Fitting PDFs with Monte Carlo solutions of evolution equations in QCD

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Introduction

New parton shower MC project is currently constructed by IFJ-PAN group from Krakow, based on evolution in rapidity space

- Such parton shower with non-standard evolution (CCFM-like,$\alpha(Q(1-z))$,$\alpha(Q(x\frac{1-z}{z}))$ etc) requires dedicated PDF fits as a part of the project
- MC is slow by its nature (permille accuracy $\approx$ 3 days of running). Therefore MC cannot be used directly for fitting - fast procedure must be designed

Evolution equation

$$\partial_t D_A(x,t) = P_{AB}(x,t) \otimes^x D_B(x,t), \quad (1)$$

$$D_A(x,t=0) = D_A^0(x,\alpha_1^A,\ldots,\alpha_k^A). \quad (2)$$

$D_A(x,t)$ denotes PDF of the type A with $A = q_i, \bar{q}_i, g;\ i = 1\ldots, n$. and t is the evolution time. The splitting kernels $P_{AB}(x,t)$ include also coupling constants and the convolution symbol $\otimes^x$ stands for

$$\big(f(x,\alpha)\otimes^x g(x,\beta)\big)(x) = \int_x^1 dy dz \delta(x-yz) f(y,\alpha) g(z,\beta). \quad (3)$$

Decomposition

The decomposition of the solution $D^A(x,t)$ can be written as follows

$$D_A(x,t) = D^{\delta}_{AB}(x,t) \otimes^x D^0_B(x, \alpha^B_1, \ldots, \alpha^B_k) \quad (4)$$

$$\partial_t D^{\delta}_{AB}(x,t) = P_{AC}(x,t) \otimes^x D^{\delta}_{CB}(x,t), \quad (5)$$

$$D^{\delta}_{AB}(x,0) = \delta_{AB}\delta(1-x). \quad (6)$$

- Whole dependence of fitted patemeters (α^B_j) is limited to D^0_B
- Only single MC run needed for D^{δ}_{AB} (independent of α^B_j)

Integration procedure

- Convolution $\otimes^x$ is a 1-dim integral, can be done numerically in a fast and accurate way.
- MC histogram of D^δ is parametrized by polynominal second order functions.
- Because we use log-scale for x, value at x close to 1 must be extrapolated from neighbouring bins
- This extrapolation could be avoided if linear scale were used, but it is not necessary for 10^{-3} precision

Test of integration procedure

We use 2-dim system of gluon (G) and singlet (Σ) PDF-s

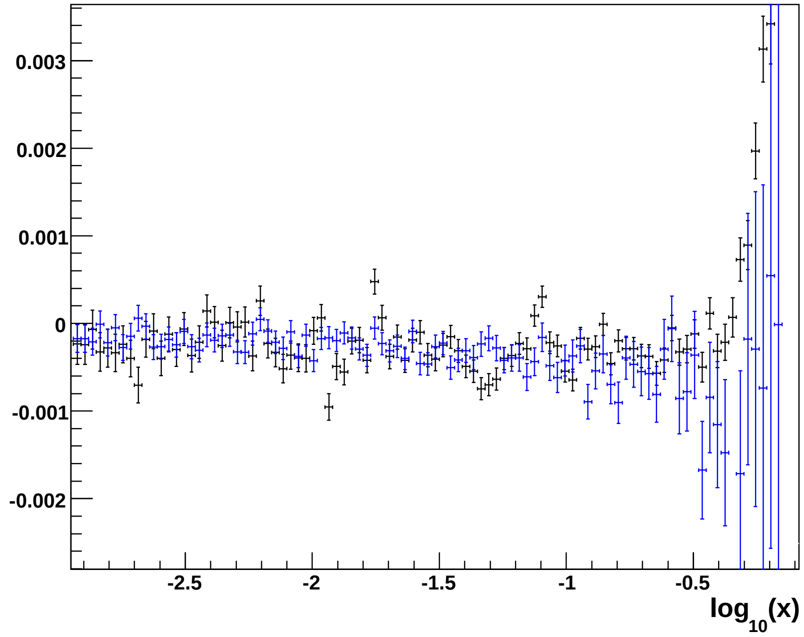
$$
\begin{aligned}
D_G(x,t) &= D^{\delta}_{GG}(x,t) \otimes D^0_G(x) + D^{\delta}_{G\Sigma}(x,t) \otimes D^0_{\Sigma}(x), \\
D_{\Sigma}(x,t) &= D^{\delta}_{\Sigma G}(x,t) \otimes D^0_G(x) + D^{\delta}_{\Sigma\Sigma}(x,t) \otimes D^0_{\Sigma}(x).
\end{aligned}
\tag{7}
$$

Initial conditions

$$
\begin{aligned}
D^0_G(x) &= 1.908 \cdot x^{-1.2}(1-x)^{5.0}, \\
D^0_{\Sigma}(x) &= D^0_{sea}(x) + D^0_u(x) + D^0_d(x), \\
D^0_{sea}(x) &= 0.6733 \cdot x^{-1.2}(1-x)^{7.0}, \\
D^0_u(x) &= 2.187 \cdot x^{-0.5}(1-x)^{3.0}, \\
D^0_d(x) &= 1.230 \cdot x^{-0.5}(1-x)^{4.0}.
\end{aligned}
\tag{8}
$$

Test of convolution Gluon and Singlet (LO)

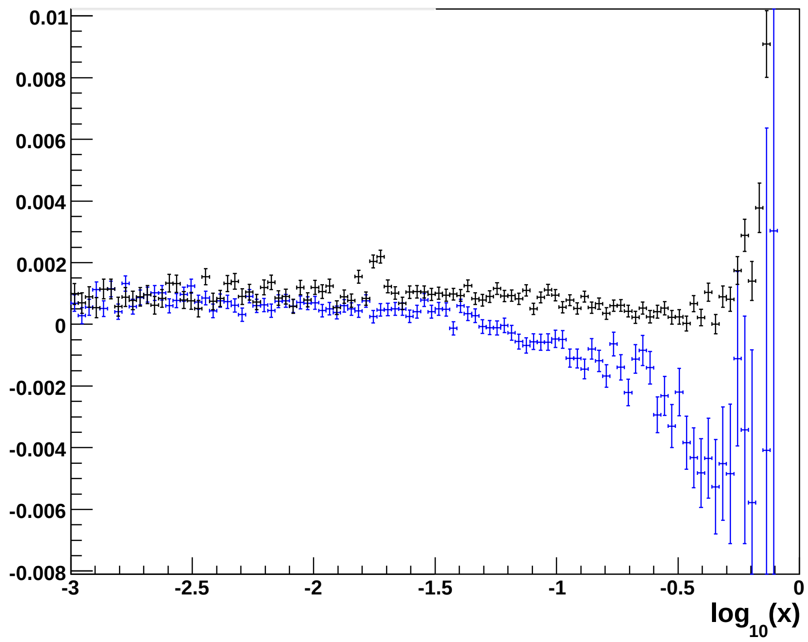
Ratio of gluon (blue) and singlet (black) generated directly from initial distribution and obtained from convolution of D_δ. Evolution from $Q = 1$ to $Q = 100 GeV$, LO type



Agreement $5 \cdot 10^{-4}$ except of $x \sim 1$, where D is very small any way

Test of convolution Gluon (NLO)

Ratio of gluon (blue) and singlet (black) generated directly from initial distribution and obtained from convolution of D_δ. Evolution from $Q = 1$ to $Q = 100 GeV$, NLO type



Agreement 1×10^{-3} except of $x \sim 1$, where D is very small anyway

Fitting (PDFs)

- To check correctness of the procedure we fitted PDFs to PDFs
- We used the same $G - \Sigma$ system
- We used MINUIT and minimized χ^2

$$\chi^2(\alpha_1^G, \ldots, \alpha_k^G; \alpha_1^\Sigma, \ldots, \alpha_k^\Sigma) = \sum_{A=G,\Sigma} \sum_i \frac{(D_A^X(x_i, t, \alpha_1^A, \ldots, \alpha_k^A) - D_A^Y(x_i, t))^2}{e_A^Y(x_i, t)^2} \quad (9)$$

X and Y denote evolution type: if $X = Y$ - technical test, if $X \neq Y$ - physical difference

Fitting PDF to PDF-test (LO)

	α_1^G	α_2^G	α_3^G	α_1^u	α_2^u	α_3^u	α_1^d	α_2^d	α_3^d
I	1.908	1.200	5.000	2.187	0.500	3.000	1.230	0.500	4.000
F	1.907	1.199	4.994	2.186	0.501	3.008	1.229	0.505	3.991

	α_1^{sea}	α_2^{sea}	α_3^{sea}
I	0.67	1.200	7.00
F	0.68	1.199	7.06

Comparison of fitted values of coefficients with the original ones used for generation for the LO evolution at $Q = 100$ GeV. I – input values, F – fitted values. The initial distributions are $\alpha_1^A x^{-\alpha_2^A}(1-x)^{\alpha_3^A}$. Accuracy mostly within 2σ.

Fitting PDF to PDF-test (NLO)

	α_1^G	α_2^G	α_3^G	α_1^u	α_2^u	α_3^u	α_1^d	α_2^d	α_3^d
I	1.908	1.200	5.000	2.187	0.500	3.000	1.230	0.500	4.000
F	1.905	1.200	5.002	2.187	0.503	3.013	1.230	0.513	3.997

	α_1^{sea}	α_2^{sea}	α_3^{sea}
I	0.673	1.200	7.000
F	0.678	1.198	7.094

Comparison of fitted values of coefficients with the original ones used for generation for the NLO evolution at $Q = 100$ GeV. I – input values, F – fitted values. The initial distributions are $\alpha_1^A x^{-\alpha_2^A}(1-x)^{\alpha_3^A}$. Accuracy mostly within 2σ.

Fitting PDF to PDF

Fitting different evolutions: Generated LO DGLAP with $\alpha(Q(1-z))$, fitted by NLO DGLAP

	α_1^G	α_2^G	α_3^G	α_1^u	α_2^u	α_3^u	α_1^d	α_2^d	α_3^d
I	1.908	1.200	5.000	2.187	0.500	3.000	1.230	0.500	4.000
F	1.801	1.202	4.724	1.992	0.359	2.625	0.46	0.234	2.991

	α_1^{sea}	α_2^{sea}	α_3^{sea}
I	0.673	1.200	7.000
F	0.605	1.47	8.011

$Q = 100$ GeV. I – input values, F – fitted values. The initial distributions are $\alpha_1^A x^{-\alpha_2^A}(1-x)^{\alpha_3^A}$ The change in parameters is big (up to 50%)

Fitting F2

In LO we have

$$F_2(x,t) = \sum_{A=q,\bar{q}} \int_0^1 d\xi D_A(\xi,t) x e_A^2 \delta(x-\xi) = \sum_{A=q,\bar{q}} D_A(x,t) x e_A^2. \tag{10}$$

$$F_2(x,t,\vec{\alpha}_1,\ldots,\vec{\alpha}_k) = x \sum_{A=q,\bar{q}} e_A^2 D_{AB}^{\delta}(x,t) \otimes^x D_B^0(x,\alpha_1^B,\ldots,\alpha_k^B). \tag{11}$$

and we fit with MINUIT with χ^2

$$\chi^2(a,b,\cdots) = \sum_n \sum_i \frac{\left(F_2(x_i,t_n,\vec{\alpha}_1,\ldots,\vec{\alpha}_k) - F_{2exp}(x_i,t_n)\right)^2}{e_{F_{2exp}}(x_i,t_n)^2}. \tag{12}$$

Fitting F2

In NLO nontrivial coefficent functions appear

$$F_2^{NLO}(x,t) = x \sum_{A=q,\bar{q}} e_A^2 D_A(x,t) + \Delta F_2, \tag{13}$$

$$\Delta F_2 = x \sum_{A=q,\bar{q}} e_A^2 D_A(x,t) \otimes^x \frac{\alpha_s}{2\pi} C_A^{\overline{MS}}(x) + x \sum_{A=q,\bar{q}} e_A^2 D_g(x,t) \otimes^x \frac{\alpha_S}{2\pi} C_g^{\overline{MS}}(x)\,, \tag{14}$$

where

$$\begin{aligned} C_q^{\overline{MS}} =& C_F \Bigg(2 \left(\frac{\ln(1-z)}{1-z} \right)_+ - \frac{3}{2} \left(\frac{1}{1-z} \right)_+ - (1+z)\ln(1-z) - \frac{1+z^2}{1-z} \ln(z) \\ & + 3 + 2z - \left(\frac{\pi^2}{3} + \frac{9}{2} \right) \delta(1-z) \Bigg) \end{aligned} \tag{15}$$

$$C_g^{\overline{MS}} = T_R \left(((1-z)^2 + z^2)) \ln\left(\frac{1-z}{z} \right) - 8z^2 + 8z - 1 \right). \tag{16}$$

Fitting F2 to F2 obtained from QCDNum16 - test (LO)

Fitting two structure functions: F2 (LO) obtained from Monte Carlo to F2 from QCDNum16

	α_1^G	α_2^G	α_3^G	α_1^u	α_2^u	α_3^u	α_1^d	α_2^d	α_3^d
I	1.908	1.200	5.000	2.187	0.500	3.000	1.230	0.500	4.000
F	1.908	1.199	4.988	2.187	0.502	3.07	1.241	0.507	4.08

	α_1^{sea}	α_2^{sea}	α_3^{sea}
I	0.673	1.200	7.000
F	0.674	1.200	6.944

$Q = 100$ GeV. I – input values, F – fitted values. The initial distributions are $\alpha_1^A x^{-\alpha_2^A}(1-x)^{\alpha_3^A}$

Fitting F2 to F2 obtained from QCDNum16 - test (NLO)

Fitting two structure functions: F2 (NLO) obtained from Monte Carlo to F2 from QCDNum16

	α_1^G	α_2^G	α_3^G	α_1^u	α_2^u	α_3^u	α_1^d	α_2^d	α_3^d
I	1.908	1.200	5.000	2.187	0.500	3.000	1.230	0.500	4.000
F	1.908	1.199	4.979	2.196	3.06	0.503	1.25	0.51	4.04

	α_1^{sea}	α_2^{sea}	α_3^{sea}
I	0.673	1.200	7.000
F	0.675	1.200	6.86

$Q = 100$ GeV. I – input values, F – fitted values. The initial distributions are $\alpha_1^A x^{-\alpha_2^A}(1-x)^{\alpha_3^A}$

Summary

- We have developed framework for fitting PDFs from Monte Carlo solutions of evolution equations.
- It is based on factorization property of the evolution equations.
- It is very fast and numerically precise.
- Numerical tests have been succefully performed at the level of PDFs and at the level of F2.