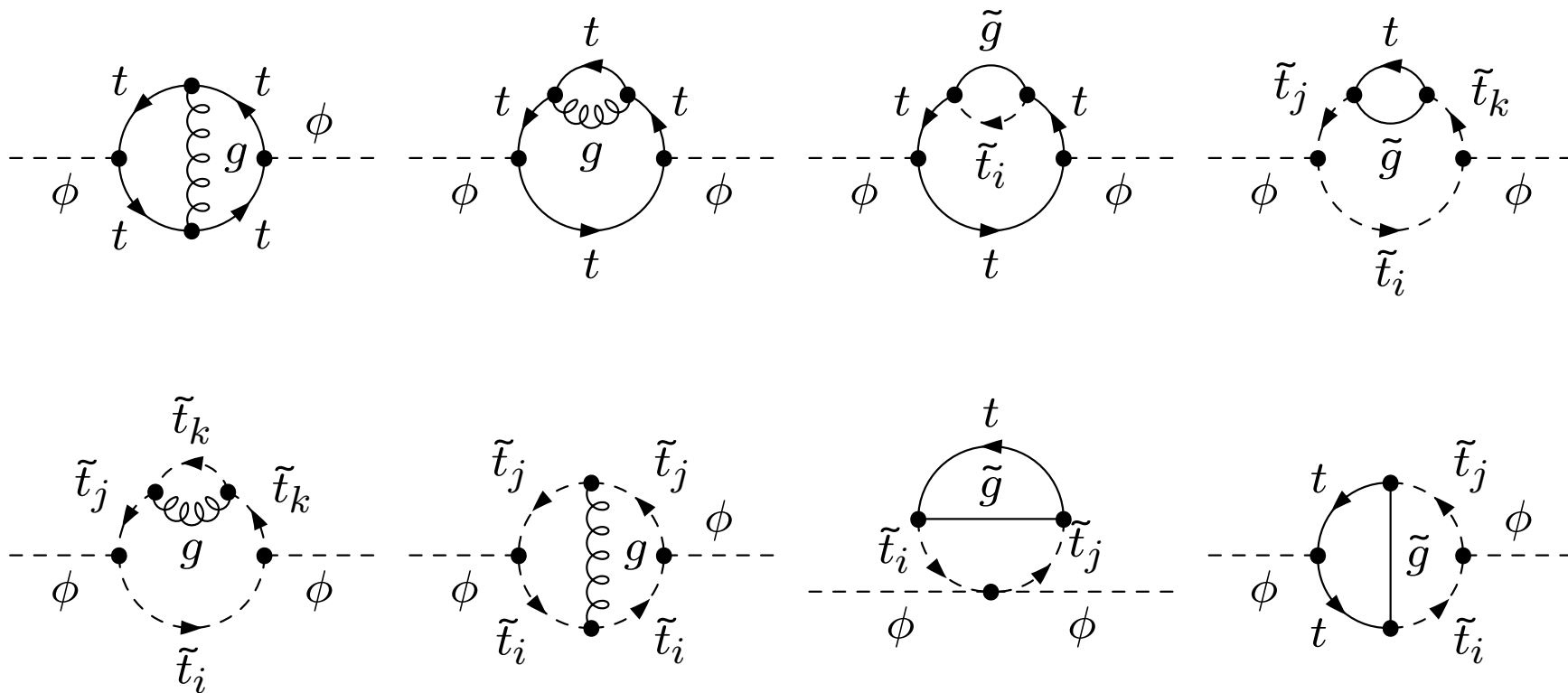
*[Rzehak, PhD thesis]*

for Higgs phenomenology with CP violation see

CERN 2006-009, hep-ph/0608079 *[S. Kraml et al. (Conv.)]*

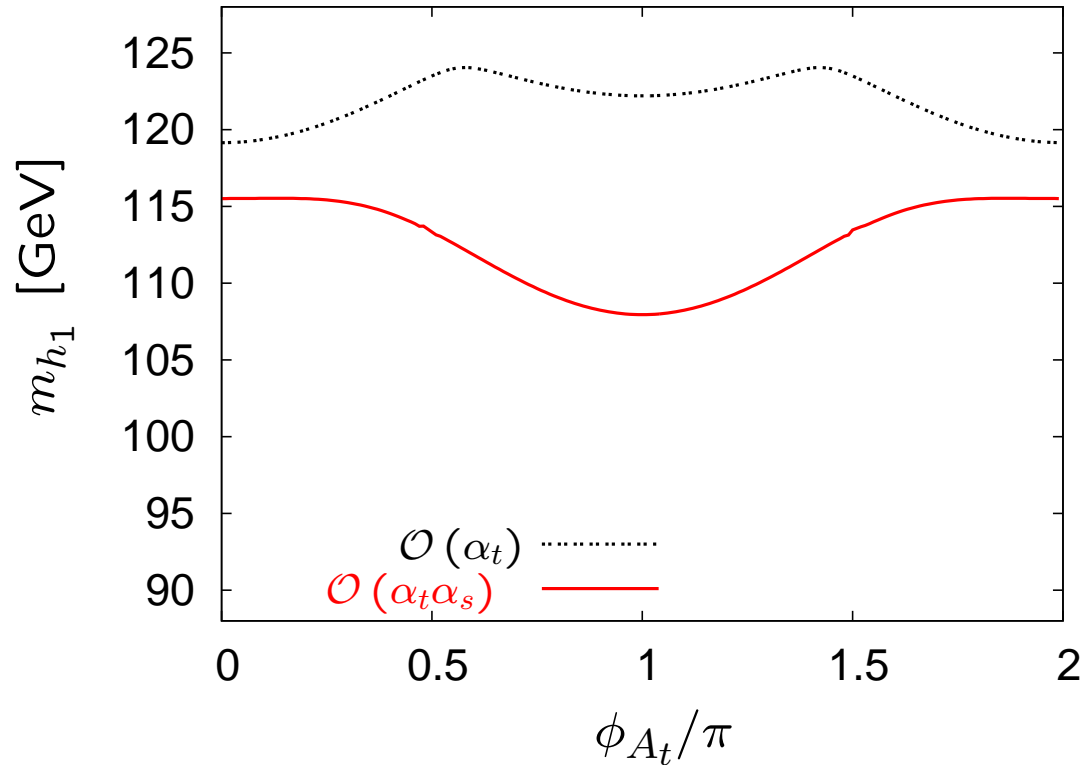
## Contributions to the 2-loop self-energy:

2-loop self-energy diagrams:



$\phi = h, H, A$

$m_{h_1}$  as a function of  $\phi_{A_t}$ :



$M_{\text{SUSY}} = 1000 \text{ GeV}$

$|A_t| = 2000 \text{ GeV}$

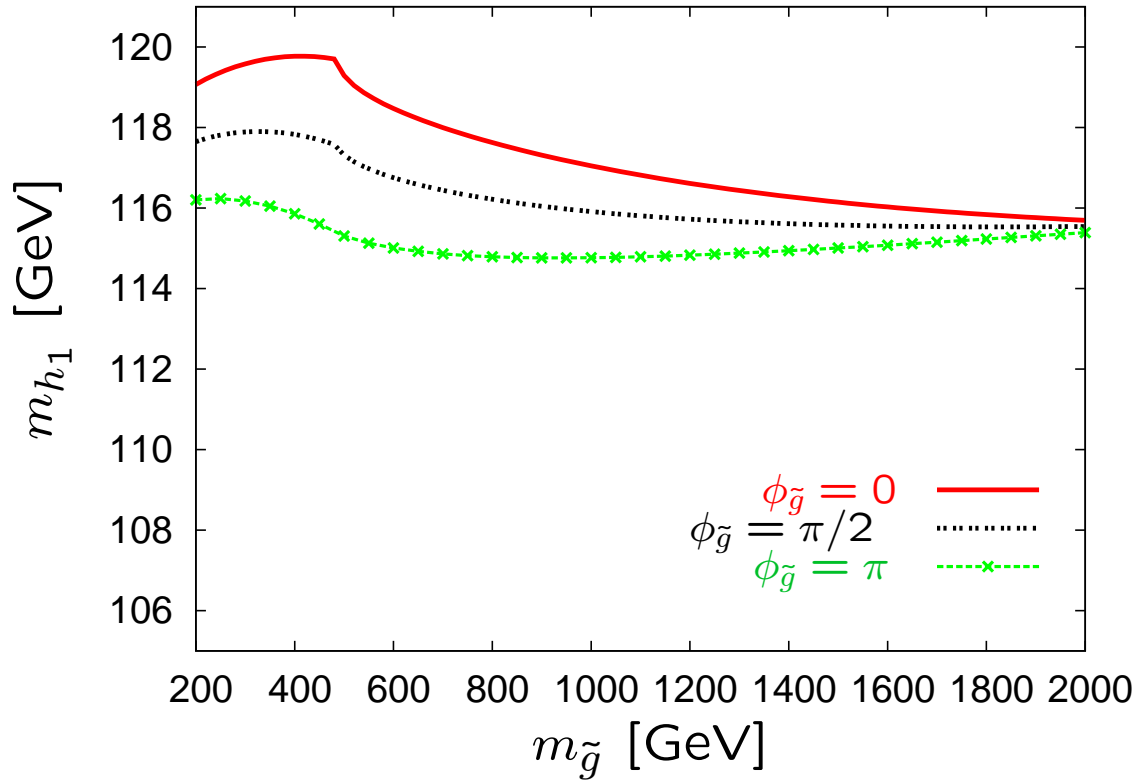
$\tan \beta = 10$

$M_{H^\pm} = 150 \text{ GeV}$

OS renormalization

$\Rightarrow$  modified dependence  
on  $\phi_{A_t}$  at the 2-loop level

$m_{h_1}$  as a function of  $\phi_{\tilde{g}}$ :



$M_{\text{SUSY}} = 500 \text{ GeV}$

$A_t = 1000 \text{ GeV}$

$\tan \beta = 10$

$M_{H^\pm} = 500 \text{ GeV}$

OS renormalization

$\Rightarrow$  threshold at  $m_{\tilde{g}} = m_{\tilde{t}} + m_t$

$\Rightarrow$  large effects around threshold

$\Rightarrow$  phase dependence has to be taken into account

# SUSY particles

from experiment:

→ precision analyses of masses and couplings **LHC⊕ILC**

from theory:

- accurate theoretical predictions to match exp. data
- loop contributions to **Lagrangian param** ↔ **observables**
- reconstruction of fundamental SUSY parameters and breaking mechanism
- RGEs for extrapolation to high scales

- **chargino/neutralino sector**  
 complete at one loop:  
 renormalization and mass spectrum  
 pair production and decay processes  
*[Fritzsche, WH] [Eberl, Majerotto, Öller]*
- **sfermion sector**  
 renormalization and mass spectrum  
*[WH, Rzehak]*  
 sfermion pair production in  $e^+e^-$  collisions  
 complete at one-loop
 

<i>[Arhrib, WH]</i>	squarks, sleptons
<i>[Kovarik, Weber, Eberl, Majerotto]</i>	squarks
<i>[Freitas, Miller, von Manteuffel, Zerwas]</i>	sleptons
- **sfermion decays into fermions and -inos**  
 complete at one-loop  
*[Guasch, WH, Solà]*

## Basis for precision calculations

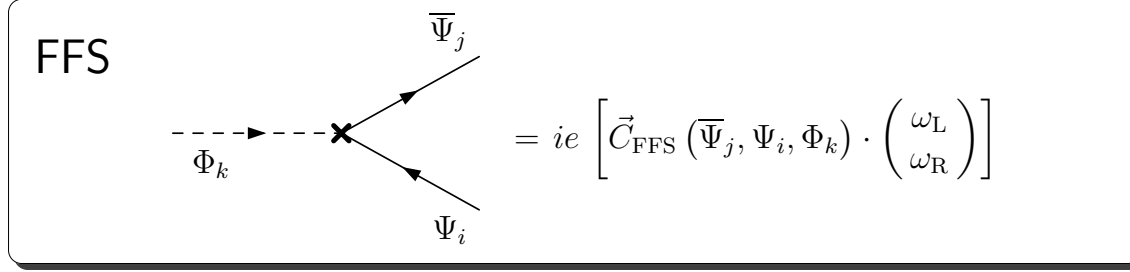
- complete Feynman rules → FeynArts  
*[Hahn, Schappacher]*
- complete set of counter terms  
automatic generation → FeynArts *[Fritzsche]*
- real photon bremsstrahlung

## Renormalization schemes

- on-shell scheme:  
renormalization conditions for pole masses  
*[WH, Kraus, Roth, Rupp, Sibold, Stöckinger]*
- $\overline{\text{DR}}$  scheme:  
CTs = singular parts in dimensional reduction

SUSY parameters different in  $\overline{\text{DR}}$  and on-shell

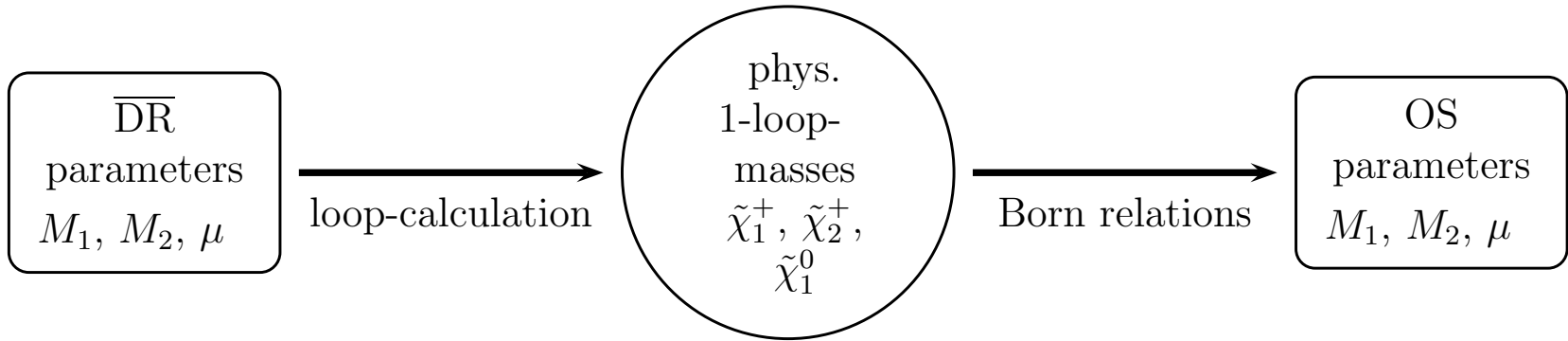
# example:



►  $\vec{C}_{\text{FFS}} (\bar{e}_j, \tilde{\chi}_i^0, \tilde{e}_k^s) = \frac{\delta_{jk}}{2\sqrt{2}c_\beta^2 c_w^3 M_W^3 s_w^2} \times$

$$\left[ \begin{aligned} & -c_\beta c_w^2 M_W^2 s_w \sum_{n=1}^4 [\delta Z_{\tilde{\chi}^0}]_{ni} \left( c_w m_{e_j} N_{n3}^* U_{s1}^{\tilde{e}_j^*} + 2c_\beta M_W s_w N_{n1}^* U_{s2}^{\tilde{e}_j^*} \right) - \\ & c_\beta c_w^2 M_W^2 s_w \sum_{n=1}^2 [\delta Z_{\tilde{e}^k}]_{ns} \left( c_w m_{e_j} N_{i3}^* U_{n1}^{\tilde{e}_j^*} + 2c_\beta M_W s_w N_{i1}^* U_{n2}^{\tilde{e}_j^*} \right) - \\ & \left( c_w^3 N_{i3}^* U_{s1}^{\tilde{e}_j^*} \right) \left\{ 2c_\beta M_W^2 s_w \delta m_j^e + m_{e_j} \left[ -2\delta c_\beta M_W^2 s_w + c_\beta M_W^2 s_w [\delta Z_e^R]_{jj}^* + \right. \right. \\ & \quad \left. \left. c_\beta (-\delta M_W^2 s_w - 2M_W^2 (\delta s_w - \delta Z_e s_w)) \right] \right\} - \\ & \left( 2c_w^2 \delta Z_e + 2\delta s_w s_w + c_w^2 [\delta Z_e^R]_{jj}^* \right) \left( 2c_\beta^2 M_W^3 s_w^2 N_{i1}^* U_{s2}^{\tilde{e}_j^*} \right) \\ & \times \\ & c_\beta c_w^2 M_W^2 s_w \sum_{n=1}^4 [\delta Z_{\tilde{\chi}^0}]_{ni}^* \left( (s_w N_{n1} + c_w N_{n2}) \left( c_\beta M_W U_{s1}^{\tilde{e}_j^*} \right) - c_w m_{e_j} N_{n3} U_{s2}^{\tilde{e}_j^*} \right) + \\ & c_\beta c_w^2 M_W^2 s_w \sum_{n=1}^2 [\delta Z_{\tilde{e}^k}]_{ns} \left( (s_w N_{i1} + c_w N_{i2}) \left( c_\beta M_W U_{n1}^{\tilde{e}_j^*} \right) - c_w m_{e_j} N_{i3} U_{n2}^{\tilde{e}_j^*} \right) - \\ & \left( c_w^3 N_{i3} U_{s2}^{\tilde{e}_j^*} \right) \left\{ 2c_\beta M_W^2 s_w \delta m_j^e + m_{e_j} \left[ -2\delta c_\beta M_W^2 s_w + c_\beta M_W^2 s_w [\delta Z_e^L]_{jj}^* + \right. \right. \\ & \quad \left. \left. c_\beta (-\delta M_W^2 s_w - 2M_W^2 (\delta s_w - \delta Z_e s_w)) \right] \right\} + \\ & \left( c_\beta^2 M_W^3 U_{s1}^{\tilde{e}_j^*} \right) \left[ \left( 2c_w^2 \delta Z_e + 2\delta s_w s_w + c_w^2 [\delta Z_e^L]_{jj}^* \right) (s_w^2 N_{i1}) + \right. \\ & \quad \left. \left( s_w [\delta Z_e^L]_{jj}^* - 2(\delta s_w - \delta Z_e s_w) \right) (c_w^3 N_{i2}) \right] \end{aligned} \right]$$





pole masses  $\leftrightarrow$  on-shell parameters

$$\begin{aligned}
 M_2^2 + \mu^2 + 2M_W^2 &= m_{\tilde{\chi}_1^+}^2 + m_{\tilde{\chi}_2^+}^2 \\
 (M_2 \mu - 2M_W^2 \sin \beta \cos \beta)^2 &= m_{\tilde{\chi}_1^+}^2 m_{\tilde{\chi}_2^+}^2 .
 \end{aligned}$$

$$\begin{aligned}
 M_1 &= \left[ -M_2 \mu M_Z^2 \sin 2\beta + [\mu M_Z^2 \sin 2\beta - M_2(\mu^2 + M_Z^2 s_W^2)] m_{\tilde{\chi}_1^0} \right. \\
 &\quad \left. + [\mu^2 + M_Z^2] m_{\tilde{\chi}_1^0}^2 + M_2 m_{\tilde{\chi}_1^0}^3 - m_{\tilde{\chi}_1^0}^4 \right] \\
 &\quad \times \left[ \mu M_Z^2 c_W^2 \sin 2\beta - M_2 \mu^2 + [\mu^2 + M_Z^2 c_W^2] m_{\tilde{\chi}_1^0} + M_2 m_{\tilde{\chi}_1^0}^2 - m_{\tilde{\chi}_1^0}^3 \right]^{-1}
 \end{aligned}$$

## $\overline{\text{DR}}$ parameters (SPS1a')

$$\begin{aligned}\tan\beta &= 10 & ; & \quad \mu = 402.87 \text{ GeV} \\ M_{A^0} &= 431.02 \text{ GeV} & ; & \quad M_1 = 103.22 \text{ GeV} \\ M_3 &= 572.33 \text{ GeV} & ; & \quad M_2 = 193.31 \text{ GeV}\end{aligned}$$

$$\begin{aligned}A_{u,c} &= -784.7 \text{ GeV} & ; & \quad A_t = -535.4 \text{ GeV} \\ A_{d,s} &= -1025.7 \text{ GeV} & ; & \quad A_b = -938.5 \text{ GeV} \\ A_{e,\mu} &= -449.0 \text{ GeV} & ; & \quad A_\tau = -445.5 \text{ GeV}\end{aligned}$$

$$\begin{aligned}m_{\tilde{t}}^{1,2} &= 181.3 \text{ GeV} & ; & \quad m_{\tilde{t}}^3 = 179.5 \text{ GeV} \\ m_{\tilde{e},\tilde{\mu}} &= 115.6 \text{ GeV} & ; & \quad m_{\tilde{\tau}} = 109.8 \text{ GeV} \\ m_{\tilde{q}}^{1,2} &= 526.9 \text{ GeV} & ; & \quad m_{\tilde{q}}^3 = 471.3 \text{ GeV} \\ m_{\tilde{u},\tilde{c}} &= 507.7 \text{ GeV} & ; & \quad m_{\tilde{t}} = 384.6 \text{ GeV} \\ m_{\tilde{d},\tilde{s}} &= 505.5 \text{ GeV} & ; & \quad m_{\tilde{t}} = 501.3 \text{ GeV}\end{aligned}$$

## on-shell parameters

$$\begin{aligned}\tan\beta &= 10 & ; & \quad \mu = 399.26 \text{ GeV} \\ M_{A^0} &= 431.02 \text{ GeV} & ; & \quad M_1 = 100.11 \text{ GeV} \\ M_3 &= 612.85 \text{ GeV} & ; & \quad M_2 = 197.55 \text{ GeV}\end{aligned}$$

$$\begin{aligned}A_{u,c} &= -784.7 \text{ GeV} & ; & \quad A_t = -535.4 \text{ GeV} \\ A_{d,s} &= -1025.7 \text{ GeV} & ; & \quad A_b = -938.5 \text{ GeV} \\ A_{e,\mu} &= -449.0 \text{ GeV} & ; & \quad A_\tau = -445.5 \text{ GeV}\end{aligned}$$

$$\begin{aligned}m_{\tilde{t}}^1 &= 184.12 \text{ GeV} & ; & \quad m_{\tilde{t}}^2 = 184.11 \text{ GeV} & ; & \quad m_{\tilde{t}}^3 = 182.18 \text{ GeV} \\ m_{\tilde{e}} &= 118.02 \text{ GeV} & ; & \quad m_{\tilde{\mu}} = 117.99 \text{ GeV} & ; & \quad m_{\tilde{\tau}} = 111.29 \text{ GeV} \\ m_{\tilde{q}}^1 &= 565.97 \text{ GeV} & ; & \quad m_{\tilde{q}}^2 = 565.91 \text{ GeV} & ; & \quad m_{\tilde{q}}^3 = 453.05 \text{ GeV} \\ m_{\tilde{u}} &= 546.78 \text{ GeV} & ; & \quad m_{\tilde{c}} = 546.84 \text{ GeV} & ; & \quad m_{\tilde{t}} = 460.52 \text{ GeV} \\ m_{\tilde{d}} &= 544.95 \text{ GeV} & ; & \quad m_{\tilde{s}} = 544.97 \text{ GeV} & ; & \quad m_{\tilde{t}} = 538.13 \text{ GeV}\end{aligned}$$



The SPA project is a joint study of theorists and experimentalists working on LHC and Linear Collider phenomenology. The study focuses on the supersymmetric extension of the Standard Model. The main targets are

- High-precision determination of the supersymmetry Lagrange parameters at the electroweak scale
- Extrapolation to a high scale to reconstruct the fundamental parameters and the mechanism for supersymmetry breaking

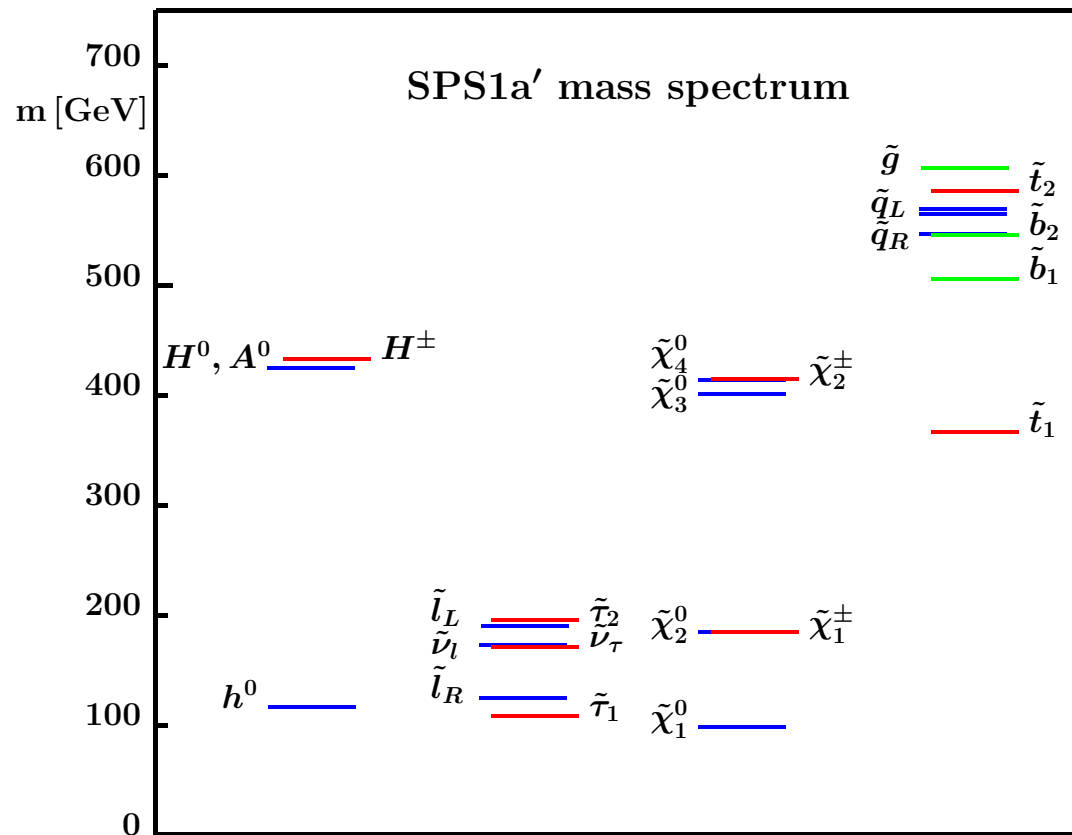
<http://spa.desy.de/spa>

P. Zerwas, J. Kalinowski, H.U. Martyn,  
W. Hollik, W. Kilian, W. Majerotto, W. Porod, ...

hep-ph/0511344, EPJC 46(2006)43

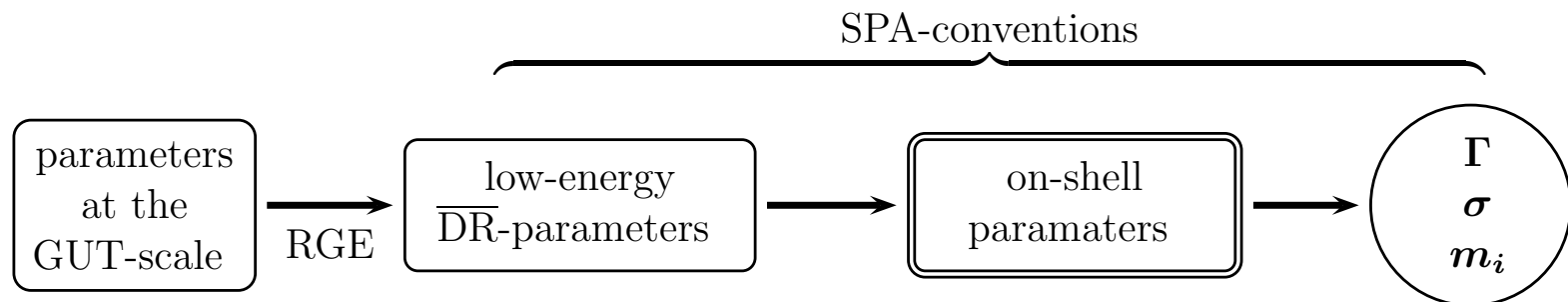
# SPS1a' scenario

$$\begin{aligned}
 M_{1/2} &= 250 \text{ GeV} & \text{sign}(\mu) &= +1 \\
 M_0 &= 70 \text{ GeV} & \tan \beta(\tilde{M}) &= 10 \\
 A_0 &= -300 \text{ GeV}
 \end{aligned}$$



# SPA CONVENTION

- The masses of the SUSY particles and Higgs bosons are defined as pole masses.
- All SUSY Lagrangian parameters, mass parameters and couplings, including  $\tan \beta$ , are given in the  $\overline{\text{DR}}$  scheme and defined at the scale  $\tilde{M} = 1 \text{ TeV}$ .
- Gaugino/higgsino and scalar mass matrices, rotation matrices and the corresponding angles are defined in the  $\overline{\text{DR}}$  scheme at  $\tilde{M}$ , except for the Higgs system in which the mixing matrix is defined in the on-shell scheme, the momentum scale chosen as the light Higgs mass.
- The Standard Model input parameters of the gauge sector are chosen as  $G_F$ ,  $\alpha$ ,  $M_Z$  and  $\alpha_s^{\overline{\text{MS}}}(M_Z)$ . All lepton masses are defined on-shell. The  $t$  quark mass is defined on-shell; the  $b$ ,  $c$  quark masses are introduced in  $\overline{\text{MS}}$  at the scale of the masses themselves while taken at a renormalization scale of 2 GeV for the light  $u$ ,  $d$ ,  $s$  quarks.
- Decay widths/branching ratios and production cross sections are calculated for the set of parameters specified above.



# DR masses $\rightarrow$ pole masses (SPS1a')

$m$	$\delta m$	$m_{\text{phys}}$	$m$	$\delta m$	$m_{\text{phys}}$
• $m_{\tilde{\chi}_1^+} = 181.026 +$	$3.178 =$	$184.204$	• $m_{\tilde{\chi}_1^0} = 100.706 + (-2.958) =$	$97.748$	
• $m_{\tilde{\chi}_2^+} = 423.420 + (-2.181) =$	$421.239$		$m_{\tilde{\chi}_2^0} = 181.404 + 3.022 =$	$184.425$	
• $m_{\tilde{g}} = 572.330 + 40.524 =$	$612.854$		$m_{\tilde{\chi}_3^0} = 408.579 + (-1.626) =$	$406.952$	
			$m_{\tilde{\chi}_4^0} = 422.991 + (-3.310) =$	$419.681$	
• $m_{\tilde{\nu}_1} = 169.890 + 2.804 =$	$172.695$		• $m_{\tilde{u}_1^1} = 506.424 + 39.255 =$	$545.680$	
• $m_{\tilde{e}_1^1} = 123.574 + 1.878 =$	$125.452$		• $m_{\tilde{u}_1^2} = 524.275 + 39.157 =$	$563.433$	
$m_{\tilde{e}_1^2} = 186.905 + 3.082 =$	$189.986$		• $m_{\tilde{d}_1^1} = 506.097 + 39.409 =$	$545.506$	
			$m_{\tilde{d}_1^2} = 530.033 + 38.826 =$	$568.859$	
• $m_{\tilde{\nu}_2} = 169.884 + 2.804 =$	$172.688$		• $m_{\tilde{u}_2^1} = 506.410 + 39.254 =$	$545.664$	
• $m_{\tilde{e}_2^1} = 123.510 + 1.877 =$	$125.387$		• $m_{\tilde{u}_2^2} = 524.285 + 39.158 =$	$563.444$	
$m_{\tilde{e}_2^2} = 186.929 + 3.080 =$	$190.009$		• $m_{\tilde{d}_2^1} = 506.092 + 39.408 =$	$545.500$	
			$m_{\tilde{d}_2^2} = 530.034 + 38.825 =$	$568.859$	
• $m_{\tilde{\nu}_3} = 168.001 + 2.629 =$	$170.630$		• $m_{\tilde{u}_3^1} = 333.171 + 35.334 =$	$368.504$	
• $m_{\tilde{e}_3^1} = 106.080 + 1.595 =$	$107.674$		• $m_{\tilde{u}_3^2} = 549.649 + 34.223 =$	$583.872$	
$m_{\tilde{e}_3^2} = 192.418 + 2.786 =$	$195.203$		$m_{\tilde{d}_3^1} = 470.247 + 34.711 =$	$504.958$	
			• $m_{\tilde{d}_3^2} = 506.244 + 38.129 =$	$544.374$	

# “QED corrections”

Full calculation inevitable

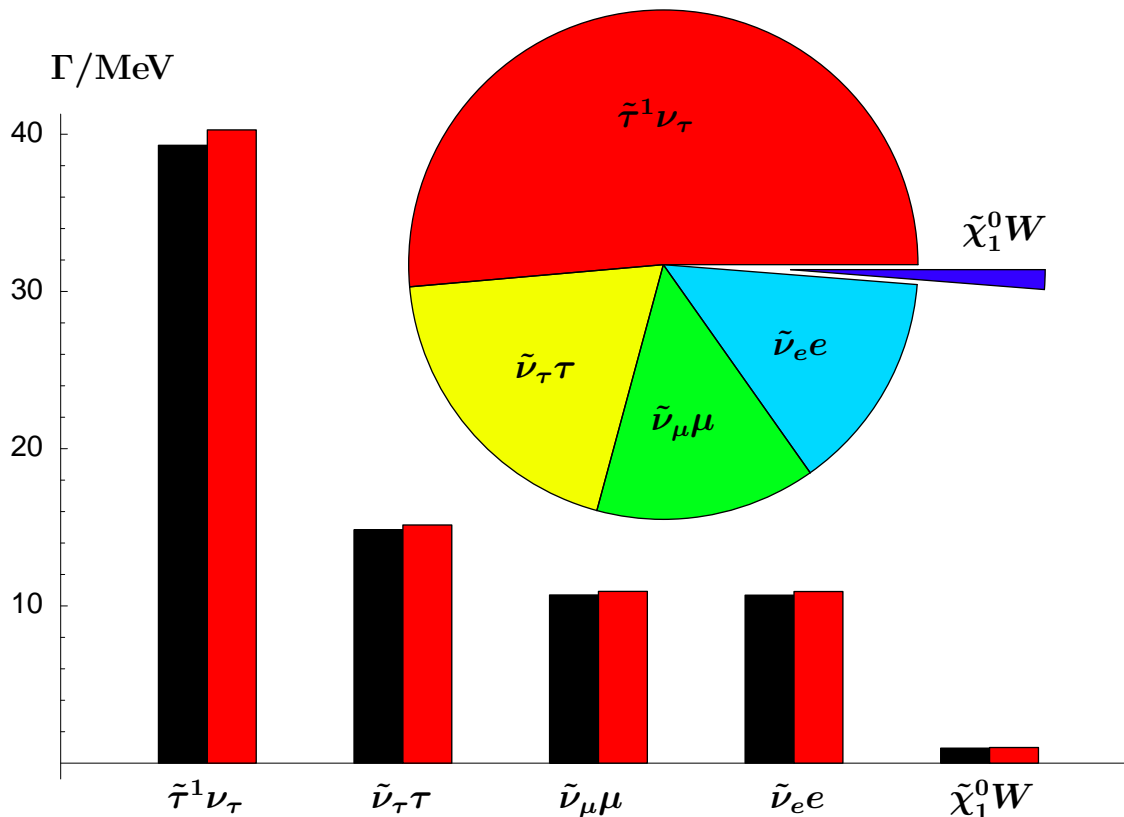
- separation of diagrams with virtual photons not UV-finite
- soft-photon bremsstrahlung necessary for getting an IR-finite result
- hard bremsstrahlung needed for realistic treatments

# SUSY particle decay rates

2-particle decays of  $\tilde{\chi}_{1,2}^{\pm}$  and  $\tilde{\chi}_{2,3,4}^0$

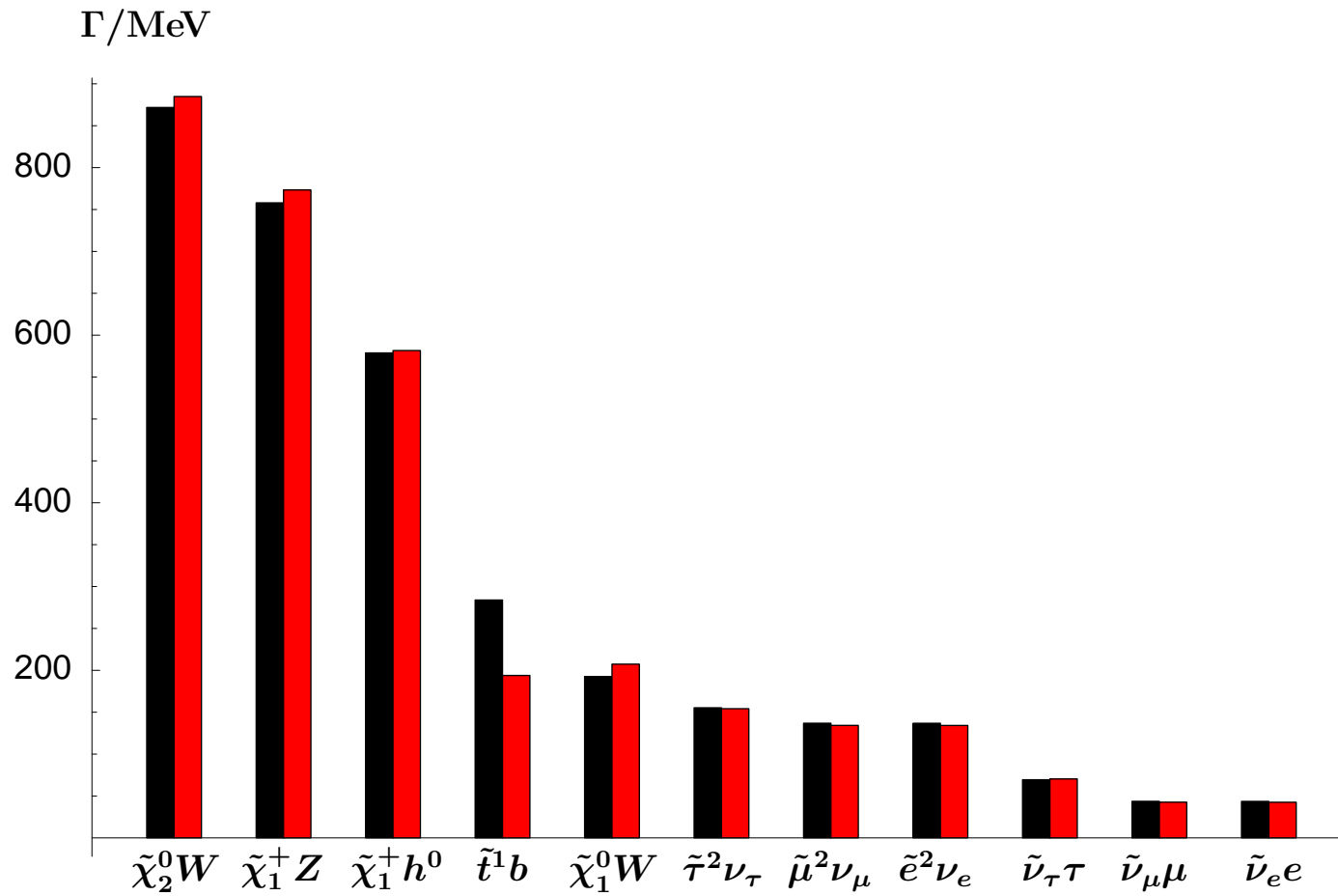
tree level (black) and 1-loop (red) [Fritzsche, WH]

$\tilde{\chi}_1^-$  decay modes (SPS1a')

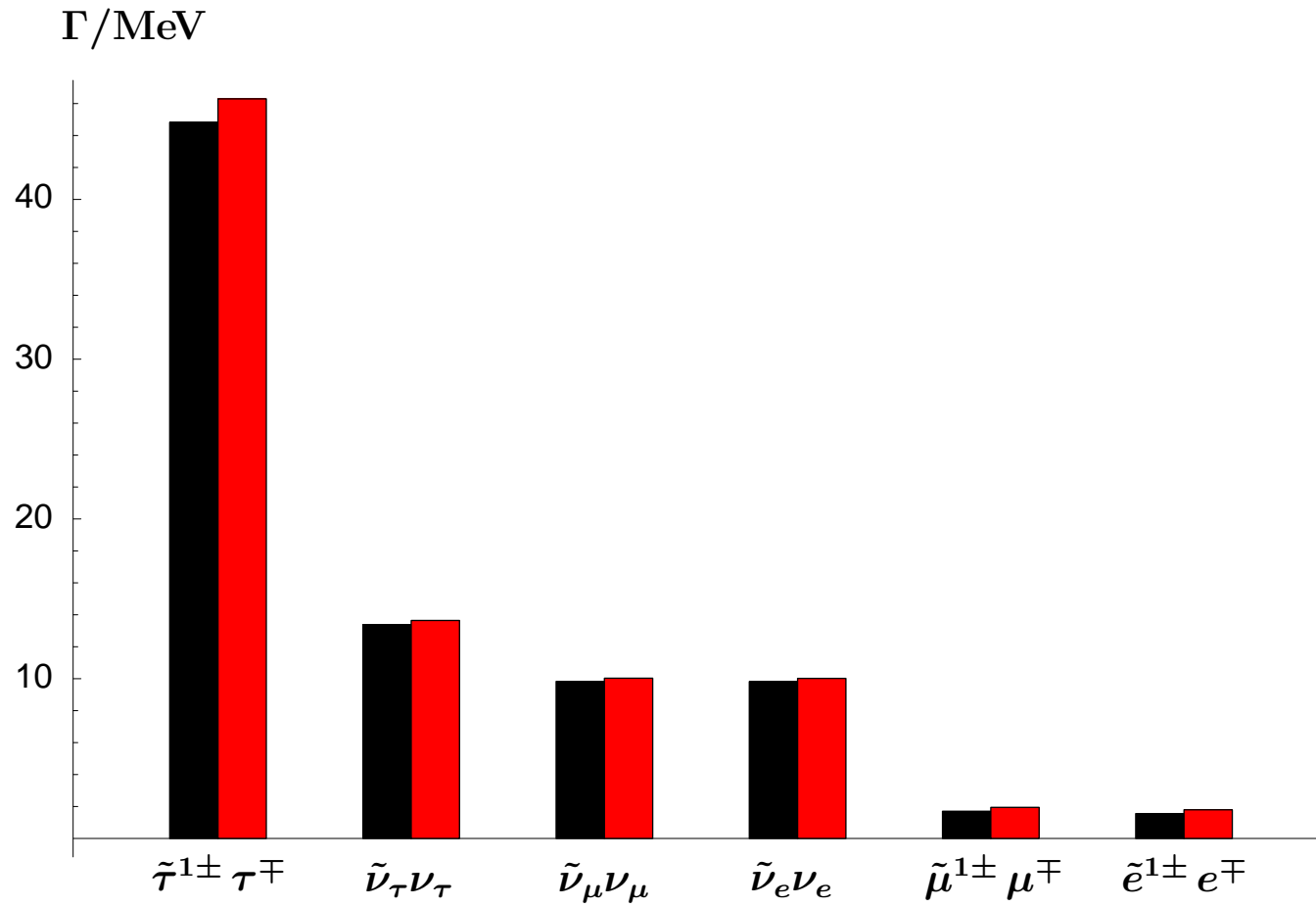




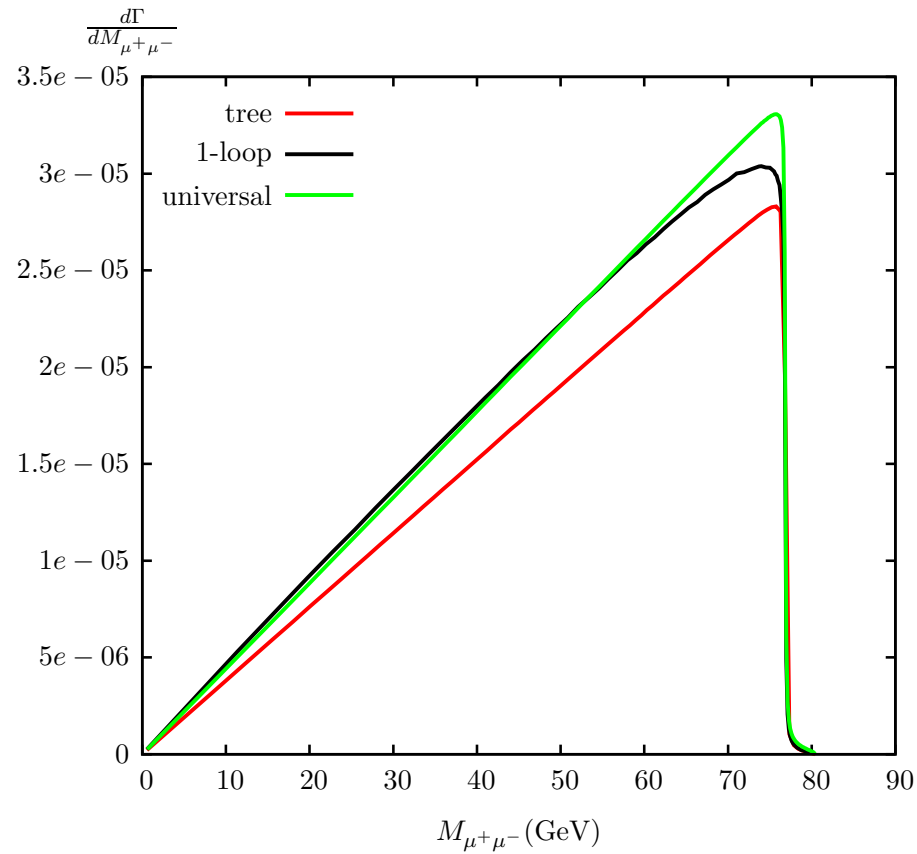
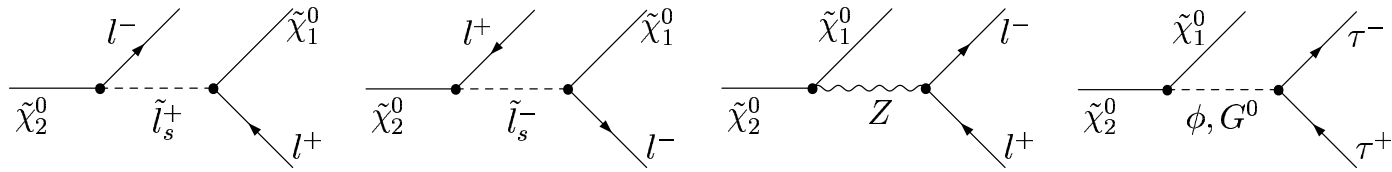
# $\tilde{\chi}_2^-$ decay modes



# $\tilde{\chi}_2^0$ decay modes



# important 3-body decay $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^-$



[Drees, WH, Qingjun Xu]

# Reconstructing Lagrangian parameters

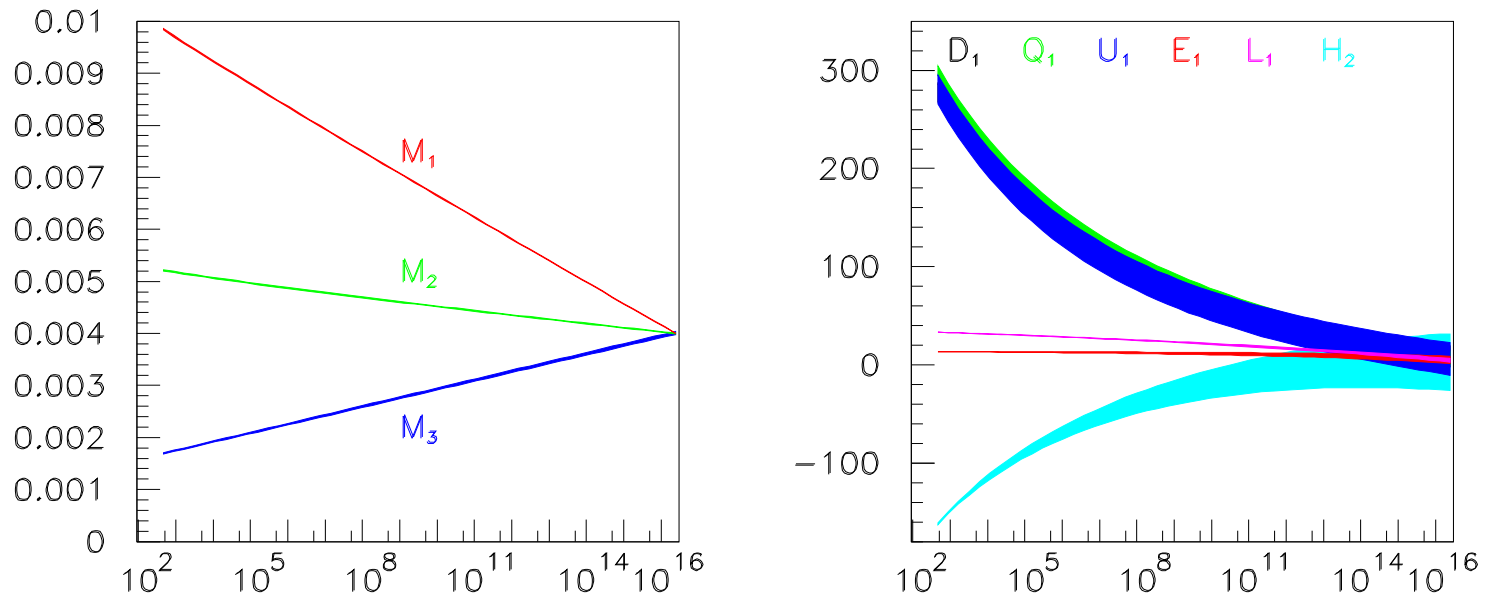
based on 82 simulated measurements at LHC and ILC

Parameter	SPS1a' value	Fit error [exp]
$M_1$	103.3	0.1
$M_2$	193.4	0.1
$M_3$	568.9	7.8
$\mu$	400.4	1.1
$M_{\tilde{e}_L}$	181.3	0.2
$M_{\tilde{e}_R}$	115.6	0.4
$M_{\tilde{\tau}_L}$	179.5	1.2
$M_{\tilde{u}_L}$	523.2	5.2
$M_{\tilde{u}_R}$	503.9	17.3
$M_{\tilde{t}_L}$	467.7	4.9
$m_A$	374.9	0.8
$A_t$	-525.6	24.6
$\tan \beta$	10.0	0.3

# Accuracy of measurements

	Mass	“LHC”	“LC”	“LHC+LC”
$h^0$	115.4	0.25	0.05	0.05
$H^0$	431.1		1.5	1.5
$\tilde{\chi}_1^0$	97.75	4.8	0.05	0.05
$\tilde{\chi}_2^0$	184.4	4.7	1.2	0.08
$\tilde{\chi}_4^0$	419.6	5.1	3 – 5	2.5
$\tilde{\chi}_1^\pm$	184.2		0.55	0.55
$\tilde{e}_R$	125.2	4.8	0.05	0.05
$\tilde{e}_L$	190.1	5.0	0.18	0.18
$\tilde{\tau}_1$	107.4	5 – 8	0.24	0.24
$\tilde{q}_R$	547.7	7 – 12	–	5 – 11
$\tilde{q}_L$	565.7	8.7	–	4.9
$\tilde{t}_1$	368.9		1.9	1.9
$\tilde{b}_1$	506.3	7.5	–	5.7
$\tilde{g}$	607.6	8.0	–	6.5

# High Scale Extrapolations



**Fig. 1.** Running of the gaugino and scalar mass parameters in  $SPS1a'$  [SPHeno 2.2.2]. Only experimental errors are taken into account; theoretical errors are assumed to be reduced to the same size in the future.

# Conclusions

- precision calculations for SUSY (MSSM) are well advanced
- electroweak precision observables → 2-loop level  
global fits of similar quality as in standard model  
indirect sensitivity to SUSY parameters
- $m_{h^0}$  is another precision observable
  - dependent on all SUSY sectors
  - accurate theoretical evaluation ( $\delta m_{h^0} \simeq 4 \text{ GeV}$ ),  
to be further improved
- progress for loop contributions to SUSY processes  
future precision allows tests of breaking scenarios

**Detailed analysis for SPS1a benchmark scenario: potential of LHC ( $300 \text{ fb}^{-1}$ ) alone and LHC + LC**

	LHC	LHC+LC
$\Delta m_{\tilde{\chi}_1^0}$	4.8	0.05 (input)
$\Delta m_{\tilde{l}_R}$	4.8	0.05 (input)
$\Delta m_{\tilde{\chi}_2^0}$	4.7	0.08
$\Delta m_{\tilde{q}_L}$	8.7	4.9
$\Delta m_{\tilde{q}_R}$	11.8	10.9
$\Delta m_{\tilde{g}}$	8.0	6.4
$\Delta m_{\tilde{b}_1}$	7.5	5.7
$\Delta m_{\tilde{b}_2}$	7.9	6.2
$\Delta m_{\tilde{l}_L}$	5.0	0.2 (input)
$\Delta m_{\tilde{\chi}_4^0}$	5.1	2.23

LHC+LC accuracy limited by LHC jet energy scale resolution

SPS 1a benchmark scenario:

favorable scenario for both LHC and LC

⇒ LC input improves accuracy significantly



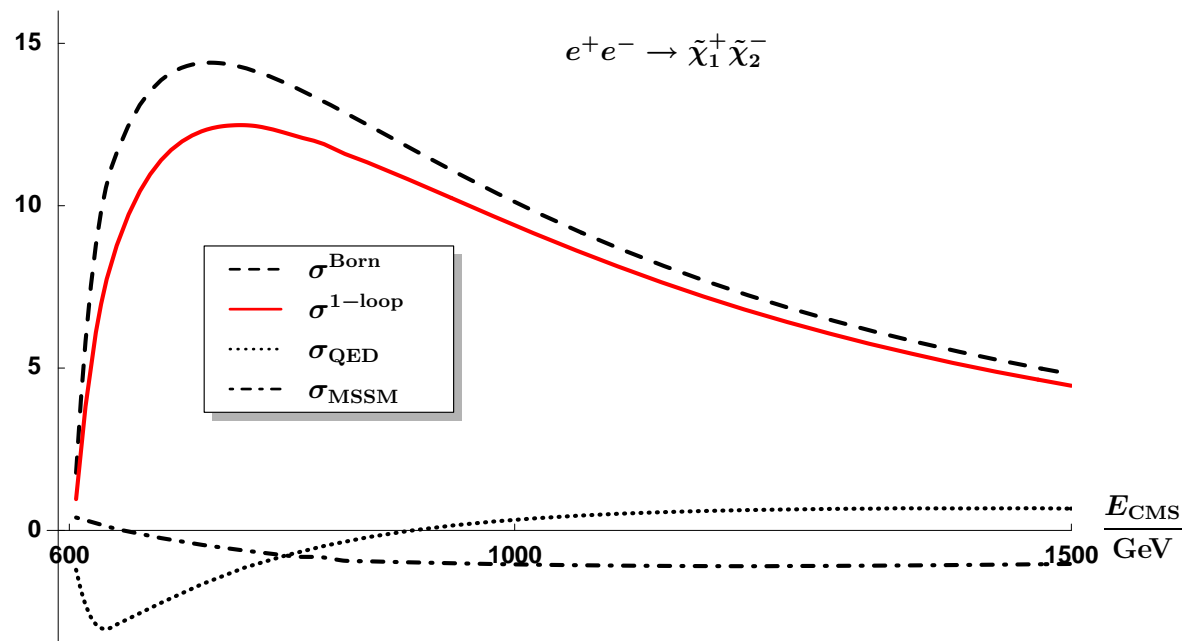
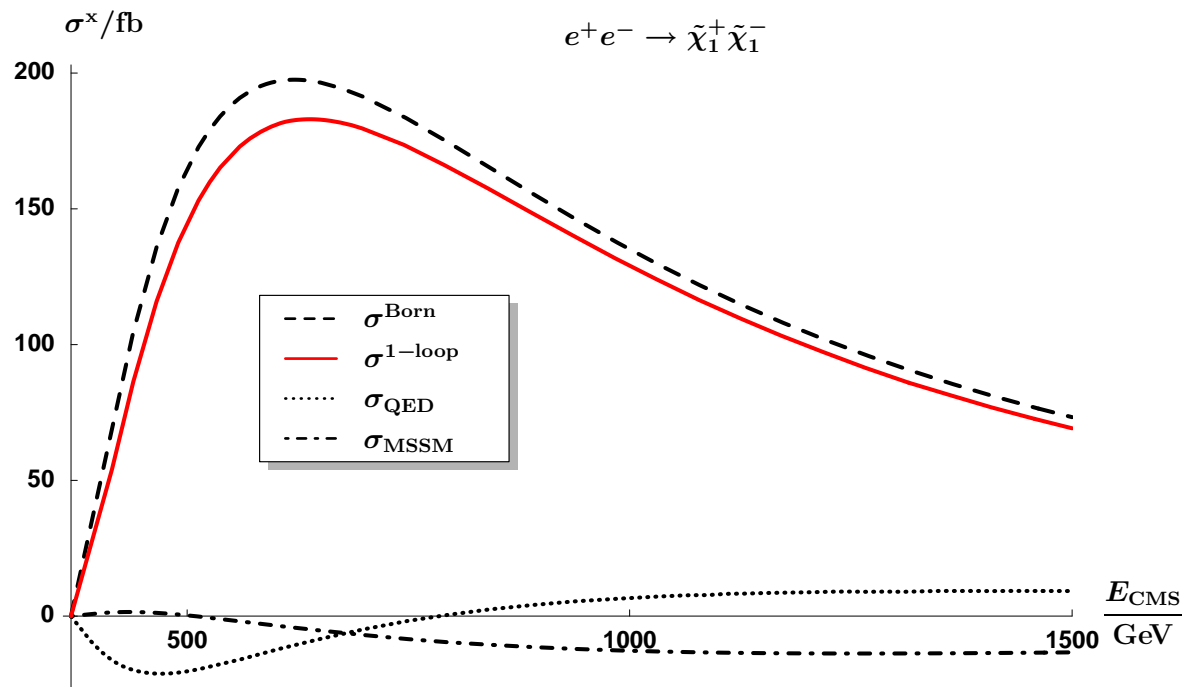
**Reasonable separation** ( $L_e = \log \frac{s}{m_e^2}$ ,  $\Delta E = E_{\gamma \text{ soft}}^{\max}$ ):

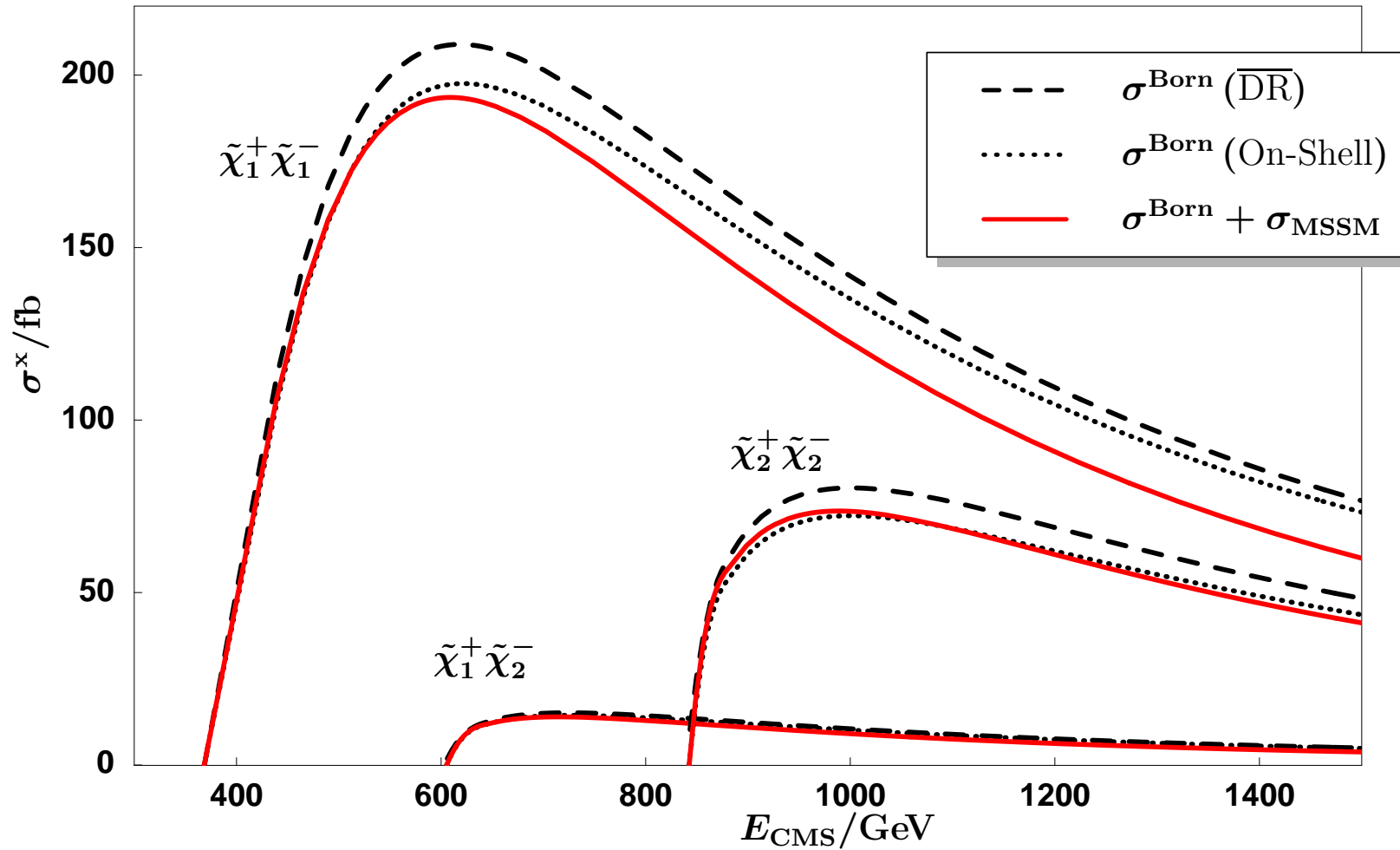
$$\sigma(1 - \text{loop}) = \sigma_{\text{QED}} + \sigma_{\text{MSSM}},$$

$$\sigma_{\text{QED}} = \sigma^{\text{hard}} + \frac{\alpha}{\pi} \left[ (L_e - 1) \log \frac{4 \Delta E^2}{s} + \frac{3}{2} L_e \right] \sigma_0$$

$$\sigma_{\text{MSSM}} = \sigma^{\text{v+s}} - \frac{\alpha}{\pi} \left[ (L_e - 1) \log \frac{4 \Delta E^2}{s} + \frac{3}{2} L_e \right] \sigma_0$$

- gauge invariant
- $\sigma_{\text{MSSM}}$  free of large soft and collinear photon terms





$$e^+e^- \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0$$

