Recursive equations for arbitrary scattering processes at tree order *and beyond*

Costas G. Papadopoulos

Epiphany Conference, January 2007, Krakow

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  hep-ph/0012004 and Tokyo 2001,(CPP2001) Computational particle physics, p. 20-25

  T. Gleisberg, *et al.* Eur. Phys. J. C 34 (2004) 173

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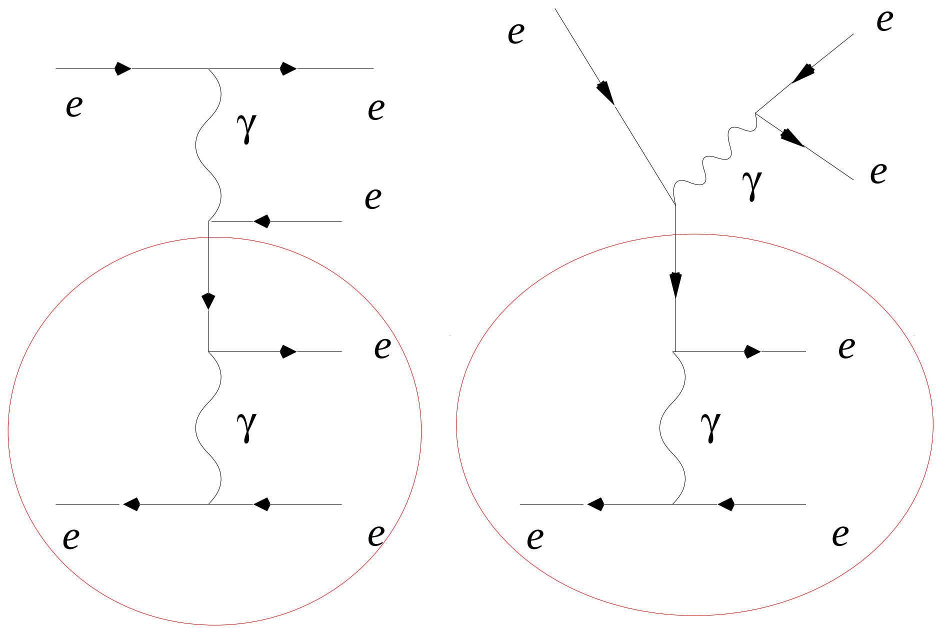
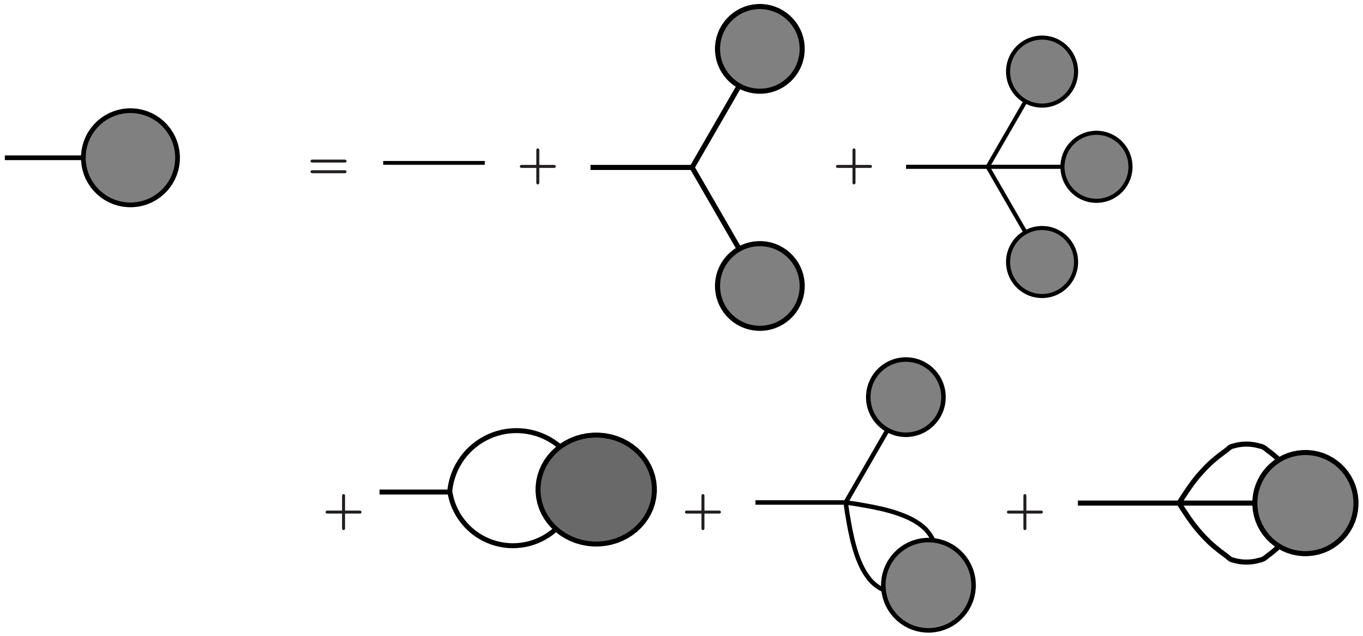
- ***Example:*** $e^- e^+ \longrightarrow e^- e^+ e^- e^+$ **in QED:**



The Dyson-Schwinger recursion

- **Imagine a theory with 3- and 4- point vertices and just one field. Then it is straightforward to write an equation that gives the amplitude for 1 $\longrightarrow$ n**



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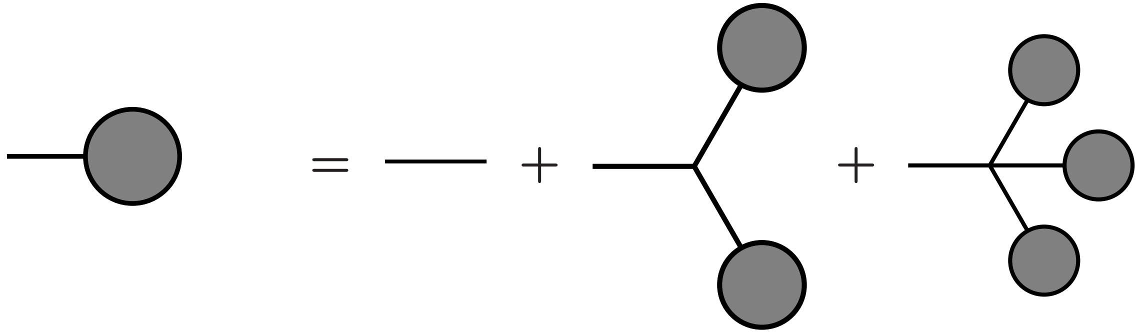

$$a(n) = \delta_{n,1} + \sum \frac{n!}{n_1!n_2!} a(n_1)a(n_2)\delta_{n_1+n_2,n}$$
$$+ \frac{n!}{n_1!n_2!n_3!} \sum a(n_1)a(n_2)a(n_3)\delta_{n_1+n_2+n_3,n}$$

HELAC

- **Construction of the skeleton solution of the Dyson-Schwinger equations. At this stage only integer arithmetic is performed. This is part of the initialization phase.**
- **Dressing-up the skeleton with momenta, provided by `PHEGAS` and wave functions, propagators, n-point functions in general.**
- **Unitary and Feynman gauges implemented. Due to multi-precision arithmetic,tests of gauge invariance can be extended to arbitrary precision.**
- **All fermions masses can be non-zero.**
- **All Electroweak and QCD vertices are implemented, including Higgs and would-be Goldstone bosons.**

Colour Configuration - EWK$\bigoplus$QCD

Colour Configuration - EWK⊕QCD

- **Ordinary approach $SU(N)$-type**

$$\mathcal{A}^{a_1 \ldots a_n} = \sum Tr(T^{a_{\sigma_1}} \ldots T^{a_{\sigma_n}}) \quad A(\sigma_1 \ldots \sigma_n)$$

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Quarks and gluons treated differently

Colour Configuration - EWK$\oplus$QCD

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- **New approach $U(N)$-type**

Each color-configuration amplitude is proportional to

$$D_i = \delta_{1,\sigma_i(1)}\delta_{2,\sigma_i(2)}\ldots\delta_{n,\sigma_i(n)}$$

where σ_i represents the i-th permutation of the set $1, 2, \ldots, n$.

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⋆ **quarks** $1 \ldots n$

⋆ **antiquarks** $\sigma_i(1 \ldots n)$ **and**

⋆ **gluons** $= q\bar{q}$

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$$\mathcal{C}_{ij} = \sum D_i D_j = N_c^{\alpha} \ , \ \ \alpha = \langle \sigma_1, \sigma_2 \rangle$$

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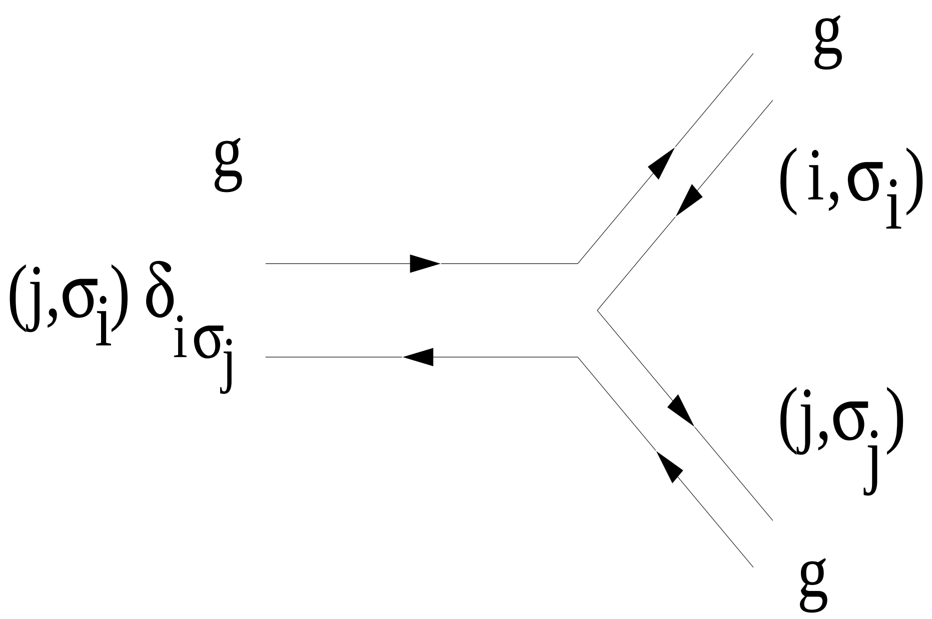
$$\mathcal{C}_{ij} = \sum D_i D_j = N_c^{\alpha} \;,\;\; \alpha = \langle \sigma_1, \sigma_2 \rangle$$

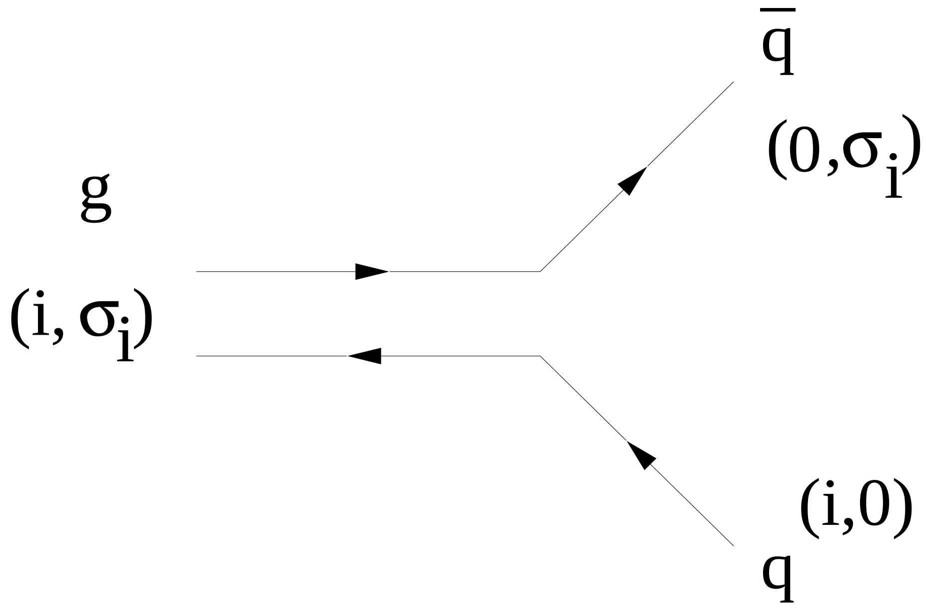
♠ exact color treatment ⇒ low color charge

Problem: number of colour connection configurations: $\sim n!$ where n is the number of gluons or $q\bar{q}$ pairs. ⇒ Monte-Carlo over continuous colour-space.



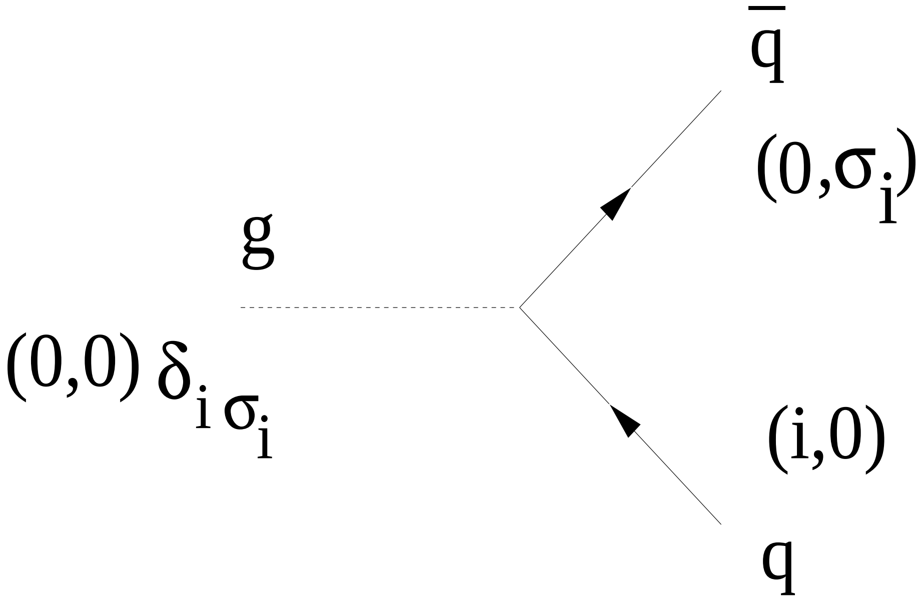
$$\sum f^{abc} t^a_{AB} t^b_{CD} t^c_{EF} = -\frac{i}{4}(\delta_{AD}\delta_{CF}\delta_{EB} - \delta_{AF}\delta_{CB}\delta_{ED})$$

$$\delta_{1\sigma_2}\delta_{2\sigma_3}\delta_{3\sigma_1}$$



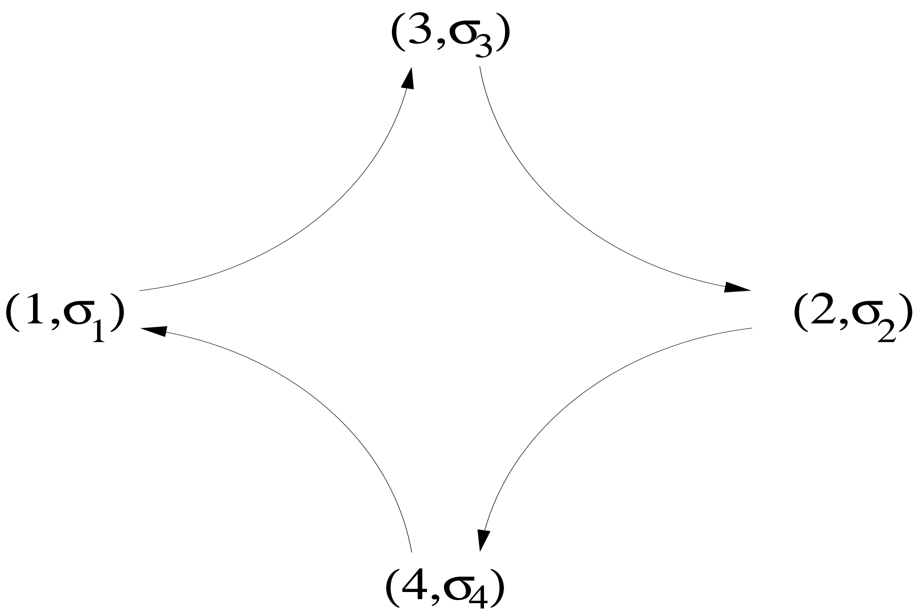
$$\sum t^{a}_{AB} t^{b}_{CD} = \frac{1}{2}\left(\delta_{AD}\delta_{CB} - \frac{1}{N_c}\delta_{AB}\delta_{AC}\right)$$

$$\frac{1}{\sqrt{2}}$$



$$\sum t^a_{AB} t^b_{CD} = \frac{1}{2}(\delta_{AD}\delta_{CB} - \frac{1}{N_c}\delta_{AB}\delta_{AC})$$

$$\frac{1}{\sqrt{2N_c}}$$



$$\delta_{1\sigma_3}\delta_{3\sigma_2}\delta_{2\sigma_4}\delta_{4\sigma_1}$$

$$2g_{12}g_{34} - g_{13}g_{24} - g_{14}g_{23}$$

The high-colour processes

The idea is to replace colour summation with integration and then follow a MC approach

$$G^{\mu}_{AB}(P_i) = \sum_{a=1}^{8} G^a(P_i)\eta^a(z) = \sqrt{6}\left(z_{iA} z^*_{iB} - \frac{1}{3}\delta_{AB}\right)\epsilon^{\mu}_{\lambda}(P_i)$$

$$\psi_A(P_i) = \sqrt{3}\, u(P_i)\, z_{iA}$$

$$\bar{\psi}_A(P_i) = \sqrt{3}\, \bar{u}(P_i)\, z^*_{iA}$$

In such a representation the amplitude can be seen

$$\mathcal{M}(z_1, z_2, \ldots) = \sum z_1 \cdot z_{\sigma(i)_1} z_2 \cdot z_{\sigma(i)_2} \ldots \mathcal{A}_i$$

and the MC is over

$$\int [dz] \equiv \int \left(\prod_{i=1}^{3} dz_i \, dz_i^* \right) \delta(\sum_{i=1}^{3} z_i z_i^* - 1)$$

where

$$\int [dz] G_{AB} G_{CD} = \int [dz] \sqrt{6} \left(z_A z_B^* - \frac{1}{3} \delta_{AB} \right) \sqrt{6} \left(z_C z_D^* - \frac{1}{3} \delta_{AB} \right)$$

$$= \frac{1}{2} \left(\delta_{AD} \delta_{CB} - \frac{1}{3} \delta_{AB} \delta_{CD} \right)$$

Multi-jet processes

Beyond any colour treatment a summation over different flavours is also needed.

Up to now the most straightforward way was to count the distinct processes and then multiply with a multiplicity factor, i.e.

process	Flavour
$gg \to ggg$	1
$q\bar{q} \to ggg$	8
$qg \to qgg$	8
$qg \to qgg$	8
$gg \to q\bar{q}g$	5
$q\bar{q} \to q\bar{q}g$	8
$q\bar{q} \to r\bar{r}g$	32
$qq \to qqg$	8
$q\bar{r} \to q\bar{r}g$	24
$qr \to qrg$	24
$qg \to qq\bar{q}$	8
$qg \to qr\bar{r}$	32
$gq \to qq\bar{q}$	8
$gq \to qr\bar{r}$	32

initial-state type		distinct processes	multiplicity factor
A	(gg)	$C_1(n)$	$\chi(n_0, n_1, \ldots, n_f; f)$
B	$(q\bar{q})$	$C_2(n)$	$\chi(n_0, n_2, \ldots, n_f; f-1)$
C	$(gq$ and $qg)$	$C_2(n-1)$	$\chi(n_0, n_2, \ldots, n_f; f-1)$
D	(qq)	$C_2(n-2)$	$\chi(n_0, n_2, \ldots, n_f; f-1)$
E	$(qq'$ and $q\bar{q}')$	$C_3(n-2)$	$\chi(n_0, n_3, \ldots, n_f; f-2)$

In order to clarify what we mean we consider the example of the type A initial state. Each distinct process is defined by an array $(n_0, n_1, \ldots, n_f)$. For instance, in the case of four-jet production we have

$$
\begin{array}{ll}
(4,0,0,0,0,0) & gg \to gggg \\
(2,1,0,0,0,0) & gg \to ggq\bar{q} \\
(0,2,0,0,0,0) & gg \to q\bar{q}q\bar{q} \\
(0,1,1,0,0,0) & gg \to q\bar{q}r\bar{r}
\end{array}
$$

$$C_1(n) = \sum_{n_0+2n_1+\ldots+2n_f=n} \Theta(n_1 \geq n_2 \geq \ldots \geq n_f)$$

$$C_2(n) = \sum_{n_0+2n_1+\ldots+2n_f=n} \Theta(n_2 \geq n_3 \geq \ldots \geq n_f)$$

and

$$C_3(n) = \sum_{n_0+2n_1+\ldots+2n_f=n} \Theta(n_3 \geq n_4 \geq \ldots \geq n_f)$$

A distinct process, given by the array $(n_0, n_1, \ldots, n_f)$ has a multiplicity factor :

$$\chi(n_0, n_1, \ldots, n_f; f) = n_f(n_f - 1)\ldots(n_f - j + 1)/j!$$

$$\begin{array}{ccc}
j = f & \text{if} & \prod_{i=1}^{f} n_i \neq 0 \\
j = f - 1 & \text{if} & \prod_{i=1}^{f-1} n_i \neq 0 \\
\ldots & & \\
j = 1 & \text{if} & n_1 \neq 0 \\
j = 0 & \text{otherwise} &
\end{array}$$

Now we can think of a flavour-MC, so the wave function is multiplied by an N_f-dimensional array representing flavour , $\vec{f} = \sqrt{N_f}(f_1, f_2, \ldots)$ such that $\langle f_i f_j \rangle = \delta_{ij}$ with a weight proportional to the relevant pdf for initial state flavours

In that case a process like

$$gg \to ggq\bar{q}q\bar{q}$$

will actually represent a plethora of processes.

The number of distinct processes is now given by

$$9k + 3 \;\; \text{if} \;\; n = 2k \quad \text{and} \quad 9k + 7 \;\; \text{if} \;\; n = 2k + 1$$

# of jets	2	3	4	5	6	7	8	9	10
# of D-processes	12	16	21	24	30	34	39	43	48
# of dist.processes	10	14	28	36	64	78	130	154	241
total # of processes	126	206	621	861	1862	2326	4342	5142	8641

Multi-jet rates

$$p_{T\ i} > 60\ GeV, \quad \theta_{ij} > 30^o \quad |\eta_i| < 3$$

# jets	3	4	5	6	7	8
$\sigma(nb)$	91.41	6.54	0.458	2.97×10^{-2}	2.21×10^{-3}	2.12×10^{-4}
% Gluon	45.7	39.2	35.7	35.1	33.8	26.6

A new code ⇒ `JetI`

- anybody to tell us how many Feynman graphs in $gg \longrightarrow 8g$?
- or $gg \longrightarrow 2g3u3\bar{u}$?

- **Feynman graphs in $gg \longrightarrow 8g$** **10,525,900 !!**

- **or $gg \longrightarrow 2g3u3\bar{u}$** **946,050!**

Summation/Integration over color

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$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n) \sim \sum_{P(2,\ldots,n)} Tr(t^{a_1} \ldots t^{a_n}) \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

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$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{I_i, J_i\}_1^n) \sim \sum_{P(2,\ldots,n)} \delta_{I_1, P(J_1)} \ldots \delta_{I_n, P(J_n)} \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

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$$\sum_{\{a_i\}_1^n\{\varepsilon_i\}_1^n} |\mathcal{M}(\{p_i\}_1^n,\{\varepsilon_i\}_1^n,\{a_i\}_1^n)|^2 = g^{2n-4}\sum_{\varepsilon}\sum_{ij}\mathcal{A}_i\mathcal{C}_{ij}\mathcal{A}_j^*$$

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$$\sum_{P(2,\ldots,n)} \sim n!$$

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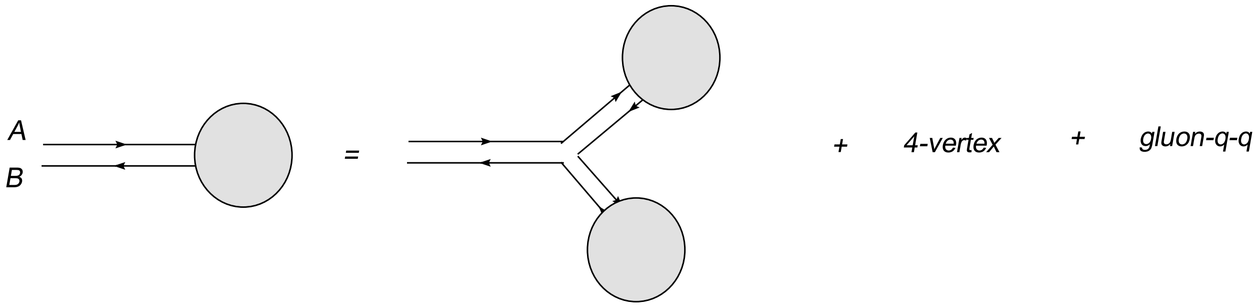
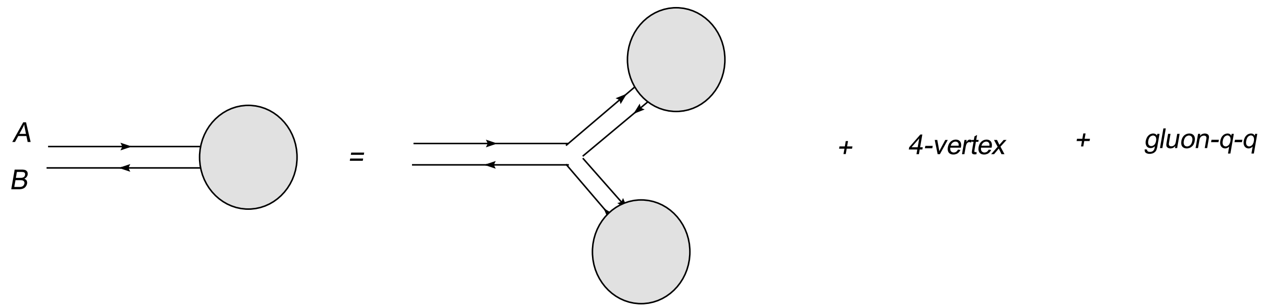
$$\sum_{P(2,\dots,n)} \sim n!$$

$$\sum_{\{I_i,J_i\}_1^n} \sim 3^n \times 3^n$$


A
B
=
+
4-vertex
+
gluon-q-q



$$[A^\mu(P);(A,B)] = \sum_{i=1}^{n}[\ \delta(P-p_i)\ A^\mu(p_i);(A,B)_i]+$$

$$\sum[\ (ig)\ \Pi^\mu_\rho\ V^{\rho\nu\lambda}(P,p_1,p_2)A_\nu(p_1)A_\lambda(p_2)\sigma(p_1,p_2);(A,B)=(C,D)_1\otimes(E,F)_2]$$

$$-\sum[\ (g^2)\ \Pi^\mu_\sigma\ G^{\sigma\nu\lambda\rho}(P,p_1,p_2,p_3)A_\nu(p_1)A_\lambda(p_2)A_\rho(p_3)\sigma(p_1,p_2+p_3);$$

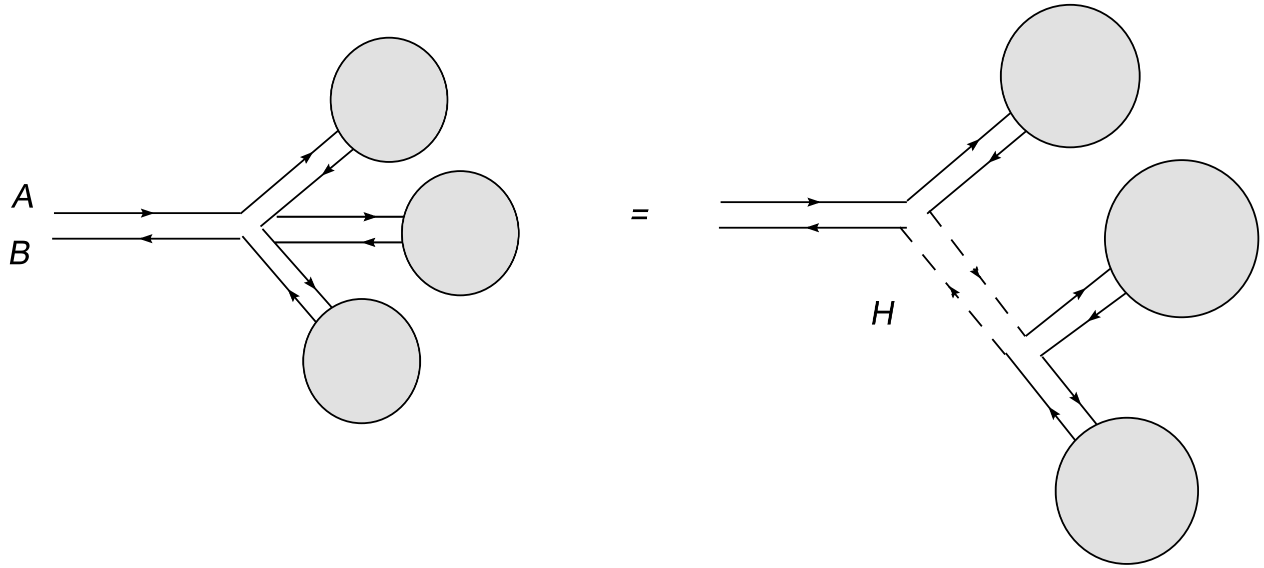
$$(A,B)=(C,D)_1\otimes(E,F)_2\otimes(G,H)_3]$$

$$+\sum_{P=p_1+p_2}[\ (ig)\ \Pi^\mu_\nu\ \bar\psi(p_1)\gamma^\nu\psi(p_2)\sigma(p_1,p_2);(A,B)=(0,D)_1\otimes(C,0)_2]$$

where $A,B,C,D,E,F,G,H = 1,2,3.$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$\mathcal{L} = -\frac{1}{2} H^a_{\mu\nu} H^{\mu\nu a} + \frac{1}{4} H^a_{\mu\nu} F^{\mu\nu a}.$$



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$$[\ (ig)\ \Pi^{\mu}_{\rho}\ \ V^{\rho\nu\lambda}(P,p_1,p_2)A_{\nu}(p_1)A_{\lambda}(p_2)\sigma(p_1,p_2);(A,B)=(C,D)_1\otimes(E,F)_2]$$

$$+$$

$$[\ (ig)\ \Pi^{\mu}_{\sigma}\ (g^{\sigma\lambda}g^{\nu\rho}-g^{\nu\lambda}g^{\sigma\rho})\ A_{\nu}(p_1)H_{\lambda\rho}(p_2)\sigma(p_1,p_2);(A,B)=(C,D)_1\otimes(E,F)_2]$$

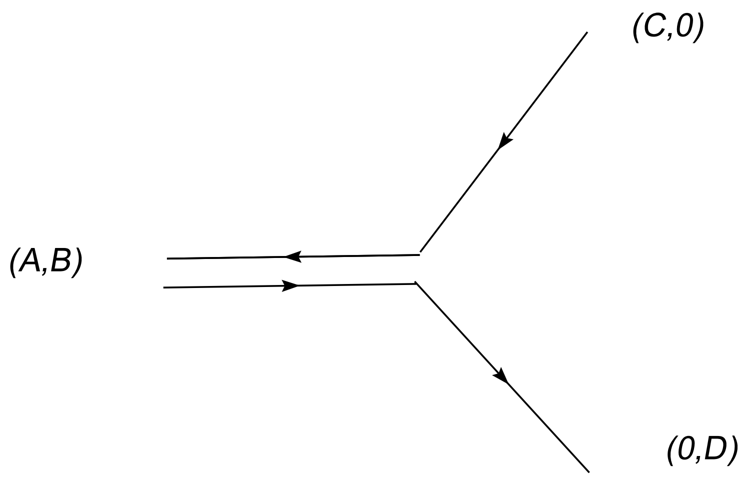
$$+$$

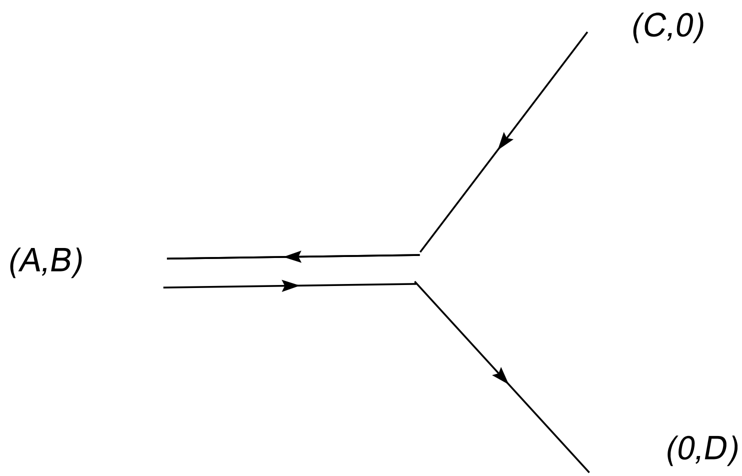
$$[\ (ig)\ \Pi^{\mu}_{\nu}\ \ \bar{\psi}(p_1)\gamma^{\nu}\psi(p_2)\sigma(p_1,p_2);(A,B)=(0,D)_1\otimes(C,0)_2]$$

and

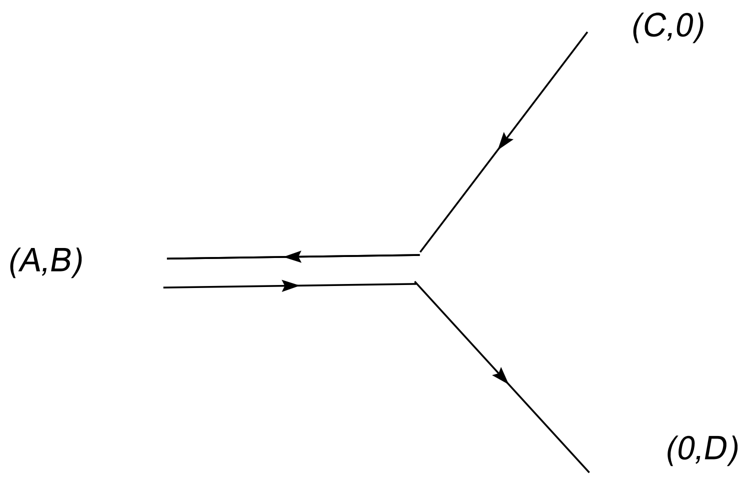
$$[H^{\mu\nu}(P);(A,B)] = \sum_{P=p_1+p_2} [\ (ig)\ (g^{\mu\lambda}g^{\nu\rho}-g^{\nu\lambda}g^{\mu\rho})\ A_{\lambda}(p_1)A_{\rho}(p_2)\sigma(p_1,p_2);$$

$$(A,B)=(C,D)_1\otimes(E,F)_2].$$


(C,0)
(A,B)
(0,D)



$$(A,B) = (C,0) \otimes (0,D) = (C,D)_{w=1}, \qquad \text{if } C \neq D.$$



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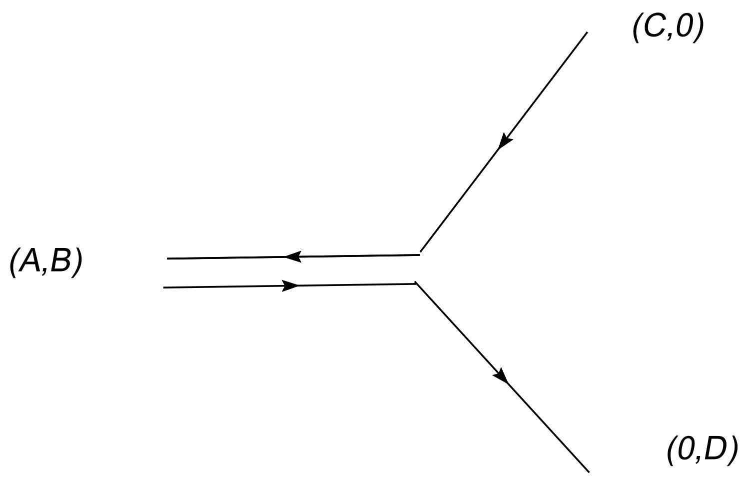
$$(A,B) = (C,0) \otimes (0,D) = (1,1)_{w1} \oplus (2,2)_{w2} \oplus (3,3)_{w3}, \qquad \text{if } C = D.$$



$$(\boldsymbol{A},\boldsymbol{B}) = (\boldsymbol{C},\boldsymbol{0}) \otimes (\boldsymbol{0},\boldsymbol{D}) = (\boldsymbol{C},\boldsymbol{D})_{w=1}, \qquad \text{if } \boldsymbol{C} \neq \boldsymbol{D}.$$

$$(\boldsymbol{A},\boldsymbol{B}) = (\boldsymbol{C},\boldsymbol{0}) \otimes (\boldsymbol{0},\boldsymbol{D}) = (\boldsymbol{1},\boldsymbol{1})_{w1} \oplus (\boldsymbol{2},\boldsymbol{2})_{w2} \oplus (\boldsymbol{3},\boldsymbol{3})_{w3}, \qquad \text{if } \boldsymbol{C} = \boldsymbol{D}.$$

$$(\boldsymbol{1},\boldsymbol{0}) \otimes (\boldsymbol{0},\boldsymbol{1}) = (\boldsymbol{1},\boldsymbol{1})_{2/3} \oplus (\boldsymbol{2},\boldsymbol{2})_{-1/3} \oplus (\boldsymbol{3},\boldsymbol{3})_{-1/3}$$

$$N_{\mathrm{CC}} = \sum_{A=0}^{n_q} \sum_{B=0}^{n_q-1} \sum_{C=0}^{n_q-A-B} \left(\frac{n_q!}{A!B!C!} \right)^2 \delta(n_q = A + B + C)$$

$$N_{\mathrm{CC}} = \sum_{A=0}^{n_q} \sum_{B=0}^{n_q-1} \sum_{C=0}^{n_q-A-B} \left(\frac{n_q!}{A!B!C!} \right)^2 \delta(n_q = A + B + C)$$

Process	$\mathrm{N}_{\mathrm{CC}}^{\mathrm{ALL}}$	N_{CC}	$\mathrm{N}_{\mathrm{CC}}^{\mathrm{F}}$ (%)
$gg \to 2g$	6561	639	59.1
$gg \to 3g$	59049	4653	68.4
$gg \to 4g$	531441	35169	77.4
$gg \to 5g$	4782969	272835	85.0
$gg \to 6g$	43046721	2157759	90.4
$gg \to 7g$	387420489	17319837	94.0
$gg \to 8g$	3486784401	140668065	96.4

Process	N_{CC}^{ALL}	N_{CC}	N_{CC}^{F} (%)
$gg \longrightarrow u\bar{u}$	729	93	93.5
$gg \longrightarrow gu\bar{u}$	6561	639	91.6
$gg \longrightarrow 2gu\bar{u}$	59049	4653	92.6
$gg \longrightarrow 3gu\bar{u}$	531441	35169	94.6
$gg \longrightarrow 4gu\bar{u}$	4782969	272835	96.4
$gg \longrightarrow 5gu\bar{u}$	43046721	2157759	97.8
$gg \longrightarrow 6gu\bar{u}$	387420489	17319837	98.6
$gg \longrightarrow c\bar{c}c\bar{c}$	6561	639	99.1
$gg \longrightarrow gc\bar{c}c\bar{c}$	59049	4653	98.8
$gg \longrightarrow 2gc\bar{c}c\bar{c}$	531441	35169	99.0
$gg \longrightarrow 3gc\bar{c}c\bar{c}$	4782969	272835	99.3
$gg \longrightarrow 4gc\bar{c}c\bar{c}$	43046721	2157759	99.6

Process	$\sigma_{\mathrm{MC}} \pm \varepsilon$ (nb)	ε (%)
$gg \to 7g$	$(0.53185 \pm 0.01149)\times 10^{-2}$	2.1
$gg \to 8g$	$(0.33330 \pm 0.00804)\times 10^{-3}$	2.4
$gg \to 9g$	$(0.17325 \pm 0.00838)\times 10^{-4}$	4.8
$gg \to 5gu\bar{u}$	$(0.38044 \pm 0.01096)\times 10^{-3}$	2.8
$gg \to 3gc\bar{c}c\bar{c}$	$(0.95109 \pm 0.02456)\times 10^{-5}$	2.6
$gg \to 4gc\bar{c}c\bar{c}$	$(0.81400 \pm 0.02583)\times 10^{-6}$	3.2

Process	$\sigma_{\mathrm{MC}} \pm \varepsilon$ (nb)	ε (%)
$gg \to Zu\bar{u}gg$	$(0.18948 \pm 0.00344)\times 10^{-3}$	1.8
$gg \to W^{+}\bar{u}dgg$	$(0.62704 \pm 0.01458)\times 10^{-3}$	2.3
$gg \to ZZu\bar{u}gg$	$(0.16217 \pm 0.00420)\times 10^{-6}$	2.6
$gg \to W^{+}W^{-}u\bar{u}gg$	$(0.27526 \pm 0.00752)\times 10^{-5}$	2.7
$d\bar{d} \to Zu\bar{u}gg$	$(0.38811 \pm 0.00569)\times 10^{-5}$	1.5
$d\bar{d} \to W^{+}\bar{c}sgg$	$(0.18765 \pm 0.00453)\times 10^{-5}$	2.4
$d\bar{d} \to ZZgggg$	$(0.99763 \pm 0.02976)\times 10^{-7}$	2.9
$d\bar{d} \to W^{+}W^{-}gggg$	$(0.52355 \pm 0.01509)\times 10^{-6}$	2.9

PHEGAS

- Phase space

$$d\Phi_n = (2\pi)^{4-3n} \prod_{i=1}^{n} \frac{d^3 p_i}{2E_i} \delta\left(\sum E_i - w\right) \delta^3\left(\sum \vec{p}_i\right)$$

- RAMBO, VEGAS-based nice but completely inefficient!

$$d\sigma_n = \mathrm{FLUX} \times |\mathcal{M}_{2\to n}|^2 d\Phi_n$$

  need appropriate mappings of peaking structures, plus optimization!

- Efficiency $\Rightarrow$ to a large number of generators, each one for a specific class of processes.

Multichannel approach

$$\mathcal{I} = \int f(\vec{x})d\mu(\vec{x}) = \int \frac{f(\vec{x})}{p(\vec{x})}p(\vec{x})d\mu(\vec{x})$$

$$p(\vec{x}) = \sum_{i=1}^{M_{ch}} \alpha_i \, p_i(\vec{x}) \qquad \sum_{i=1}^{M_{ch}} \alpha_i = 1$$

$$\mathcal{I} \rightarrow \left\langle \frac{f(\vec{x})}{p(\vec{x})} \right\rangle \qquad \mathcal{E}^2 N \rightarrow \left\langle \left(\frac{f(\vec{x})}{p(\vec{x})} \right)^2 - \mathcal{I}^2 \right\rangle$$

$\star$ **Optimize $\alpha_i \Rightarrow$ Minimize $\mathcal{E}$** $\star$

R.Kleiss and R.Pittau, Comput. Phys. Commun. 83, 141 (1994).

New **Dyson-Schwinger equations: subamplitude is a combination of several peaking structures!**

problem unsolved?

QCD antennas

P.D.Draggiotis, A.van Hameren and R.Kleiss, hep-ph/0004047.

New **Dyson-Schwinger equations: subamplitude is a combination of several peaking structures!**

problem unsolved?

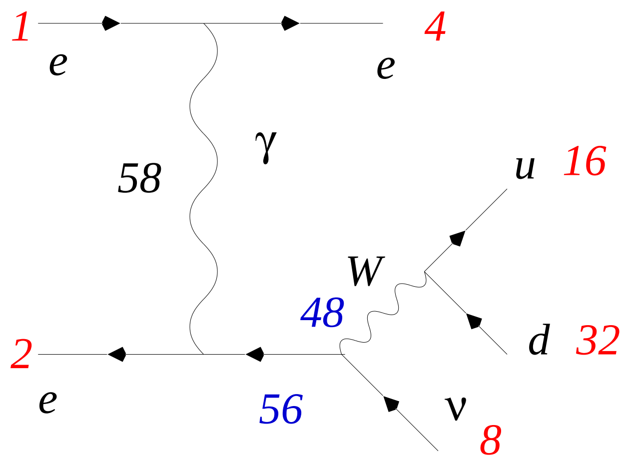
QCD antennas

P.D.Draggiotis, A.van Hameren and R.Kleiss, hep-ph/0004047.

Old **Feynman graphs: exhibit single peaking structure!**

problem solved

Back to Feynman graphs:



The corresponding intrinsic representation looks like

62	**-2**	**4**	**-2**	**58**	**31**
58	**31**	**2**	**-2**	**56**	**2**
56	**2**	**48**	**33**	**8**	**1**
48	**33**	**16**	**-3**	**32**	**4**

Current Status

`http://www.cern.ch/helac-phegas/`

- **Complete generation for pp and $p\bar{p}$ collisions, including all sub-processes. We do not exclude any processes!**
- **Interfacing with Pythia, including CKKW-like reweighting and use of `UPVETO` à la MLM.**
- **The new version, to become publicly available very soon. Suitable for multi-color final states (more than 7 equivalent gluons at the final state, i.e. 14 quarks (antiquarks) !)**

Higher-order corrections

- **Fermion-loop corrections have been implemented and studied for up to 3-point vertices.**
- **We have started the computation of 4-point contributions.**
  - `FORM` **has been used to reduce the expressions to Passarino-Veltman coefficient functions**
  - `FF` **has been updated to include level-4 tensor coefficient functions for 4-point integrals.**
  - **Implementation and checking in** `HELAC` **is in progress.**
- **This implementation will allow to study 4 fermion+γ and 6 fermion production including the running of electroweak couplings.**

- **Fermion-loop corrections to six-fermion production process**

$$e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d} \tau^- \tau^+$$

- **Number of Feynman Graphs: 208**
- **Number of DS vertices: 140**
- **Cuts:** $E_l, E_q > 5\text{GeV}$ **and** $m_{ll}, m_{qq} > 10\text{GeV}$
- **Results:** $E = 500\text{GeV}$
- $\sigma_0/ab = 54,96(26)$ $\sigma_1/ab = 57,31(28)$ $K/100 = 4.28(2)$
- **MC data: generated: 1M(961792) used: 404842 time:6 1/2 h**

Ossola, Papadopoulos, Pittau hep-ph/0609007

$$
\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} \bar{D}_i \\
& + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} \bar{D}_i \\
& + \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i \neq i_0, i_1}^{m-1} \bar{D}_i \\
& + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i \neq i_0}^{m-1} \bar{D}_i \\
& + \tilde{P}(q) \prod_{i}^{m-1} \bar{D}_i \, .
\end{aligned}
$$

$$q^\mu = -p_0^\mu + y_1 k_1^\mu + y_n n^\mu + y_7 \ell_7^\mu + y_8 \ell_8^\mu \,.$$

$$n \cdot k_1 = 0\,, \quad \text{and} \quad n^2 = -k_1^2$$

$$k_1{}^\mu = \bar{k}_1{}^\mu + \frac{k_1{}^2}{2k_1 \cdot r} r^\mu \qquad\qquad n^\mu = \bar{k}_1{}^\mu - \frac{k_1{}^2}{2k_1 \cdot r} r^\mu$$

$$\ell_7{}^\mu = \bar{u}_-(\bar{k}_1)\gamma^\mu u_-(r) \qquad\qquad \ell_8{}^\mu = \bar{u}_-(r)\gamma^\mu u_-(\bar{k}_1)$$

$$q^\mu = -p_0^\mu + \frac{[(q+p_0)\cdot k_1]}{k_1^2} k_1^\mu - \frac{[(q+p_0)\cdot n]}{k_1^2} n^\mu + \frac{[(q+p_0)\cdot \ell_8]}{(\ell_7 \cdot \ell_8)} \ell_7^\mu + \frac{[(q+p_0)\cdot \ell_7]}{(\ell_7 \cdot \ell_8)} \ell_8^\mu$$

$$\int d^d q \frac{q^\mu q^\nu}{\overline{D}_0 \overline{D}_1}$$

$$\frac{(-\mathrm{m0}^2 + \mathrm{m1}^2 + x^2)\, g^{m\,n}\, x^2 + 4\,(\mathrm{m0}^2 - \mathrm{m1}^2 + 2\,x^2)\, p_1^m\, p_1^n}{12\,x^4}$$

$$\frac{(\mathrm{m0}^2 - \mathrm{m1}^2 + x^2)\,(x^2\, g^{m\,n} - 4\, p_1^m\, p_1^n)}{12\,x^4}$$

$$\frac{4\left(\mathrm{m0}^4 + (x^2 - 2\,\mathrm{m1}^2)\,\mathrm{m0}^2 + (\mathrm{m1}^2 - x^2)^2\right) p_1^m\, p_1^n - x^2 \left(\mathrm{m0}^4 - 2\,(\mathrm{m1}^2 + x^2)\,\mathrm{m0}^2 + (\mathrm{m1}^2 - x^2)^2\right) g^{m\,n}}{12\,x^4}$$

$$\frac{D_0}{\overline{D}_0} \to \frac{\overline{D}_0 - \tilde{q}^2}{\overline{D}_0} \to \frac{\tilde{q}^2}{\prod \overline{D}_i}$$

$$\int d^n\bar{q} \frac{\tilde{q}^4}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\epsilon)\,,$$

$$\int d^n\bar{q} \frac{\tilde{q}^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon)\,,$$

$$\int d^n\bar{q} \frac{\tilde{q}^2}{\bar{D}_i \bar{D}_j} = -\frac{i\pi^2}{2} \left[m_i^2 + m_j^2 - \frac{(p_i - p_j)^2}{3} \right] + \mathcal{O}(\epsilon)$$

- Nice procedure to numerically attack the problem of computing arbitrary scattering amplitude at the one-loop level

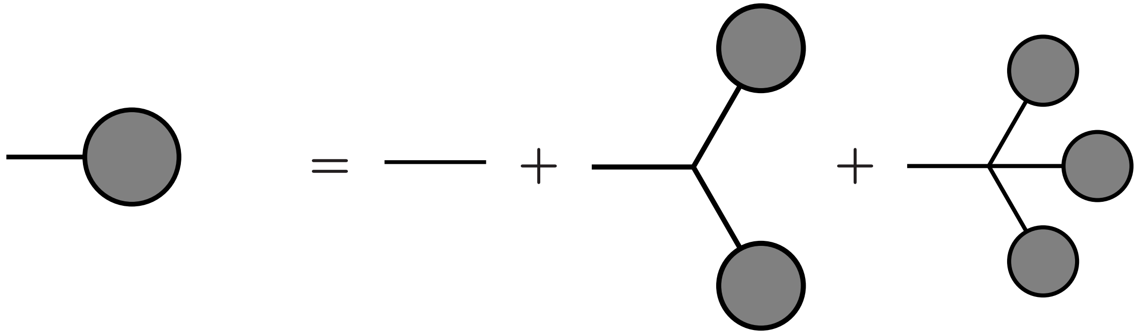
- Tests already completed with 4-photon amplitude at one loop

- Next-to-trivial the 6-photon amplitude

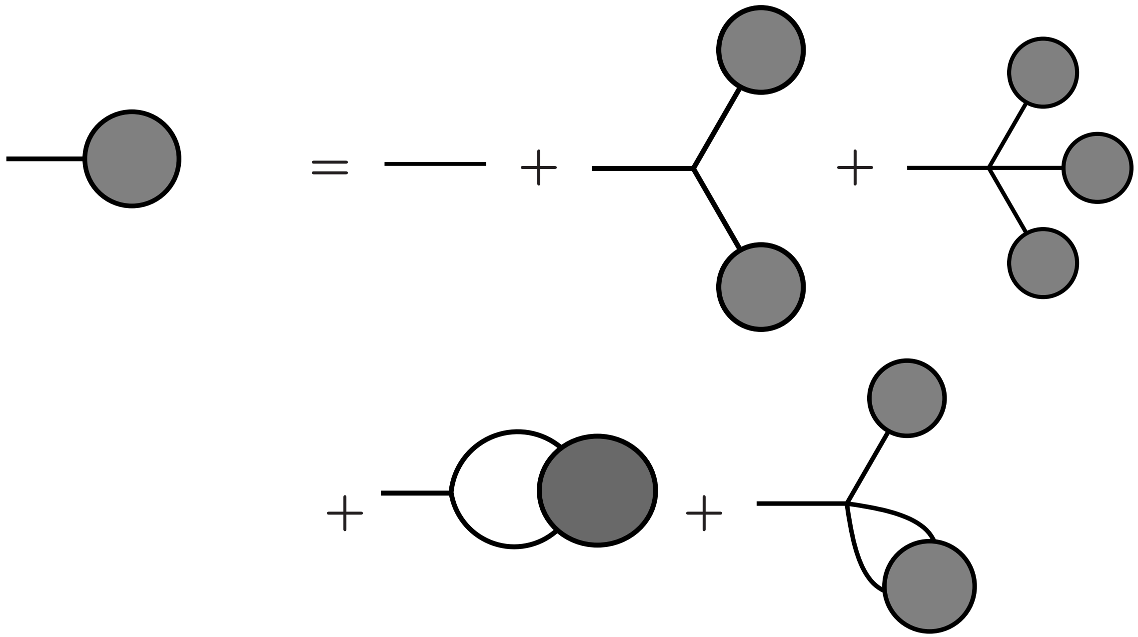
Higher-order corrections - NEW

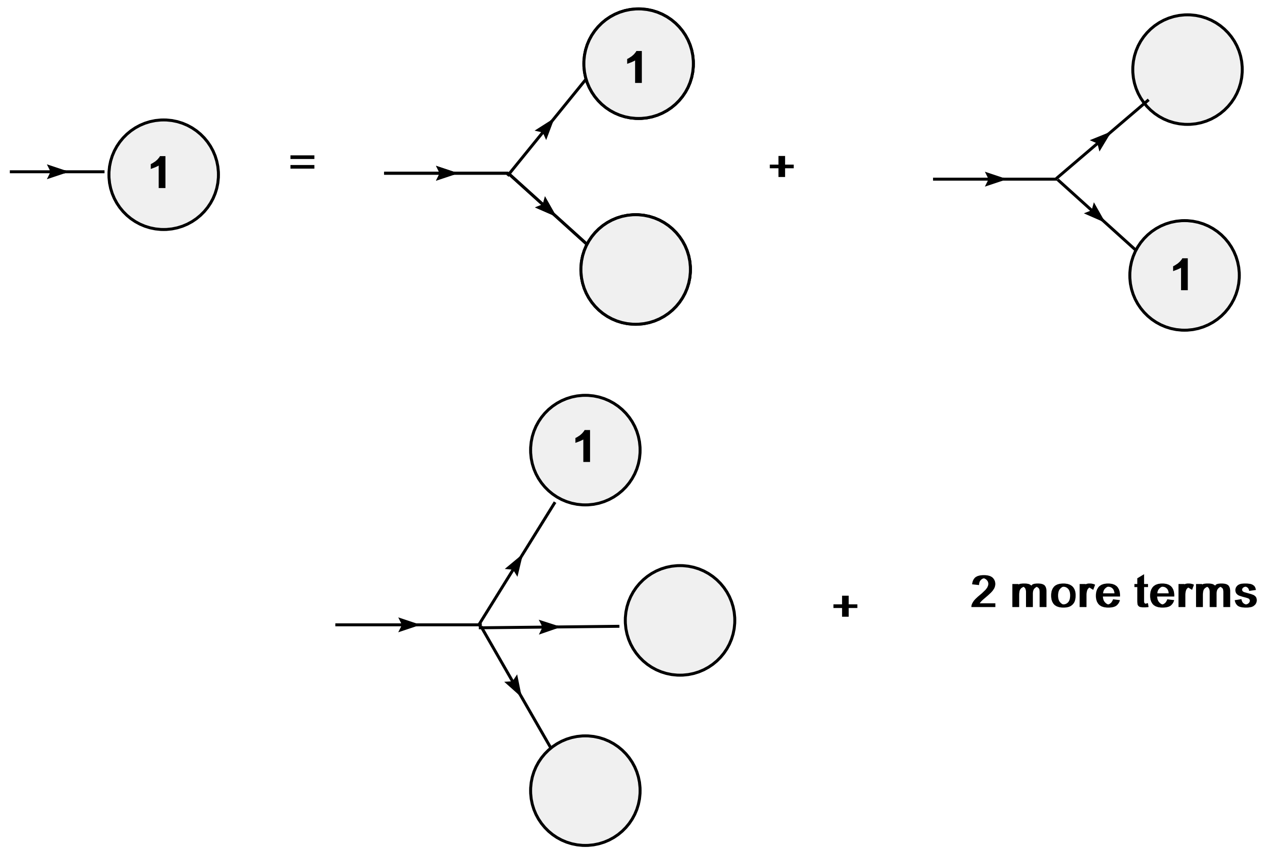
Lets go back to the DS equation

- **Tree order:**



- **One loop:**




1
=
1
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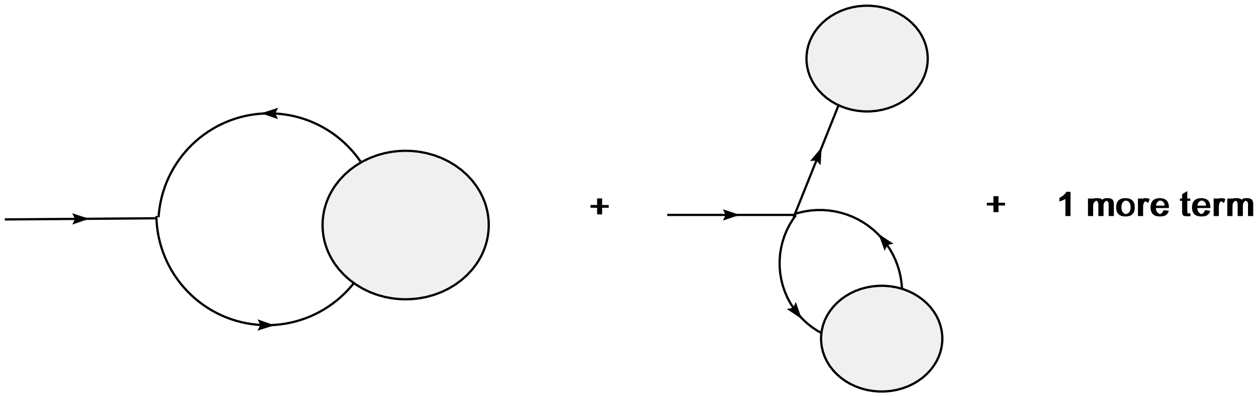
2 more terms


+
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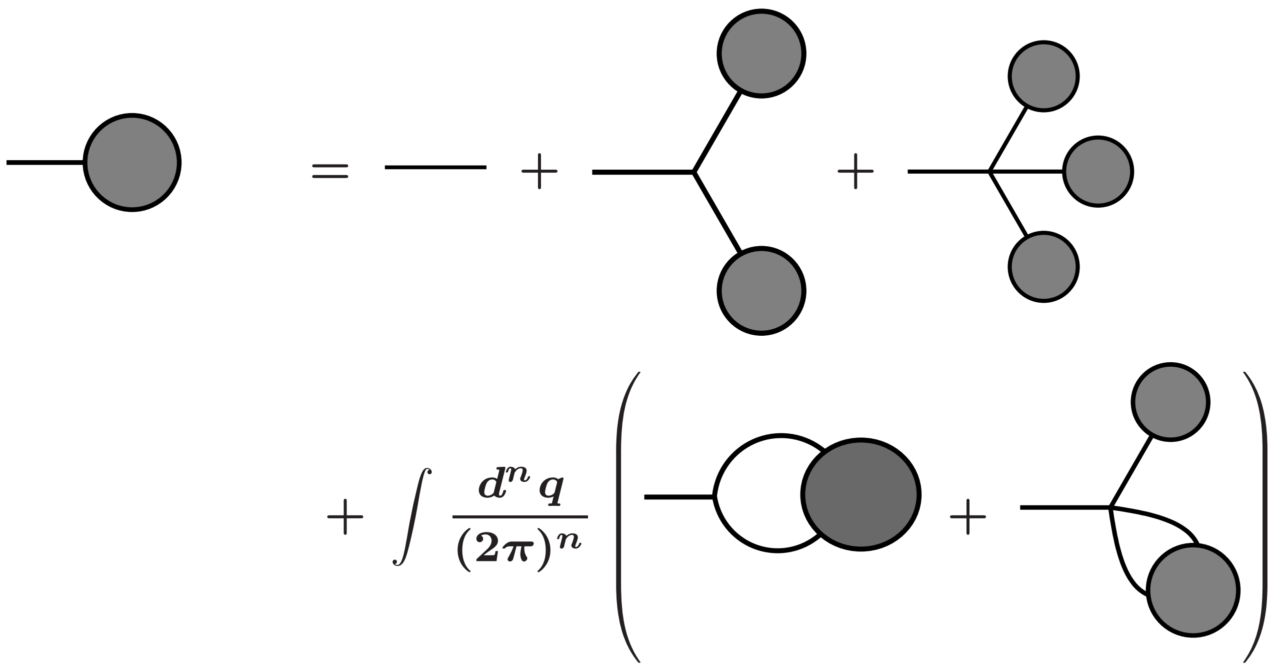
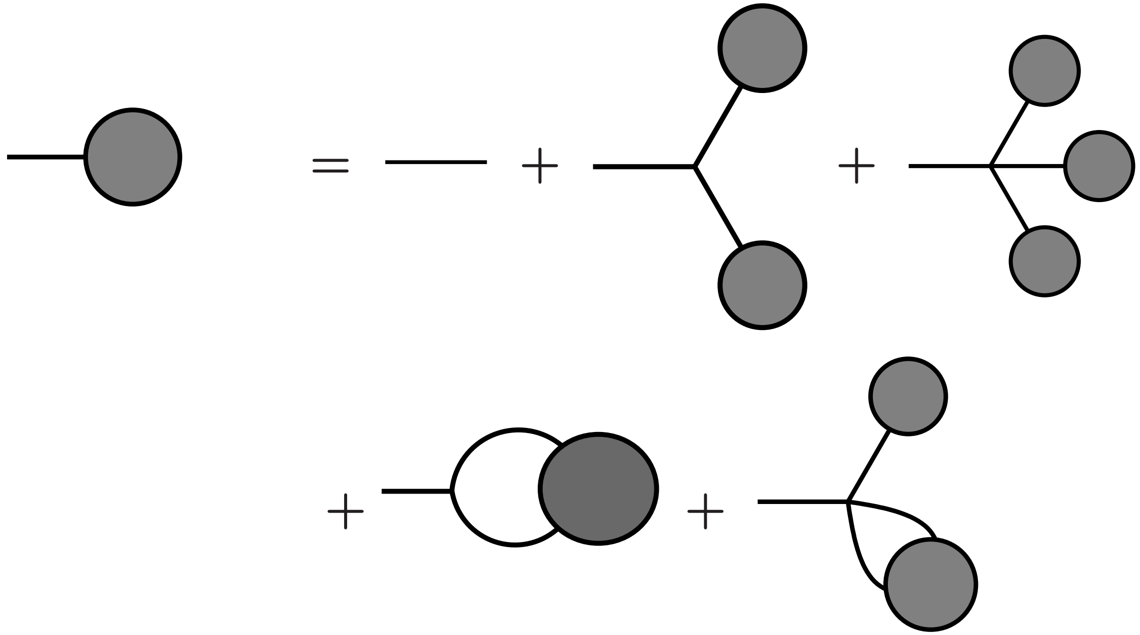
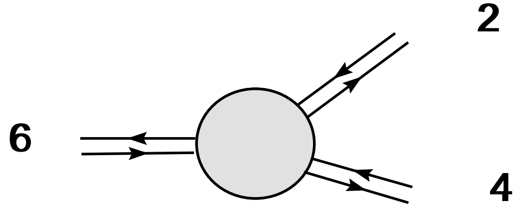
1 more term

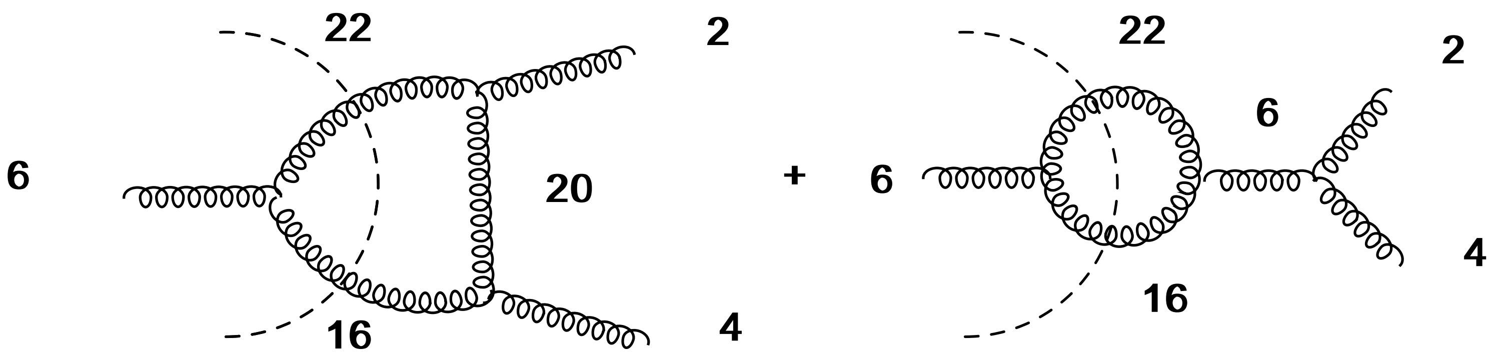

= + +
+ $\int \frac{d^n q}{(2\pi)^n}$ (+)

$$\left(\int \frac{d^n q}{(2\pi)^n} \right)$$



g(1)+g(2) $\longrightarrow$ g(4)+g(8)   q=16



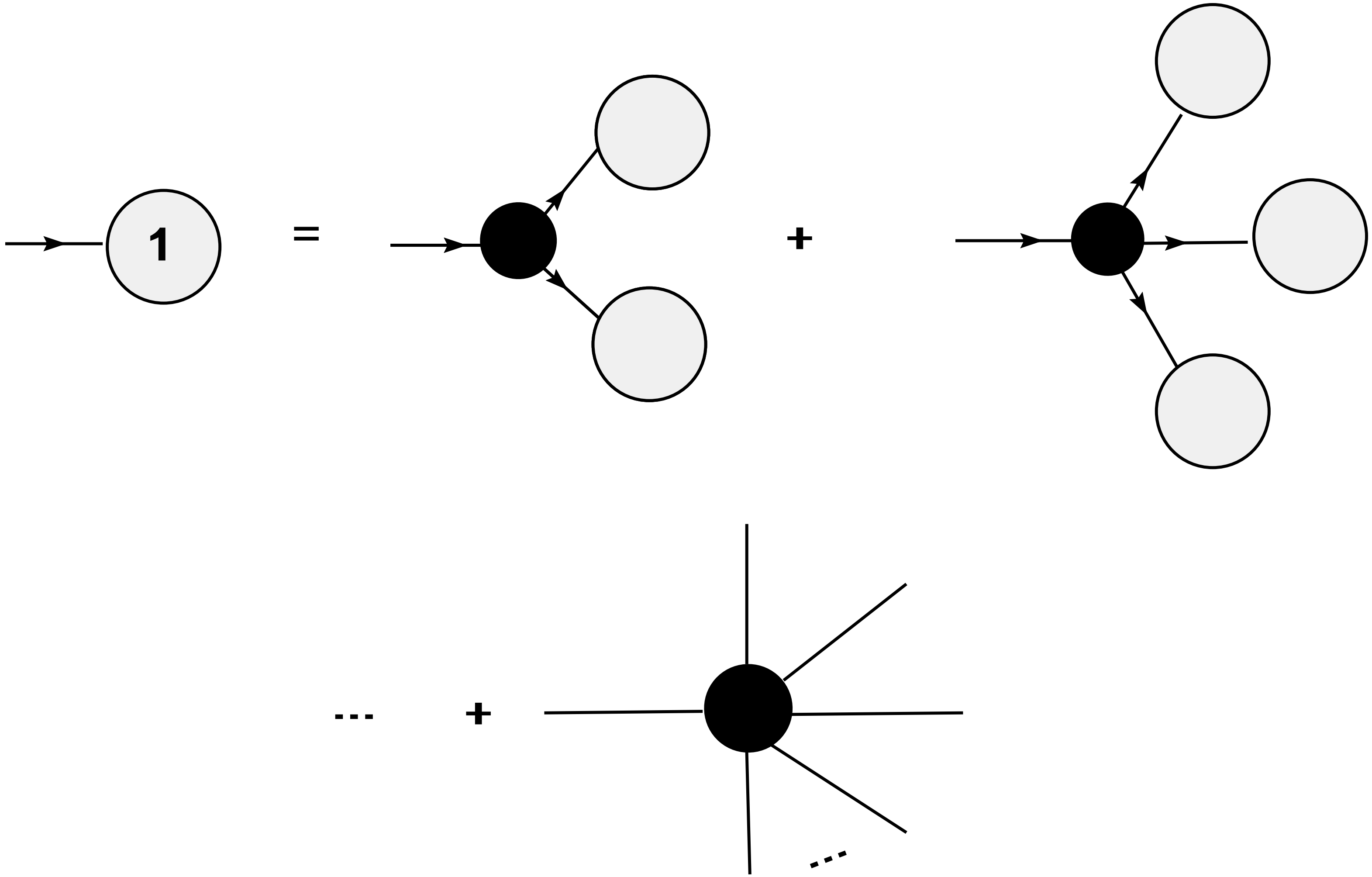


$N(q) = N_1(q) + N_2(q) \times D(20)$

Generalized vertices approach

- **Tree structure with effective quantum action vertices**
- **Re-normalized vertices**

One-particle irreducible (1PI) graphs !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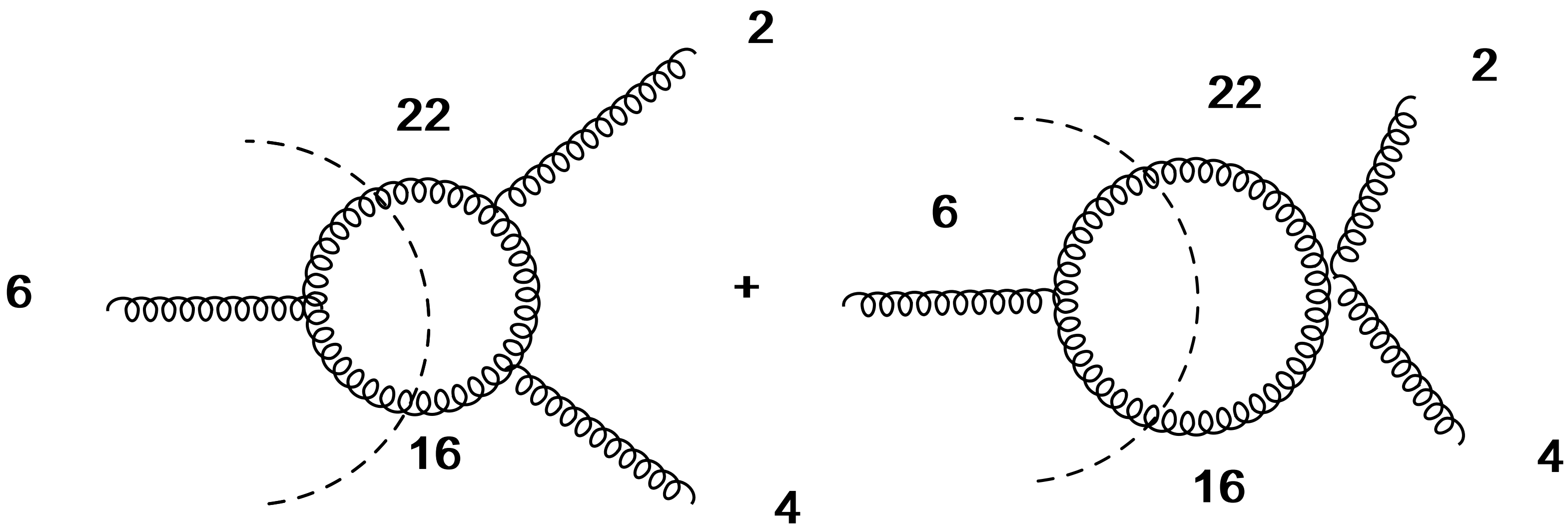

6
22
16
2
4
+
6
22
16
2
4

Outlook

- **PHEGAS / HELAC: a framework for high-energy phenomenology**
- **Standard Model fully included**

⋆ **High color charge processes: multijet production**

P.Draggiotis, R.H.Kleiss and C.G.Papadopoulos, Phys. Lett. B439 (1998) 157;
Eur. Phys. J. C 24 (2002) 447 hep-ph/0202201
C. G. Papadopoulos and M. Worek, arXiv:hep-ph/0512150

⋆ **Higher order corrections**

  - **Direct approach. Ongoing work to better understand Dyson-Schwinger equations and loop calculations: stepping equations, recursive actions, etc.**
  - **Running couplings and masses: 4-point FL contributions and BBC non-local approch to go beyond 4-fermion final states.**

- **SUSY and new particles**

wait and see