The three-loop calculation of DIS and its LHC applications

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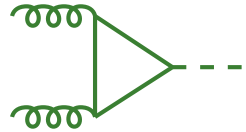
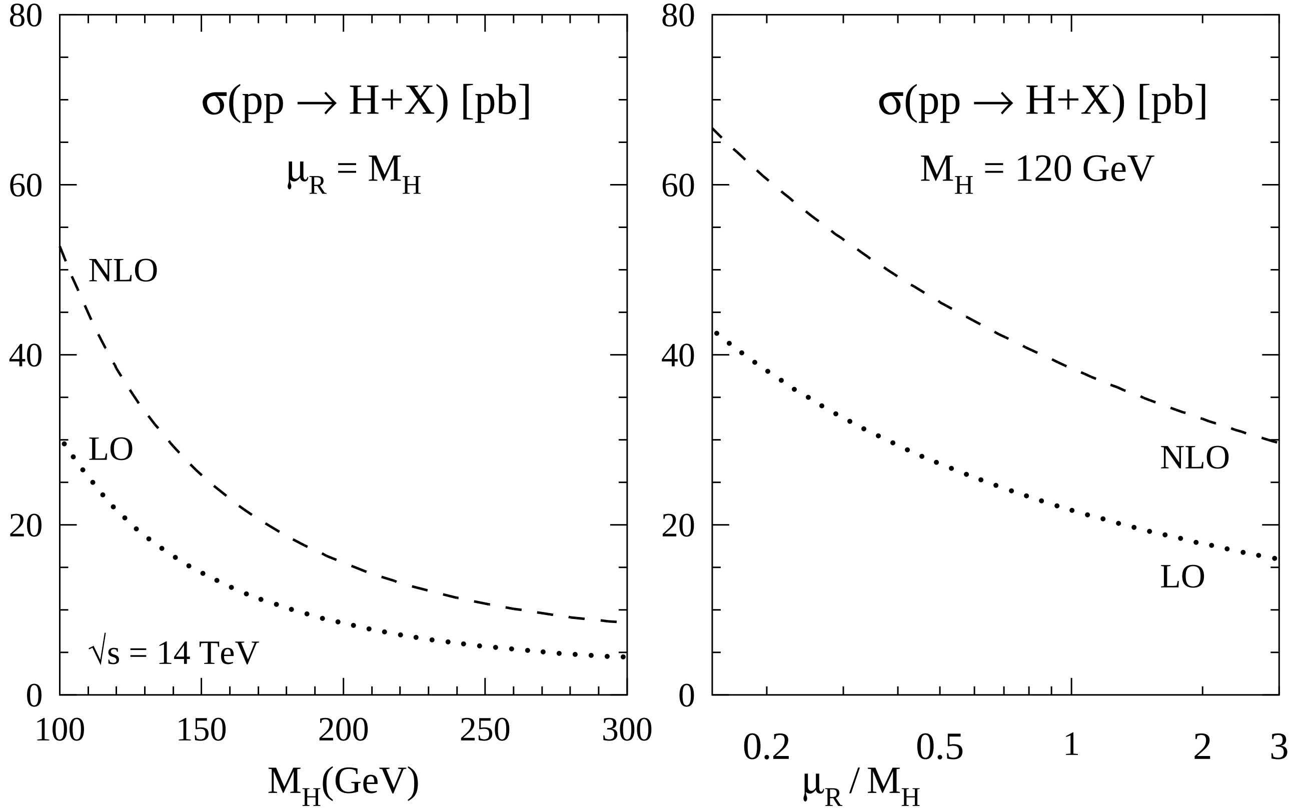
Collaborations with Sven Moch, Jos Vermaseren and Gavin Salam

Higgs boson production at the LHC (I)

Dominant channel: $gg \to H + X$ via top-quark loop



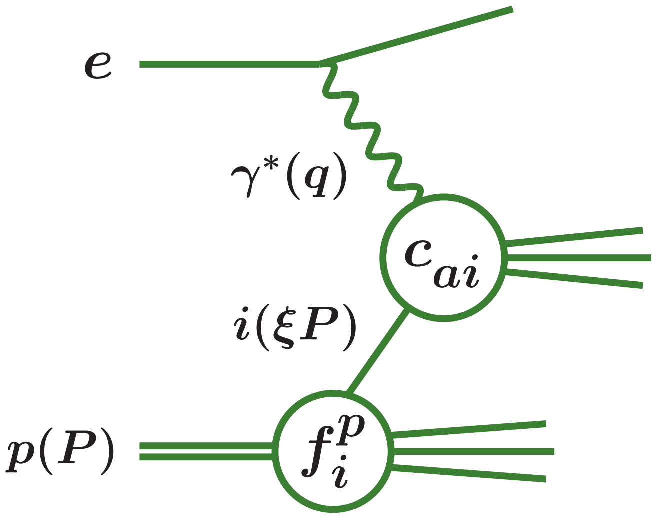


Error guess / estimate: apparent convergence, variation of scale μ

Next-to-leading order (NLO) insufficient for reliable predictions

Hard processes in perturbative QCD

Example: inclusive deep-inelastic scattering (DIS)



Kinematic variables

$$Q^2 = -q^2$$
$$x = Q^2/(2P\cdot q)$$

Lowest order : $x = \xi$

Hard processes in perturbative QCD

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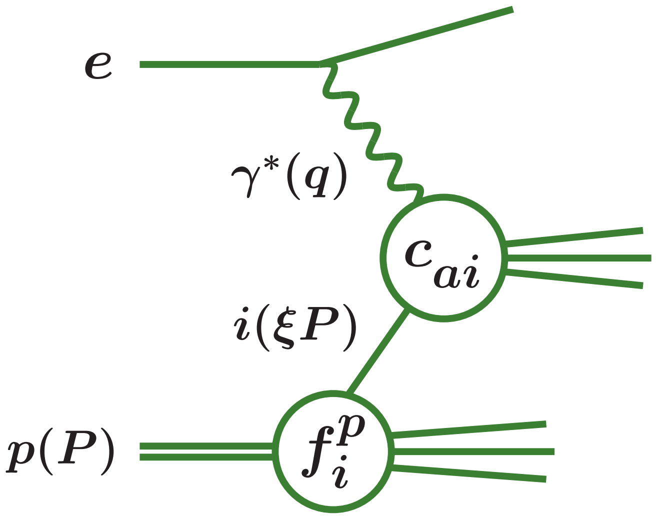

Kinematic variables

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Lowest order : $x = \xi$

Structure function F_2 **[up to** $\mathcal{O}(1/Q^2)$ **]**

$$x^{-1}F_2^p(x,Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi}\, c_{2,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) f_i^p(\xi,\mu^2)$$

Coefficient functions: renormalization/factorization scale $\mu = \mathcal{O}(Q)$

Hard processes in perturbative QCD

Parton distributions f_i: evolution equations

$$\frac{d}{d\ln\mu^2} f_i(\xi,\mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2)\right](\xi)$$

Initial conditions incalculable in pert. QCD. Lattice: low moments

$\Rightarrow$ predictions: fit-analyses of reference processes, universality

Hard processes in perturbative QCD

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$\Rightarrow$ **predictions: fit-analyses of reference processes, universality**

Splitting functions P, coefficient functions c_a

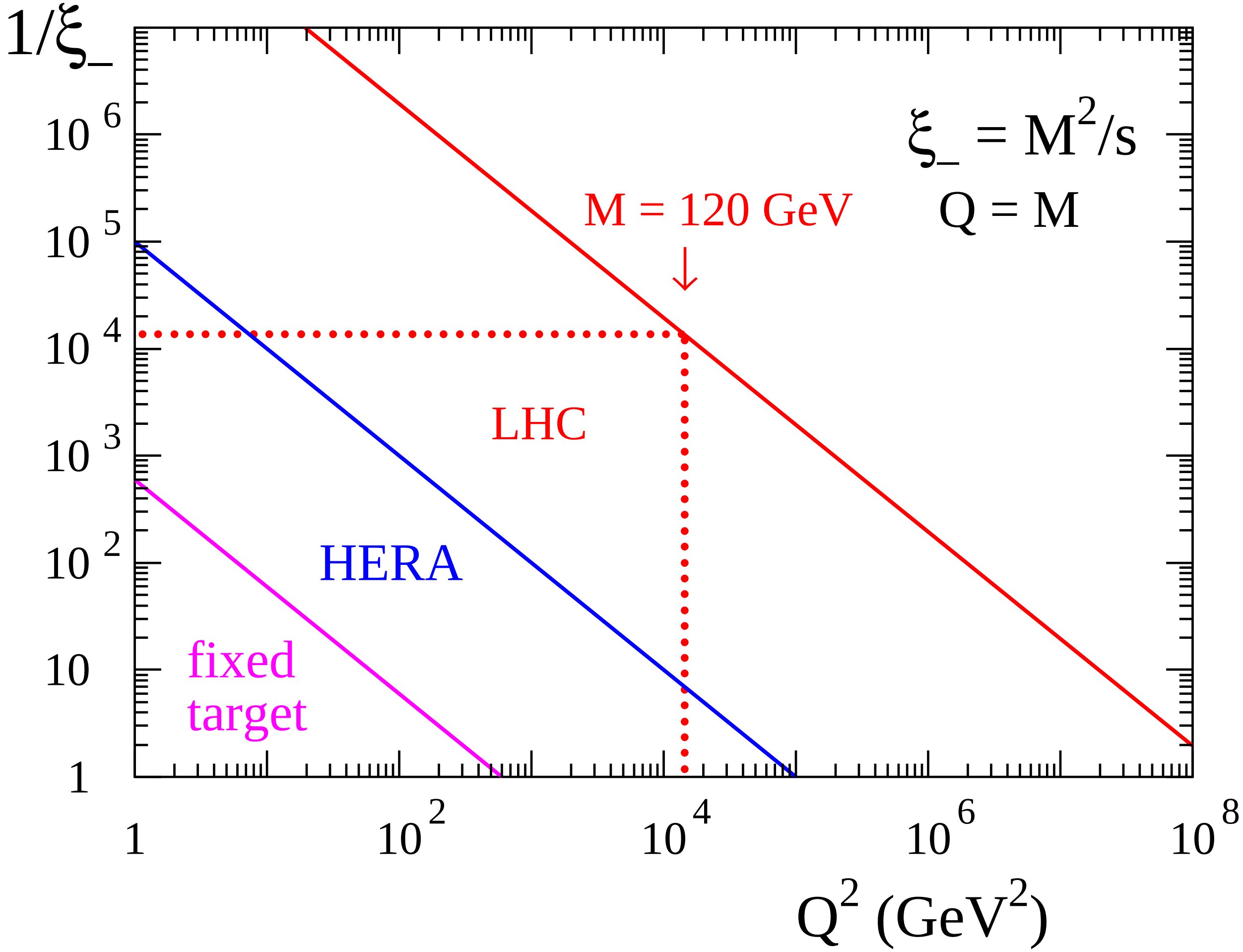
$$\begin{aligned} P &= \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots \\ c_a &= \underbrace{\alpha_s^{n_a}\Big[c_a^{(0)} + \alpha_s c_a^{(1)}}_{} + \alpha_s^2 c_a^{(2)} + \ldots \Big] \end{aligned}$$

NLO: standard, but no serious error estimate, ...

Next-to-next-to-leading order (NNLO): $P^{(2)}$, $c_a^{(2)}$

Parton evolution from HERA to LHC

Kinematics: partons with momentum fractions $\xi_- < \xi < 1$ contribute

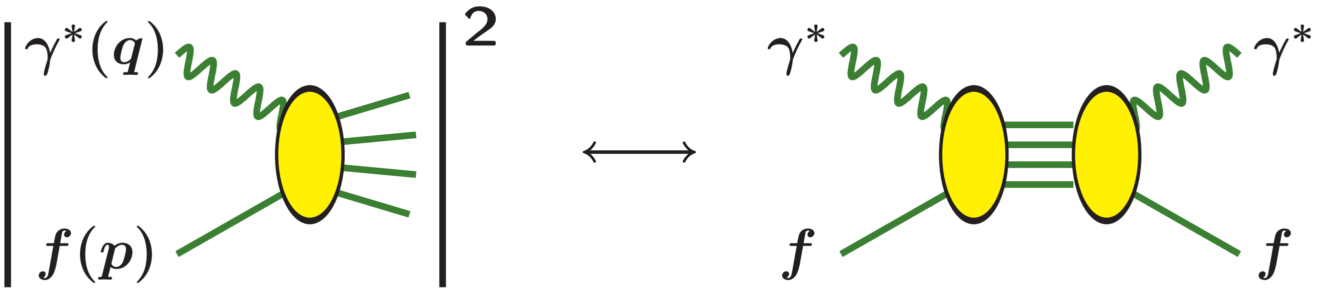


HERA $\longrightarrow$ LHC: Q^2 evolution across up to three orders of magnitude

Three-loop calculation of DIS

Moch, Vermaseren, A.V. (2001-05)

Optical theorem: $\gamma^* f$ **total cross section** $\leftrightarrow$ **forward amplitude**



Dispersion relation in x **: coefficient of** $(2p\cdot q)^N$ $\leftrightarrow$ N**-th moment**

$$A^N = \int_0^1 dx\, x^{N-1} A(x)$$

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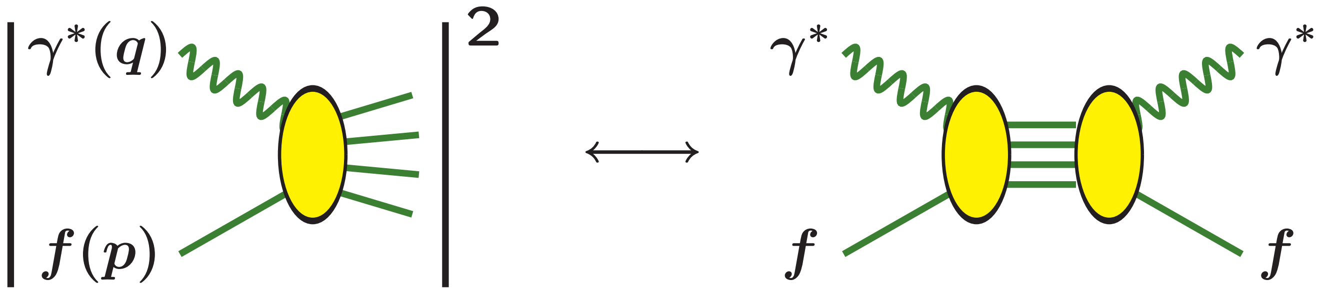


Dispersion relation in x **: coefficient of** $(2p\cdot q)^N$ $\leftrightarrow$ N**-th moment**

$$A^N = \int_0^1 dx\, x^{N-1} A(x)$$

UV and mass singularities : dimensional regularization, $D = 4 - 2\epsilon$

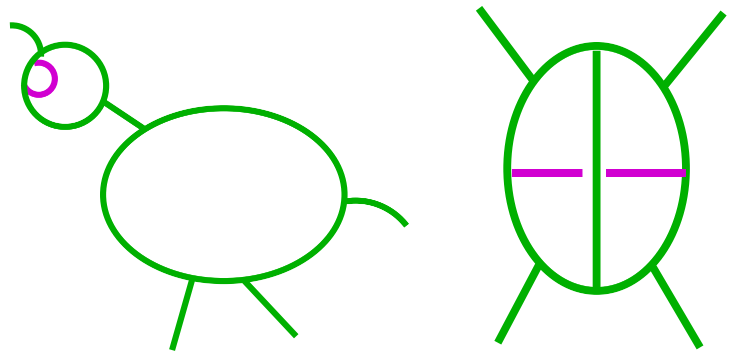
$1/\epsilon$ **poles : splitting functions,** ϵ^0 **part : coefficient functions**

Three-loop calculation of DIS

$P_{\rm gi}$: DIS with scalar ϕ coupling to $G^a_{\mu\nu}G^{\mu\nu}_a$ ($\leftrightarrow$ Higgs for large m_t)

	tree	1-loop	2-loop	3-loop
qγ	1	3	25	359
gγ		2	17	345
hγ			2	56
qW	1	3	32	589
qϕ		1	23	696
gϕ	1	8	218	6378
hϕ		1	33	1184
sum	3	18	350	9607

A benign and an evil 3-loop topology



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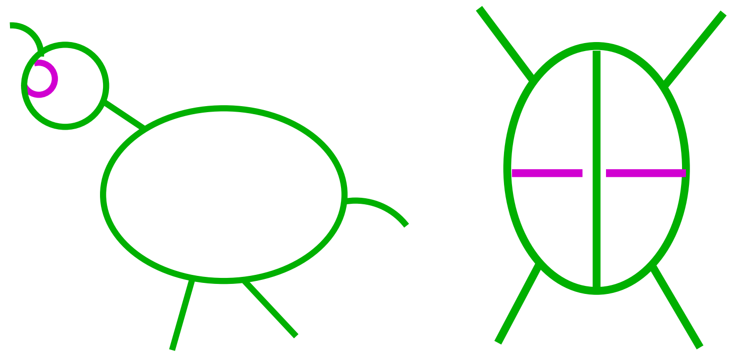


Highly optimised symbolic treatment: FORM Vermaseren

>10 person years, several CPU years, update to FORM 3.1

$\gtrsim 10^5$ tabulated symbolic integrals ($>$ 3 GB)

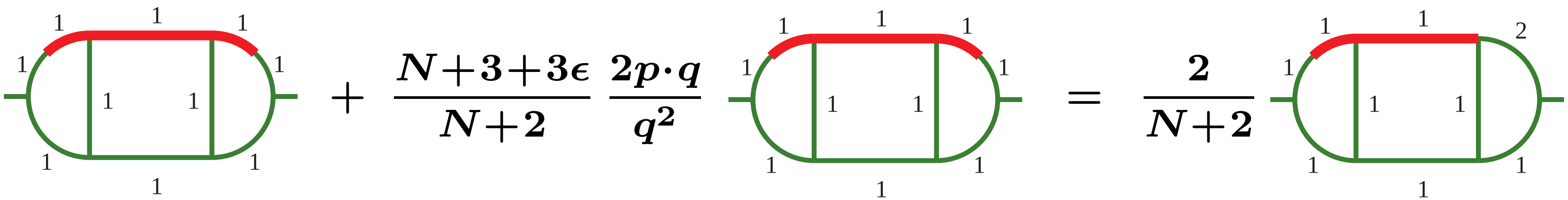
Treatment of the forward-Compton integrals

Combine identities: integration by parts, scaling, Passarino-Veltman

$\Rightarrow$ Difference equations for $I(N)$ [recall: coefficient of $(2p\cdot q)^N$]

$$a_0(N)I(N) - \ldots - a_n(N)I(N-n) = I_0(N)$$

Simple example [red line: flow of massless parton momentum p]



Successive reduction to simpler (less ‘red’) integrals

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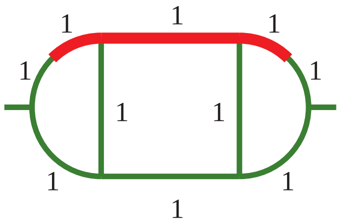
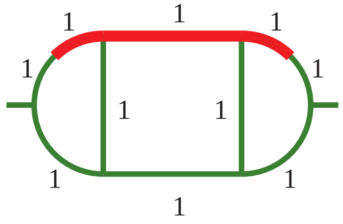
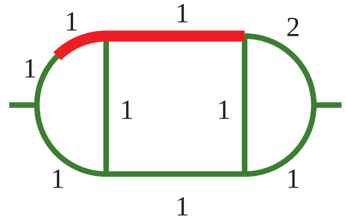


$$+ \; \frac{N+3+3\epsilon}{N+2}\,\frac{2p\cdot q}{q^2}$$



$$= \; \frac{2}{N+2}$$



Successive reduction to simpler (less ‘red’) integrals

Essential: non-symbolic case for low N done before via Mincer

Larin et al. (94,97); Retey, Vermaseren (00)

Check of new code and results at all stages: $I(N=2,3,4,\ldots) = ?$

The NNLO gluon-gluon splitting function

$$P_{gg}^{(2)}(x) =$$

$$16C_AC_Fn_f\Big(x^2\Big[\frac{4}{9}H_2 + 3H_{1,0} - \frac{97}{12}H_1 + \frac{8}{3}H_{-2,0} - \frac{2}{3}H_0\zeta_2 + \frac{103}{27}H_0 - \frac{16}{3}\zeta_2 + 2H_3$$

$$-6H_{-1,0} + 2H_{2,0} + \frac{127}{18}H_{0,0} - \frac{511}{12}\Big] + p_{gg}(x)\Big[2\zeta_3 - \frac{55}{24}\Big] + \frac{4}{3}(\frac{1}{x} - x^2)\Big[\frac{17}{24}H_{1,0} - \frac{43}{18}H_0$$

$$-\frac{521}{144}H_1 - \frac{6923}{432} - \frac{1}{2}H_{2,1} + 2H_1\zeta_2 + H_0\zeta_2 - 2H_{1,0,0} + \frac{1}{12}H_{1,1} - H_{1,1,0} - H_{1,1,1}\Big] - \frac{175}{12}H_2$$

$$+6H_{-1,0} + 8H_0\zeta_3 - 6H_{-2,0} - \frac{53}{6}H_0\zeta_2 - \frac{49}{2}H_0 + \frac{185}{4}\zeta_2 + \frac{511}{12} - \frac{1}{2}H_{2,0} - 3H_{1,0} - 4H_{0,0,0,0}$$

$$-\frac{67}{12}H_{0,0} + \frac{43}{2}\zeta_3 - H_{2,1} + \frac{97}{12}H_1 - 4\zeta_2^2 - \frac{9}{2}H_3 - 8H_{-3,0} + \frac{33}{2}H_{0,0,0} + \frac{4}{3}(\frac{1}{x} + x^2)\Big[\frac{1}{2}H_2 - H_{2,0}$$

$$+\frac{11}{3}H_{-1,0} + H_{-2,0} + \frac{19}{6}\zeta_2 + 2\zeta_3 - H_{-1}\zeta_2 - 4H_{-1,-1,0} - \frac{1}{2}H_{-1,0,0} - H_{-1,2}\Big] + (1-x)\Big[9H_1\zeta_2$$

$$+12H_{0,0,0,0} - \frac{293}{108} + \frac{61}{6}H_0\zeta_2 - \frac{7}{3}H_{1,0} - \frac{857}{36}H_1 - 9H_0\zeta_3 + 16H_{-2,-1,0} - 4H_{-2,0,0} + 8H_{-2}\zeta_2$$

$$-\frac{13}{2}H_{1,0,0} + \frac{3}{4}H_{1,1} - H_{1,1,0} - H_{1,1,1}\Big] + (1+x)\Big[\frac{1}{6}H_{2,0} - \frac{95}{3}H_{-1,0} - \frac{149}{36}H_2 + \frac{3451}{108}H_0$$

$$-7H_{-2,0} + \frac{302}{9}H_{0,0} + \frac{19}{6}H_3 - \frac{991}{36}\zeta_2 - \frac{163}{6}\zeta_3 - \frac{35}{3}H_{0,0,0} + \frac{17}{6}H_{2,1} - \frac{43}{10}\zeta_2^2 + 13H_{-1}\zeta_2$$

$$+18H_{-1,-1,0} - H_{3,1} - 6H_4 - 4H_{-1,2} + 6H_{0,0}\zeta_2 + 8H_2\zeta_2 - 7H_{2,0,0} - 2H_{2,1,0} - 2H_{2,1,1} - 4H_{3,0}$$

$$-9H_{-1,0,0}\Big] - \frac{241}{288}\delta(1-x)\Big) + 16C_An_f^2\Big(\frac{19}{54}H_0 - \frac{1}{24}xH_0 - \frac{1}{27}p_{gg}(x) + \frac{13}{54}(\frac{1}{x} - x^2)\Big[\frac{5}{3} - H_1\Big]$$

$$+(1-x)\Big[\frac{11}{72}H_1 - \frac{71}{216}\Big] + \frac{2}{9}(1+x)\Big[\zeta_2 + \frac{13}{12}xH_0 - \frac{1}{2}H_{0,0} - H_2\Big] + \frac{29}{288}\delta(1-x)\Big)$$

$$+16C_A^2n_f\Big(x^2\Big[\zeta_3 + \frac{11}{9}\zeta_2 + \frac{11}{9}H_{0,0} - \frac{2}{3}H_3 + \frac{2}{3}H_0\zeta_2 + \frac{1639}{108}H_0 - 2H_{-2,0}\Big] + \frac{1}{3}p_{gg}(x)\Big[\frac{10}{3}\zeta_2$$

$$-\frac{209}{36} - 8\zeta_3 - 2H_{-2,0} - \frac{1}{2}H_0 - \frac{10}{3}H_{0,0} - \frac{20}{3}H_{1,0} - H_{1,0,0} - \frac{20}{3}H_2 - H_3\Big] + \frac{10}{9}p_{gg}(-x)\Big[\zeta_2$$

$$+2H_{-1,0} + \frac{3}{10}H_0\zeta_2 - H_{0,0}\Big] + \frac{1}{3}(\frac{1}{x} - x^2)\Big[H_3 - H_0\zeta_2 - \frac{13}{3}H_2 + \frac{5443}{108} - 3H_1\zeta_2 + \frac{205}{36}H_1$$

$$-\frac{13}{3}H_{1,0} + H_{1,0,0}\Big] + (\frac{1}{x} + x^2)\Big[\frac{151}{54}H_0 - \frac{8}{3}\zeta_2 + \frac{1}{3}H_{-1}\zeta_2 - \zeta_3 + 2H_{-1,-1,0} - \frac{2}{3}H_{-1,0,0}$$

$$-\frac{37}{9}H_{-1,0} + \frac{2}{3}H_{-1,2}\Big] + (1-x)\Big[\frac{5}{6}H_{-2,0} + H_{-3,0} + 2H_{0,0,0} - \frac{269}{36}\zeta_2 - \frac{4097}{216} - 3H_{-2}\zeta_2$$

$$-6H_{-2,-1,0} + 3H_{-2,0,0} - \frac{7}{2}H_1\zeta_2 + \frac{677}{72}H_1 + H_{1,0} + \frac{7}{4}H_{1,0,0}\Big] + (1+x)\Big[\frac{193}{36}H_2 - \frac{11}{2}H_{-1}\zeta_2$$

$$+\frac{39}{20}\zeta_2^2 - \frac{7}{12}H_3 - \frac{53}{9}H_{0,0} + \frac{7}{12}H_0\zeta_2 - \frac{5}{2}H_{0,0}\zeta_2 + 5\zeta_3 - 7H_{-1,-1,0} + \frac{77}{6}H_{-1,0} + \frac{9}{2}H_{-1,0,0}$$

$$+2H_{-1,2} - 3H_2\zeta_2 - \frac{2}{3}H_{2,0} + \frac{3}{2}H_{2,0,0} + \frac{3}{2}H_4\Big] + \frac{1}{9}\zeta_2 + 7H_{-2,0} + 2H_2 + \frac{458}{27}H_0 + H_{0,0}\zeta_2$$

$$+\frac{3}{2}\zeta_2^2 + 4H_{-3,0} - x\Big[\frac{131}{12}H_{0,0} - \frac{8}{3}H_0\zeta_2 + \frac{7}{2}H_3 - H_{0,0,0,0} + \frac{7}{6}H_{0,0,0} + \frac{1943}{216}H_0 + 6H_0\zeta_3\Big]$$

$$-\delta(1-x)\Big[\frac{233}{288} + \frac{1}{6}\zeta_2 + \frac{1}{12}\zeta_2^2 + \frac{5}{3}\zeta_3\Big]\Big) + 16C_A^3\Big(x^2\Big[33H_{-2,0} + 33H_0\zeta_2 - \frac{1249}{18}H_{0,0}$$

$$-44H_{0,0,0} - \frac{110}{3}H_3 - \frac{44}{3}H_{2,0} + \frac{85}{6}\zeta_2 + \frac{6409}{108}H_0\Big] + p_{gg}(x)\Big[\frac{245}{24} - \frac{67}{9}\zeta_2 - \frac{3}{10}\zeta_2^2 + \frac{11}{3}\zeta_3$$

$$-4H_{-3,0} + 6H_{-2}\zeta_2 + 4H_{-2,-1,0} + \frac{11}{3}H_{-2,0} - 4H_{-2,0,0} - 4H_{-2,2} + \frac{1}{6}H_0 - 7H_0\zeta_3 + \frac{67}{9}H_{0,0}$$

$$-8H_{0,0}\zeta_2 + 4H_{0,0,0,0} - 6H_1\zeta_3 - 4H_{1,-2,0} + 10H_{2,0,0} - 6H_{1,0}\zeta_2 + 8H_{1,0,0,0} + 8H_{1,1,0,0} + 8H_4$$

$$+\frac{134}{9}H_{1,0} + \frac{11}{6}H_{1,0,0} + 8H_{1,2,0} + 8H_{1,3} + \frac{134}{9}H_2 - 4H_2\zeta_2 + 8H_{3,1} + 8H_{2,2} + \frac{11}{6}H_3 + 10H_{3,0}$$

$$+8H_{2,1,0}\Big] + p_{gg}(-x)\Big[\frac{11}{2}\zeta_2^2 - \frac{11}{6}H_0\zeta_2 - 4H_{-3,0} + 16H_{-2}\zeta_2 - 12H_{-2,2} - \frac{134}{9}H_{-1,0} + 2H_2\zeta_2$$

$$+8H_{-2,-1,0} + 12H_{-1}\zeta_3 - 18H_{-2,0,0} + 8H_{-1,-2,0} - 16H_{-1,-1}\zeta_2 + 24H_{-1,-1,0,0} + 16H_{-1,-1,2}$$

$$+18H_{-1,0}\zeta_2 - 16H_{-1,0,0,0} - 4H_{-1,2,0} - 16H_{-1,3} - 5H_0\zeta_3 - 8H_{0,0}\zeta_2 + 4H_{0,0,0,0} + 2H_{3,0}$$

$$-\frac{67}{9}\zeta_2 + \frac{67}{9}H_{0,0} + 8H_4\Big] + \Big(\frac{1}{x} - x^2\Big)\Big[\frac{16619}{162} + \frac{22}{3}H_{2,0} - \frac{55}{2}\zeta_3 - \frac{11}{2}H_0\zeta_2 - \frac{67}{9}H_2 - \frac{67}{9}H_{1,0}$$

$$-\frac{413}{108}H_1 - \frac{11}{2}H_1\zeta_2 + \frac{33}{2}H_{1,0,0}\Big] + 11(\frac{1}{x} + x^2)\Big[\frac{71}{54}H_0 - \frac{1}{6}H_3 - \frac{389}{198}\zeta_2 - \frac{2}{3}H_{-2,0} - \frac{1}{2}H_{-1}\zeta_2$$

$$+H_{-1,-1,0} - \frac{523}{198}H_{-1,0} + \frac{8}{3}H_{-1,0,0} + H_{-1,2}\Big] + (1-x)\Big[\frac{31}{36}H_1 + \frac{27}{2}H_{1,0} - \frac{25}{2}H_{1,0,0} - 4H_{-3,0}$$

$$-\frac{263}{12}H_{0,0} - \frac{29}{3}H_{0,0,0} - \frac{19}{3}H_{-2,0} - \frac{11317}{108} - 4H_{-2}\zeta_2 - 8H_{-2,-1,0} - 12H_{-2,0,0} - \frac{3}{2}H_1\zeta_2\Big]$$

$$+(1+x)\Big[\frac{27}{2}H_0\zeta_2 - \frac{43}{6}H_3 + \frac{29}{3}H_{2,0} + \frac{4651}{216}H_0 - \frac{329}{18}\zeta_2 + \frac{11}{2}(1+x)\zeta_3 - \frac{43}{5}\zeta_2^2 - \frac{215}{6}H_{-1,0}$$

$$-22H_{0,0}\zeta_2 - 8H_0\zeta_3 - 3H_{-1,-1,0} + 38H_{-1,0,0} + 25H_{-1,2} + 10H_{2,0,0} - 4H_2\zeta_2 + 16H_{3,0} + 26H_4$$

$$-\frac{158}{9}H_2 - \frac{53}{2}H_{-1}\zeta_2\Big] - 29H_{0,0} - \frac{40}{3}H_{0,0,0} + 27H_{-2,0} + \frac{41}{3}H_0\zeta_2 - 20H_3 - 24H_{2,0} + \frac{53}{6}\zeta_2$$

$$+\frac{601}{12}H_0 + 24\zeta_3 + 2\zeta_2^2 + 27H_2 - 4H_{0,0}\zeta_2 - 16H_0\zeta_3 - 16H_{-3,0} + 28xH_{0,0,0,0} + \delta(1-x)\Big[\frac{79}{32}$$

$$-\zeta_2\zeta_3 + \frac{1}{6}\zeta_2 + \frac{11}{24}\zeta_2^2 + \frac{67}{6}\zeta_3 - 5\zeta_5\Big]\Big) + 16C_Fn_f^2\Big(\frac{2}{9}x^2\Big[\frac{11}{6}H_0 + H_2 - \zeta_2 + 2H_{0,0} - 9\Big] + \frac{1}{3}H_2$$

$$-\frac{1}{3}\zeta_2 - \frac{10}{3}H_0 - \frac{1}{3}H_{0,0} + 2 + \frac{2}{9}(\frac{1}{x} - x^2)\Big[\frac{8}{3}H_1 - 2H_{1,0} - H_{1,1} - \frac{77}{18}\Big] - (1-x)\Big[\frac{1}{3}H_{1,0} + \frac{1}{6}H_{1,1}$$

$$+\frac{4}{9} + \frac{13}{6}H_1 + xH_1\Big] + \frac{1}{3}(1+x)\Big[\frac{68}{9}H_0 - \frac{4}{3}H_2 + \frac{4}{3}\zeta_2 + \frac{29}{6}H_{0,0} - \zeta_3 + 2H_0\zeta_2 - H_{0,0,0} - 2H_3$$

$$-H_{2,1} - 2H_{2,0}\Big] + \frac{11}{144}\delta(1-x)\Big) + 16C_F^2n_f\Big(\frac{4}{3}x^2\Big[\frac{163}{16} + \frac{27}{8}H_0 + \frac{7}{2}H_{0,0} - H_{2,0} - \zeta_2 + \frac{9}{4}H_{1,0}$$

$$-H_{2,1} + \frac{1}{2}H_{0,0,0} + \frac{85}{16}H_1 + H_2 - 2H_{-2,0} - \frac{3}{2}\zeta_3\Big] + \frac{4}{3}(\frac{1}{x} - x^2)\Big[\frac{31}{16}H_1 - \frac{11}{16} - \frac{5}{4}H_{1,0} + \frac{1}{2}H_{1,0,0}$$

$$-H_1\zeta_2 - H_{1,1} + H_{1,1,0} + H_{1,1,1} + \zeta_3\Big] + \frac{4}{3}(\frac{1}{x} + x^2)\Big[H_{-1}\zeta_2 + 2H_{-1,-1,0} - H_{-1,0,0}\Big] + \frac{215}{12}H_{0,0}$$

$$+\frac{20}{3}H_0 - \frac{131}{6} + 3H_{2,0} + \frac{205}{12}x\zeta_2 - 3H_{1,0} + H_{2,1} - \frac{85}{12}H_1 + \frac{11}{4}H_2 + 8H_{-2,0} + 2\zeta_2^2 - H_0\zeta_2$$

$$+H_3 + 6H_0\zeta_3 + 8H_{-3,0} - 4xH_{0,0,0} + (1-x)\Big[\frac{107}{12}H_1 - \frac{5}{6}H_{1,0} - 4\zeta_2 + H_0\zeta_3 - 8H_{-2,-1,0}$$

$$-4H_{-2}\zeta_2 + 4H_{-2,0,0} - 4H_1\zeta_2 + \frac{7}{2}H_{1,0,0} - \frac{7}{12}H_{1,1} + H_{1,1,0} + H_{1,1,1}\Big] + (1+x)\Big[\frac{5}{4}H_2 + \frac{33}{8}$$

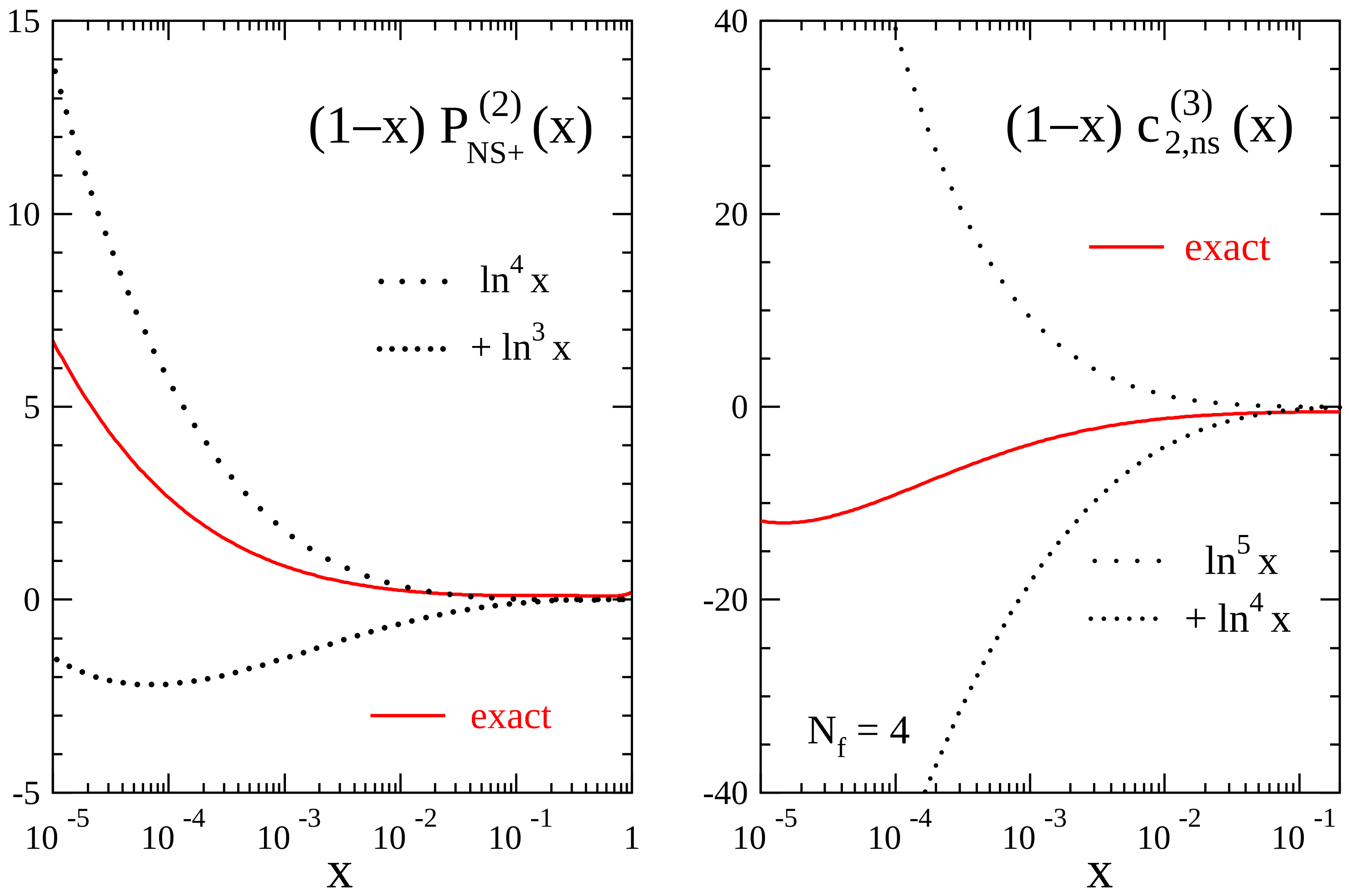
$$-\frac{99}{4}H_{0,0} - 8H_{2,0} - \frac{541}{24}H_0 - 4H_{2,1} - \frac{3}{2}H_{0,0,0} - 2x\zeta_3 + \frac{9}{2}\zeta_2^2 + 5H_0\zeta_2 - 5H_3 - 4H_{-1}\zeta_2$$

$$-8H_{-1,-1,0} + \frac{67}{3}H_{-1,0} + 4H_{-1,0,0} + 2H_{0,0}\zeta_2 - 2H_{0,0,0,0} - 4H_2\zeta_2 + 3H_{2,0,0} + 2H_{2,1,0}$$

$$+2H_{2,1,1} + H_{3,1} - 2H_4\Big] + \frac{1}{16}\delta(1-x)\Big)$$

MVV (2004)

Non-singlet three-loop quantities at small x



20% accuracy by leading $x \to 0$ logarithm of $c^{(3)}_{2,\mathrm{ns}}$: $x < 10^{-50}$ ...

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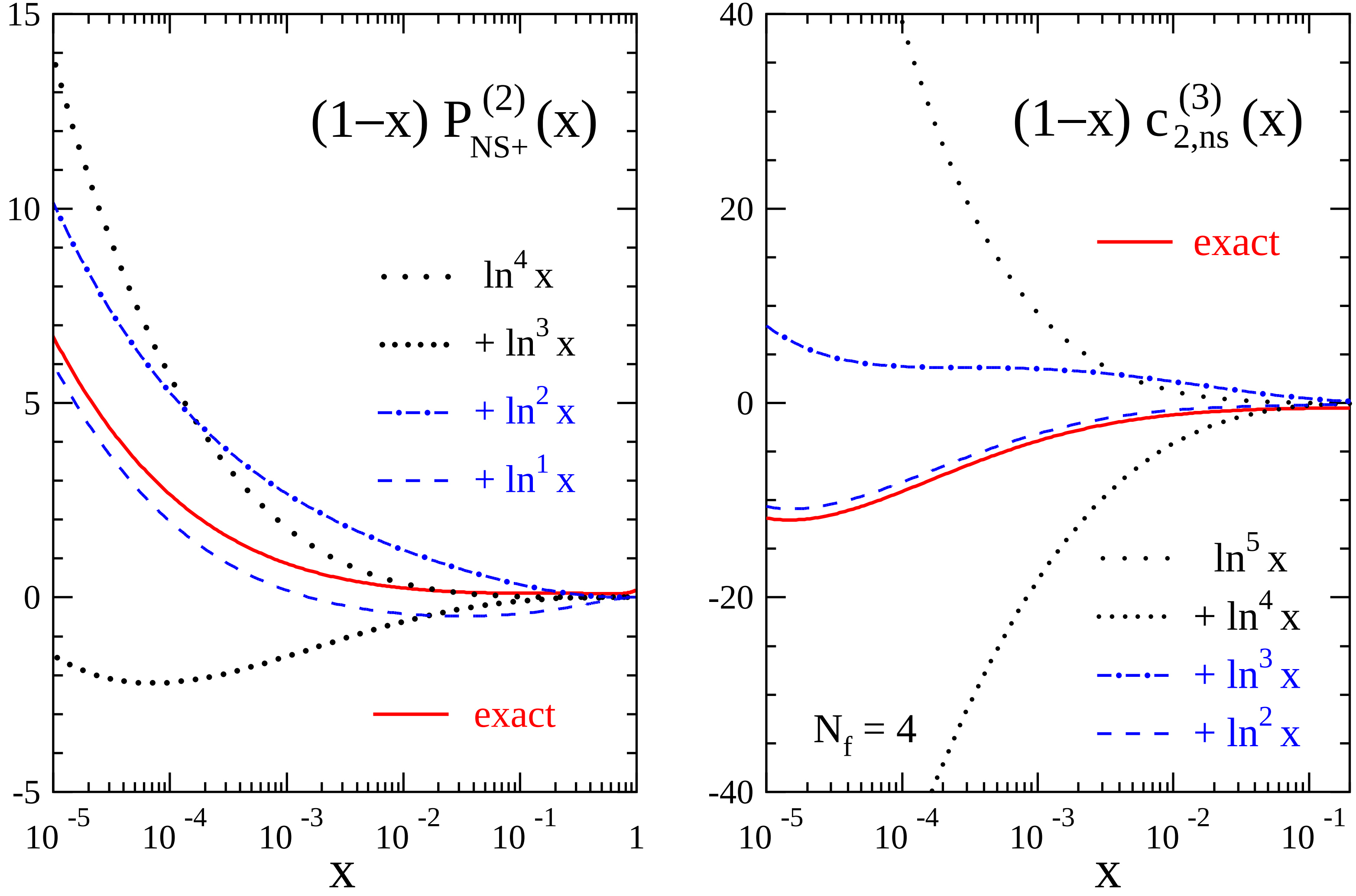
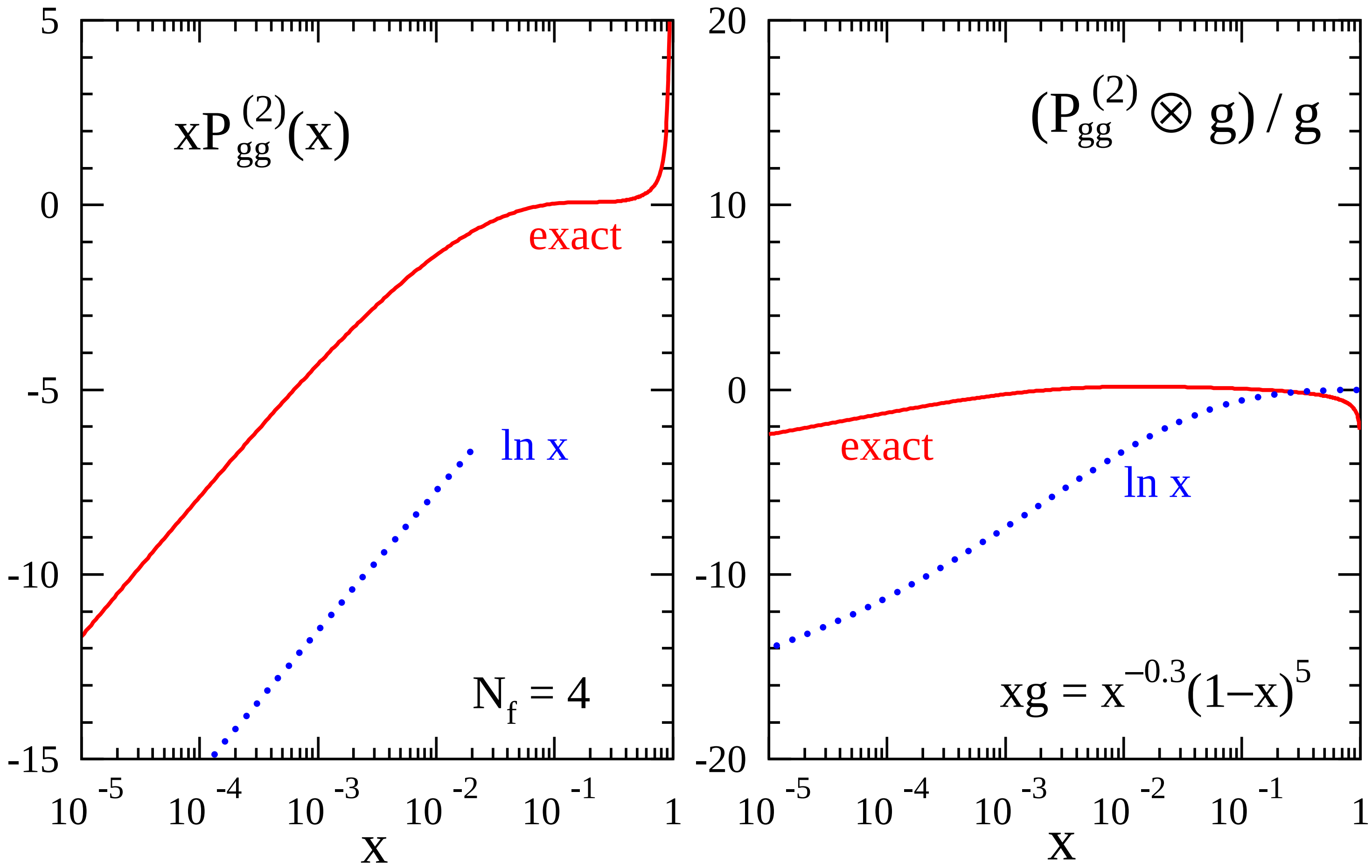


20% accuracy by leading $x \to 0$ logarithm of $c^{(3)}_{2,ns}$: $x < 10^{-50}$ ...

Splitting functions and evolution at small x

Splitting functions $\longrightarrow$ observables : **convolutions,** $\int_x^1 \frac{dy}{y} P(y) f\left(\frac{x}{y}\right)$



Leading $x \longrightarrow 0$ term (BFKL) confirmed but insufficient at colliders

Splitting functions and evolution at small x

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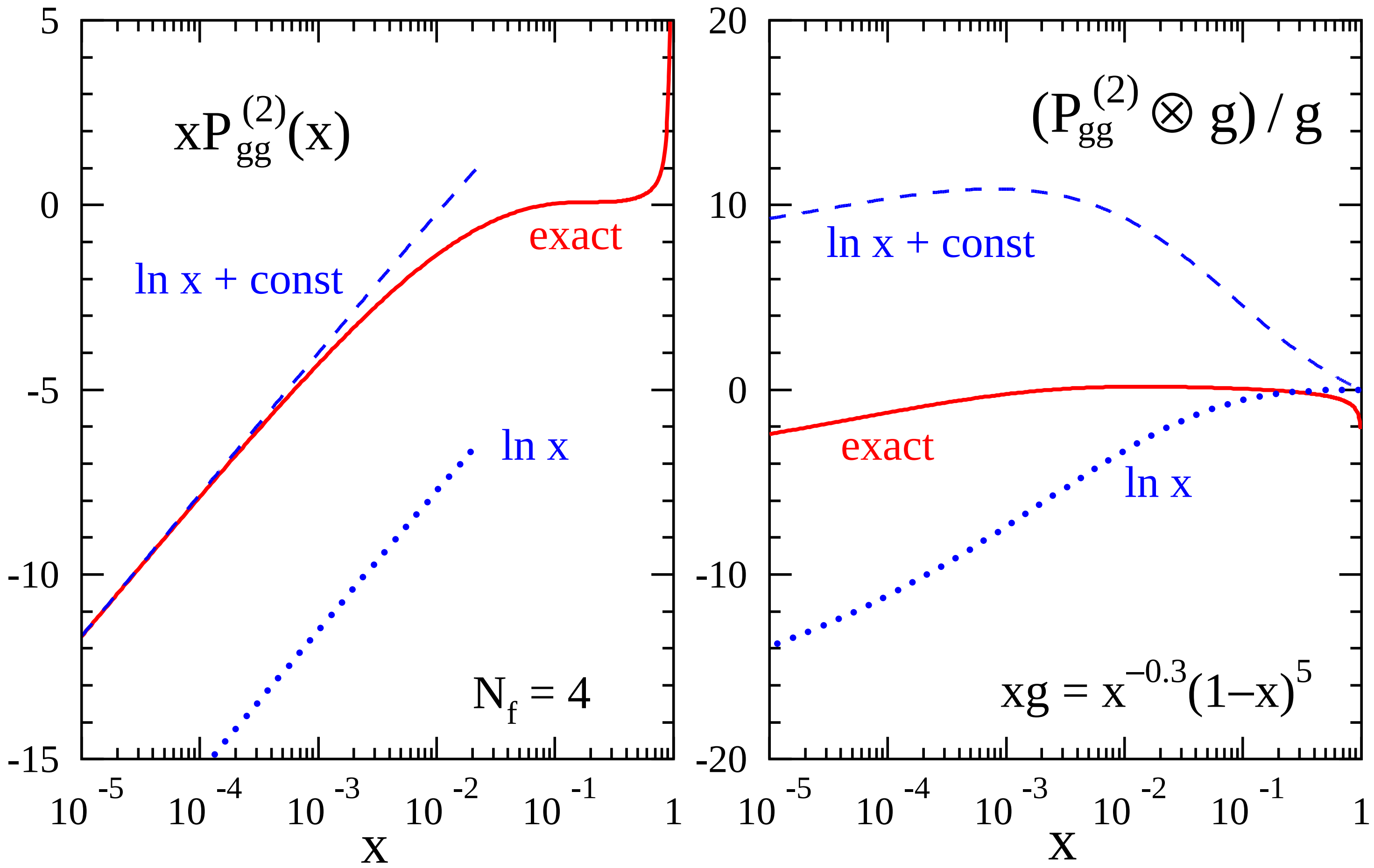
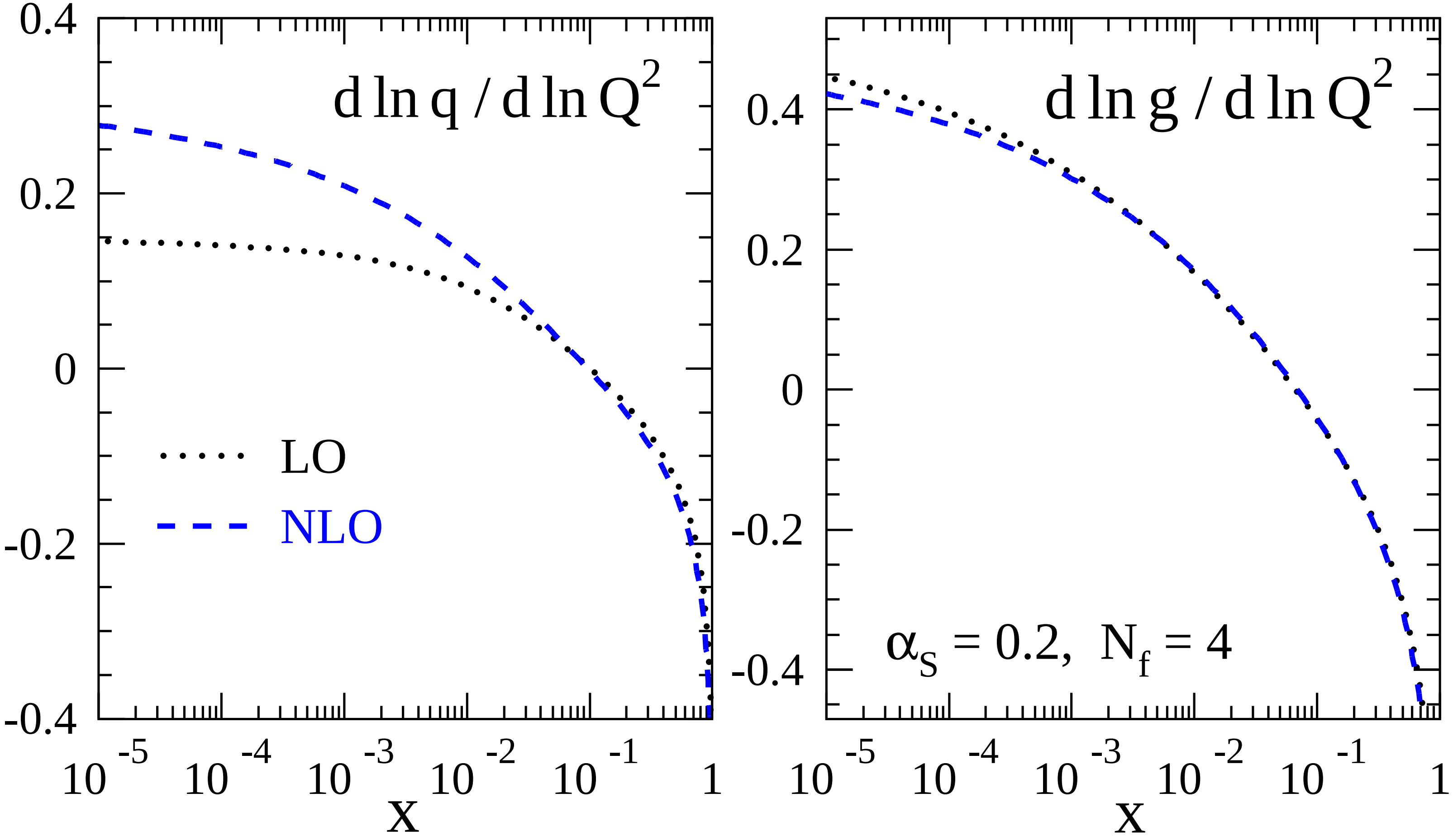


Small-x limits of splitting functions insufficient for small-x physics

Evolution of flavour singlet distributions

Scale derivatives of quark and gluon distributions at $Q^2 \approx 30$ GeV^2



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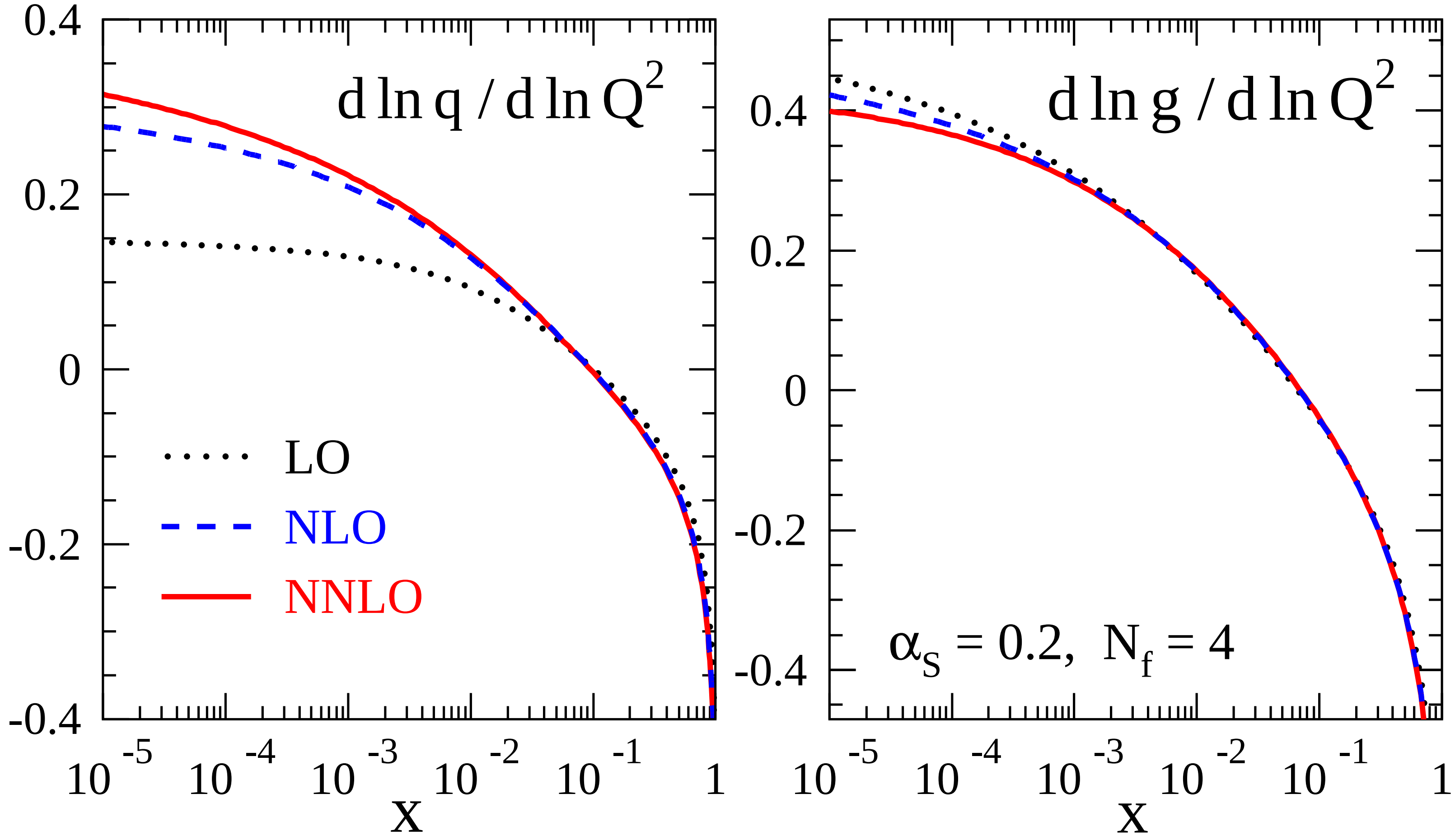
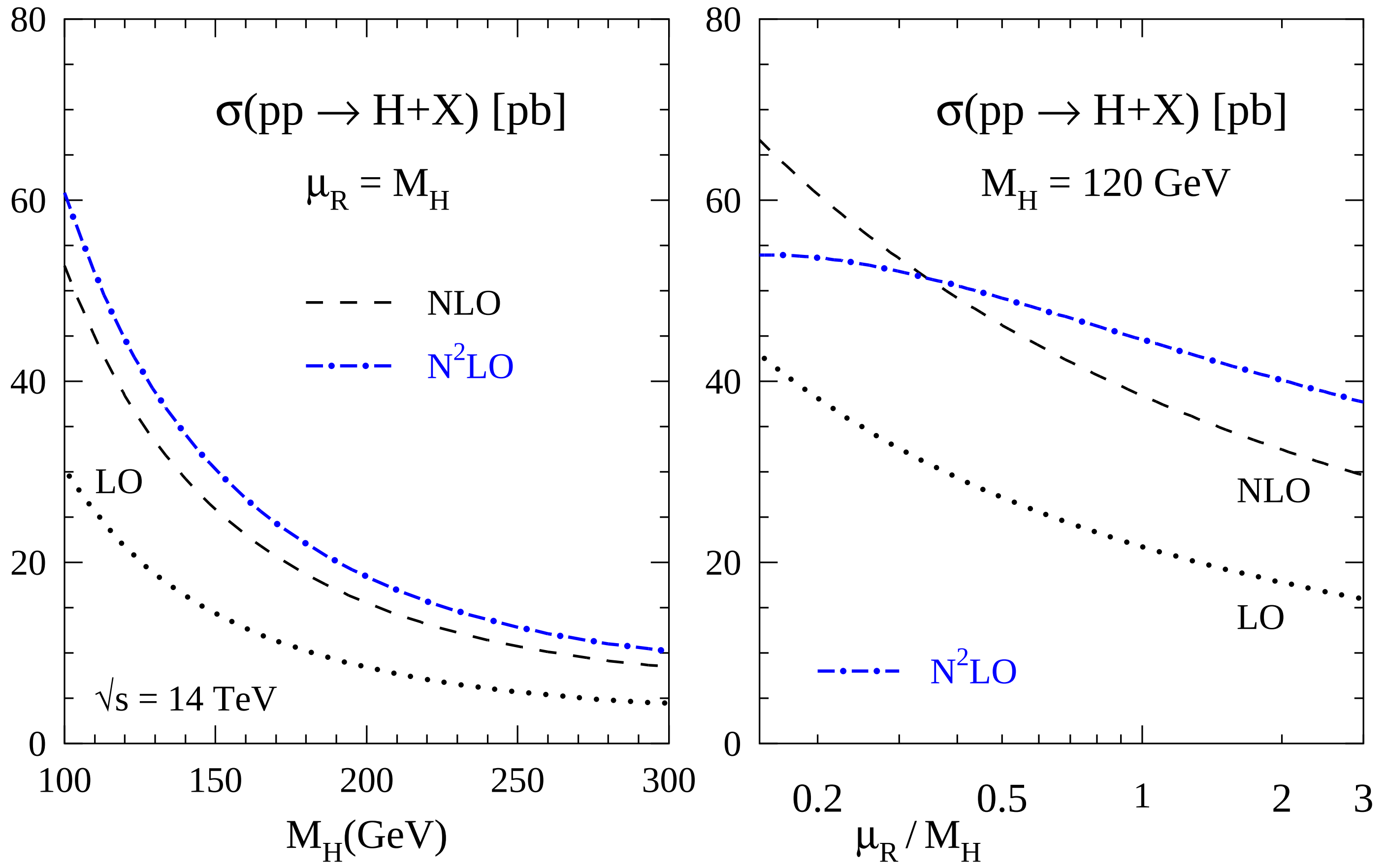


Expansion very stable except for very small momenta $x \lesssim 10^{-4}$

Higgs boson production at the LHC (II)

$\hat{\sigma}_{NNLO}$: Harlander, Kilgore (02); Anastasiou, Melnikov (02, 05 [σ_{diff}])

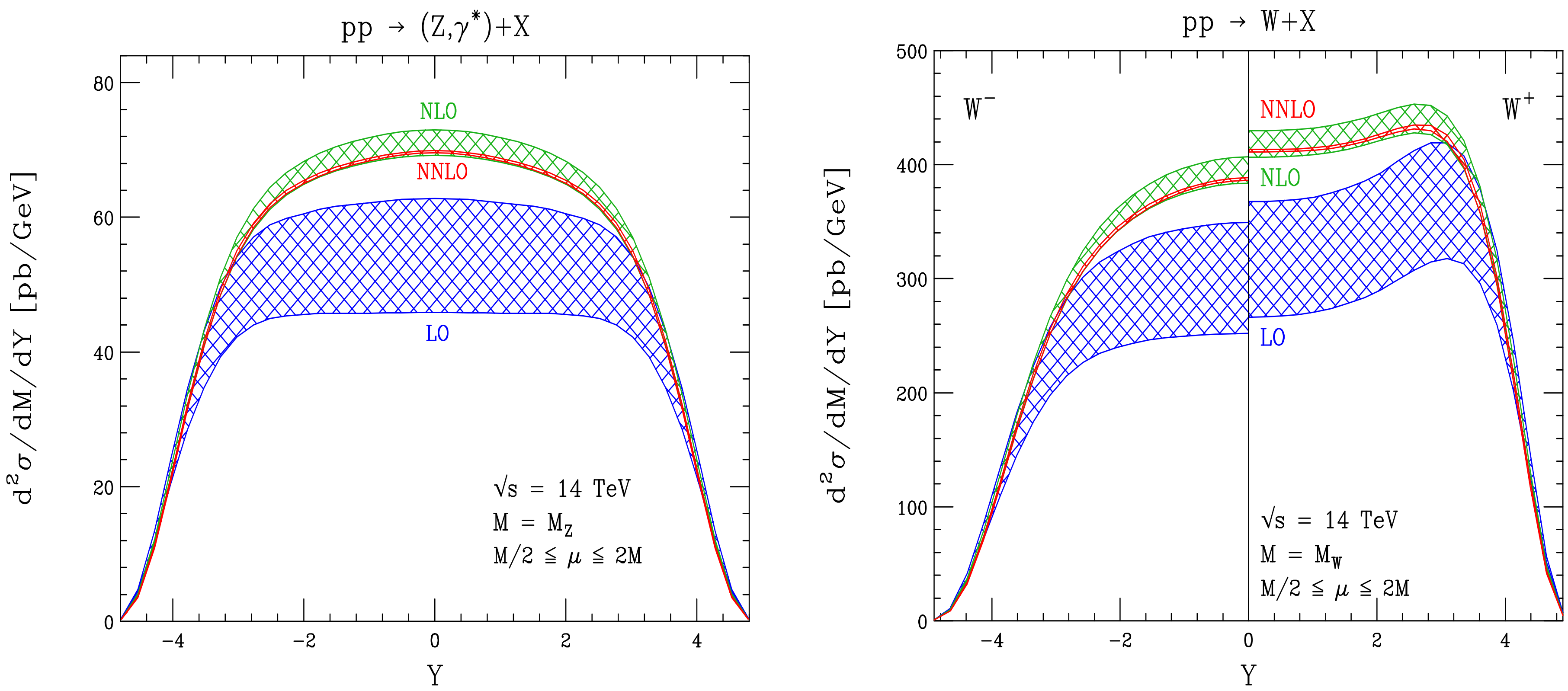
Partons including NNLO: Martin, Roberts, Stirling, Thorne; Aljechin



Stabilization, but still sizeable ($\sim 15\%$) higher-order uncertainties

NNLO gauge boson production at the LHC

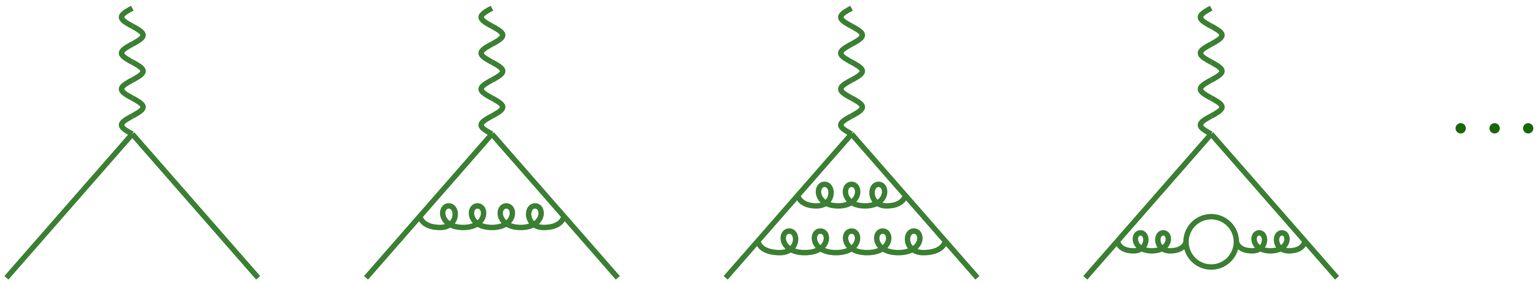
diff. $\hat{\sigma}_{\rm NNLO}$: Anastasiou, Dixon, Melnikov, Petriello (03)



'Gold-plated' processes: NNLO perturbative accuracy better than 1%

⇒ use to determine (at least parton-parton) luminosities at the LHC

Form factors of massless quarks and gluons



On-shell $m = 0$ quark form factor $\mathcal{F}_q$: QCD corr's to $\gamma^* qq$ vertex

$$\Gamma_\mu \;=\; i e_q \, (\bar{u} \gamma_\mu u) \, \mathcal{F}_q(\alpha_s, Q^2)$$

Gauge invariant, divergent: dimensional regularization, $D = 4 - 2\varepsilon$

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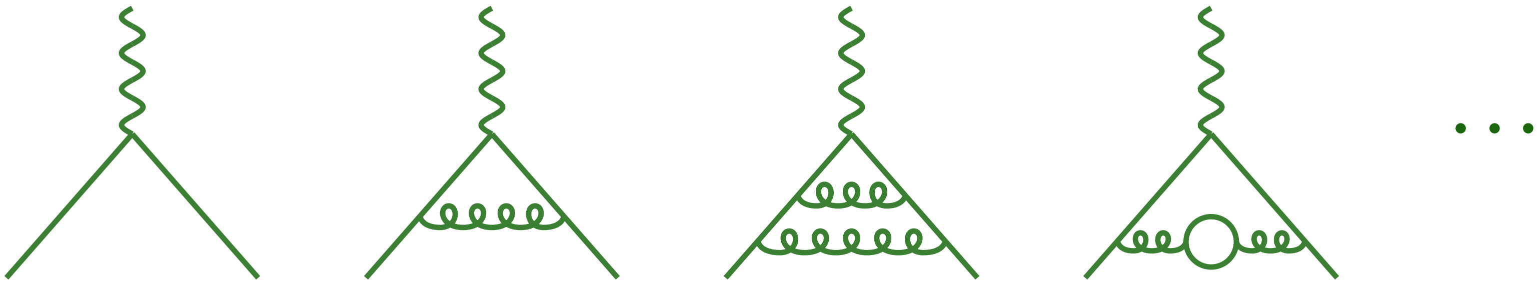

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Gauge invariant, divergent: dimensional regularization, $D = 4 - 2\varepsilon$

Gluon form factor $\mathcal{F}_g$: effective Hgg vertex in heavy top-quark limit

$$\mathcal{L}_{\text{eff}} \;=\; -\frac{1}{4} \, C_H \, H G^a_{\mu\nu} G^{a,\mu\nu}$$

Coefficient C_H known to N^{3}LO — Chetyrkin, Kniehl, Steinhauser (97)

Renormalization of $G^a_{\mu\nu} G^{a,\mu\nu}$:

$$Z_{G^2} \;=\; \left[1 - \beta(a_s)/(a_s \varepsilon)\right]^{-1}$$

Extraction of $\mathcal{F}_3$ from (ϕ)DIS at third order

a_s expansion of the bare structure functions at large Bjorken-x

$$
\begin{aligned}
F_0^{\mathrm{b}} &= \delta(1-x) \\
F_1^{\mathrm{b}} &= 2\,\mathcal{F}_1\,\delta(1-x) + \mathcal{S}_1 \\
F_2^{\mathrm{b}} &= (2\,\mathcal{F}_2 + \mathcal{F}_1^{\,2})\delta(1-x) + 2\,\mathcal{F}_1\mathcal{S}_1 + \mathcal{S}_2 \\
F_3^{\mathrm{b}} &= (2\,\mathcal{F}_3 + 2\,\mathcal{F}_1\mathcal{F}_2)\delta(1-x) + (2\,\mathcal{F}_2 + \mathcal{F}_1^{\,2})\mathcal{S}_1 + 2\,\mathcal{F}_1\mathcal{S}_2 + \mathcal{S}_3
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$\mathcal{F}_l$: bare l-loop space-like q or g form factor. $\mathcal{S}_l$: soft real emissions

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\begin{aligned}
\mathcal{S}_k &= \mathsf{S}_k(\varepsilon)\cdot\varepsilon[\,(1-x)^{-1-k\varepsilon}\,]_+ \\
&= \mathsf{S}_k(\varepsilon)\Big\{-\frac{1}{k}\,\delta(1-x) + \sum_{i=0}\frac{(-k\varepsilon)^i}{i\,!}\,\varepsilon\,\mathcal{D}_i\Big\}\ , \quad \mathcal{D}_i \equiv \left[\frac{\ln^i(1-x)}{(1-x)}\right]_+
\end{aligned}
$$

Calculation of $F_3^{\rm b}$ to order ε^m $\Rightarrow$ $\mathcal{F}_3$ and $\mathcal{S}_3$ to order ε^{m-1}

MVV(2005) : coefficient fct's for (ϕ)DIS + dedicated n_f calc. to $\mathcal{O}(\varepsilon)$

Explicit three-loop contribution to $\mathcal{F}_\mathbf{g}$

$$\begin{aligned}
\mathcal{F}_3^{\mathbf{g}} = \; & \mathbf{C_A^3}\Big\{ -\frac{4}{3\epsilon^6} + \frac{11}{3\epsilon^5} + \frac{361}{81\epsilon^4} + \frac{1}{\epsilon^3}\left(-\frac{3506}{243} - \frac{517}{54}\zeta_2 + \frac{22}{3}\zeta_3\right) \\
& + \frac{1}{\epsilon^2}\left(-\frac{17741}{243} + \frac{481}{162}\zeta_2 - \frac{209}{27}\zeta_3 + \frac{247}{90}\zeta_2^2\right) \\
& + \frac{1}{\epsilon}\left(-\frac{145219}{2187} + \frac{20329}{243}\zeta_2 + \frac{241}{9}\zeta_3 - \frac{3751}{360}\zeta_2^2 - \frac{\mathbf{85}}{\mathbf{9}}\boldsymbol{\zeta_2\zeta_3} - \frac{\mathbf{878}}{\mathbf{15}}\boldsymbol{\zeta_5}\right)\Big\} \\
& + \mathbf{C_A^2 n_f}\Big\{ -\frac{2}{3\epsilon^5} - \frac{2}{81\epsilon^4} + \frac{1}{\epsilon^3}\left(\frac{1534}{243} + \frac{47}{27}\zeta_2\right) + \frac{1}{\epsilon^2}\left(\frac{4280}{243} - \frac{425}{81}\zeta_2 + \frac{518}{27}\zeta_3\right) \\
& + \frac{1}{\epsilon}\left(-\frac{92449}{2187} - \frac{7561}{243}\zeta_2 + \frac{1022}{81}\zeta_3 + \frac{2453}{180}\zeta_2^2\right)\Big\} \\
& + \mathbf{C_A C_F n_f}\Big\{ \frac{20}{9\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{526}{27} - \frac{160}{9}\zeta_3\right) + \frac{1}{\epsilon}\left(\frac{2783}{81} - \frac{22}{3}\zeta_2 - \frac{224}{27}\zeta_3 - \frac{176}{15}\zeta_2^2\right)\Big\} \\
& + \mathbf{C_A n_f^2}\Big\{ -\frac{8}{81\epsilon^4} - \frac{80}{243\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{8}{9} + \frac{20}{27}\zeta_2\right) + \frac{1}{\epsilon}\left(\frac{34097}{2187} + \frac{200}{81}\zeta_2 + \frac{664}{81}\zeta_3\right)\Big\} \\
& + \mathbf{C_F^2 n_f}\Big\{\frac{2}{3\epsilon}\Big\} + \mathbf{C_F n_f^2}\Big\{ \frac{8}{9\epsilon^2} + \frac{1}{\epsilon}\left(\frac{424}{27} - \frac{32}{3}\zeta_3\right)\Big\}
\end{aligned}$$

$\zeta_2\zeta_3,\ \zeta_5 \leftrightarrow$ **MSYM**

Bern, Dixon, Smirnov (05)

Pole structure of $q\bar{q} \longrightarrow \gamma^*$ and $gg \longrightarrow H$

α_s^n expansion coefficients of bare partonic cross sections to $n = 3$

cf. Matsuura, van Neerven (88)

$$W_0^{\mathrm{b}} = \delta(1-x)$$

$$W_1^{\mathrm{b}} = 2\,\mathrm{Re}\,\mathcal{F}_1\,\delta(1-x) + \mathcal{S}_1$$

$$W_2^{\mathrm{b}} = (2\,\mathrm{Re}\,\mathcal{F}_2 + |\mathcal{F}_1|^2)\delta(1-x) + 2\,\mathrm{Re}\,\mathcal{F}_1\mathcal{S}_1 + \mathcal{S}_2$$

$$W_3^{\mathrm{b}} = (2\,\mathrm{Re}\,\mathcal{F}_3 + 2\,|\mathcal{F}_1\mathcal{F}_2|)\delta(1-x) + (2\,\mathrm{Re}\,\mathcal{F}_2 + |\mathcal{F}_1|^2)\mathcal{S}_1 + 2\,\mathrm{Re}\,\mathcal{F}_1\mathcal{S}_2 + \mathcal{S}_3$$

$\mathcal{F}_l$: bare l-loop time-like q or g form factor, $\mathcal{S}_l$: soft real emissions

$$\mathcal{S}_k = \mathsf{S}_k(\varepsilon)\cdot\varepsilon[\,(1-x)^{-1-2k\varepsilon}\,]_+ = \mathsf{S}_k(\varepsilon)\left[-\frac{1}{2k}\,\delta(1-x) + \sum_{i=0}\frac{(-2k\varepsilon)^i}{i\,!}\,\varepsilon\,\mathcal{D}_i\right]$$

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$$

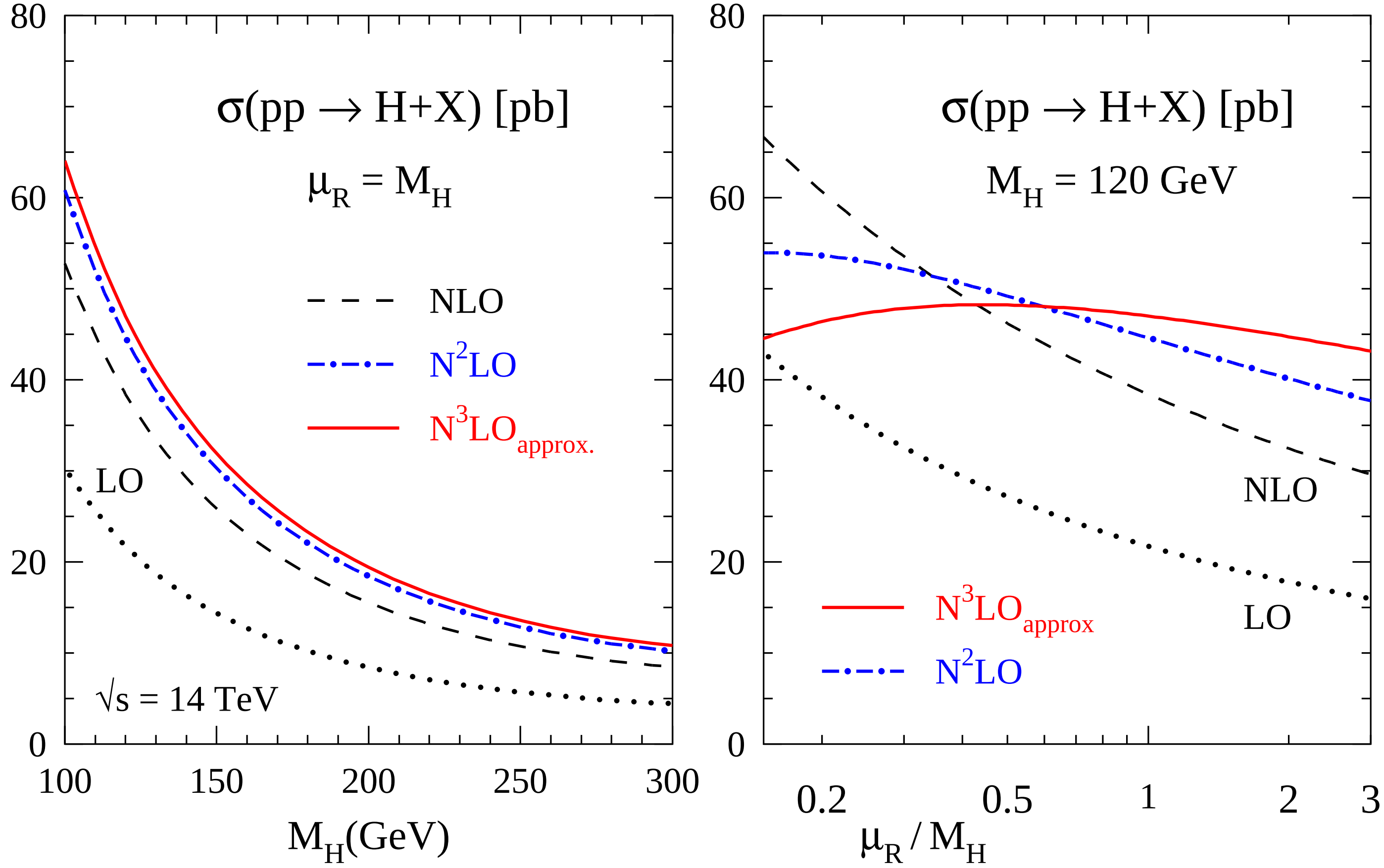
Poles in $\varepsilon = 2 - D/2$: KLN, renormalization, mass factorization

$1/\varepsilon$ pieces of $\mathcal{F}_n$ + n-loop splitting fct's $\longrightarrow$ $1/\varepsilon$ coefficients of S_n

$\longrightarrow$ all soft-enhanced $\mathcal{D}_{2n-1,\ldots,0}$ terms of N$^{\rm n}$LO coefficient fct's c_n

Higgs boson production at the LHC (III)

$N^3LO_{approx.}$: trf. $\mathcal{D}_k$ to N, drop $1/N$ terms (10% error for $N^{(2)}LO$ corr.)



N^3LO increase at $\mu_r = M_H$: 5% (NNLO pdf's). μ_r variation: 4%

⇒ 5% accuracy reached by approx. N^3LO **Moch, A.V. (2005)**

Coefficient functions at large x / large N

Threshold: inhibited radiation

$\Rightarrow$ log^2-enhanced corrections

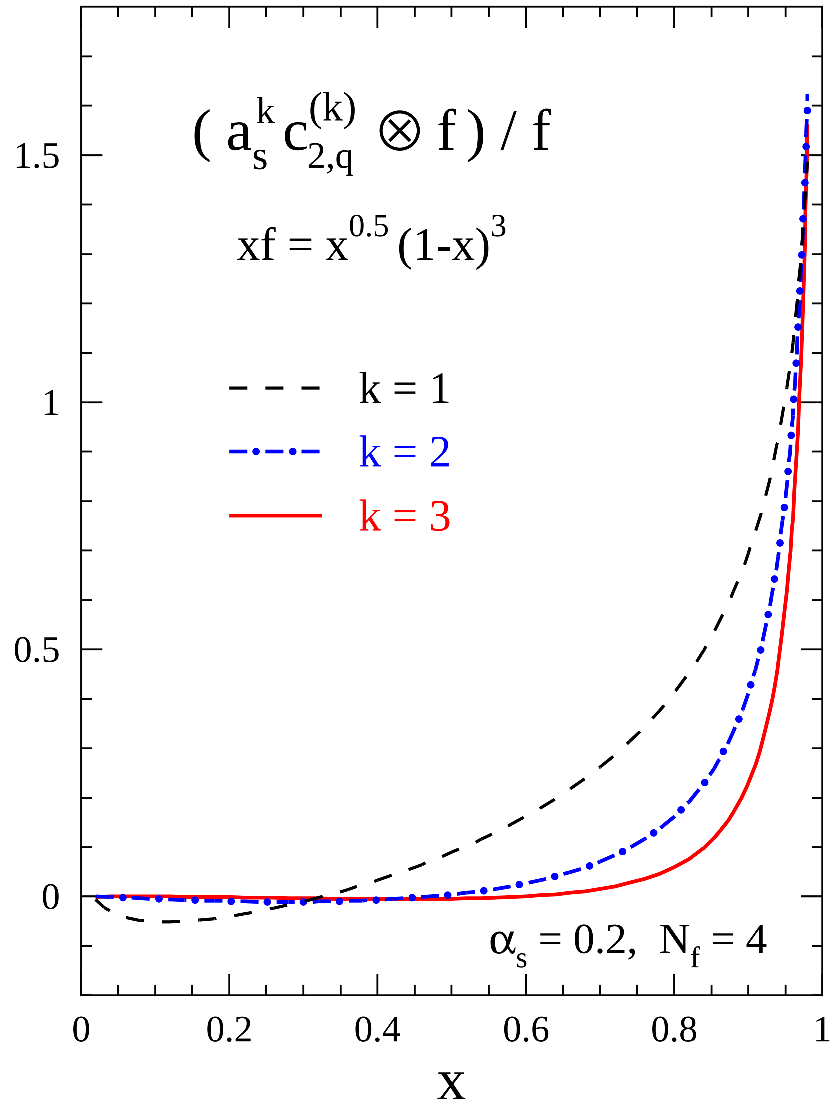
Scaling variable x, moments N

$$\alpha_s^k \left[\frac{\ln^{2l-1}(1-x)}{1-x} \right]_+ \; ,$$

$$\alpha_s^k \; \ln^{2l} N \quad , \quad l = 1, \ldots, k$$

α_s expansion spoiled for $x \to 1$, $N \to \infty$ $\;\Rightarrow\;$ resummation

Example: DIS, only x-dep. case fully known to N^{3}LO **MVV(05)**



Soft gluon exponentiation

$\overline{\text{MS}}$ coefficient functions for few-parton cases, large Mellin-N

$$C^N / C^N_{\text{LO}} \; = \; g_0 \cdot \exp \mathcal{G}^N \; + \; \mathcal{O}(N^{-1} \ln^n N)$$

g_0 : N^0 contributions, $\mathcal{G}^N$: resummation of $\ln^n N$ terms

Sterman (87); Catani, Trentadue (89); ...

Drell-Yan, DIS

$$\mathcal{G}^N_{\text{DY}} \;=\; 2 \ln \Delta_q + \ln \Delta^{\text{int}}_{\text{DY}}$$

$$\mathcal{G}^N_{\text{DIS}} \;=\; \ln \Delta_q + \ln J_q + \underbrace{\ln \Delta^{\text{int}}_{\text{DIS}}}_{=0}$$

Forte, Ridolfi; Gardi, Roberts (02)

$gg \longrightarrow H$ for large m_{top} $\Leftrightarrow$ Drell-Yan

Direct photons in pp, $ab \longrightarrow c\,\gamma$ Catani, Mangano, Nason (98)

The radiative factors

- **Soft collinear radiation off initial-state parton $p = q,\ g$**

$$\ln \Delta_{\rm p} \;=\; \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \int_{\mu_f^2}^{(1-z)^2 Q^2} \frac{dq^2}{q^2}\, A_{\rm p}(\alpha_{\rm s}(q^2))$$

- **Collinear emission off ‘unobserved’ final-state parton**

$$\ln J_{\rm p} \;=\; \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \left[\int_{(1-z)^2 Q^2}^{(1-z) Q^2} \frac{dq^2}{q^2}\, A_{\rm p}(\alpha_{\rm s}(q^2)) + B_{\rm p}(\alpha_{\rm s}([1-z]Q^2)) \right]$$

- **Large-angle soft gluons, process-dependent**

$$\ln \Delta^{\rm int} \;=\; \int_0^1 dz\, \frac{z^{N-1}-1}{1-z}\, D(\alpha_{\rm s}([1-z]^2 Q^2))$$

Integrands of $\Delta_{\rm p}$, $J_{\rm p}$, $\Delta^{\rm int}$: power expansions in $a_{\rm s} = \dfrac{\alpha_{\rm s}}{4\pi}$

$$F(\alpha_{\rm s}) \;=\; \sum_{l=1} F_l\, a_{\rm s}^l \;, \quad F = A,\ B,\ D$$

The resummation exponents

Up to next-to-next-to-next-to-leading logarithmic (N^3LL) accuracy

$$\mathcal{G}^N \;=\; \ln N \, g_1(\lambda) + g_2(\lambda) + a_s \, g_3(\lambda) + a_s^2 \, g_4(\lambda) \;\; , \quad \lambda \;=\; \beta_0 a_s \ln N$$

Integrations for g_3, g_4, . . . : **A.V. (00); Catani, de Florian, Grazzini, Nason (03)**
MVV (05) [⟵ XSUMMER package: Moch, Uwer (05)]

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MVV (05) [$\longleftarrow$ XSUMMER package: Moch, Uwer (05)]

Dependence on coefficients

$$\begin{aligned} g_1 &= g_1(\lambda, A_1, \beta_0) \\ g_2 &= g_1(\ldots, A_2, B_1, D_1, \beta_1) \\ &\vdots \\ g_k &= g_k(\ldots, A_k, B_{k-1}, D_{k-1}, \beta_{k-1}) \end{aligned}$$

N^{n}LO calculation (B_n, D_n) $\Longrightarrow$ N^{n}LL resummation (mod. A_{n+1})

M[V]V (05) : N^{3}LL for incl. (ϕ)DIS, lepton-pair and Higgs production

Coeff's D_l for Drell-Yan and Higgs production

Maximally non-abelian, $C_I = C_F$ (DY), $C_I = C_A$ (Higgs) with

$$\mathbf{D_1} = 0$$

$$\mathbf{D_2} = C_I \left[C_A \left(-\frac{1616}{27} + \frac{176}{3}\,\zeta_2 + 56\,\zeta_3 \right) + n_f \left(\frac{224}{27} - \frac{32}{3}\,\zeta_2 \right) \right]$$

Catani, Trentadue (89) [D_1]; A.V. (00); Catani, de Florian, Grazzini, Nason (03)

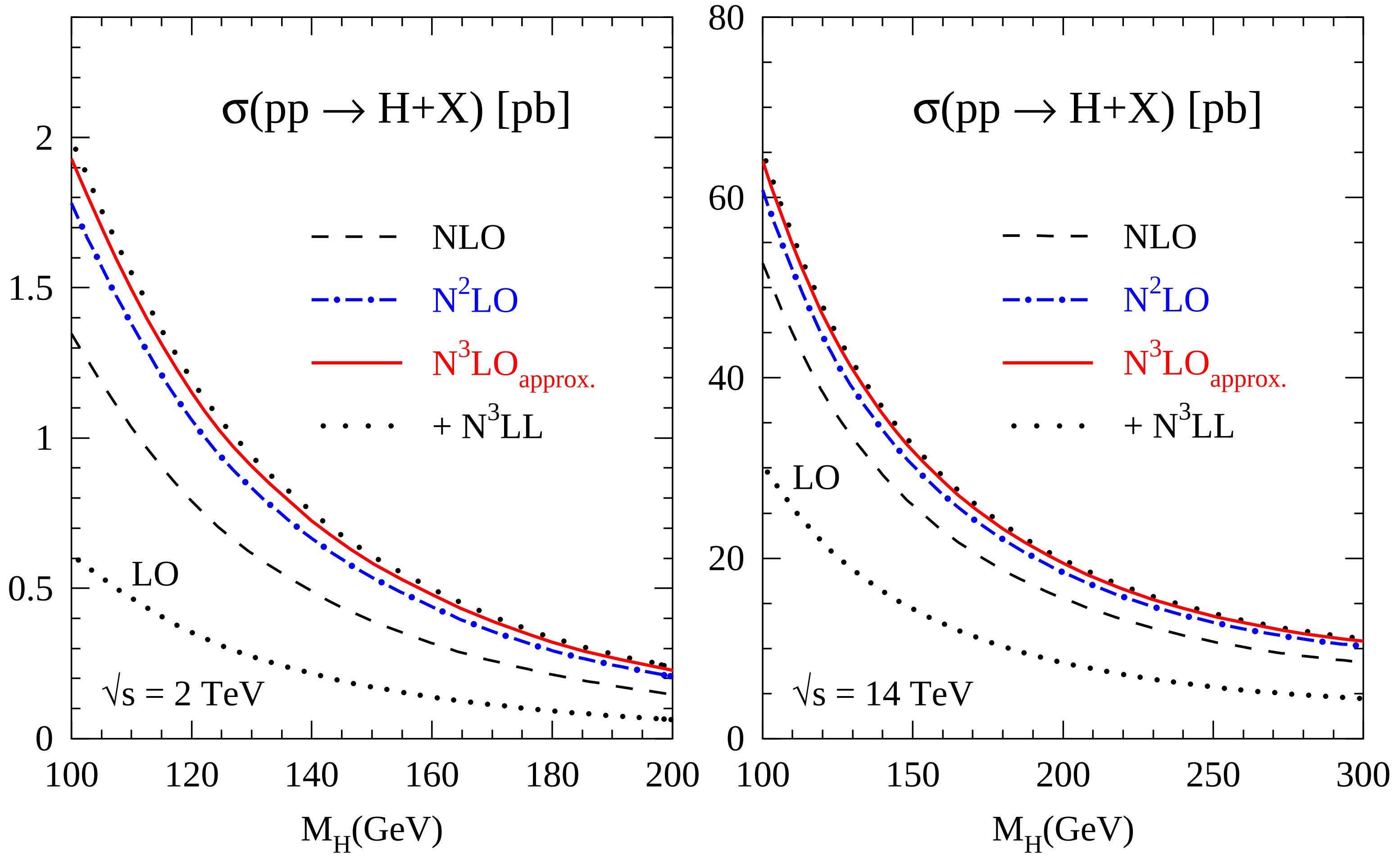
$$\begin{aligned}\mathbf{D_3} = {} & C_I C_A^2 \left[-\frac{594058}{729} + \frac{98224}{81}\,\zeta_2 + \frac{40144}{27}\,\zeta_3 - \frac{2992}{15}\,\zeta_2^2 - \frac{352}{3}\,\zeta_2\zeta_3 - 384\,\zeta_5 \right] \\ & + C_I C_A n_f \left[\frac{125252}{729} - \frac{29392}{81}\,\zeta_2 - \frac{2480}{9}\,\zeta_3 + \frac{736}{15}\,\zeta_2^2 \right] \\ & + C_I C_F n_f \left[\frac{3422}{27} - 32\,\zeta_2 - \frac{608}{9}\,\zeta_3 - \frac{64}{5}\,\zeta_2^2 \right] - C_I n_f^2 \left[\frac{3712}{729} - \frac{640}{27}\,\zeta_2 - \frac{320}{27}\,\zeta_3 \right]\end{aligned}$$

Moch, A.V. (05); Laenen, Magnea (05) [DY]; ...

Simple relation of D_n with form-factor resummation coefficients f_n

Higgs production at LHC (IV) and Tevatron

Parameters ($m_{top} = 173.4$ GeV etc), as before: **Ravindran, Smith, van Neerven (03)**



N^3LL resummation confirms N^3LO error estimate **Moch, A.V. (2005)**

Parton evolution via Mellin N-space

Evolution equations $\longrightarrow$ ordinary matrix differential eqn. for each N

$$a(N) \;=\; \int_0^1 dx\, x^{N-1} a(x) \quad \Rightarrow \quad (a \otimes b)(N) \;=\; a(N)\, b(N)$$

Solution by time-ordered exponential, expanded around LO result

$$q(N,\mu_f^2) = \left[1 + \sum a_s^k\, U_k(N)\right] \left(\frac{a_s}{a_0}\right)^{-R_0(N)} \left[1 + \sum a_0^k\, U_k(N)\right]^{-1} q(N,\mu_0^2)$$

$R_0 = P_0/\beta_0\,,\; U_k = f(P_{i\leq k}, \beta_{i\leq k})$ **iterative (commutation relations)**

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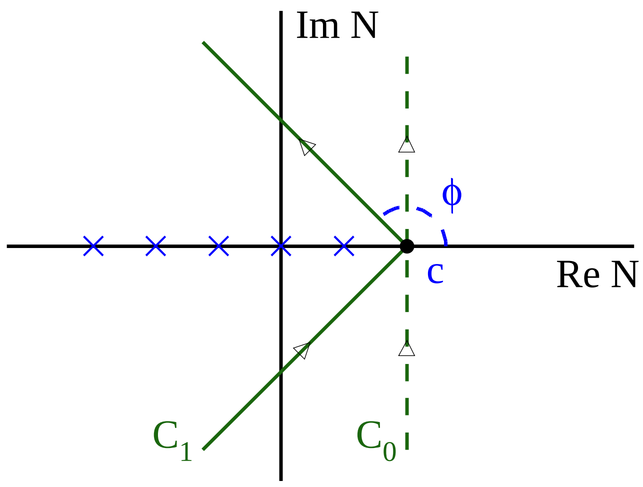
$R_0 = P_0/\beta_0\,,\; U_k = f(P_{i\le k}, \beta_{i\le k})$ **iterative (commutation relations)**

Inverse Mellin transformation

$$a(x) \;=\; \frac{1}{2\pi i} \int_{\mathcal{C}} dN\, x^{-N}\, a(N)$$

Contour $\mathcal{C}_1$: exponential damping

$$\sim\; \exp\left(z \ln(1/x) \cos\phi\right)$$



Published package (rigid contour): **QCD-Pegasus** **A.V. (2004)**

Benchmark results for parton evolution codes

G. Salam, A.V. (2002, 05)

Evolution of Les Houches (2001) reference input

$$xu_v(x, \mu_{f,0}^2) = 5.1072 \; x^{0.8} \; (1-x)^3 \quad , \quad \ldots$$
$$xg\,(x, \mu_{f,0}^2) = 1.7000\, x^{-0.1}\, (1-x)^5$$

with

$$\alpha_s(\mu_r^2 = 2\ \text{GeV}^2) = 0.35$$

at LO, NLO, NNLO, for $\mu_r = \{0.5,\ 1,\ 2\}\, \mu_f$, with fixed / variable N_f

Two completely different codes. G.S.: discretization in x and μ_f, f90

Five-figure agreement over wide range in x, $\mu_f^2 \rightarrow$ reference tables

Example: NNLO, $\mu_r = 2\mu_f$, $N_f = 4$ at $x = 10^{-4}$, $\mu_f^2 = 10^4$ GeV^2

$$xu_v = 1.3206 \cdot 10^{-2} \;, \quad \ldots \;, \quad xg = 9.0162 \cdot 10^{1}$$

Observables via Mellin N-space

Direct method requires analytic coefficient-function moments, e.g.,

$$\frac{1}{x} F_2(x, Q^2) \;=\; \frac{1}{2\pi i} \int_{\mathcal{C}} dN \, x^{-N} \, C_{a,p}(N, a_{\rm s}, Q^2, \mu_{\rm f}^2) \, f_p(N, \mu_{\rm f}^2)$$

General DIS observables: $\widehat{\sigma}$ with more integrations, exp. cuts, . . .

$$\sigma(x, Q^2) \;=\; \int_{y_{\rm min}}^{1} dy \, f_p(y, \mu_{\rm f}^2) \, \widehat{\sigma}_p(x, y, Q^2, \mu_{\rm f}^2)$$

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Pseudo-moment method: insert inverse $N \rightarrow x$ trf. for f_p, rearrange

$$\sigma(x, Q^2) \;=\; \frac{1}{2\pi i} \int_{\mathcal{C}} dN\, f_p(N, \mu_{\rm f}^2)\, \widetilde{\sigma}_p(x, N, Q^2, \mu_{\rm f}^2)$$

with

$$\widetilde{\sigma}_p(x, N, Q^2, \mu_{\rm f}^2) \;=\; \int_{y_{\rm min}}^{1} dy\, y^{-N}\, \widehat{\sigma}_p(x, y, Q^2, \mu_{\rm f}^2)$$

[Berger, Graudenz, Hampel, A.V. (95)], Kosower (97); Stratmann, Vogelsang (00)

$\widetilde{\sigma}$: pre-calculate / store for each N, data bin, parton p, order, scale $\mu_{\rm f}$

$pp\,/\,p\bar{p}$ observables : corresponding procedure (double integrations)

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$O(\alpha_s^3)$ splitting functions for evolution of parton distributions

Full NNLO for crucial LHC processes: Drell-Yan, $pp \to H + X$

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ε^0 terms of three-loop deep-inelastic scattering

- N^{3}LO coefficient functions for inclusive structure functions
  $\Delta\alpha_s(M_Z^2) \simeq 1\%$ from truncation of DIS perturbation series
- On-shell form factors of quarks and gluons to higher order
  approx. N^{3}LO for (total) cross section for Higgs production
- Threshold resummation of soft gluons to higher accuracy
  universal coefficients $B_{q,g}$, $D_{\mathrm{DY/Higgs}}$ to $O(\alpha_s^3) \to$ N^{3}L logs

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Partons and observables via complex Mellin space

Precision benchmarks, efficient higher-order analyses of data