Strange, Charm and Bottom masses at NNLO and N^{3}LO

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SFB TR-9

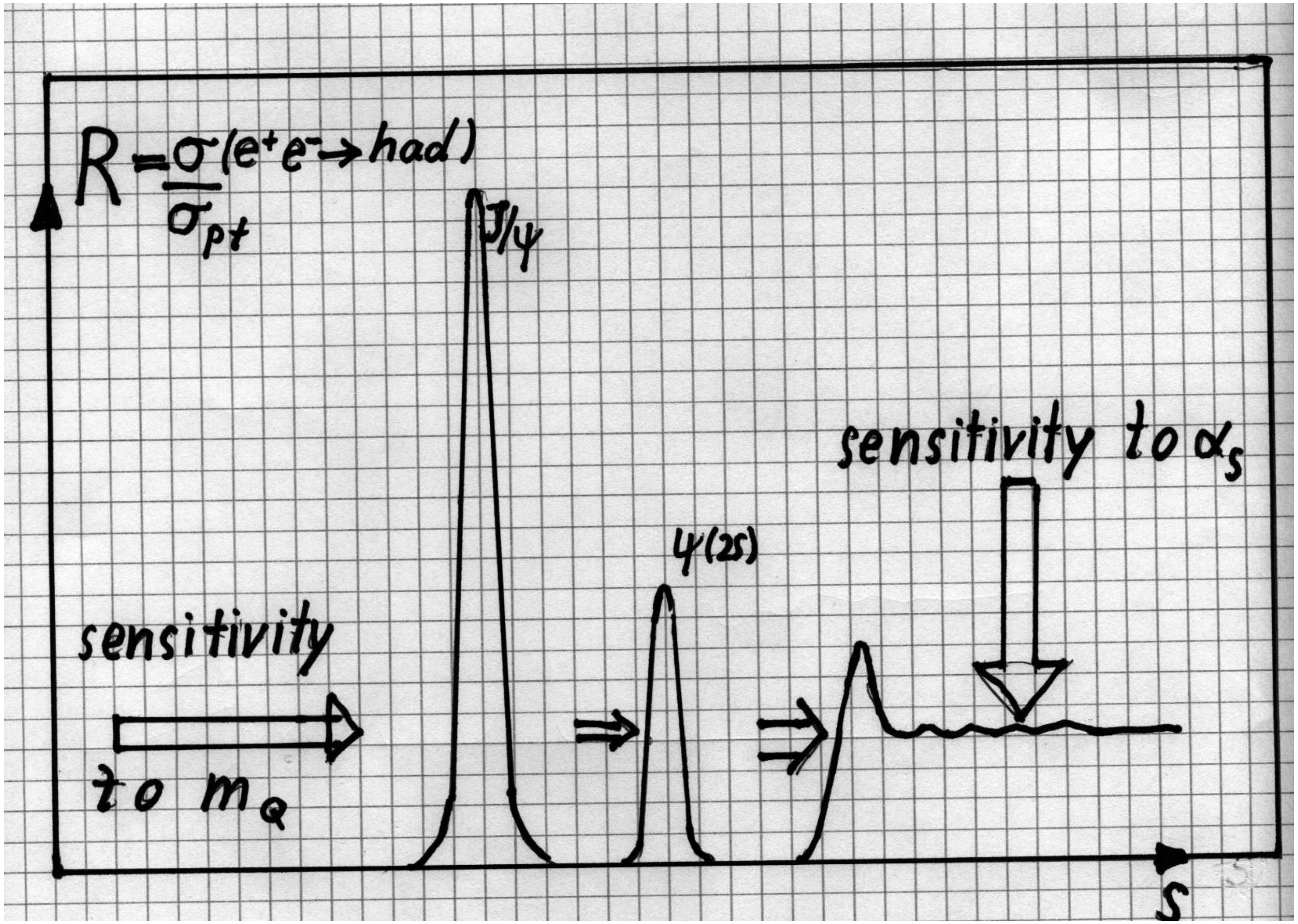
I. Sum Rules with Charm and Bottom Quarks: recent data and N^3LO calculations (Chetyrkin, JK, Steinhauser, Sturm)

II. m_c and m_b: overview

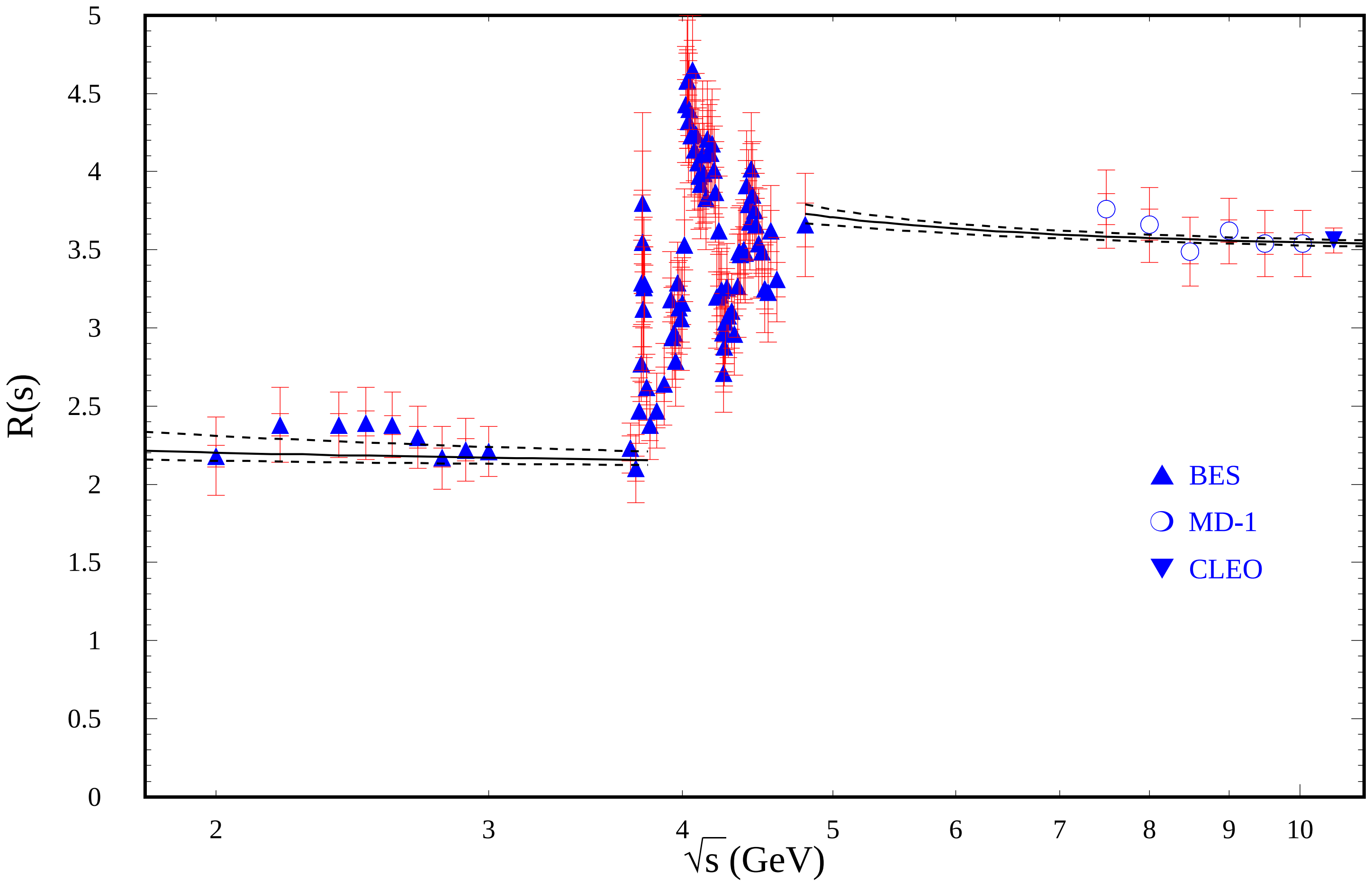
III. m_s from τ-decays and sum rules

I Sum Rules with Charm and Bottom Quarks

Main Idea



Data



pQCD and data agree well in the regions

2 — 3.73 GeV and 5 — 10.52 GeV

experiment	energy [GeV]	date	systematic error
BES	2 — 5	2001	4%
MD-1	7.2 — 10.34	1996	4%
CLEO	10.52	1998	2%
PDG	J/ψ		(7%) 2%
PDG	ψ'		(9%) 3.7%
PDG	ψ''		(15%)
BES	ψ'' region	2006	4%

m_Q from SVZ Sum Rules, Moments and Tadpoles

Some definitions

$$R(s) = 12\pi\, \mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu\, q_\nu\right)\, \Pi(q^2) \equiv i \int \mathrm{d}x\, e^{iqx} \langle T j_\mu(x) j_\nu(0)\rangle$$

with the electromagnetic current j_μ

Taylor expansion: $\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n\, z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

Coefficients $\bar{C}_n$ up to $n = 8$ known analytically in order α_s^2

(Chetyrkin, JK, Steinhauser, 1996

recently up to $n = 30$!

(Boughezal, Czakon, Schutzmeier)

recently also $\bar{C}_0$ and $\bar{C}_1$ in order α_s^3 (four loops!)

➪ reduction to master integrals through Laporta algorithm

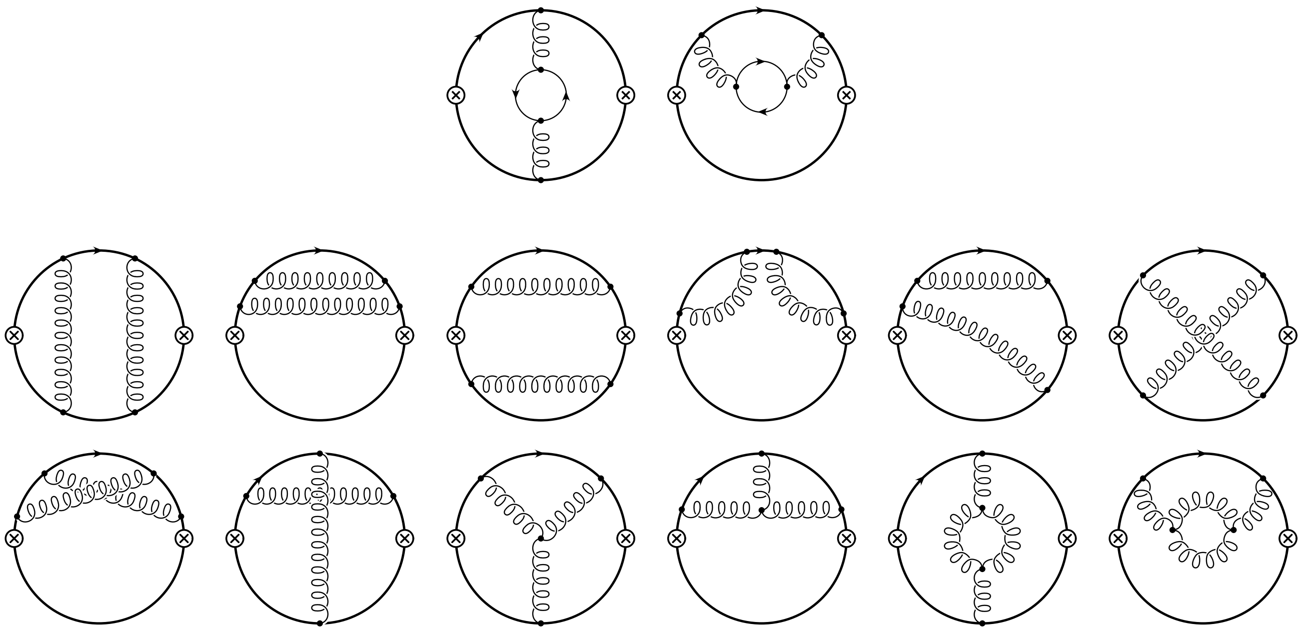
(Chetyrkin, JK, Sturm; confirmed by Boughezal, Czakon, Schutzmeier)

evaluation of master integrals numerically through difference equations (30 digits) or Padé method or analytially in terms of transzendentals (Schröder + Vuorinen, Chetyrkin et al., Schröder + Steinhauser, Laporta, Broadhurst, Kniehl et al.

Analysis in NNLO

Coefficients $\bar{C}_n$ from three-loop one-scale tadpole amplitudes with "arbitrary" power of propagators;

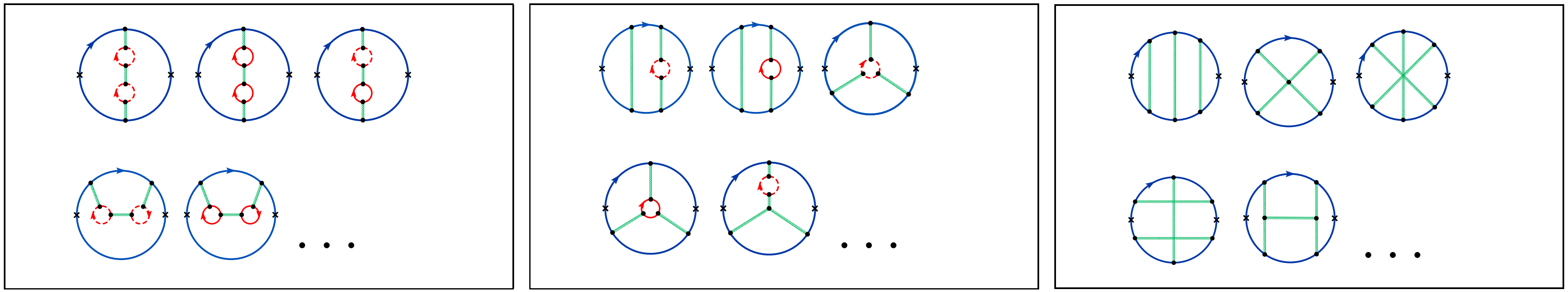
FORM-program MATAD



Analysis in N^{3}LO

Algebraic reduction to 13 master integrals (Laporta algorithm);
numerical evaluation of master integrals

n_f^2-contributions n_f^1-contributions n_f^0-contributions



: heavy quarks, : light quarks,

n_f: number of active quarks

$\Longrightarrow$ About 700 Feynman-diagrams

$\bar{C}_n$ depend on the charm quark mass through $l_{m_c} \equiv \ln(m_c^2(\mu)/\mu^2)$

$$
\begin{aligned}
\bar{C}_n \; = \; & \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c}\right) \\
& + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2\right) \\
& + \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 \left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{mc} + \bar{C}_n^{(32)} l_{mc}^2 + \bar{C}_n^{(33)} l_{ms}^3\right)
\end{aligned}
$$

n	$\bar{C}_n^{(0)}$	$\bar{C}_n^{(10)}$	$\bar{C}_n^{(11)}$	$\bar{C}_n^{(20)}$	$\bar{C}_n^{(21)}$	$\bar{C}_n^{(22)}$	$\bar{C}_n^{(30)}$	$\bar{C}_n^{(31)}$	$\bar{C}_n^{(32)}$	$\bar{C}_n^{(33)}$
1	1.0667	2.5547	2.1333	2.4967	3.3130	−0.0889	−5.6404	4.0669	0.9590	0.0642
2	0.4571	1.1096	1.8286	2.7770	5.1489	1.7524	—	6.7216	6.4916	−0.0974
3	0.2709	0.5194	1.6254	1.6388	4.7207	3.1831	—	7.5736	13.1654	1.9452
4	0.1847	0.2031	1.4776	0.7956	3.6440	4.3713	—	4.9487	17.4612	5.5856

Coefficients of the photon polarization function in the $\overline{\mathrm{MS}}$ scheme.
$n_f = 4$ **has been adopted which is appropriate for the charm threshold.**

Define the moments

$$\mathcal{M}_n^{\rm th} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Big|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

Perturbation theory: $\bar{c}_n$ is function of α_s and $\ln\frac{m_c^2}{\mu^2}$

dispersion relation:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s\,\frac{R_c(s)}{s(s-q^2)} + \text{subtraction}$$

➪ $\mathcal{M}_n^{\rm exp} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$

constraint: $\mathcal{M}_n^{\rm exp} = \mathcal{M}_n^{\rm th}$

➪ m_c

SVZ:

$\mathcal{M}_n^{\text{th}}$ can be reliably calculated in pQCD:

low n: dominated by scales of $\mathcal{O}(2m_Q)$

- fixed order in α_s is sufficient, in particular no resummation of $1/v$ - terms from higher orders required
- condensates are unimportant
- pQCD in terms of short distance mass : $m_c(3\text{ GeV}) \Rightarrow m_c(m_c)$ stable expansion : no pole mass or closely related definition (1S-mass, potential-subtracted mass) involved
- moments available in NNLO
- and $\bar{C}_0$, $\bar{C}_1$ in N^{3}LO

update compared to NPB619 (2001)

experiment:

$\alpha_s = 0.1187 \pm 0.0020$

$\Gamma_e(J/\psi, \psi')$ from BES & CLEO & Babar

$\psi(3770)$ from BES

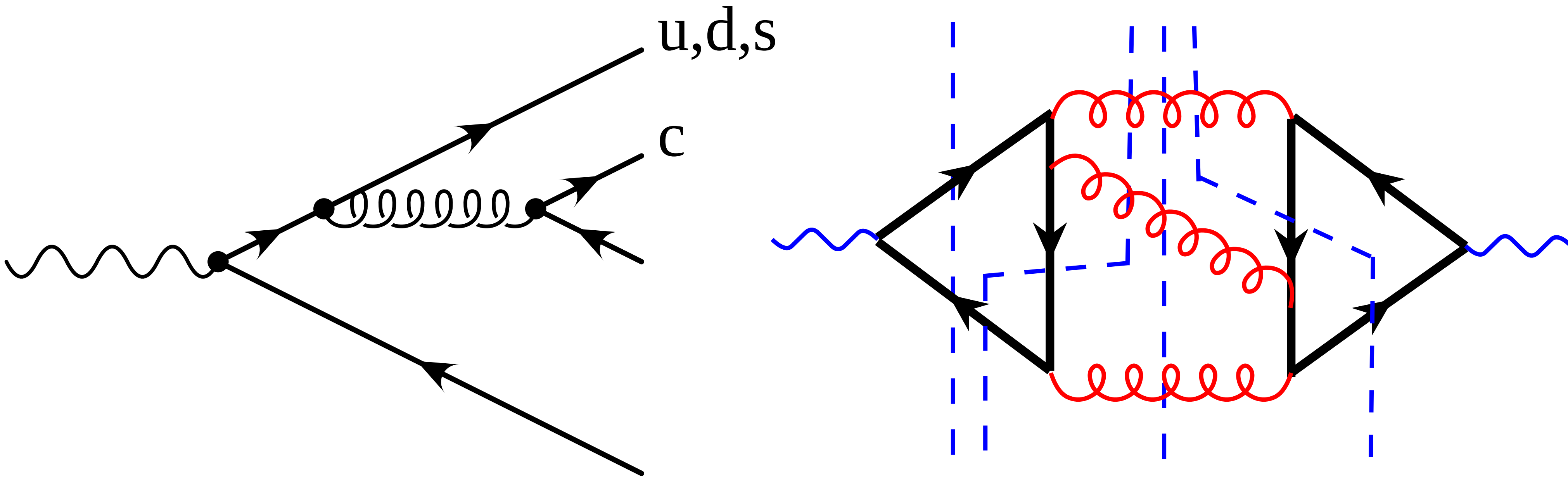
theory: N^3LO for n=1

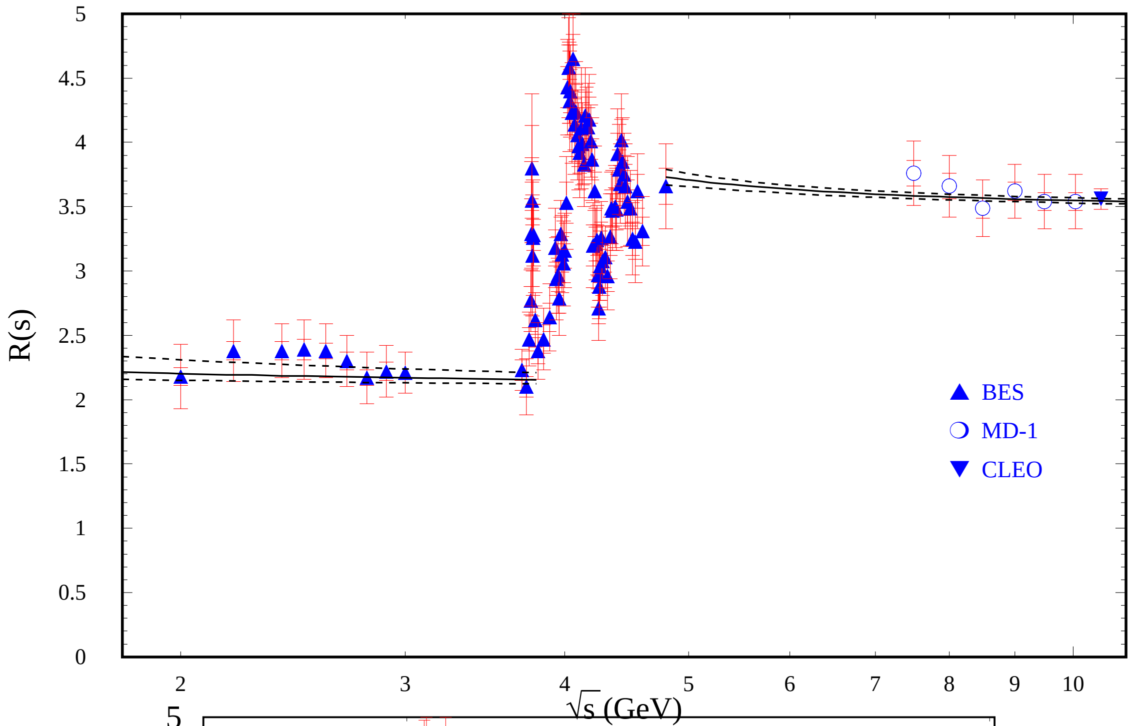
N^3LO - estimate for n =2,3,4

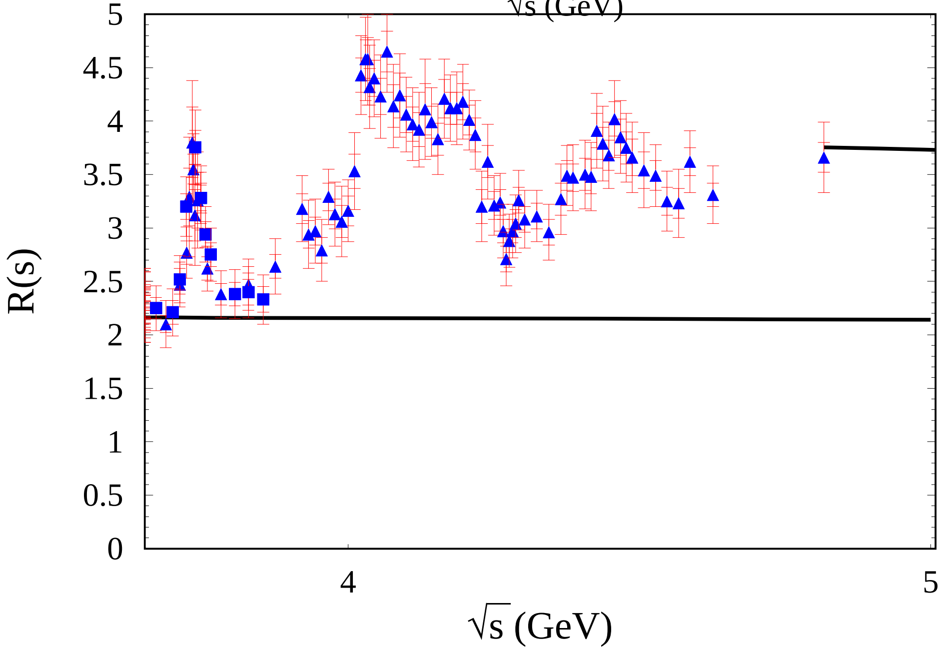
include condensates

$$\delta\mathcal{M}_n^{\text{np}} = \frac{12\pi^2 Q_c^2}{(4m_c^2)^{(n+2)}} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle a_n \left(1 + \frac{\alpha_s}{\pi}\bar{b}_n\right) \tag{1}$$

careful extrapolation of R_{uds} including m_c dependent terms


u,d,s
c


R(s)
√s (GeV)
BES
MD-1
CLEO


R(s)
√s (GeV)

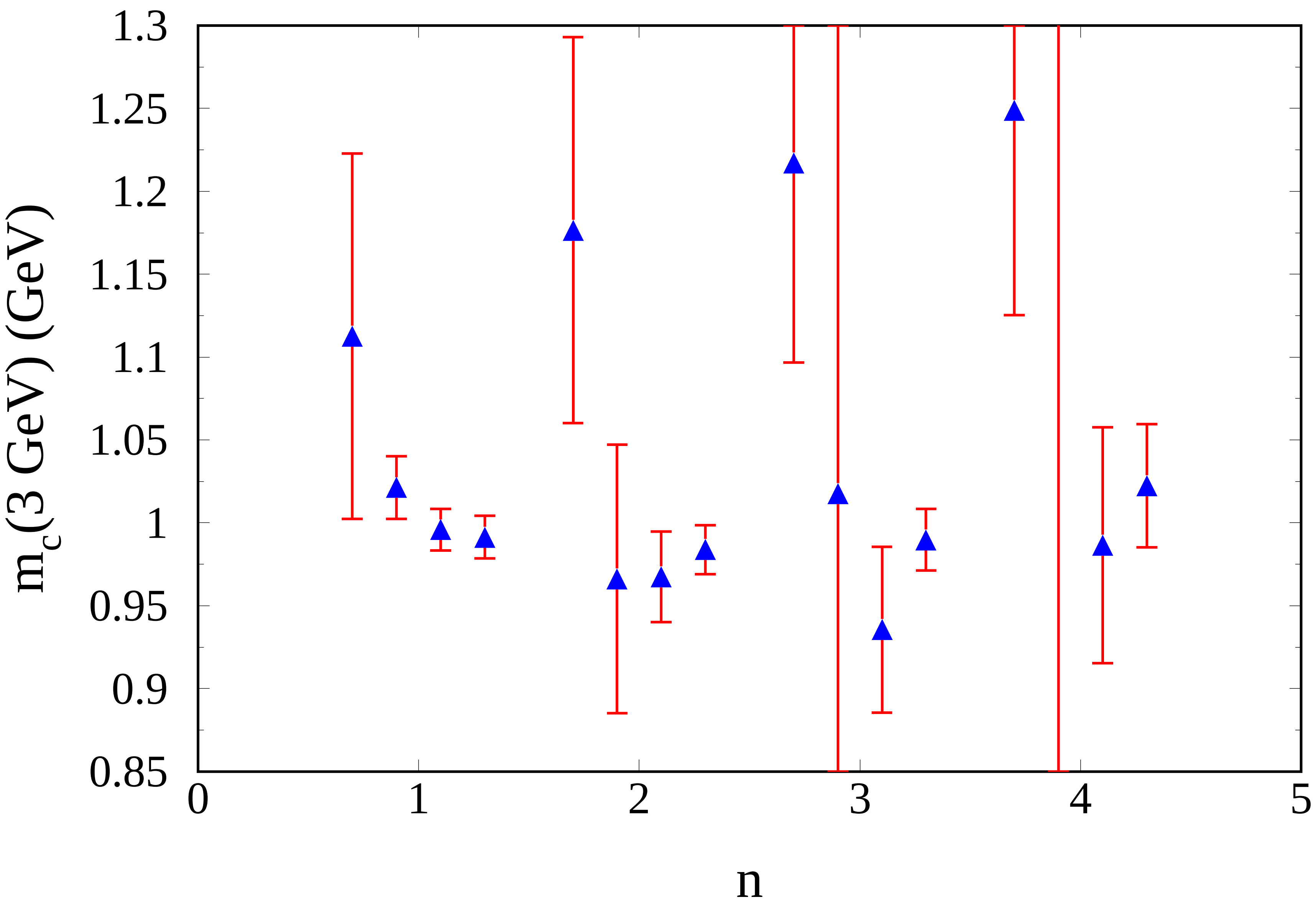
Preliminary results (m_c)

n	$m_c(3\text{ GeV})$	exp	α_s	μ	np	total	$\delta\bar{C}_n^{30}$
1	0.989	0.010	0.008	0.001	0.001	0.013	—
2	0.984	0.006	0.013	0.003	0.000	0.015	0.006
3	0.990	0.005	0.013	0.012	0.002	0.019	0.010
4	1.022	0.003	0.007	0.036	0.007	0.037	0.014

$n = 1$:

$$m_c(3\text{GeV}) = 989 \pm 13\,\text{MeV}$$

$$m_c(m_c) = 1288 \pm 11\,\text{MeV}$$



1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
n
m_c(3 GeV) (GeV)

update on m_b

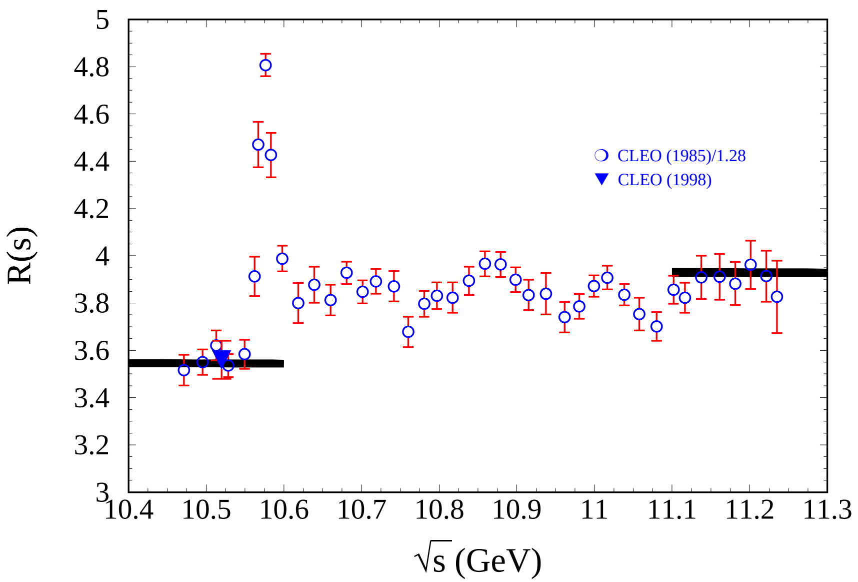
theory: N^3LO

experiment:

$\alpha_s = 0.1187 \pm 0.0020$

$\Gamma_e(\Upsilon, \Upsilon', \Upsilon'')$ from CLEO

improved analysis of threshold region



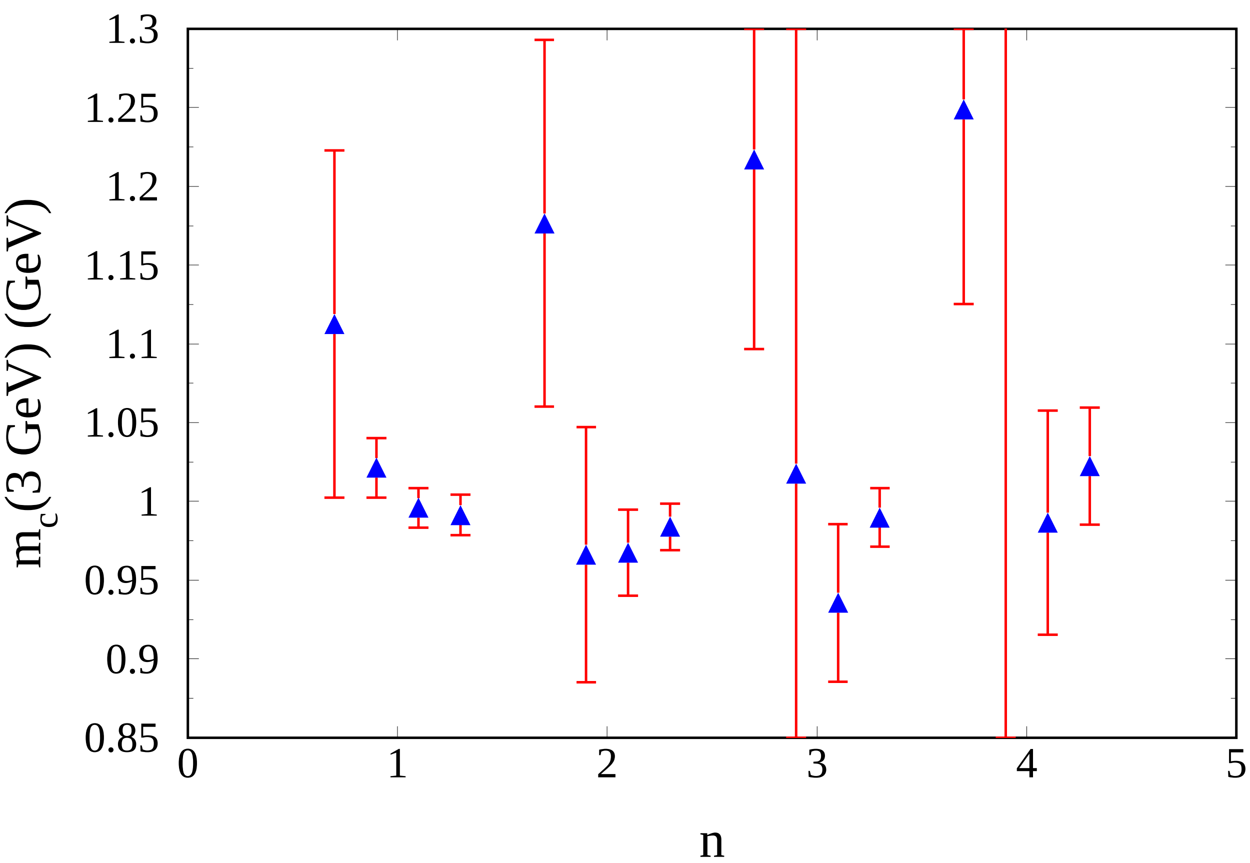
preliminary results (m_b)

n	$m_b(10\text{ GeV})$	exp	α_s	μ	np	total	$\delta\bar{C}_n^{30}$
1	3.596	0.019	0.007	0.001	0.000	0.020	—
2	3.613	0.013	0.012	0.001	0.000	0.017	0.005
3	3.623	0.010	0.014	0.010	0.000	0.020	0.008
4	3.638	0.008	0.014	0.026	0.000	0.031	0.012

$n = 1$:

$$m_b(10\text{GeV}) = 3596 \pm 20\,\text{MeV}$$

$$m_b(m_b) = 4151 \pm 21\,\text{MeV}$$



1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
m_c(3 GeV) (GeV)
n

Summary on m_c and m_b

- drastic improvement in δm_c, δm_b from moments with low n in N^{2}LO
- direct determination of short-distance mass

improved measurements of $\Gamma_e(J/\psi, \psi')$ and $\Gamma_e(\Upsilon, \Upsilon', \Upsilon'')$

improved measurement of charm threshold region

reanalysis of bottom threshold region

and new N^3LO results lead to significant improvements

preliminary results (based on $n = 1$)q:

$$m_c(3\text{ GeV}) = 0.989(13)\text{ GeV}$$
$$m_c(m_c) = 1.288(11)\text{ GeV}$$

$$m_b(10\text{ GeV}) = 3.596(20)\text{ GeV}$$
$$m_b(m_b) = 4.151(21)\text{ GeV}$$

(old result: $m_c(m_c) = 1.304(27) GeV$ $m_b(m_b) = 4.191(51) GeV$)

II m_c and m_b: other characteristic results

no review

charm

- moments of B-decay distributions (hadron mass, lepton energy)

  HQE up to $\mathcal{O}(1/m_b^3)$, pQCD up to $\mathcal{O}(\alpha_s^2 \beta_0)$

  1240 ± 70 O. Buchmüller, Flächer

  $1224 \pm 17 \pm 54$ Hoang, Manohar

- Lattice, from D_s (quenched ➪ $\pm(40 - 60)$)

  $1260 \pm 40 \pm 120$ Becirevic, Lubicz, Martinelli

  1301 ± 34 Rolf, Sint

bottom

- moments of B-decay distributions (hadron mass, lepton energy)
  HQE up to $\mathcal{O}(1/m_b^3)$, pQCD up to $\mathcal{O}(\alpha_s^2\beta_0)$
  4200 ± 40 Buchmüller, Flächer
  4170 ± 30 Bauer et al.
- Υ-spectroscopy (1S-state), pNRQCD + nonperturbative effects
  4346 ± 70 Penin, Steinhauser (N^3LO)
  $4210 \pm 90 \pm 25$ Pineda (N^2LO)
- Lattice (HQET + $1/m_b$ terms, quenched)
  4301 ± 70 ALPHA-Coll. Dellta Morte et al.

III m_s from τ-decays and sum rules

$\tau \to \nu s \bar{d}$

input: moments of $m(s\bar{d})$ (ALEPH, OPAL)

V_{us} (Czarnecki, Marciano, Sirlin)

phenomenology (Gamiz et al)

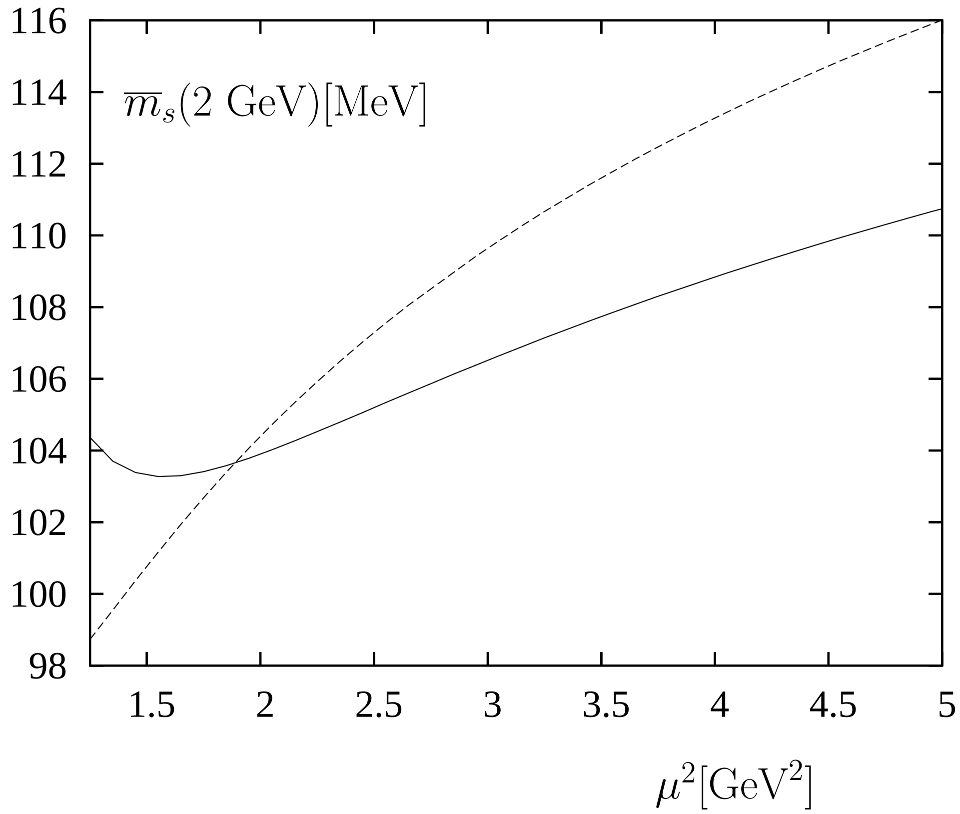
pQCD in $\mathcal{O}(\alpha^3)$ (Baikov, Chetyrkin, JK)

(finite part of massless four-loop correlator)

➪ $ms(M_\tau) = 100 \pm \left({+5 \atop -3}\right)_{\text{theo}} \pm \left({+17 \atop -19}\right)_{\text{rest}}$

pseudoscalar sum rules in $\mathcal{O}(\alpha_s^4)$

$\bar{m}_s(2\,\text{GeV}) = 105 \pm 6(\text{param}) \pm 7(\text{hadr})$ Chetyrkin, Khodjamirian



Method	$\overline{m}_s(2\text{ GeV})$ [MeV]	Ref.
Pseudoscalar Borel sum rule	$105 \pm 6 \pm 7$	Chetyrkin
Pseudoscalar FESR	100 ± 12	Maltman
Scalar Borel sum rule	99 ± 16	Jamin
Vector FESR	139 ± 31	Eidemüller
Spectral function	> 77	Baikov
Hadronic τ decays	81 ± 22	Gamiz
	96^{+5+16}_{-3-18}	Baikov
	104 ± 28	Narison
τ decays $\oplus$ sum rules	99 ± 28	Narison
Lattice QCD ($n_f = 2$)	97 ± 22	Della Morte
	100 -130	Gockeler
	$101 \pm 8^{+25}_{-0}$	Becirevic
Lattice QCD ($n_f = 3$)	$76 \pm 3 \pm 7$	Aubin
	86.7 ± 5.9	Ishikawa
	$87 \pm 4 \pm 4$	Mason
PDG04 average	80 -130	Eidelman

Summary

new multiloop results from pQCD + improved data (preliminary analysis)

$$m_c(3\text{ GeV}) = 989 \pm 13\text{ MeV} \qquad m_c(m_c) = 1288 \pm 11\text{ MeV}$$

$$m_b(10\text{ GeV}) = 3596 \pm 20\text{ MeV} \qquad m_b(m_b) = 4151 \pm 21\text{ MeV}$$

significantly reduced errors, consistent with other determinations, but more precise

$$m_s(2\text{ GeV}) = 105 \pm 10\text{ MeV}$$

on the basis of N^3LO pseudoscalar sumrules