Transverse momentum distributions for Standard Model boson production at the LHC

Anna Kulesza (DESY)

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Introduction

- LHC as W, Z factory
  - large cross sections for W/Z production processes

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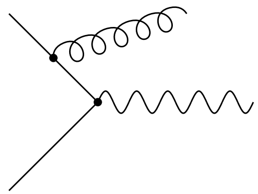
  - later: W/Z production as a possible luminosity monitor
- LHC as Higgs discovery machine
  - important to fully exploit the physics potential of the production processes
    $\Rightarrow$ need to know differential distributions

Introduction

Theoretical status

Drell-Yan type processes:



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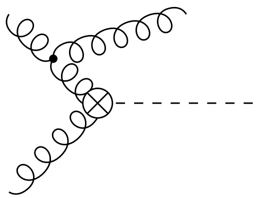
$$q\bar{q}' \rightarrow W^\pm + X$$



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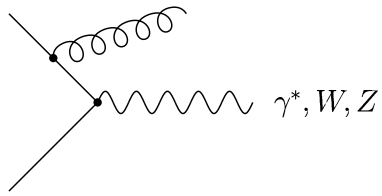

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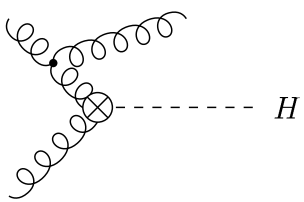

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- QCD corrections $-$ p_{T} distribution known up to NLO $\left(\mathcal{O}(\alpha_s^2)\right)$ correction
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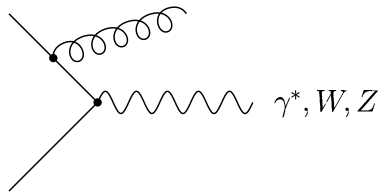

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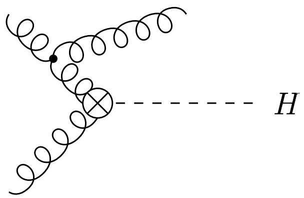

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- EW corrections – NLO ($\mathcal{O}(\alpha)$) virtual corrections to p_T distributions of γ/Z
  [*Maina,Moretti, Ross'04*] [*Kühn, A.K., Pozzorini, Schulze'05*]

  recently, also effects of real weak boson emission analysed [*Baur'06*]

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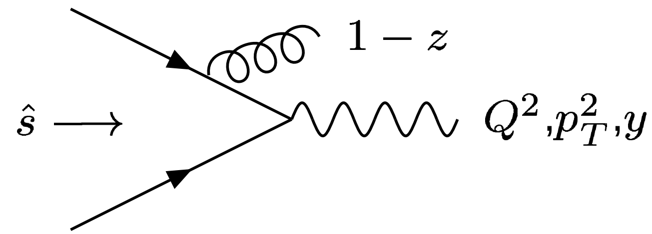
- At large p_{T}, EW effects also important – powers of $\log\left(M_W^2/\hat{s}\right)$

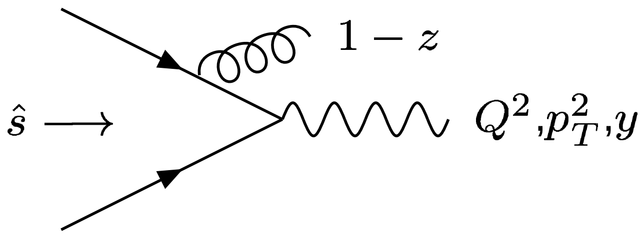
Small p_{T}



$$\left.\frac{d\sigma^{q\bar{q}\to\gamma^*}}{dQ^2\,dp_{\mathrm{T}}^2\,dy}\right|_{p_{\mathrm{T}}\ll Q} = \sigma_0\left(\frac{\alpha_s C_F}{\pi}\right)\frac{1}{p_{\mathrm{T}}}\log\frac{Q^2}{p_{\mathrm{T}}^2}\left[f_q\left(x_1=\sqrt{\frac{Q^2}{\hat{s}}}e^{y},m^2\right)f_{\bar{q}}\left(x_2=\sqrt{\frac{Q^2}{\hat{s}}}e^{-y},m^2\right)+(q\leftrightarrow\bar{q})\right]$$

[*Ellis, Martinelli, Petronzio'81*]

Small p_T



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[*Ellis, Martinelli, Petronzio'81*]

At higher orders:

Structure:	$\mathcal{O}(\alpha_s^n)$	$\alpha_s^n \log^m \left(\frac{p_T^2}{Q^2}\right)$	$0 \le m \le 2n-1$
Origin:	soft/collinear gluon emission		

if $\alpha_s \log^2 \left(p_T^2/Q^2\right) \gtrsim 1$ then traditional fixed-order perturbation theory breaks down

$\Rightarrow$ leads to explicitly divergent cross section in $p_T \to 0$ limit

Small $p_{\rm T}$

Systematic treatment of higher order corrections provided by resummation techniques

[Parisi, Petronzio'79] [Collins, Soper'83] [Collins, Soper, Sterman'85]

$$\delta\left(\mathbf{p}_{\rm T} - \sum_i \mathbf{k}_{\mathbf{T}}^{\mathbf{i}}\right) = \frac{1}{2\pi^2}\int d^2b e^{i\mathbf{b}(\mathbf{p}_{\rm T} - \sum_i \mathbf{k}_{\mathbf{T}}^{\mathbf{i}})}$$

Higher order corrections are summed in the Fourier conjugate to $p_{\rm T}$, b space

$$\ln(p_{\rm T}^2/Q^2) \leftrightarrow \ln(b^2 Q^2)$$

Small p_{T}: resummation

CSS formalism: resummation of all terms in the perturbation series which are as singular as $1/p_{\mathrm{T}}^2$ when $p_{\mathrm{T}} \to 0$. [Collins,Soper,Sterman'85]

$$\frac{d\sigma^{\mathrm{res}}}{dp_T^2} = \frac{\tau}{2} \int db J_0(p_{\mathrm{T}} b)\, W(b,Q) + Y_{\mathrm{fin.}} \qquad \tau = Q^2/s$$

$$W(b,Q) = \sum_{a,b} \sigma_0 \int_0^1 dx_a dx_b \delta(x_a x_b - \tau) \int_{x_a}^1 \frac{dz_a}{z_a} \int_{x_b}^1 \frac{dz_b}{z_b} \mathcal{C}_{i/a}(\frac{x_a}{z_a}, b_0/b) f_{a/A}(z_a, b_0/b)$$
$$\times\, \mathcal{C}_{j/b}(\frac{x_b}{z_b}, b_0/b) f_{b/B}(z_b, b_0/b)\ \exp\left[\,\mathcal{S}_{ij}(b,Q)\,\right]$$

with $\mathcal{S}_{ij}(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) A_i(\alpha_S(\bar{\mu}^2)) + B_i(\alpha_S(\bar{\mu}^2))\right]$

Functions $\mathcal{F} = A, B, \mathcal{C}$ have perturbative expansion $\mathcal{F} = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{F}^{(n)}$

Known coefficients (NLL):

$$A_i^{(1)} = C_i \qquad C_i = C_{F/A} \text{ for } i = q/g$$
$$A_i^{(2)} = \frac{1}{2} C_i K \qquad K = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9} T_R N_F$$
$$B_q^{(1)} = -\frac{3}{2} C_F \qquad B_g^{(1)} = -\frac{1}{6}(11 C_A - 4 T_R N_f)$$

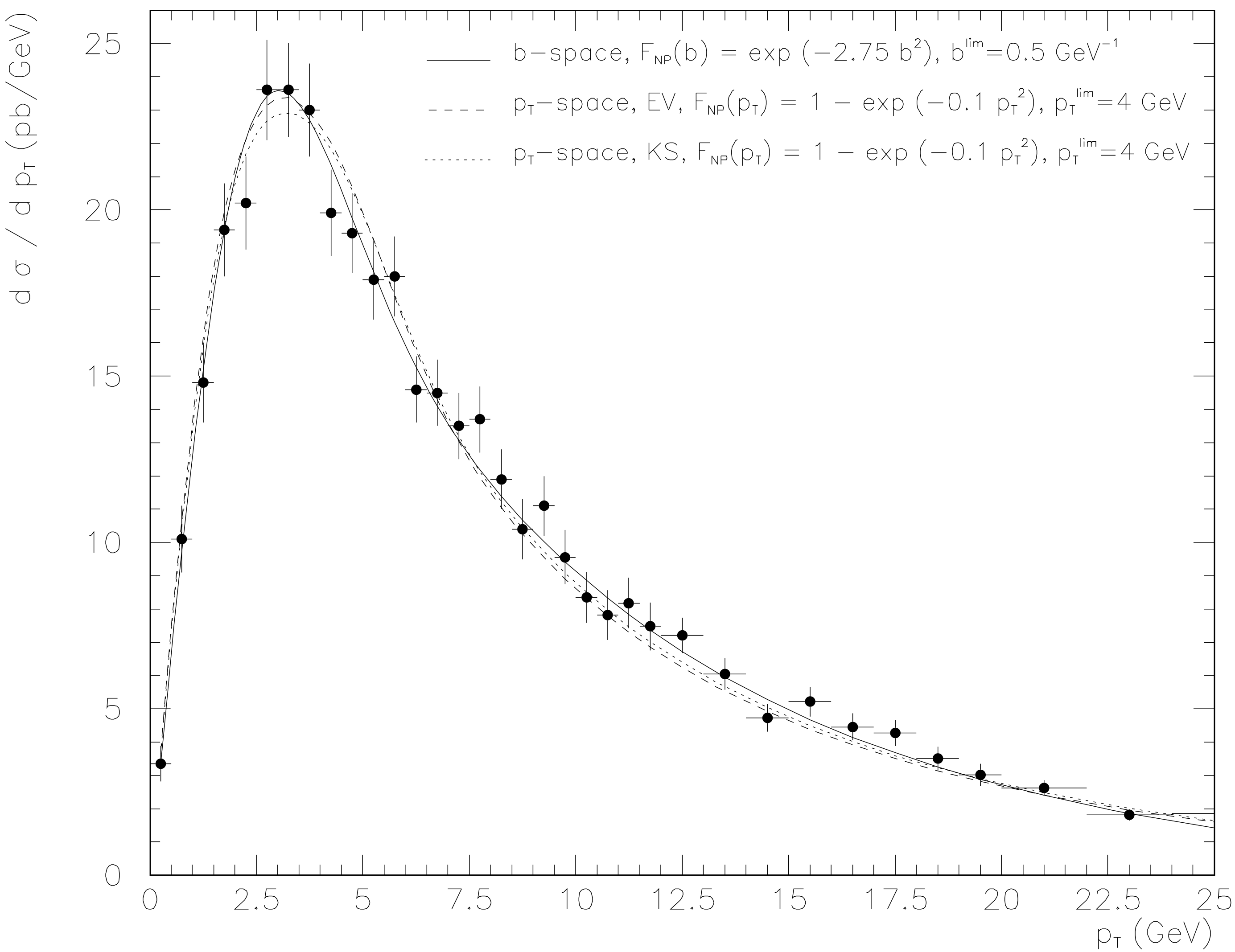
[Kodaira, Trentadue '82] [Catani, D'Emilio, Trentadue '88]

Also known NNLL coefficients $B_i^{(2)},\ A_i^{(3)}$

[Davies, Stirling'84] [de Florian, Grazzini'00] [Moch, Vermaseren, Vogt'04]

Small p_T: resummation

W/Z production at the Tevatron: theory vs data (CDF) [A.K., Stirling'01]



Small p_{T}: resummation

Resummation involves integration of the running coupling over the Landau pole:
Integration over b from 0 to ∞ $\Rightarrow$ $\alpha_s(1/b)$ large when $b \to 1/\Lambda$

: non-perturbative effects

$\Rightarrow$ introduce an additional function $F^{NP}(Q, b, x_A, x_B)$ in the integrand to suppress the Sudakov factor

$$\exp(\mathcal{S}(b,Q)) \to \exp(\mathcal{S}(b,Q))F^{NP}(Q,b,x_A,x_B)$$
$$F^{NP}(Q,b,x_A,x_B) = e^{-g(Q,b,x_A,x_B)b^2}$$

[Collins, Soper, Sterman'85]

determined from fits to Drell-Yan/Z data

$\Rightarrow$Ambiguity in definition of perturbative series: pdf's and the Sudakov factor defined at

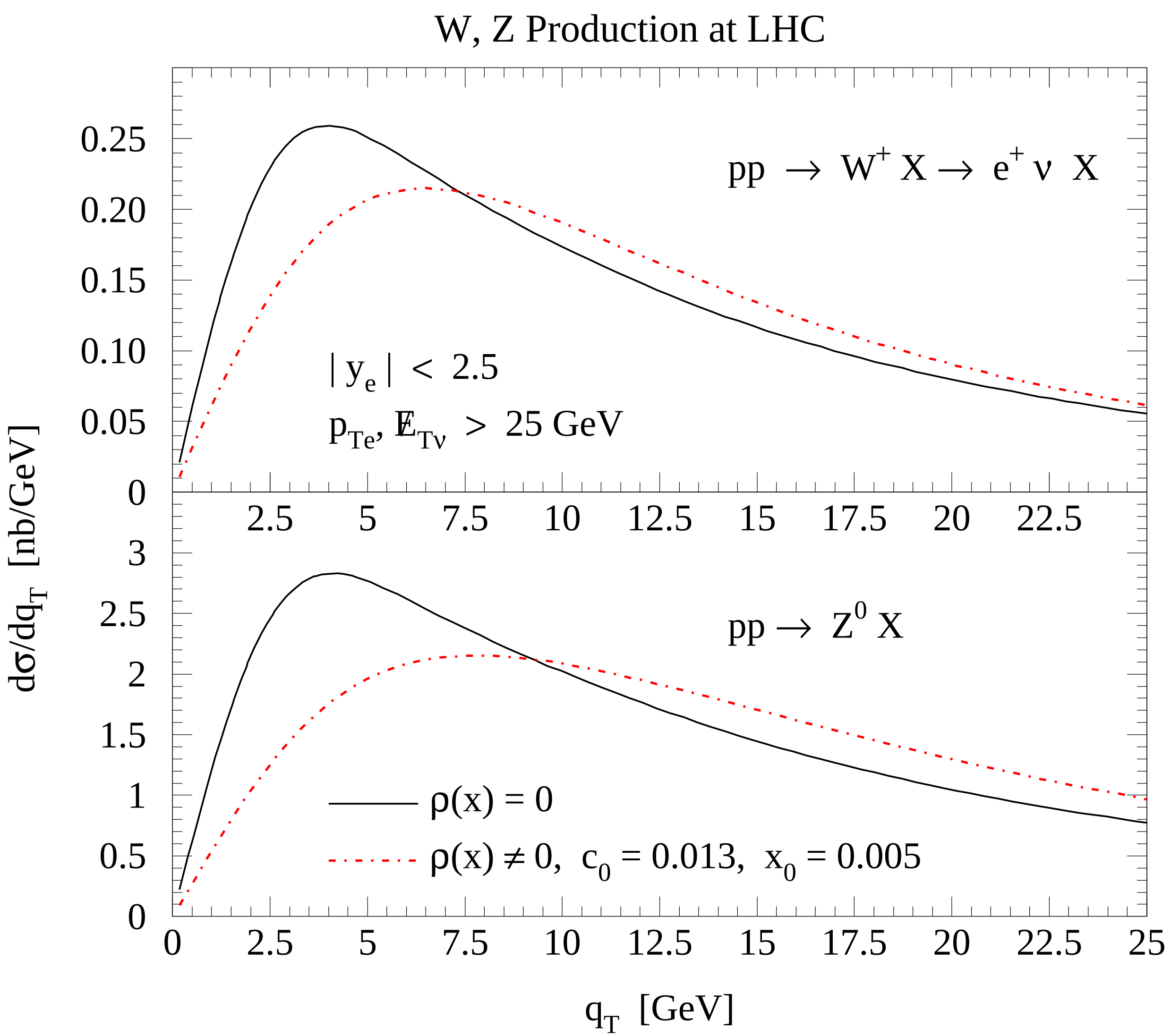
$$b_* = \frac{b}{\sqrt{1+(b/b_{\mathrm{lim}})^2}} \qquad\qquad b_* < b_{\mathrm{lim}}$$

Other ways to avoid the Landau pole

- return to p_{T}-space [Ellis, Veseli'97] [A.K., Stirling'99-'01]
- deforming b-integration into contour over the complex b-plane [Laenen, Sterman, Vogelsang'00] [A.K., Sterman, Vogelsang'02] [Bozzi, Catani, de Florian, Grazzini'05]
- smooth extrapolation of the perturbative result to large b region [Qiu, Zhang'01]

W/Z production at the LHC

[*Berge, Nadolsky, Olness, Yuan'05*]



W/Z production at the LHC

Recent proposal:

$$F^{NP} \to F^{NP} e^{-\left(\rho(x_a)+\rho(x_b)\right)b^2}$$

with

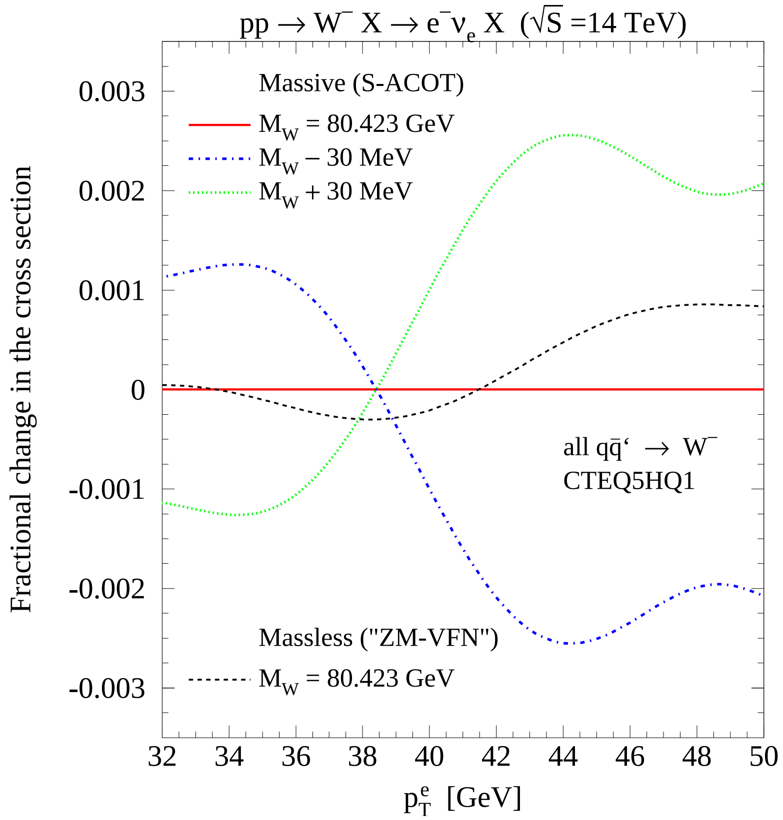
$$\rho(x) = c_0 \sqrt{\frac{1}{x^2} + \frac{1}{x_0^2}} - \frac{1}{x_0} \qquad c_0 \sim 0.013 \qquad x_0 \sim 0.005$$

$$\rho(x) \sim \frac{c_0}{x} \text{ for } x \ll x_0$$

- Motivated by SIDIS data - description requires broadening of this form (Theoretical motivation: BFKL-like $\log(\frac{1}{x})$ terms)
- For the Tevatron: change in M_W
  - by 10-20 MeV in the central region $|y| < 1$
  - by 50 MeV in the forward region $|y| > 1$
- More analysis needed: coming soon [*Dasgupta et al.'06*]

W/Z production at the LHC

Effect of the quark masses (c, b) on the resummed distributions for W, Z at the LHC
[*Berge et al.'05*]



- $M_T^{e\nu}$ method of determining M_W - not sensitive
- p_T^e method - shift up to 10 MeV

Joint resummation

Threshold logarithms $z = Q^2/\hat{s}$

$$\hat{\sigma}(z) = \alpha_S\,[c^z_{11}\,\mathcal{D}_1(z) + c^z_{10}\,\mathcal{D}_0 + c^z_{1\delta}\,\delta(1-z) + c^z_1] + \alpha_S^2\,[c^z_{23}\,\mathcal{D}_3(z) + \ldots]$$

$$\mathcal{D}_i(z) = \left(\frac{\ln^i(1-z)}{1-z}\right)_+$$

Threshold resummation in Mellin moment N space, $\log(1-z) \leftrightarrow \log N$

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Threshold resummation in Mellin moment N space, $\log(1-z) \leftrightarrow \log N$

Possible to resum both recoil and threshold logarithms within one formalism

[*Laenen, Sterman, Vogelsang'00*]

$$\mathcal{S}(N, b, Q) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[A_i(\alpha_S(k_T)) \ln\left(\frac{Q^2}{k_T^2}\right) + B_i(\alpha_S(k_T)) \right]$$

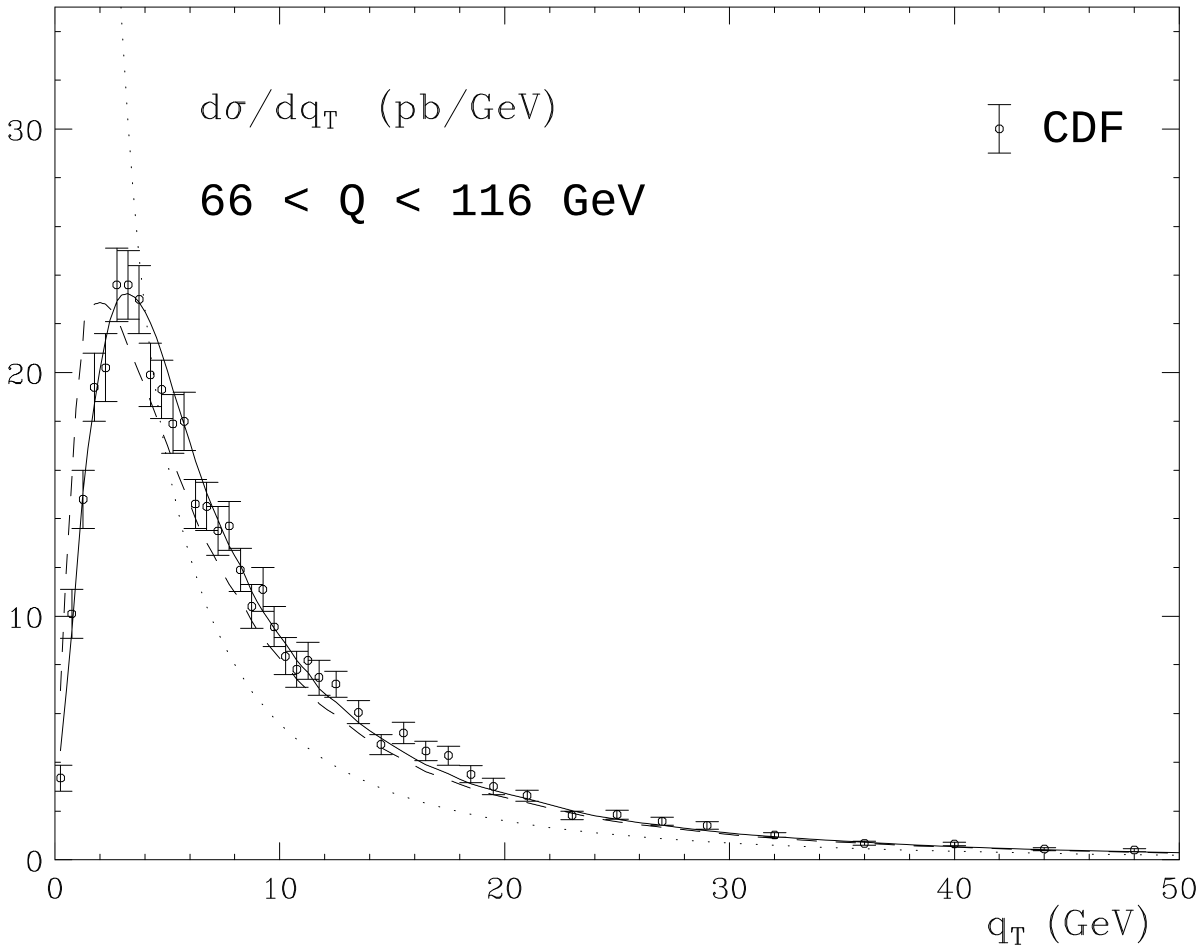
with $\chi = \chi(b, N)$

In the limit $b \gg N$ recoil resummation recovered

$N \gg 1$ ($b = 0$) threshold resummation recovered

Joint resummation

CDF data [*Affolder et al.'00*] compared to joint resummation prediction on Z production

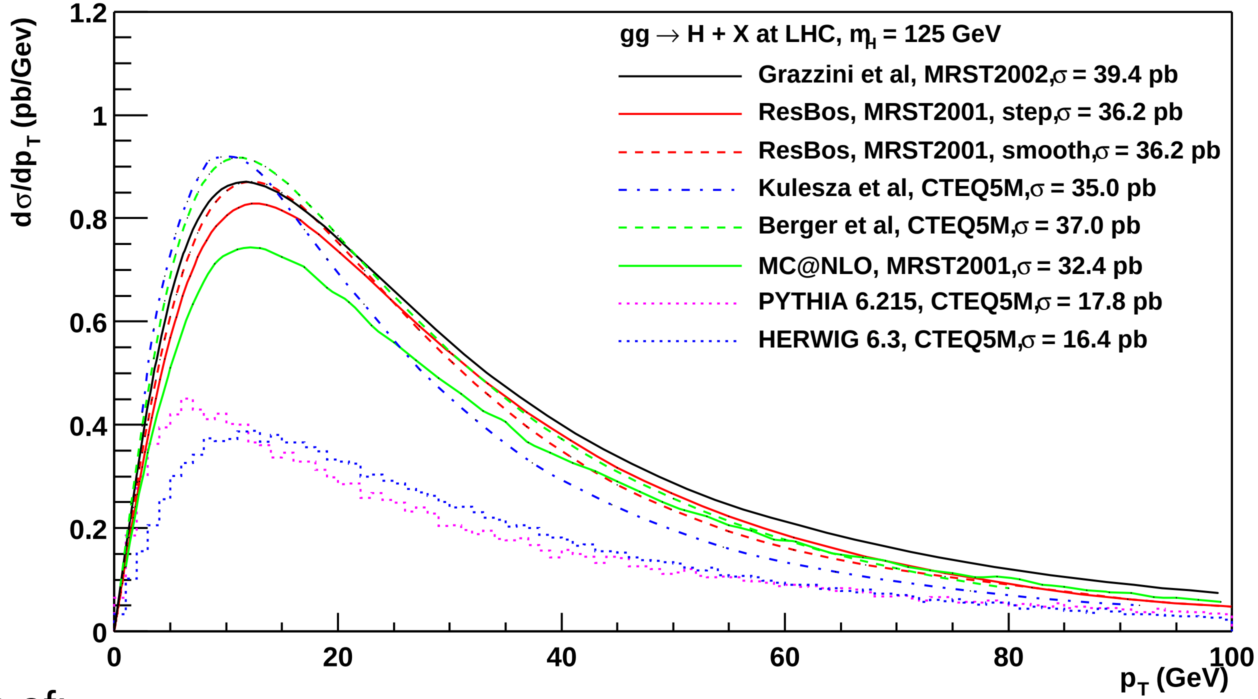


[*A.K., Sterman, Vogelsang'02*]

NLL matched to NLO perturbation theory

$\Rightarrow$ threshold effects modest

Higgs production via gluon fusion at the LHC



Comparison of:

NNLL+NLO	b-space with constraint: $\int dp_{\rm T} \frac{d\sigma^{\rm NNLO}}{dp_{\rm T}} = \sigma^{\rm NNLO}$	[*Bozzi et al.'03'05*]
"Sudakov" NNLL + LO	b-space	[*Berger, Qiu'02*]
"Sudakov" NNLL + LO	joint	[*A.K., Sterman, Vogelsang'03*]
"Sudakov" NNLL + NLO	b-space	[*Balazs, Yuan'00*]
MC@NLO	LO $p_{\rm T}$-distribution + parton shower	
PYTHIA	with hard matrix el. corrections	
HERWIG	without hard matrix el. corrections	

Resummation and unintegrated pdfs

Higgs production via gluon fusion at the LHC

[Gawron, Kwieciński'03]

$$\frac{d\sigma}{dQ^2 dy dp_{\mathrm{T}}^2} = \frac{\sigma_0 Q^2}{\tau s} \pi \delta(Q^2 - M^2) \int \frac{d^2\mathbf{k_1} d^2\mathbf{k_2}}{\pi^2} f_g(x_1, k_1, Q) f_g(x_2, k_2, Q) \delta^{(2)}(\mathbf{k_1} + \mathbf{k_2} - \mathbf{p}_{\mathrm{T}})$$

- $f(x, k_T, Q)$ obtained through numerical solution of the (approximated) CCFM equations
- Correspondance to the CSS resummed formulae shown at the level of $A^{(1)}$, $B^{(1)}$
- Subleading (remaining NLL, NNLL) effects not included
- similar analysis for W production [Kwieciński, Szczurek'03]

Large p_{T}: fixed order

Threshold corrections: here for Higgs production via gluon fusion

$$\frac{d\hat{\sigma}_{ij}}{dp_T^2 dy_H} = \frac{\sigma_0}{\hat{s}}\left[\frac{\alpha_S}{2\pi}G_{ij}^{(1)} + \left(\frac{\alpha_S}{2\pi}\right)^2 G_{ij}^{(2)} + \ldots\right]$$

$G_{ij}^{(1)}$, $G_{ij}^{(2)}$ known [*Glosser, Schmidt'02*]

$\frac{d\sigma}{dp_{\mathrm{T}}^2}$ is a function of the variable $\hat{y}_T$ [*de Florian, A.K., Vogelsang'05*]

$$\hat{y}_T = \frac{p_T + m_T}{\sqrt{\hat{s}}} \qquad\qquad m_T = \sqrt{m_H^2 + p_T^2}$$

$\rightarrow$ measures distance from partonic threshold for production of a massive particle (m_H) with transverse momentum p_T

$\hat{y}_T = 1$ partonic threshold

$$G_{ij}^{(1)} \rightarrow \frac{d\hat{\sigma}_{ab}^{(1)}}{dp_T^2} = \sigma_0 \frac{\alpha_S}{2\pi}\frac{\mathcal{N}_{ab}(\hat{y}_T, p_T)}{p_T^2\sqrt{1-\hat{y}_T^2}}$$

$\mathcal{N}_{ab}(\hat{y}_T, p_T)$ regular function of $\hat{y}_T$

$$G_{ij}^{(2)} \rightarrow \frac{d\hat{\sigma}_{ab}^{(2)}}{dp_T^2} = \frac{\alpha_S}{2\pi}\frac{d\hat{\sigma}_{ab}^{(1)}}{dp_T^2}\left[g_2(p_T)\ln^2(1-\hat{y}_T^2) + g_1(p_T)\ln(1-\hat{y}_T^2) + g_0(p_T)\right] + f(p_T, \hat{y}_T)$$

$f(p_T, \hat{y}_T)$: terms vanishing in the limit $\hat{y}_T \rightarrow 1$

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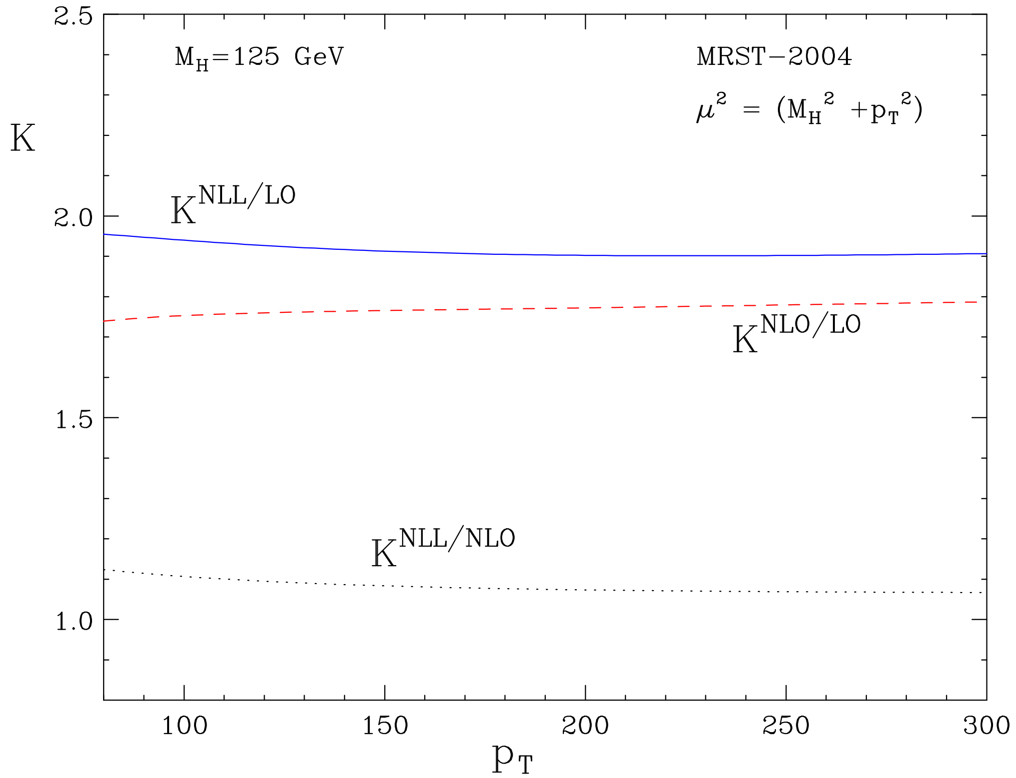
- Resummation takes place in the Mellin (N) moment space

Large p_T: Higgs p_T distribution at the LHC

NLL resummation matched to NLO fixed-order result

[de Florian, AK, Vogelsang'05]



$$K^{NLL/LO} = \frac{d\sigma^{NLL}/dp_T}{d\sigma^{LO}/dp_T}$$

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- 10% correction in the considered p_T range
- Reduced scale dependence for the resummed result

EW corrections at large p_T

- EW corrections: naively expected to be small

$$\begin{array}{ccc} \mathcal{O}(\alpha) & \sim & \mathcal{O}(\alpha_s^2) \\ \mathrm{NLO(EW)} & \sim & \mathrm{NNLO(QCD)} \end{array}$$

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- Systematic enhancements due to logarithmic (Sudakov) terms of the structure

  $\mathcal{O}(\alpha)$: $\alpha \log^2\left(\frac{\hat{s}}{M_W^2}\right)$ leading log (LL)

  $\alpha \log\left(\frac{\hat{s}}{M_W^2}\right)$ next-to-leading log (NLL)

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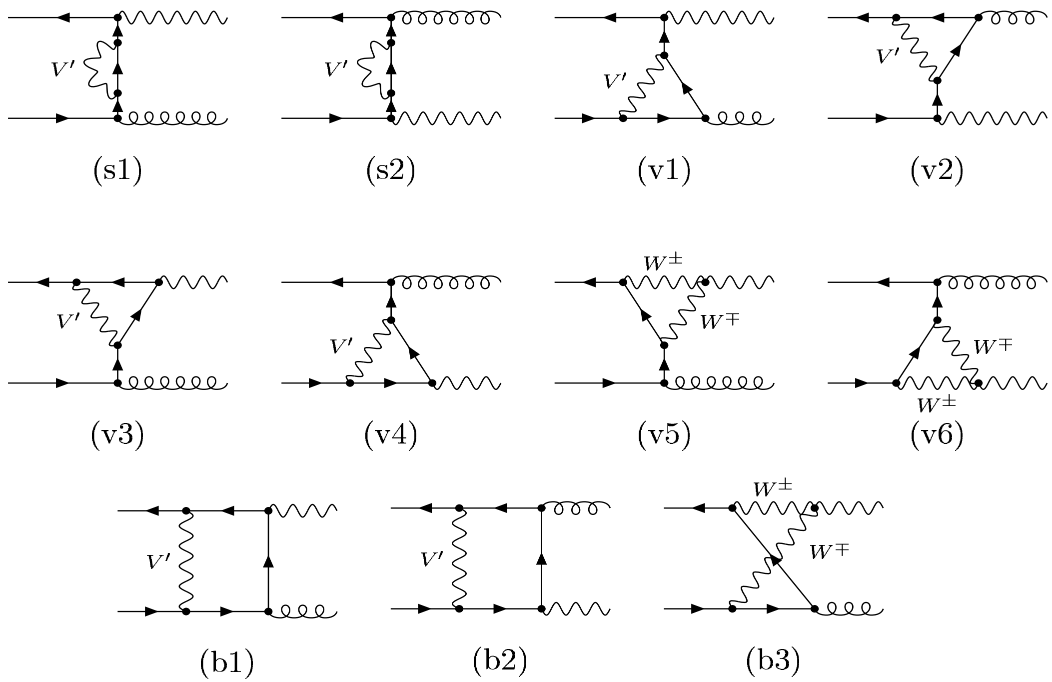
- Origin: soft/collinear emission of virtual *massive* gauge bosons (W, Z)
- Real radiation possible to observe $\Longrightarrow$ no compensation of virtual emission by real radiation
- Finite logarithmic corrections $\Longrightarrow$ different from massless gauge theories such as QCD or QED

$\mathcal{O}(\alpha)$ corrections to $pp \to Z + 1$ jet at the LHC

Nucl. Phys. B727 (2005) 368, hep-ph/0508253



- Analytic, compact expressions for $\mathcal{O}(\alpha)$ weak corrections

$\Rightarrow$ ready to be put into a numerical code!

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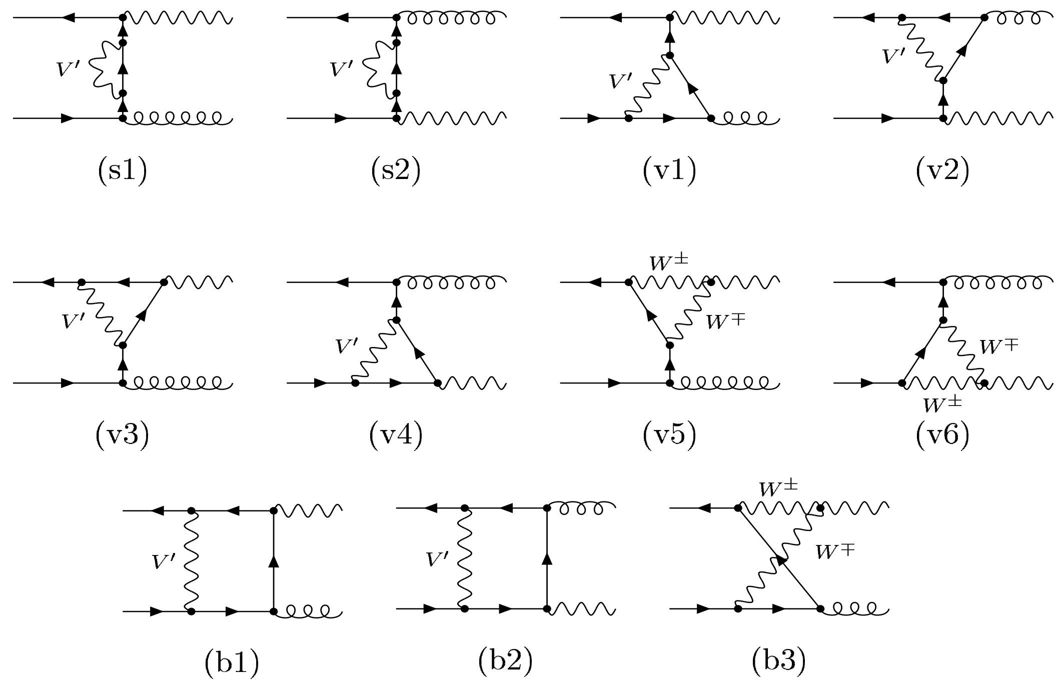
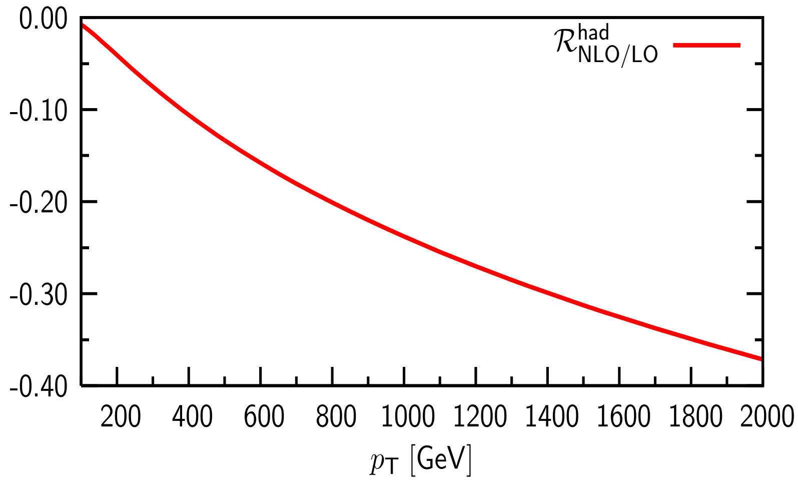


- Analytic, compact expressions for $\mathcal{O}(\alpha)$ weak corrections
- Super-compact high-energy (NNLL) approximation of this result (permille accuracy)

$\Rightarrow$ ready to be put into a numerical code!

$\mathcal{O}(\alpha)$ corrections to $pp \to Z + 1$ jet at the LHC

$$\mathcal{R}^{had}_{\mathrm{NLO/LO}} = \frac{d\sigma_{\mathrm{NLO}}/dp_T}{d\sigma_{\mathrm{LO}}/dp_T} - 1$$



LO MRST2001 pdf's, $\alpha_s(M_Z) = 0.13$, $\mu_F = \mu_R = p_T$
$\overline{MS}$ scheme, $\alpha(M_Z)$=1/127.9, ${s_W}^2 = 0.231$, M_Z= 91.19 GeV, $M_W = c_W M_Z$,
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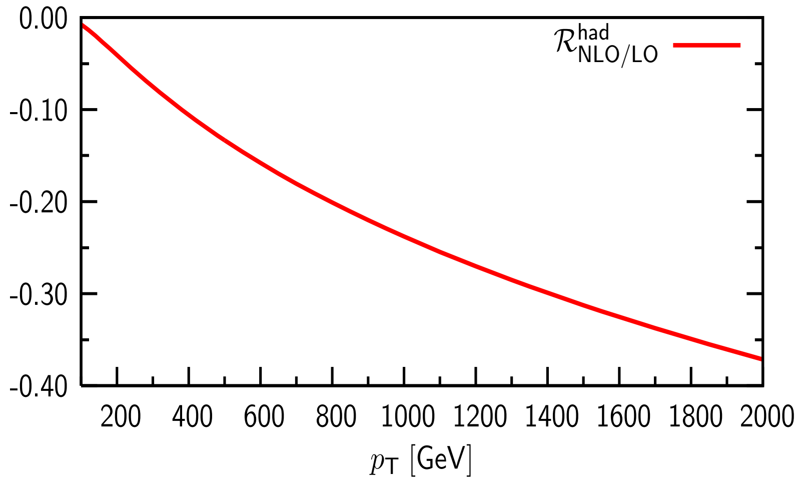
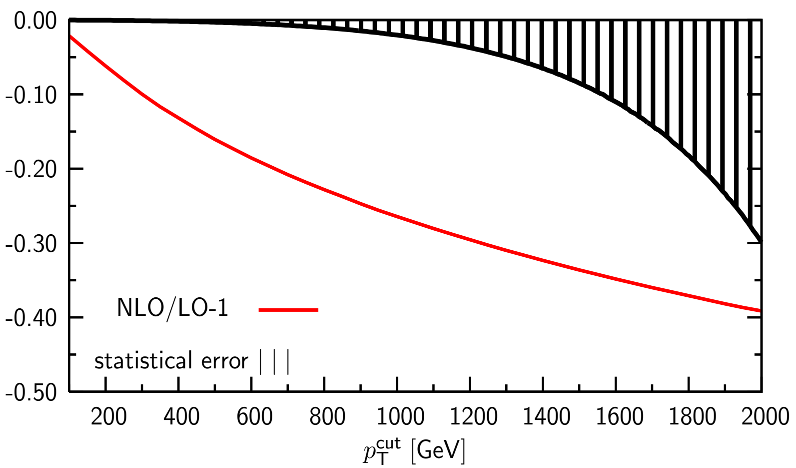


Integrated $\Delta\sigma(p_\mathrm{T}^{\mathrm{cut}})$ vs. $\Delta\sigma_{\mathrm{stat}} = \frac{\sigma}{\sqrt{N}}$

$N = \mathcal{L} \times \mathrm{BR}(Z \to l,\ \nu_l) \times \sigma_{\mathrm{LO}}$

$\mathrm{BR}(Z \to l,\ \nu_l) = 30.6\%$, $\mathcal{L} = 300$ fb^{-1}



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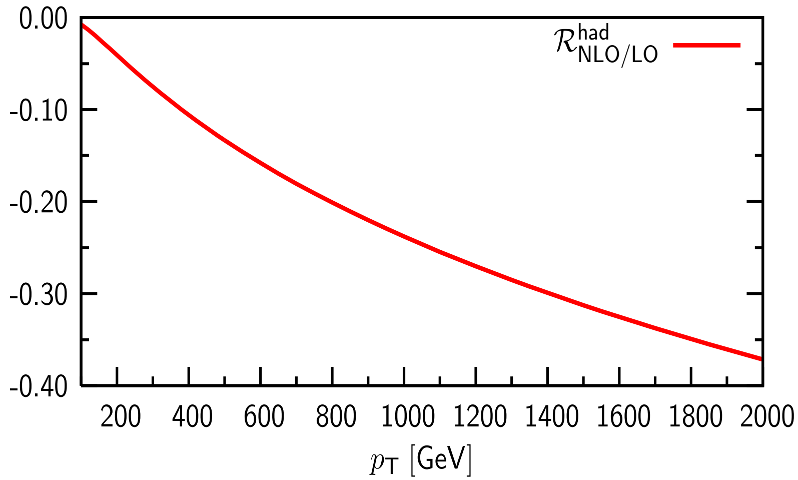

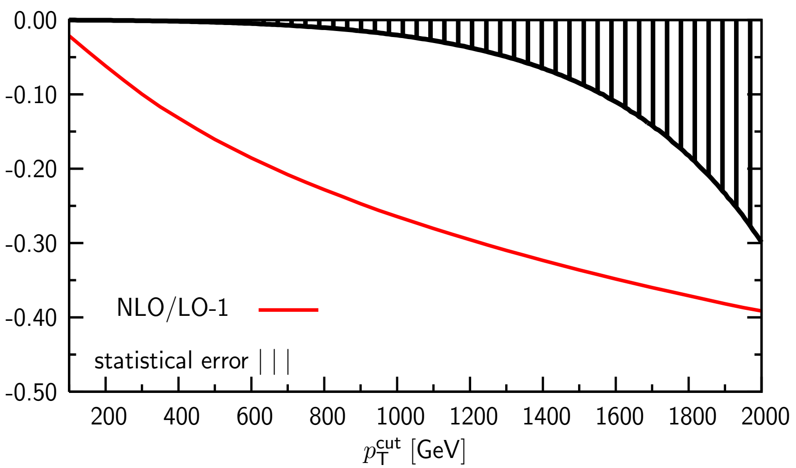

- Corrections negative; range from -13% at 500 GeV up to -37 % at 2 TeV
- Size of the integrated corrections much bigger than the statistical error!

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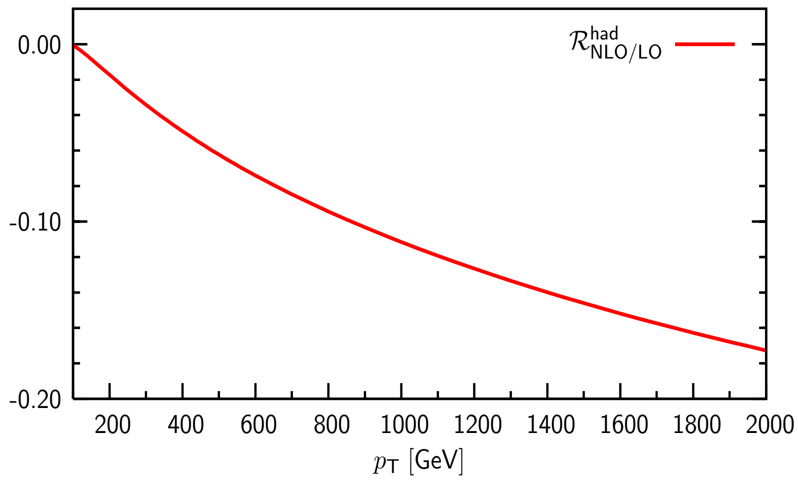

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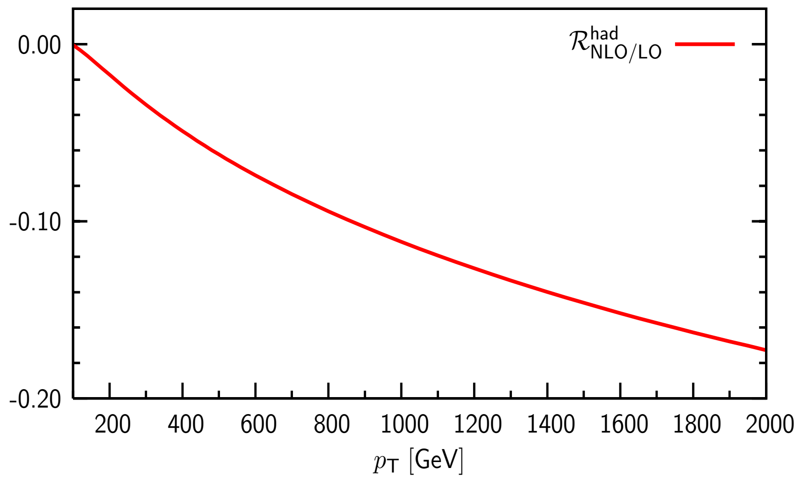

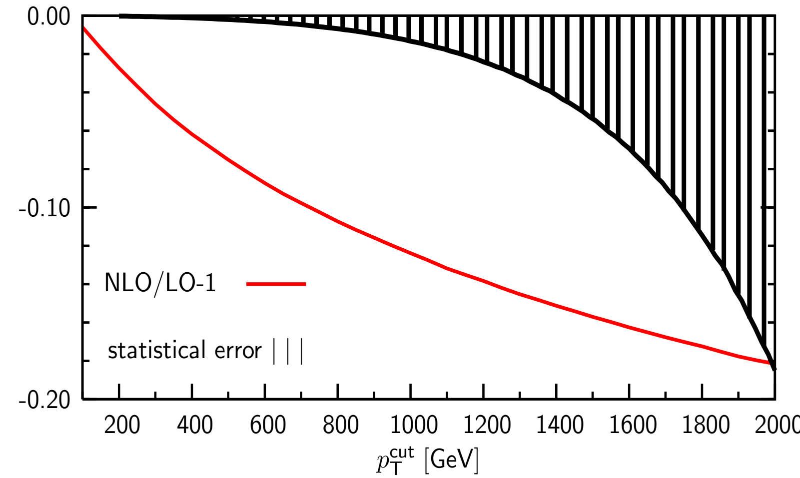

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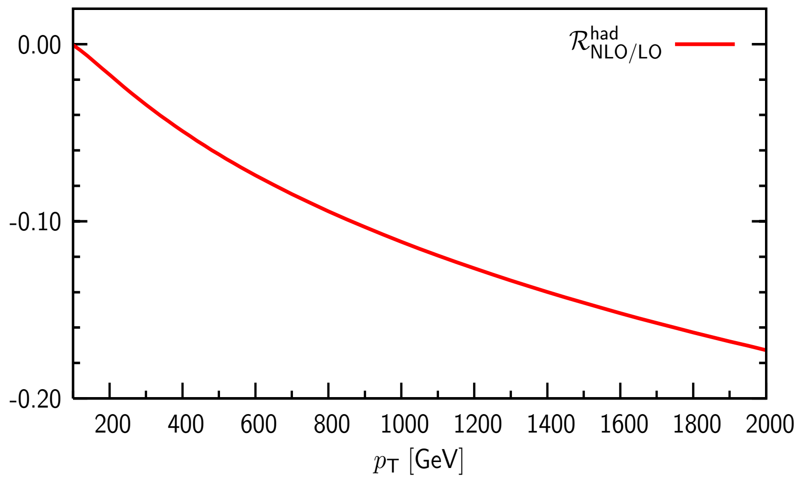

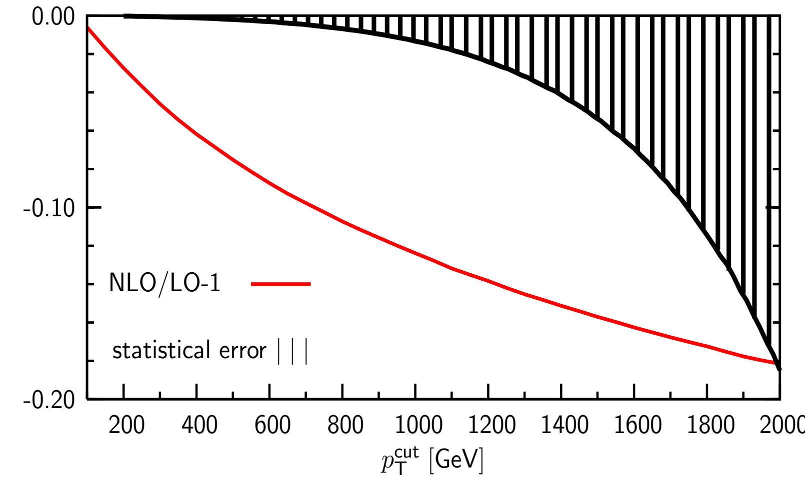

- Corrections negative; range from -6% at 500 GeV up to -17 % at 2 TeV
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NLL approximation

Phys. Lett. B 609 (2005) 277

- Electroweak corrections in the high energy region: $|\hat{r}| \gg M_W^2 \sim M_Z^2$ for $\hat{r} = \hat{s},\ \hat{t},\ \hat{u}$
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Phys. Lett. B 609 (2005) 277

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$\rightarrow$ Calculations based on results available in the literature:

1- loop

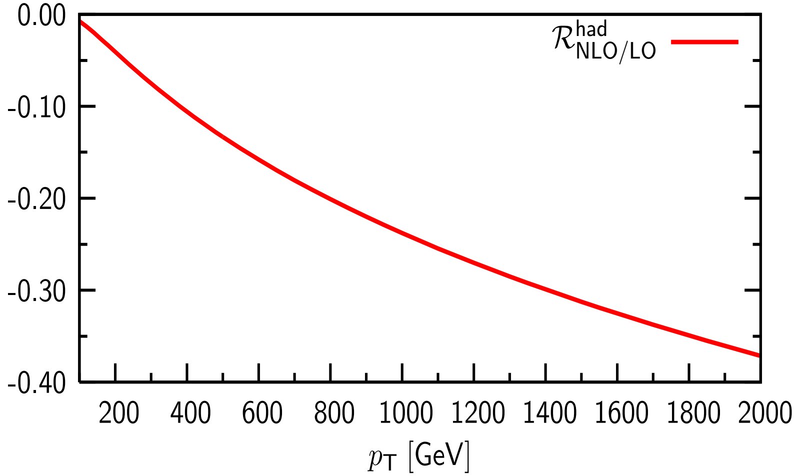
factorization and universality of 1-loop EW corrections at the LL and NLL level $\alpha \log^2\left(\frac{|\hat{r}|}{M_W^2}\right)$, $\alpha \log\left(\frac{\hat{s}}{M_W^2}\right)$ for arbitrary processes [*Denner, Pozzorini'01*]

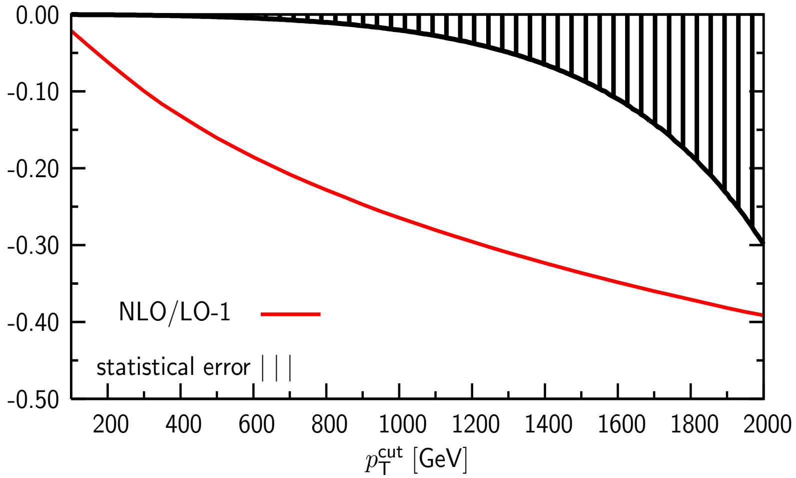
2- loop

LL+NLL terms $\alpha^2 \log^4\left(\frac{|\hat{r}|}{M_W^2}\right)$ from exponentiated 1-loop result [*Denner, Melles, Pozzorini'03*] + NLL terms $\alpha^2 \log^3\left(\frac{\hat{s}}{M_W^2}\right)$ from general resummation formula [*Melles'02,'03*]

NLL approximation: 2-loop results

Large p_T Z-boson production at the LHC





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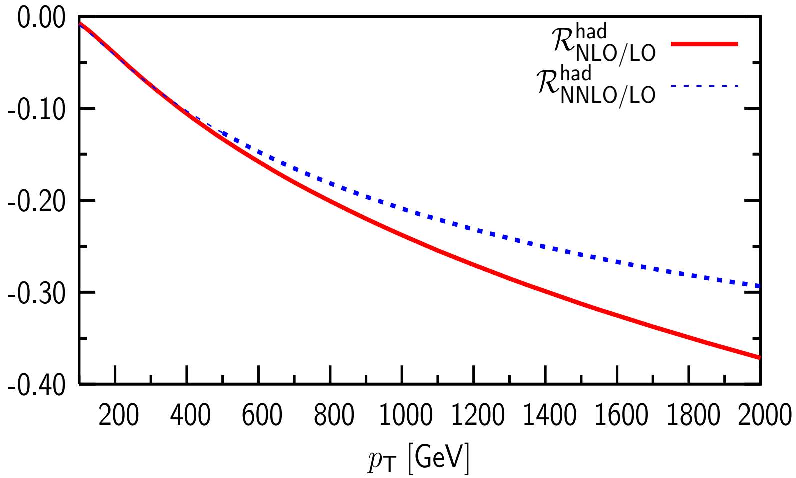

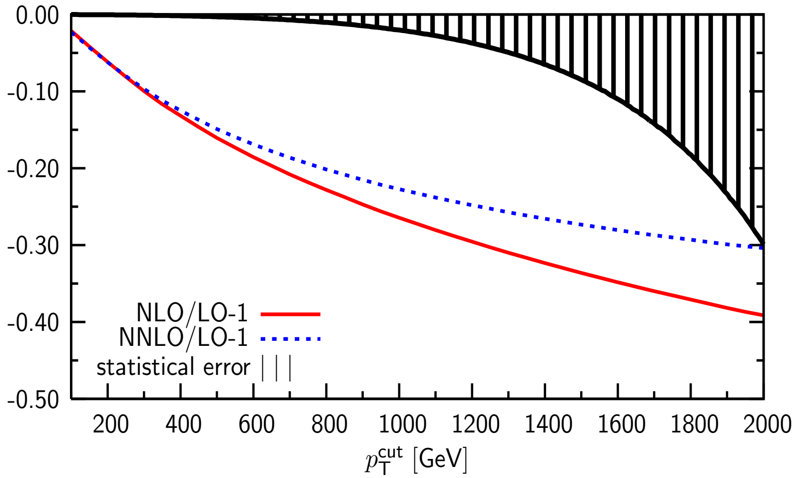

NLL approximation: 2-loop results

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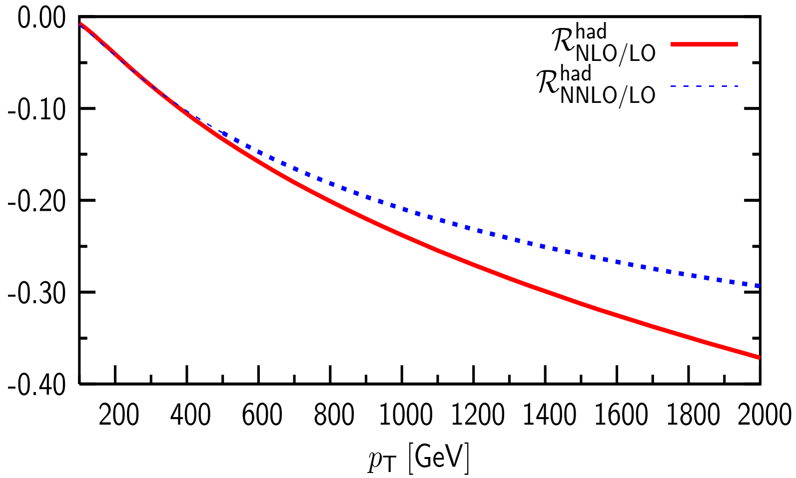
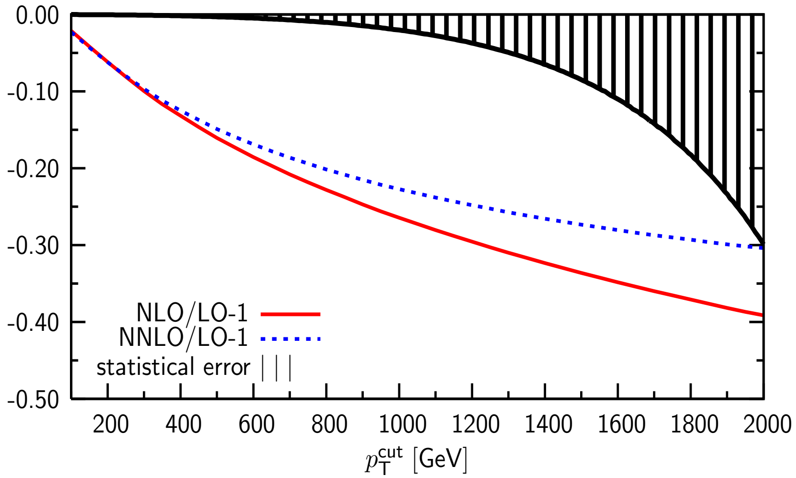
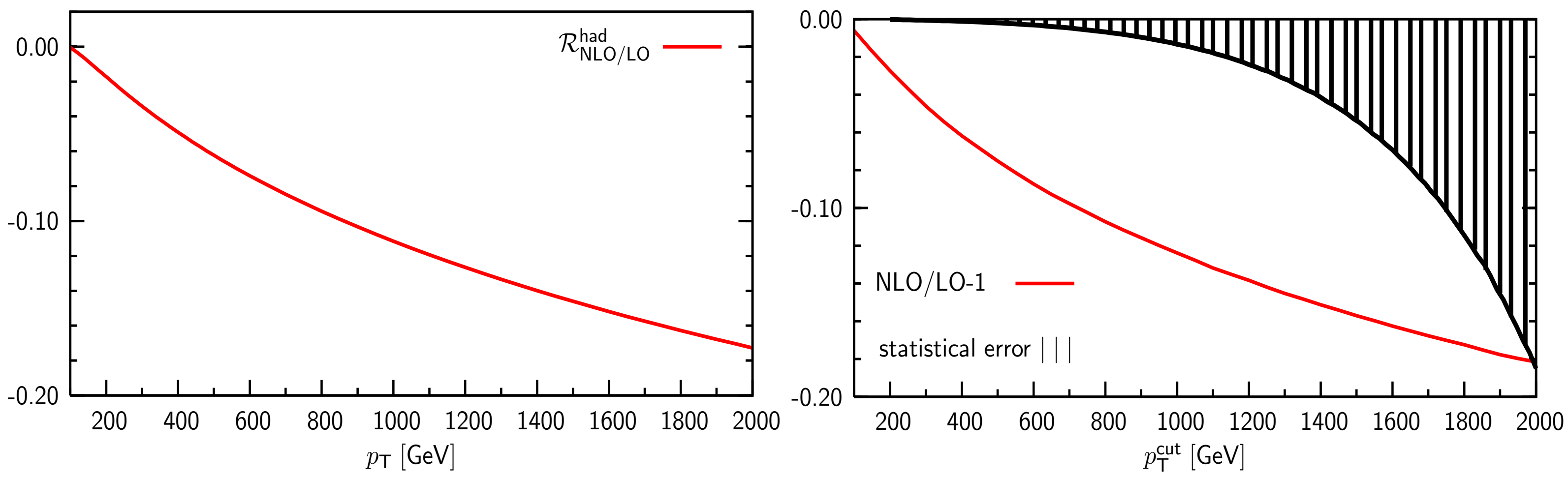




- NLL 2-loop terms positive, up to 8% contribution (at 2 TeV)
- 1-loop + 2-loop NLL amount to up to -30% correction (at 2 TeV)
- For large range of p_T values 2-loop effects comparable with statistical error!

NLL approximation: 2-loop results

Large p_T photon production at the LHC



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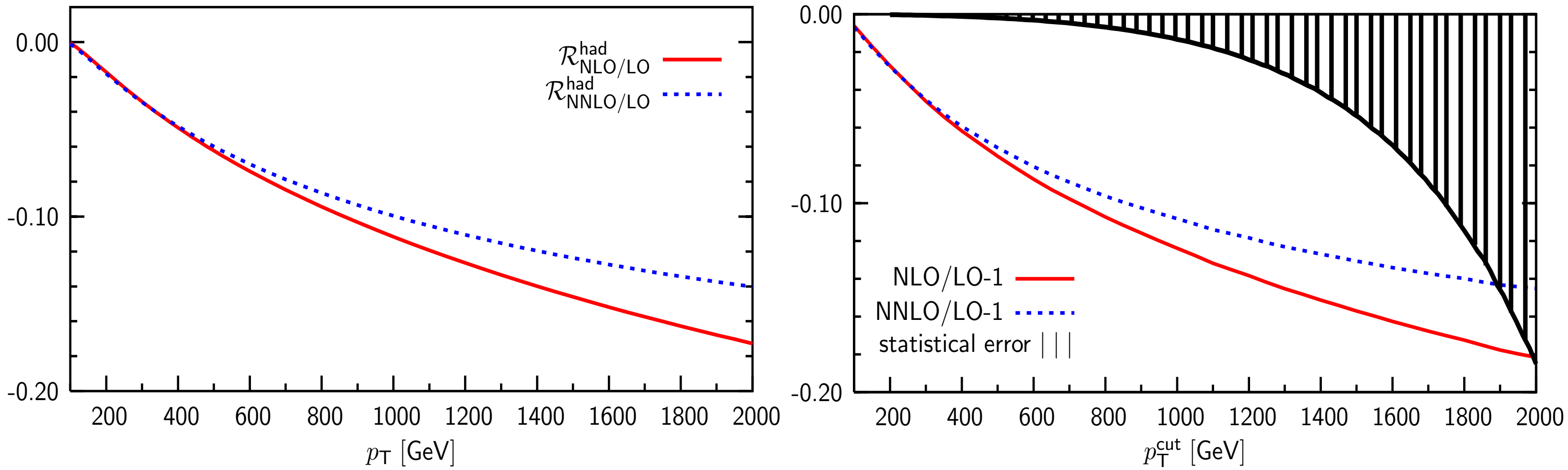

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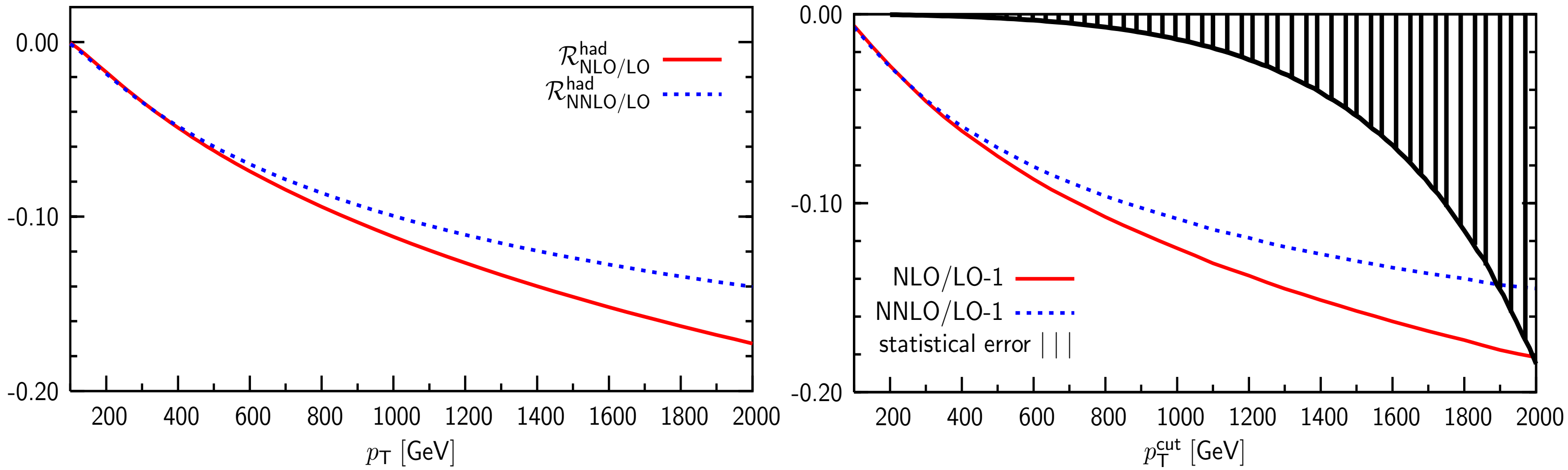
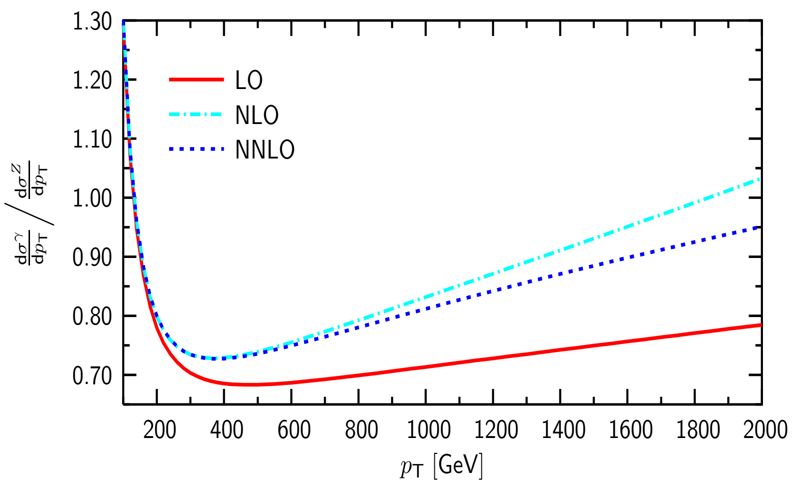


- NLL 2-loop terms positive, up to 3% contribution (at 2 TeV)
- 1-loop + 2-loop NLL amount to up to -14% correction (at 2 TeV)
- For large range of p_T values 2-loop effects comparable with statistical error!

Ratio of the p_T distributions: γ to Z

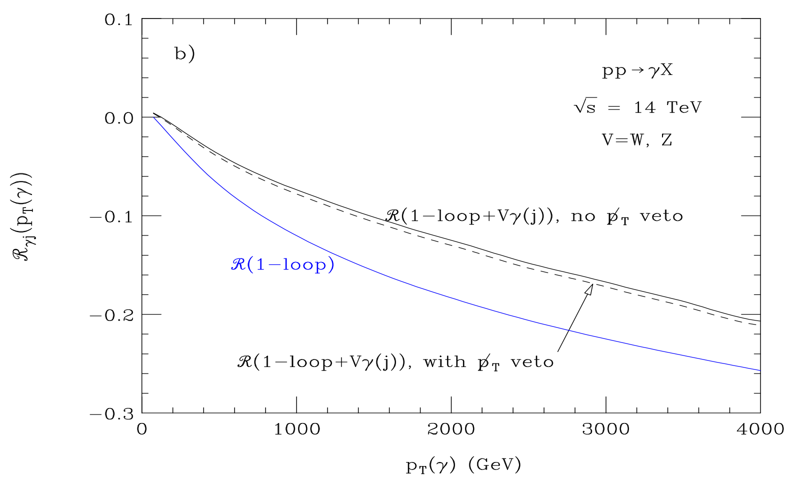


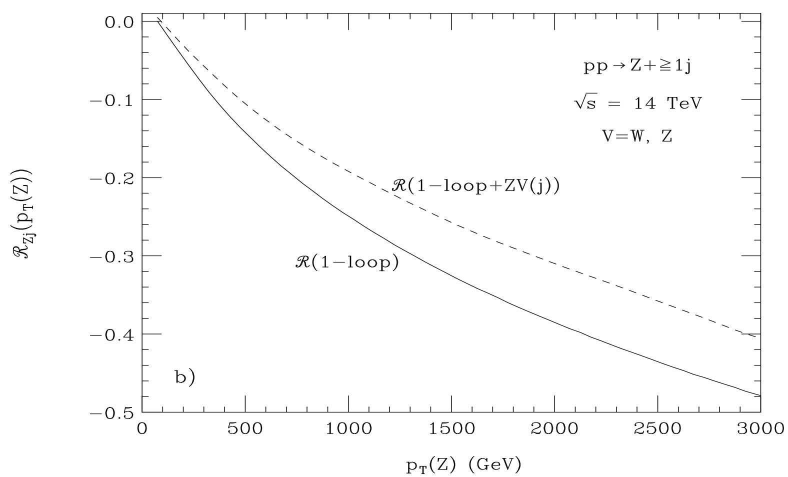
- Cancellation of theoretical uncertainties (pdf's, α_s)
- Stability wrt. QCD corrections
- Ratio of the LO distributions: $\frac{d\sigma^\gamma}{dp_T} / \frac{d\sigma^Z}{dp_T} \sim 0.7 - 0.8$
- Weak corrections modify the ratio; strongest effect at large p_T
  NLO: $\frac{d\sigma^\gamma}{dp_T} / \frac{d\sigma^Z}{dp_T} \sim 0.75 - 1$, NNLO: $\frac{d\sigma^\gamma}{dp_T} / \frac{d\sigma^Z}{dp_T} \sim 0.75 - 0.95$

Effects of real emission

[*Baur, hep-ph/0611241*]

- No real corrections for exclusive processes
- Effects from *real* weak boson emission for inclusive processes
- Violation of Bloch-Norsieck theorem for non-abelian gauge theories $\Longrightarrow$ logarithmic terms survive
- Moderate effects at the LHC





Conclusions

- In the small p_{T} regime QCD resummation techniques are essential for reliable predictions of p_{T} distributions
- Many implementations of the CSS formalism developed (subleading logs, order of perturbative predictions to which resummed calculations are matched, b-integral evaluation)
- Resummed predictions for p_{T} distribution for W/Z and Higgs production at the LHC available
- At higher p_{T} effects of threshold corrections relevant for for Higgs production; NLL resummation gives 10% correction to the NLO p_{T} distribution
- At high p_{T} EW corrections are large and must be included in predictions for vector boson p_{T} distribution at the LHC
- For p_T distribution of Z-bosons and direct photons
  - Analytic results for the full 1-loop $\mathcal{O}(\alpha)$ weak correction
  - NNLL aproximation of the NLO: compact expression, excellent approximation
  - NLL approximation: 1-loop and 2-loop corrections

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Approximate expressions at large p_T: limit $\frac{M_W^2}{\hat{s}} \to 0$

[*Roth, Denner'96*]

NNLL approximation =

LL terms $\sim \alpha \log^2\left(\frac{\hat{s}}{M_W^2}\right)$ + NLL terms $\sim \alpha \log\left(\frac{\hat{s}}{M_W^2}\right)$ + constant terms in this limit

$$H_1^{V,A/N}(M_{V'}^2) \stackrel{\text{NNLL}}{=} \text{Re}\left[g_0^{V,A/N}(M_{V'}^2)\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}} + g_1^{V,A/N}(M_{V'}^2)\frac{\hat{t}^2-\hat{u}^2}{\hat{t}\hat{u}} + g_2^{V,A/N}(M_{V'}^2)\right]$$

$$\begin{aligned}
g_0^{V,A}(M_{V'}^2) &= -\log^2\left(\frac{-\hat{s}}{M_{V'}^2}\right) + 3\log\left(\frac{-\hat{s}}{M_{V'}^2}\right) + \frac{3}{2}\left[\log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right) + \log\left(\frac{\hat{t}}{\hat{s}}\right) + \log\left(\frac{\hat{u}}{\hat{s}}\right)\right] + \frac{7\pi^2}{3} - \frac{5}{2} \\
g_0^{V,N}(M_W^2) &= 2\left[2/(4-D) - \gamma_E + \log\left(\frac{4\pi\mu^2}{M_Z^2}\right) - \delta_{V\gamma}\log\left(\frac{M_W^2}{M_Z^2}\right)\right] + \log^2\left(\frac{-\hat{s}}{M_W^2}\right) - \log^2\left(\frac{-\hat{t}}{M_W^2}\right) - \log^2\left(\frac{-\hat{u}}{M_W^2}\right) \\
&\quad + \log^2\left(\frac{\hat{t}}{\hat{u}}\right) - \frac{3}{2}\left[\log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right)\right] - 2\pi^2 + 2\delta_{VZ}\left(-\frac{\pi^2}{9} - \frac{\pi}{\sqrt{3}} + 2\right) \\
g_1^{V,N}(M_{V'}^2) &= -g_1^{V,A}(M_{V'}^2) + \frac{3}{2}\left[\log\left(\frac{\hat{u}}{\hat{s}}\right) - \log\left(\frac{\hat{t}}{\hat{s}}\right)\right] = \frac{1}{2}\left[\log^2\left(\frac{\hat{u}}{\hat{s}}\right) - \log^2\left(\frac{\hat{t}}{\hat{s}}\right)\right] \\
g_2^{V,N}(M_{V'}^2) &= -g_2^{V,A}(M_{V'}^2) = -2\left[\log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right) + \log\left(\frac{\hat{t}}{\hat{s}}\right) + \log\left(\frac{\hat{u}}{\hat{s}}\right)\right] - 4\pi^2
\end{aligned}$$

$$g_i^{\gamma,A} = g_i^{Z,A} \text{ for } i = 0,1,2 \quad g_j^{\gamma,N} = g_j^{Z,N} \text{ for } j = 1,2$$

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$$\overline{\sum}|\mathcal{M}_1^{\bar{q}q}|^2 = 8\pi^2\alpha\alpha_S(N_c^2-1)\times\sum_{\lambda=\mathrm{R,L}}\Bigg\{ \left(I^V_{q_\lambda}\right)^2\left[H_0^V\left(1+2\delta C^{V,\mathrm{A}}_{q_\lambda}\right)+\frac{\alpha}{2\pi}\sum_{V=\mathrm{Z,W^\pm}}\left(I^{V'}I^{\bar{V}'}\right)_{q_\lambda}H_1^{V,\mathrm{A}}(M^2_{V'})\right] + \frac{U_{VW^3}}{s_W}T^3_{q_\lambda}I^V_{q_\lambda}\left[2H_0^V\delta C^{V,\mathrm{N}}_{q_\lambda}+\frac{\alpha}{2\pi}\frac{1}{s_W^2}H_1^{\mathrm{N}}(M_W^2)\right]\Bigg\}$$

$$H_1^{\mathrm{A/N}}(M_V^2) = \mathrm{Re}\left[\sum_{j=0}^{14}K_j^{\mathrm{A/N}}(M_V^2)\,J_j(M_V^2)\right] \leftarrow \left(\begin{array}{l}\text{independent of}\\ \text{flavour and chirality}\end{array}\right)$$

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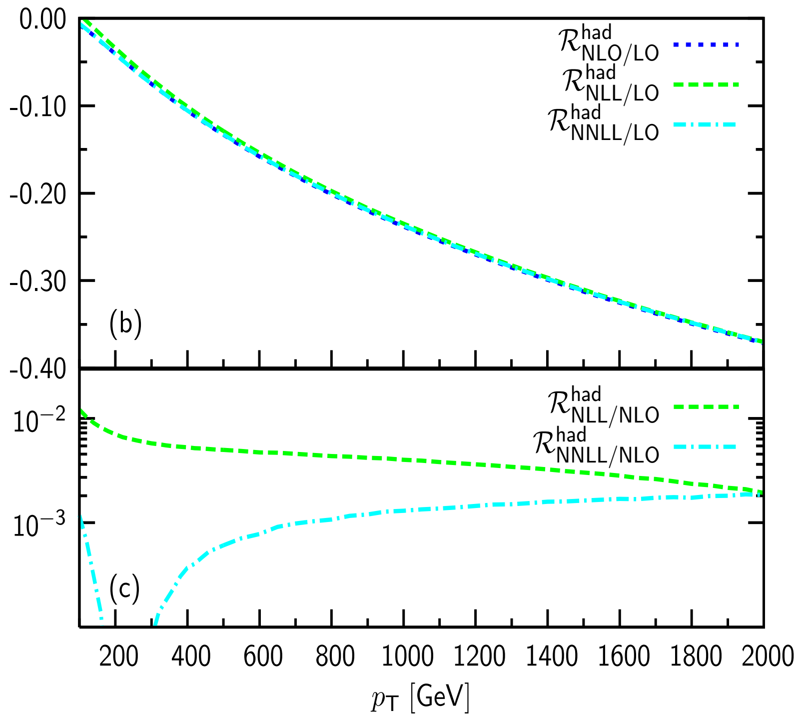
Large p_T Z-boson production at the LHC



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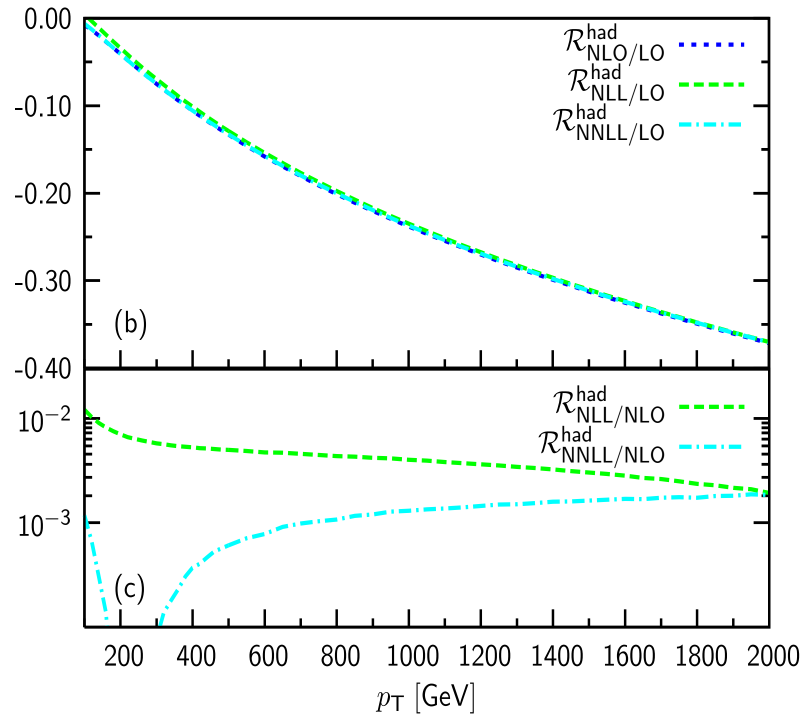
Large p_T Z-boson production at the LHC



$$\mathcal{R}^{had}_{\rm NLO/LO} = \frac{d\sigma_{\rm NLO}/dp_T}{d\sigma_{\rm LO}/dp_T} - 1$$

etc.

- NLL approximation: percent (or better) level

  $\sim$ 1% deviation from NLO at low p_T

  $\sim$ 0.2% deviation from NLO at $p_T = 2$ TeV

- NNLL approximation: permille level

Extra slides

Generic hard scattering process:

$$\hat{\sigma}(z) \sim \hat{\sigma}_0 \left[1 + \sum_{n=1}^{\infty} \int_0^1 dz_1 \ldots dz_n \frac{d\omega_n(z_1, \ldots, z_n)}{dz_1 \ldots dz_n} \Theta_{PS}^{(n)}(z, z_1, \ldots, z_n)\right]$$

* Dynamical factorization(universal, process independent)

QCD: gluon emission correlated (colour charge) but in the soft limit QED-like factorization [*Ermolaev, Fadin '81*] [*Bassetto, Ciafaloni, Marchesini '83*]

$\frac{d\omega_n(z_1,\ldots,z_n)}{dz_1 \ldots dz_n} = \frac{1}{n!} \prod_{i=1}^{n} \frac{d\omega(z_i)}{dz_i}$.

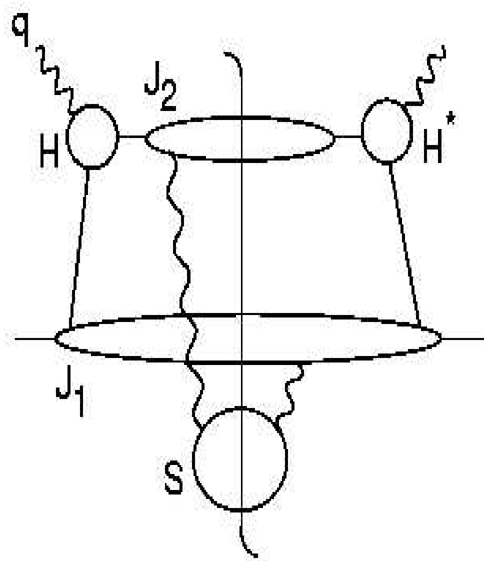
* Phase–space factorization depends on the process: Θ_{PS} contains kinematical constraints defining physical cross section $\Theta_{PS}^{(n)}(z, z_1, \ldots, z_n) = \prod_{i=1}^{n} \Theta_{PS}(z, z_i)$

$$\begin{aligned} \hat{\sigma}(z) &\sim \hat{\sigma}_0 \left\{1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left[\int_0^1 dz_i \frac{d\omega(z_i)}{dz_i} \Theta_{PS}(z, z_i)\right]^n \right\} \sim \hat{\sigma}_0 \exp\left[\int_z^1 dz' \frac{d\omega(z')}{dz'} \Theta_{PS}(z, \right. \\ &\sim \hat{\sigma}_0 \exp\left[\alpha_S L^2 + \ldots\right] \end{aligned}$$

Extra slides

$\Rightarrow$ In general, resummation follows from factorization

(factorization $\Rightarrow$ evolution eq. $\Rightarrow$ exponentiation)



Schematically:

$$\sigma(N) = H(p_1/\mu, p_2/\mu, \zeta_i) S(Q/\mu N, \zeta_i) J_1(p_1\zeta_1/\mu, Q/\mu N) J_2(p_2\zeta_2/\mu, Q/\mu$$

$$\mu\frac{d}{d\mu}\sigma = 0 \Rightarrow \mu\frac{d}{d\mu}\ln H = -\gamma_H \qquad \mu\frac{d}{d\mu}\ln J = -\gamma_J \qquad \mu\frac{d}{d\mu}\ln S = -\gamma_S$$

$$(\gamma_H + \gamma_S + \sum_i \gamma_{Ji} = 0)$$

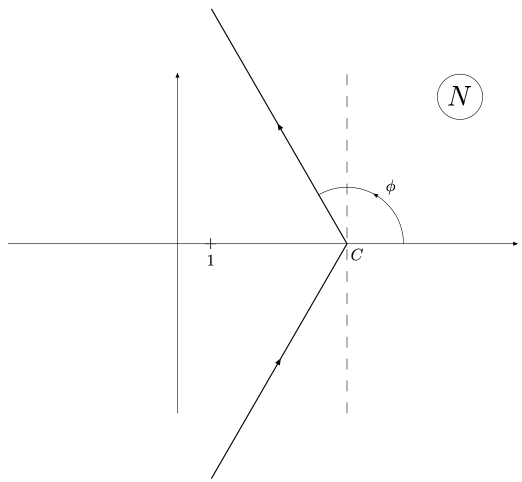
$$S(Q_S/\mu) = S(1)\exp\left[-\int_{Q_S}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_S(\bar{\mu})\right]$$

reorganizing Feynman

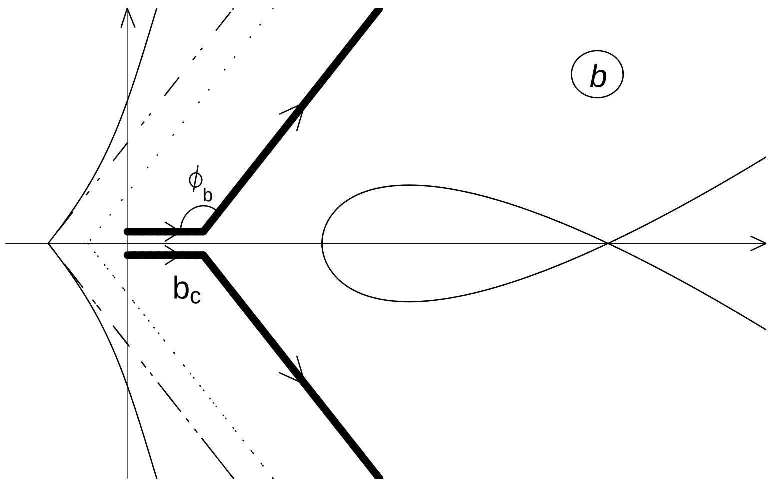
Extra slides

* Inverse $\bar{N}$ and b transforms – integrals along contours in $(\bar{N}, b)$ complex space

N contour:



b contour:



Landau pole:

$$\beta = \frac{1}{2} \quad \Rightarrow \quad \chi(\bar{N}, \bar{b}) = \exp\left[1/(2b_0\alpha_s(\mu))\right] \equiv \rho_L$$

$$\chi(\bar{N}, \bar{b}) = \bar{b} + \frac{\bar{N}}{1 + \frac{\bar{b}}{4\bar{N}}} = \exp\left[1/(2b_0\alpha_s(\mu))\right] \equiv \rho$$

$$\rightarrow \chi = 0 \Rightarrow \bar{b} = -2\bar{N}$$

$$\rightarrow 1 + \frac{\bar{b}}{4\bar{N}} = 0 \Rightarrow \bar{b} = -4\bar{N}$$

$$rightmost\ pdf\ singularity < C < \rho_L$$

$$\int d^2b\, e^{i\vec{Q}_T\cdot\vec{b}} f(b) \;=\; 2\pi \int_0^\infty db\, b\; J_0(bQ_T)\, f(b) \;=$$

$$\pi \int_0^\infty db\, b\, \left[h_1(bQ_T, v) + h_2(bQ_T, v) \right] f(b)$$