$\text{QED}\otimes\text{QCD}$ Resummation and Shower/ME Matching for LHC Physics

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Papers by S. Jadach and B.F.L. Ward, S. Jadach *et al.*, Mod. Phys. Lett **A14** (1999) 491 (hep-ph/0205062); *ibid* **A12** (1997) 2425; C. Glosser, B.F.L. Ward, and S.A. Yost, *ibid* **A19** (2004) 2113 (hep-ph/0404087); B.F.L. Ward and S.A. Yost, hep-ph/0410238, 0509003, 0508140.

Motivation

- FNAL/RHIC $t\bar{t}$ production; polarized pp processes; $b\bar{b}$ production J/Ψ production — soft $n(g)$ effects are already needed ...
  - Δm_t = 5.1 GeV with soft $n(g)$ uncertainty $\sim$ 2 – 3 GeV, ...
- For the LHC/ILC, QCD soft $n(g)$ the precision requirements on soft $n(g)$ effects will be even more demanding.
  - Desire 1% precision for $W^{\pm}, Z$ production at LHC.
  - How important are higher-order QED corrections, when QCD is controlled at the 1% level?

Proposal

We believe an important part of the necessary theory to meet the precision requirements of the LHC will be:

- an exponentiated soft $n(g)$ MC event generator using a $\mathcal{O}(\alpha_s^2)L$ cross section, in the presence of showers — without double counting and with exact multi-gluon phase space.
- The exponentiation will be motivated by the YFS (Yennie-Frautschi-Suura) calculus used in LEP physics suitably adapted to a nonabelian context. QCD and QED effects will be exponentiated simultaneously.

Cross-checks of QCD Literature

- Phase space: Compare Catani, Catani-Seymore – all initial partons massless. We will be including masses for all partons.
- Resummation: Compare Catani *et al.*, Berger *et al.*, ...
- Confront "no-go" theorem on initial state parton masses.
  - There is an uncanceled infrared divergence at $2^{\rm nd}$ order when parton masses are included in the initial state – this is one reason calculations at this level normally set these masses to zero, but it must have a physical resolution since we know quarks have masses.

Cross-checks of QED Literature

- Estimates by Spiesberger, Stirling, Roth and Weinzieri: few per mille effects in structure function evolution from QED corrections
- Well-known possible enhancement of QED effects at threshold, especially in resonance production
  - How big are these effects at the LHC?
- Treating QED and QCD simultaneously in YFS-style exponentiation will allow us to estimate the role of QED and illustrate an approach to shower/ME matching.

Vector Boson Production

We will consider processes of the form

$$pp \to V + n(\gamma) + m(g) + X \to \bar{\ell}\ell' + n'(\gamma) + m(g) + X$$

with vector bosons and leptons chosen from among the following choices:

V	ℓ	ℓ'
W^-	ν_e, ν_μ	e, μ
W^+	ν_e, ν_μ	ν_e, ν_μ
Z	e, μ	ν_e, ν_μ

Review of YFS Theory

For $e^+(p_1)e^-(q_1) \to \overline{f}(p_2)f(q_2) + n(\gamma)(k_1,\ldots,k_n)$, YFS theory gives [PRD36 (1987) 939]

$$
\begin{aligned}
d\sigma_{\rm exp} &= e^{2\alpha({\rm Re}B+\widetilde{B})}\sum_{n=0}^{\infty}\frac{1}{n!}\int\frac{d^3p_2\,d^3q_2}{p_2^0q_2^0}\prod_{j=1}^{n}\frac{d^3k_j}{k_j^0}\\
&\times\int\frac{d^4y}{(2\pi)^4}e^{iy\cdot(p_1+q_1-p_2-q_2-\sum_j k_j)+D}\,\overline{\beta}_n(k_1,\ldots,k_n)
\end{aligned}
$$

where B and $\widetilde{B}$ are the virtual and real YFS infrared functions, and the YFS hard photon residuals $\overline{\beta}_n$ are IR finite.

Review of YFS Theory

In terms of the standard YFS soft photon emission factor

$$\widetilde{S}(k) = \frac{\alpha}{4\pi^2}\left[Q_f Q_{\overline{f}'}\left(\frac{p_1}{p_1\cdot k} - \frac{q_1}{q_1\cdot k}\right)^2 + \dots\right]$$

(with Q_f, $Q_{\overline{f}'}$ the charges of fermions f, $\overline{f}'$ in units of the positron charge),

$$2\alpha\widetilde{B} = \int_{k\le K_{\rm max}} \frac{d^3k}{k^0}\widetilde{S}(k), D = \int d^3k\frac{\widetilde{S}(k)}{k^0}\left(e^{-iy\cdot k} - \theta(K_{\rm max} - k)\right).$$

With these definitions, $d\sigma_{\rm exp}$ is independent of $K_{\rm max}$.

YFS Theory for Bhabha Luminosity

- This construction gives an exact representation of the cross section. In practice the YFS hard photon residuals $\overline{\beta}_i$ are calculated to reach the desired precision.
- For Bhabha scattering ($f, \overline{f} = e^+, e^-$), the hard photon residuals $\overline{\beta}_i$ for $i = 0, 1, 2$ are given in Jadach *et al* [CPC102 (1997) 229], where they are calculated exactly at $\mathcal{O}(\alpha)$ and to leading log (LL) at $\mathcal{O}(\alpha^2)$.
- The resulting exponentiated cross section is implemented in the Monte Carlo program BHLUMI 4.04, which has been used for precision calculations of the Bhabha luminosity process at LEP.

MC Implementation

- The YFS exponentiation just described has been implemented in a host of MC programs, including YFS2, YFS3, BHLUMI, BHWIDE, KORALZ, KKMC, YFSWW3, YFSZZ, Koral W.
- Two implementations have been developed, EEX and CEEX. EEX is based on the original YFS formulation, while CEEX requires a coherent choice of spinor representation which permits the YFS subtraction to be implemented at the amplitude level.
- The "GPS" spinor conventions of Jadach/Ward/Wąs are used for the photon polarization vectors.

Extension of YFS Theory to QCD

The YFS construction has been extended to QCD [hep-ph/0210357 (ICHEP02), Acta Phys. Polon. **B33**, 1543 (2002)].

$$
\begin{aligned}
d\hat{\sigma}_{\rm exp} &= \sum_{n=0}^{\infty} d\hat{\sigma}^n \\
&= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \frac{1}{n!} \int \frac{d^3P_2 d^3Q_2}{P_2^0 Q_2^0} \prod_{j=1}^{n} \frac{d^3k_j}{k_j^0} \\
&\times \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(P_1+Q_1-P_2-Q_2-\sum_j k_j)+D_{\rm QCD}} \tilde{\bar{\beta}}_n(k_1,\ldots,k_n)
\end{aligned}
$$

with appropriately modified infrared functions.

Extension of YFS Theory to QCD

- For example, the soft gluon factor becomes

$$\widetilde{S}_{\rm QCD}(k) = \frac{\alpha_s(Q)}{4\pi^2}\Bigg\{C_F\left(\frac{P_1}{P_1\cdot k}-\frac{Q_1}{Q_1\cdot k}\right)^2 -\Delta C_S\frac{2P_1\cdot Q_1}{(P_1\cdot k)(Q_1\cdot k)}+\ldots\Bigg\}$$

with $C_F = 4/3,\qquad \Delta C_S = \begin{cases} -1 \\ -1/6 \end{cases}$ for $\begin{matrix} qq' \\ q\bar{q}' \end{matrix}$ incoming.

- The IR real and virtual functions in are similarly generalized. . .

Real and Virtual IR Functions

Real IR function: $$2\alpha_s \widetilde{B}_{\rm QCD} = \int_{k \le K_{\rm max}} \frac{d^3k}{k^0} \widetilde{S}_{\rm QCD}(k),$$

Virtual IR function: $$B_{\rm QCD} = \frac{i}{8\pi^2} \int \frac{d^4k}{k^2 - m_g^2 + i\epsilon}$$

$$\times \Big[C_F \left(\frac{2P_1 + k}{k^2 + 2P_1 \cdot k + i\epsilon} + \frac{2Q_1 - k}{k^2 - 2Q_1 \cdot k + i\epsilon} \right)$$

$$+ \Delta C_S \frac{2(2P_1 + k) \cdot (2Q_1 - k)}{(k^2 + 2P_1 \cdot k + i\epsilon)(k^2 - 2Q_1 \cdot k - i\epsilon)} + \ldots \Big]$$

Exponent: $$\mathrm{SUM_{IR}(QCD)} = 2\alpha_{\rm s}\, \mathrm{Re}\, \mathrm{B_{QCD}} + 2\alpha_{\rm s}\, \widetilde{\mathrm{B}}_{\rm QCD}.$$

Hard Gluon Residuals

- The hard gluon residuals $\bar{\tilde{\beta}}_n$ include contributions to all orders in $\alpha_s(Q)$, and may be expanded as

$$\bar{\tilde{\beta}}_n(k_1,\ldots,k_n) = \sum_{\ell=0}^{\infty} \bar{\tilde{\beta}}_n^{(\ell)}(k_1,\ldots,k_n).$$

The extra $\sim$ indicates a non-abelian modification, ...

- The finiteness of these residuals is nontrivial in QCD, due to left-over genuine non-abelian virtual and real IR singularities, not considered in some early papers on non-abelian exponentiation [DeLaney *et al.*, PRD52 (1995) 108, PLB342 (1995) 239].

Hard Gluon Residuals

- If $\overline{\beta}_n$ is defined by direct analogy with the abelian case, they will contain IR singularities due to the emission of gluons from gluon lines which are not accounted for in the abelean YFS construction.
- These non-abelian virtual and real IR singularities cancel between $\int d\mathrm{PS}\overline{\beta}_n$ and $\int d\mathrm{PS}\overline{\beta}_{n+1}$. This cancelation permits the $\tilde{\overline{\beta}}_n$ to be defined by minimal subtraction of the left-over divergences so they are IR-finite to all orders.

Another Nonabelian Exponentiation

- An expresson for the general exponentiation of the eikonal cross section for nonabelian gauge theory was obtained by Gatheral [PLB133 (1983) 90]. However, this expression is approximate, since everything that does not eikonalize and exponentiate is dropped.
- Gatheral expresses the eikonal cross section as $\exp(\sum_{r=0}^{\infty} \widetilde{F}_r)$, and expands the r-real-gluon function $\widetilde{F}_r$ in functions $\widetilde{F}_r(N)$ of order α_s^N. Only the $N = 1$ term exists in the abelian case. Our construction corresponds to this $N = 1$ term, but is not an approximation, since nothing is dropped from the cross section.

DGLAP Synthesis

- The parton-level cross section $d\hat{\sigma}_{\rm exp}$ must be combined with **parton distribution functions** to obtain the complete cross section
$$d\sigma_{\rm exp} = \sum_{ij} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{\rm exp}(x_i x_j s).$$

- But using the equations just given would lead to **double counting**, since the DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) equation for the structure function evolution already includes all leading logs from gluon ISR and related collinear emission effects.

DGLAP Synthesis

The solution is to remove the part of the soft gluon emission function corresponding to the YFS limit of the DGLAP kernel, defining "nls" functions (non-leading in the s channel)

$$\widetilde{S}^{nls}_{\rm QCD} = \widetilde{S}_{\rm QCD} - \widetilde{S}_{\rm DGLAP}$$

with an appropriate definition

$$\widetilde{S}_{\rm DGLAP}(k) = \frac{C_F\alpha_s}{4\pi^2}\frac{4(1-|z|v)^2}{k_\perp^2+m_q^2v^2z^2}\left(1-\frac{m_q^2v^2z^2}{k_\perp^2+m_q^2v^2z^2}\right)$$

with $z = k_z/k$, $v = 2k^0/\sqrt{s}$. The nls YFS IR function $\widetilde{B}^{nls}_{\rm QCD}$ defined using $S^{nls}_{\rm QCD}$ and the $\widetilde{\overline{\beta}}_n$ redefined appropriately.

Extension to QED$\otimes$QCD and QCED

The simultaneous exponentiation of higher order QED and QCD effects has also been considered [Mod. Phys. Lett. **A19** (2004) 2113 (hep-ph/0404087)]. This leads to combined QED-QCD YFS IR functions

$$
\begin{aligned}
\widetilde{S}^{nls}_{\rm QCD} &\rightarrow \widetilde{S}^{nls}_{\rm QCD} + \widetilde{S}^{nls}_{\rm QED} &\equiv \widetilde{S}^{nls}_{\rm QCED} \\
\widetilde{B}^{nls}_{\rm QCD} &\rightarrow \widetilde{B}^{nls}_{\rm QCD} + \widetilde{B}^{nls}_{\rm QED} &\equiv \widetilde{B}^{nls}_{\rm QCED} \\
B^{nls}_{\rm QCD} &\rightarrow B^{nls}_{\rm QCD} + B^{nls}_{\rm QED} &\equiv B^{nls}_{\rm QCED}
\end{aligned}
$$

We use the term **QCED** to denote the simultaneous exponentiation of QCD and QED effects.

Extension to QED⊗QCD and QCED

We then obtain a QCED-exponentiated cross section

$$
\begin{aligned}
d\widehat{\sigma}_{\rm exp} &= e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \frac{d^3P_2 d^3Q_2}{P_2^0 Q_2^0} \prod_{j=1}^{n} \frac{d^3k_j}{k_j^0} \prod_{i=1}^{m} \frac{d^3{k'}_i}{{k'}_i^0} \\
&\times \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(P_1+Q_1-P_2-Q_2-\sum_j k_j-\sum_i {k'}_i)+D_{\rm QCD}} \\
&\times \tilde{\bar{\beta}}_n(k_1,\ldots,k_n;{k'}_1,\ldots {k'}_m)
\end{aligned}
$$

where the new residuals $\tilde{\bar{\beta}}_n$ represent successive applications of the YFS expansion first for QCD and then for QED.

Extension to QED$\otimes$QCD and QCED

The IR functions in $d\hat{\sigma}_{\rm exp}$ are now

$$\mathrm{SUM_{IR}(QCED)} = 2\alpha_s \,\mathrm{Re}\, B^{nls}_{\rm QCED} + 2\alpha_s \tilde{B}^{nls}_{\rm QCED},$$

$$D_{\rm QCED} = \int \frac{dk}{k^0}\left(e^{-ik\cdot y} - \theta(K_{\max} - k^0)\right) \tilde{S}^{nls}_{\rm QCED}$$

where the same cutoff $K_{\max}$ is used for both QCD and QED.

The cross section is independent of $K_{\max}$.

Infrared Algebra of QCED

- The average x for QED and QCD may be expressed as

$$x_{\rm avg}({\rm QED}) \approx \frac{\gamma({\rm QED})}{1+\gamma({\rm QED})}, \quad x_{\rm avg}({\rm QCD}) \approx \frac{\gamma({\rm QCD})}{1+\gamma({\rm QCD})},$$

where

$$\gamma({\rm QED}) = \frac{2\alpha Q_f^2}{\pi}(L_s - 1), \qquad \gamma({\rm QCD}) = \frac{2\alpha_s C_F}{\pi}(L_s - 1).$$

- Thus, QCD-dominant corrections happen an order of magnitude earlier than QED-dominant corrections.

QED⊗QCD Threshold Corrections

- We shall apply the simultaneous QED⊗QCD exponentiation to single Z production with leptonic decay at the LHC (and FNAL), focusing on ISR alone.
- Exact $\mathcal{O}(\alpha_s)$ results are found in work of Baur *et al.*, Dittmaier and Kramer, and Zykunov, while exact $\mathcal{O}(\alpha_s^2)$ results have been obtained by Hamberg *et al.*, van Neerven and Matsuura, and Anastasiou *et al.*. Higher-order results recently reported by Moch, Vogt, *et al.*
- We shall illustrate the calculation by computing the size of threshold corrections both with and without QED corrections.

QED⊗QCD Threshold Corrections

- We calculate

$$d\sigma_{\rm exp}(pp \to V + X \to \bar{\ell}\ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{\rm exp}(x_i x_j s)$$

 using the QCED expression for $d\hat{\sigma}_{\rm exp}$ and struction functions from Martin *et al.*

- To estimate the size of the corrections, we will use semi-analytical methods and the leading-order $\bar{\beta}^{(0,0)}_{0,0}$.
- A Monte Carlo realization is under construction.

QED⊗QCD Threshold Corrections

The semi-analytical result for the integral at leading order is

$$\widehat{\sigma}_{\rm exp}(x_1x_2s) = \int_0^{v_{\rm max}} dv\ \gamma v^{\gamma-1} F_{\rm YFS}(\gamma) e^{\delta_{\rm YFS}} \widehat{\sigma}_{\rm Born}\left((1-v)x_1x_2s\right)$$

where

$$\gamma = \left\{2Q_f^2\frac{\alpha}{\pi} + 2C_F\frac{\alpha_s}{\pi}\right\} L_{nls} \text{ with } L_{nls} = \ln(x_1x_2s/\mu^2),$$

$$F_{\rm YFS}(\gamma) = \frac{e^{-c_E\gamma}}{\Gamma(1+\gamma)} \text{ with } c_E = 0.5772\ldots,$$

$$\delta_{\rm YFS}(\gamma) = \frac{\gamma}{4} + \left(Q_f^2\frac{\alpha}{\pi} + C_F\frac{\alpha_s}{\pi}\right)\left(\frac{\pi^2}{3} - \frac{1}{2}\right).$$

QED⊗QCD Threshold Corrections

Computing the ratio $r_{\rm exp} = \sigma_{\rm exp}/\sigma_{\rm Born}$ gives

$$r_{\rm exp} = \begin{cases} 1.1901 & \text{QCED} \equiv \text{QCD} + \text{QCD} & \text{LHC} \\ 1.1872 & \text{QCD} & \text{LHC} \\ 1.1911 & \text{QCED} \equiv \text{QCD} + \text{QCD} & \text{FNAL} \\ 1.1879 & \text{QCD} & \text{FNAL} \end{cases}$$

- QED is at 0.3% at both the LHC and FNAL.
- This result is stable under scale variations.
- We agree with Baur *et al.*, Hamberg *et al.*, van Neerven and Zijlstra.
- QED effect in structure functions have similar size.

Shower/ME Matching

- We do not attempt to replace **Herwig** and/or **PYTHIA**. Our exact YFS-style parton-level cross section $d\widehat{\sigma}_{\rm exp}(x_i x_j s)$ can be combined with them with appropriate matching to avoid double-counting effects.
- We could use an alternative shower algorithm, for example that of Jadach and Skrzypek [Acta Phys. Pol. **B35** (2004) 745] and talks here by S. Jadach, M. Skrzypek, W. Płacek.
- **ISAJET** cannot be considered, due to lack of color coherence.
- We are studying two approaches to matching: p_T**-matching** and **shower-subtracted residuals**.

Shower/ME Matching

- p_T **matching** uses the fact that the factorization scale μ corresponds to the largest p_T of the gluon emission included in the structure function. The shower generator can generate a parton shower starting from parameters (x_1, x_2) at factorization scale μ after this point is provided by the $\{F_i\}$.

- **Shower-subtracted residuals** $\hat{\tilde{\bar{\beta}}}_n(k_1, \ldots, k_n; k'_1, \ldots, k'_m)$ can be constructed by expanding the product of the shower formula and the QED$\otimes$ QCD exponentiation formula, requiring the terms to match the exact result to the specified order. (hep-ph/0509003)

MC Event Generator Implementation

- The exponentiated QED$\otimes$QCD structures described here provide a firm basis for implementing parton-level cross sections in an event generator for vector boson production and decay at the 1% precision level for FNAL/LHC/RHIC/ILC physics.
- A calculation of the fully exclusive $\mathcal{O}(\alpha_s^2, \alpha_s\alpha, \alpha^2)$ parton-level cross section will be needed to reach the desired precision level.
- The combination of theoretical constructs described here can be systematically improved with exact results calculated order-by-order in α_s and α, with exact n-gluon, m-photon phase space.

Conclusions

YFS Theory (both EEX and CEEX) extends to nonabelian gauge theory and allows simultaneous exponentiation of QED and QCD with proper shower/ME matching built in.

For QED$\otimes$QCD:

- A full MC event generator realization is possible.
- Semi-analytical results for QCD (and QED) threshold effects agree with literature for Z production.
- Since QED comes in at 0.3%, it is needed for 1% predictions at the LHC.