Matching CMC at NLO

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Overview

- Backward Evolution
- Constrained Monte Carlo (CMC)
- NLO Calculation
- Matching implemented for MC@NLO
- Proposal for CMC at NLO
- Unresolved Issues

Backward Evolution

- Probability of branching between scales t_1 and t_2

$$\mathcal{P}(t_1, t_2; x) \quad = \quad \exp\left\{ -\int_{t_1}^{t_2} \frac{dt}{t} \int \frac{dz}{z} P(z) \frac{f(x/z, t)}{f(x, t)} \right\}$$

- When one integrates over all emissions, we regain the original collinear pdf
- Thus we can rely on standard factorization schemes in matrix element matching

Constrained Monte Carlo

- As discussed by S. Jadach and M. Skrypek, the CMC is a forward evolution of the initial state (unintegrated pdf)
- The constraints on the evolution, imposed by the hard process, are included in the evolution through a transformation and reweighting procedure
- Inclusion of the hard process in a similar way as the YFS method in QED

CMC

$$\begin{aligned} xD_f(t,x) &= e^{-\Phi_f(t,t_0|x)} xD_f(t_0,x) + \\ & \sum_{N=0}^{\infty} \int_0^1 dx_0 \left[\prod_{i=1}^{N} \int_{t_0}^{t} dt_i \theta(t_i - t_{i-1}) \int_0^1 dy_i \int_0^{2\pi} \frac{d\phi_i}{2\pi} \right] \\ & \times e^{-\Phi_f(t,t_N|x)} \left[\prod_{i=1}^{N} \frac{x_i}{x_{i-1}} K_{ff}(t_i, x_i, x_{i-1}) e^{-\Phi_f(t_i,t_{i-1}|x_{i-1})} \right] \\ & \times x_0 D_f(t_0, x_0) \delta(x - x_0 + \sum y_j), \\ &= \int_0^1 dx_0 \, \mathcal{U}_{ff}(t, x|t_0, x_0) x_0 D_f(t_0, x_0) \end{aligned}$$

and of course, the $k_\perp$ can be reconstructed from the other kinematic variables.

CMC

- We can combine two hemispheres, and retain full phase space coverage by dividing the two hemispheres by a line of constant rapidity (that of the hard subprocess), $\eta^\star = \frac{1}{2}\ln\frac{x_F}{x_B}$
- Impose a δ to fix the s' for the hard process, with $x = s'/s$

$$\begin{aligned}\sigma &= \int dx \int dx_F dx_B \sum_{f_0^F f_0^B} \int dx_{0F} dx_{0B} D_{f_0^F}(t_0, x_{0F}) D_{f_0^B}(t_0, x_{0B}) \\ &\quad \times \mathcal{U}^F_{f^F f_0^F}(t_F, x_F | t_0, x_{0F}, \eta^\star) \mathcal{U}^B_{f^B f_0^B}(t_B, x_B | t_0, x_{0B}, \eta^\star) \sigma^{\rm Born}(s') \\ &\quad \times s\delta(sx - (q_{0F} + q_{0B} - K_F - K_B)^2)\end{aligned}$$

CMC

- In order to use the same evolution operators, $\mathcal{U}$, and to match the hard process, we must consider several issues
  - IR regularization of MEs
  - Avoid double counting (hard contribution and parton shower contribution)
  - May have a new class of object, in the purely collinear regime
  - Negative weights
  - Relationship to standard factorization theorems and schemes
- We have successfully dealt with the first four items, still working on the last one

CMC

- We can now define a β function, which will allow us to construct the NLO cross section

$$\beta(s', \{k_i^F\}, \{k_i^B\}) \approx \frac{\sigma^{\rm Born} + \sigma^{\rm NLO}(\cdot)}{\sigma^{\rm Born}(s')}$$
$$= \beta_0(s') + \beta_1(s', \{k_i\})$$
$$+\beta_1^{\rm col}(z_F) + \beta_1^{\rm col}(z_B)$$

- We must avoid double counting contributions and define an appropriate IR regulation procedure to define the β function

CMC

- We can now define a β function, which will allow us to construct the NLO cross section

$$\beta(s', \left\{k_i^F\right\}, \left\{k_i^B\right\}) \approx \frac{\sigma^{\text{Born}} + \sigma^{\text{NLO}}(\cdot)}{\sigma^{\text{Born}}(s')}$$

$$= \beta_0(s') + \beta_1(s', \{k_i\})$$

$$+\beta_1^{\text{col}}(z_F) + \beta_1^{\text{col}}(z_B)$$

$$\sigma^{\text{NLO}} = \int dx \int dx_F dx_B \sum_{f_0^F f_0^B} \int dx_{0F} dx_{0B} D_{f_0^F}(t_0, x_{0F}) D_{f_0^B}(t_0, x_{0B})$$

$$\times \mathcal{U}^F_{f^F f_0^F}(t_F, x_F | t_0, x_{0F}, \eta^\star) \mathcal{U}^B_{f^B f_0^B}(t_B, x_B | t_0, x_{0B}, \eta^\star) \sigma^{\text{Born}}(s')$$

$$\times s\delta(sx - (q_{0F} + q_{0B} - K_F - K_B)^2) \beta(s', \left\{k_i^{F,B}\right\})$$

NLO Calculation

- The NLO calculation (from Feynman diagrams) is generally of the form (in $4-2\epsilon$ dimensions)

$$
\begin{aligned}
d_{n+1}\sigma_V &= \left[\left(\frac{A_V}{\epsilon^2}+\frac{B_V}{\epsilon}\right)d_n\sigma^{\rm Born}+d_n\sigma_{V,reg}\right]\delta(p^+)\delta(p^-)dp^+dp^- \\
d_{n+1}\sigma_R &= d_{n+1}\sigma_f+\frac{\delta(p^+)\delta(p^-)}{\epsilon^2}d_n\sigma_S dp^+dp^- \\
&\quad +\frac{dp^+dp^-}{\epsilon}\left(d_n\sigma_{C+}(p^+)\delta(p^-)+d_n\sigma_{C-}(p^-)\delta(p^+)\right)
\end{aligned}
$$

  - d_n : the differential element of the Born phase space
  - the angular information is implicit
  - p^+ and p^- are the light-cone components of the gluon emission

NLO Calculation

- The soft singularities cancel with the virtual contribution
- The collinear singularities are handled by the factorization scheme, i.e. absorbed into the non-perturbative part of pdf
- We define the subtracted quantities $d\hat{\sigma} = d\sigma - d\sigma_{\mathrm{ct}}$
- So we have the following finite quantities

$$d\sigma_{\mathrm{NLO}} + \mathrm{PDF}_{ct} \;=\; d\hat{\sigma}_{SV} + d\hat{\sigma}_{C+} + d\hat{\sigma}_{C-} + d\sigma_{f}$$

MC@NLO Frixione & Webber

- The MC contribution at NLO is computed, i.e. $d\sigma_{MC}$
- The counterterms are “undone” and rearranged

$$
\begin{aligned}
2 \to 3 \quad &: \quad d\sigma_R - d\sigma_{MC} \\
2 \to 2 \quad &: \quad d\sigma_{VS} - d\sigma_{R,ct} + d\sigma_{MC} + d\sigma^{\mathrm{Born}} \\
2 \to \tilde{2} \quad &: \quad d\sigma_{C+} + d\sigma_{C-} - d\sigma_{C+,ct} - d\sigma_{C-,ct}
\end{aligned}
$$

  - The last class of events, can be treated with $2 \to 2$ kinematics via a longitudinal boost
  - The two individual classes of kinematics are used to generate the initial conditions of the shower
  - The two classes are combined with the appropriate weight (ratio of cross sections) at the end

CMC at NLO

- We propose a similar approach as MC@NLO (to avoid double counting)
- Our IR regularization procedure is universal. Only parton shower connection is due to the separation of forward and backward hemispheres
- Use this approach to defi ne the β functions, i.e.

$$\beta_0 \sim 2 \rightarrow 2 \quad ; \quad \beta_1 \sim 2 \rightarrow 3$$

  - Avoids double counting
  - One class of events - no negative weights

CMC at NLO

- Use the AP splitting kernel (plus eikonal factor), in $4 - 2\epsilon$ dimensions
- Virtual subtraction term is minus the integral of the real term
- Subtraction over full range of values (not restricted by shower cutoffs)

$$d_n\sigma_{V,ct} = -\int \frac{dp^+}{(p^+)^{1+\epsilon}} \frac{dp^-}{(p^-)^{1+\epsilon}} \left[K(p^+,\epsilon)\theta_F + K(p^-,\epsilon)\theta_B\right] d_n\sigma^{\mathrm{Born}}$$

CMC at NLO

- These counterterms lead to the definition of the β function

$$\beta_0 = 1 + \frac{d\sigma_V - d\sigma_{V,ct}}{d\sigma_B} \quad ; \quad \beta_1(k_i) = \frac{d\sigma_R - d\sigma_{R,ct}}{d\sigma_B K(k_i)}$$

- Gluon momentum for β_1 chosen to be the hardest emission; must lie next to hard process (as in Nason@NLO)
  - Numerically confi rmed 60% of hardest emissions next to hard process
  - Not necessary, but differs by subleading terms

CMC at NLO

- Subtraction term is not equal to parton shower contribution, due to ordering and IR regulator $1 - \varepsilon_z$ (resolvability)
- Difference between subtracted term and parton shower can be computed
- Missing contributions lie in collinear region
- This defines $\beta_1^{\rm col}$
- A factorization scheme must be applied to define $\beta_1^{\rm col}$

CMC at NLO

- If we use the same terms as $\overline{\mathrm{MS}}$ to define our scheme we find

$$\beta^{\mathrm{col}}_{1,\overline{\mathrm{MS}}-\mathrm{like}}(z) = (1-z) + \left(\frac{1+z^2}{1-z}\right)_{\varepsilon_z} \log\frac{\tilde{q}^2}{\mu_R^2} + \left(\frac{\log(1-z)}{1-z}\right)_{\varepsilon_z} (1+z^2)$$

with

$$\int_{1-\varepsilon_z}^{1} dz \left(\frac{1}{1-z}\right)_{\varepsilon_z} f(z) = \int_{1-\varepsilon_z}^{1} \frac{f(z)-f(1)}{1-z}$$

  - $\tilde{q} = q_{n-1}\sqrt{x_F x_B}$
  - μ_R is the renormalization scale for the emission

CMC at NLO

- Subtraction terms do not change normalization of cross section
- β function is finite
- β function gives correct cross section and differential distribution
- Discrepency between shower and counterterms is universal, i.e. does not depend on hard process
- Discrepency does not effect cross section
- Exact treatment of collinear β function still to be worked out; must be tested numerically!

Remaining issues

- Understand how our evolution differs from standard collinear factorization; can we correct in some way to all orders?
- Verify proposed treatment of β_1^{col}
- Implement for $q\bar{q} \to W^+W^- + g$
  - Expect results soon

Conclusion

- Presented a method for universal treatment of hard process
- Parton shower specific issues are treated once for all hard processes
- No negative weights (by construction)
- No double counting
- Expect to have results for W^+W^- production soon