"Conditions for detecting CP violation via neutrinoless double beta decay"

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Outline

- Motivation
- $(\beta\beta)_{0\nu}$ - general information.
- How can we determinate Majorana phases from $(\beta\beta)_{0\nu}$?
- Conditions for detecting CP violation via $(\beta\beta)_{0\nu}$ - present situation.
- Conditions for detecting CP violation via $(\beta\beta)_{0\nu}$ - future.
- Conclusions.

Motivation

- $(\beta\beta)_{0\nu}$ gives us possibility for studying the fundamental properties of neutrinos beyond the standard electroweak theory ($\Delta L = 2$).
- Studies of $(\beta\beta)_{0\nu}$ play a crucial role by probing:
    - the Majorana nature of neutrinos,
    - the neutrino mass spectrum,
    - the absolute ν-mass scale,
    - the Majorana CP phases.

CP violation in neutrino oscillations

The charged current neutrino state (ν_α) is related to mass states (ν_i) by an unitary transformation

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle,$$

where

$$U_{\alpha i} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times$$

$$\begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

CP violation in neutrino oscillations

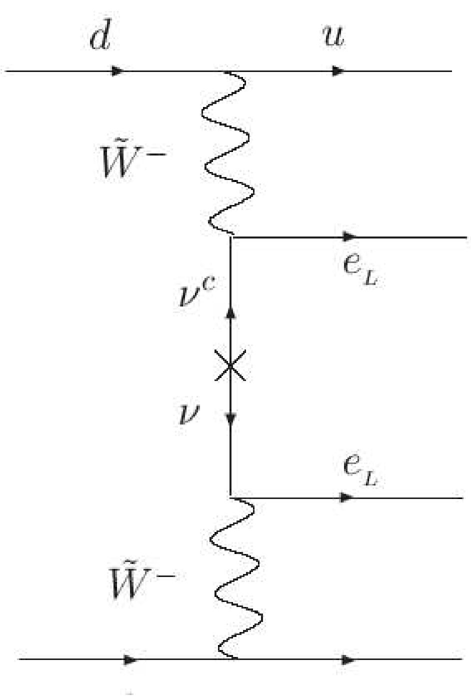
- Dirac neutrinos: A very small or vanishing CP breaking signal due to fact that $\sin\theta_{13}$ and $e^{\pm i\delta}$ always appear in a combination. From the present fits: $\sin^2\theta_{13} < 0.05$ for 99.7% C.L.

  (*A. Bandyopadhyay et al., Phys.Lett.* ***B581****,62, 2004*).

- Majorana neutrinos: Majorana phases do not affect neutrino oscillations.

Neutrinoless double beta decay

- A nuclear process changing the nuclear charge Z by two units while leaving the atomic mass A unchanged.
- It is allowed when neutrino and antyneutrino are identical particles.



Neutrinoless double beta decay

The half-life of $(\beta\beta)_{0\nu}$ decay is given by the expression:

$$\left[T^{0\nu}_{1/2}(A,Z)\right]^{-1} = |\langle m_\nu \rangle|^2 |M^{0\nu}(A,Z)|^2 G^{0\nu}(E_0,Z)$$

- $\langle m_\nu \rangle = |\sum_{i=1}^{3} U_{ei}^2 m_i|$ - the effective Majorana mass,
- $M^{0\nu}(A,Z)$ - nuclear matrix element (NME) - determined only by nuclear properties, doesn't depend on neutrino masses and mixing,
- $G^{0\nu}(E_0,Z)$ - phase-space factor.

Nuclear matrix element

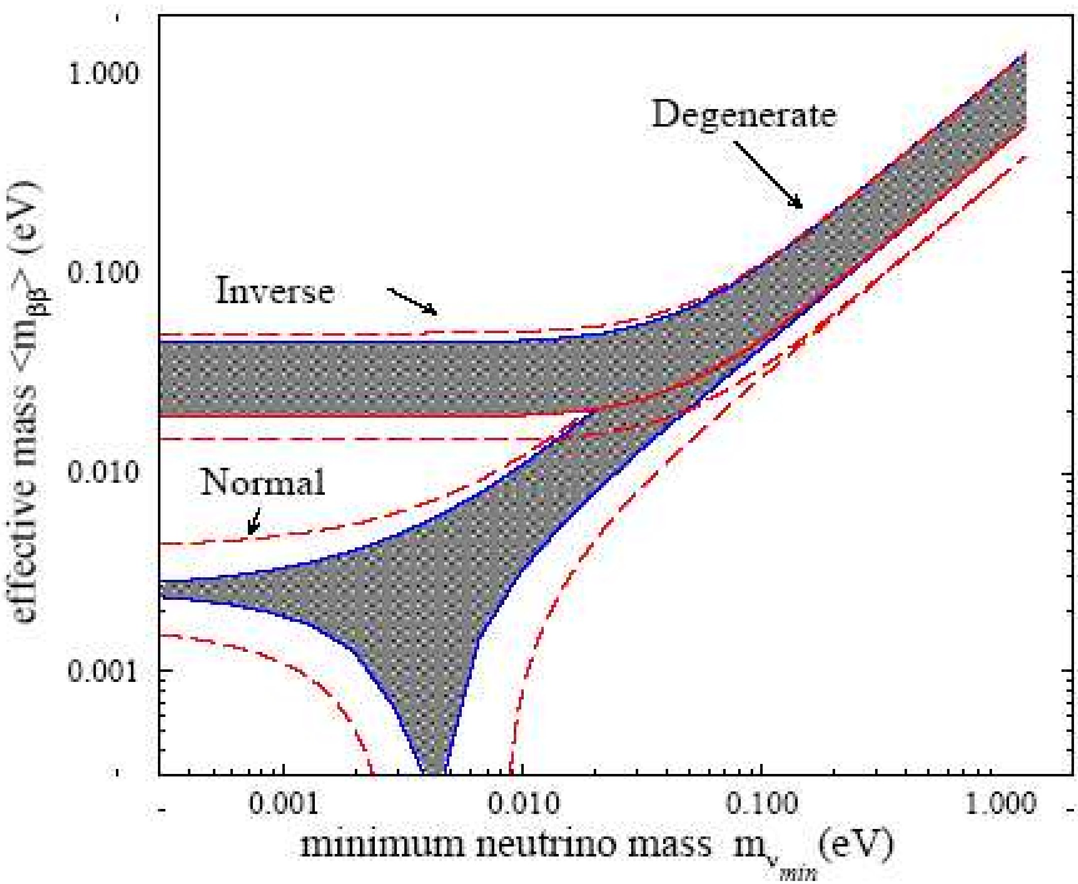
- Calculation of the NME is a complicated nuclear problem:
  - many intermediate nuclear states must be taken into account,
  - two approaches, which are based on different physical assumptions, are usually used for the calculation of NME: Nuclear Shell Model (NSM) and Quasiparticle Random Phase Approximation (QRPA),
  - different calculations of the same NME differ by factor 2-3 or even more,
- The new calculation, where the observed $(\beta\beta)_{2\nu}$ decay has been used to fix relevant parameters, has shown the great stability of the final results.

(V. Rodin et al.**Phys.Rev.C 68**, 044302, (2003))

Neutrinoless double beta decay

- The possible precision of the future experiments will give a chance to look for CP violation only for higher neutrino masses $(m_1 \gtrsim 0.1\, eV)$, where the mass spectrum starts to be degenerated $m_1 \approx m_2 \approx m_3 = m_\nu$.
- In this case the effective neutrino mass m_β measured in tritium beta decay is just equal to neutrino masses $m_\beta = \left[\sum_{i=1}^{3} |U_{ei}|^2 m_i^2\right]^{1/2} = m_\nu.$
- We can combine both measurements to find values of CP violating phases.

Neutrinoless double beta decay & beta decay



Picture from:*Neutrinoless double beta decay and direct searches for neutrino mass*, hep-ph:0412300.

Conditions for CP symmetry conservation

- For Majorana neutrinos CP symmetry holds if $\alpha_i, \delta \in \{0, \pm\frac{\pi}{2}, \pm\pi\}$.
- Then four conserving CP values of $\langle m_\nu \rangle$ can be obtained:

$$
\begin{aligned}
\langle m_\nu \rangle_{(1)} &= m_\beta, \\
\langle m_\nu \rangle_{(2)} &= m_\beta \cos 2\theta_{13}, \\
\langle m_\nu \rangle_{(3)} &= m_\beta \left(\cos^2\theta_{13} |\cos 2\theta_{12}| + \sin^2\theta_{13}\right), \\
\langle m_\nu \rangle_{(4)} &= m_\beta \left(\cos^2\theta_{13} |\cos 2\theta_{12}| - \sin^2\theta_{13}\right).
\end{aligned}
$$

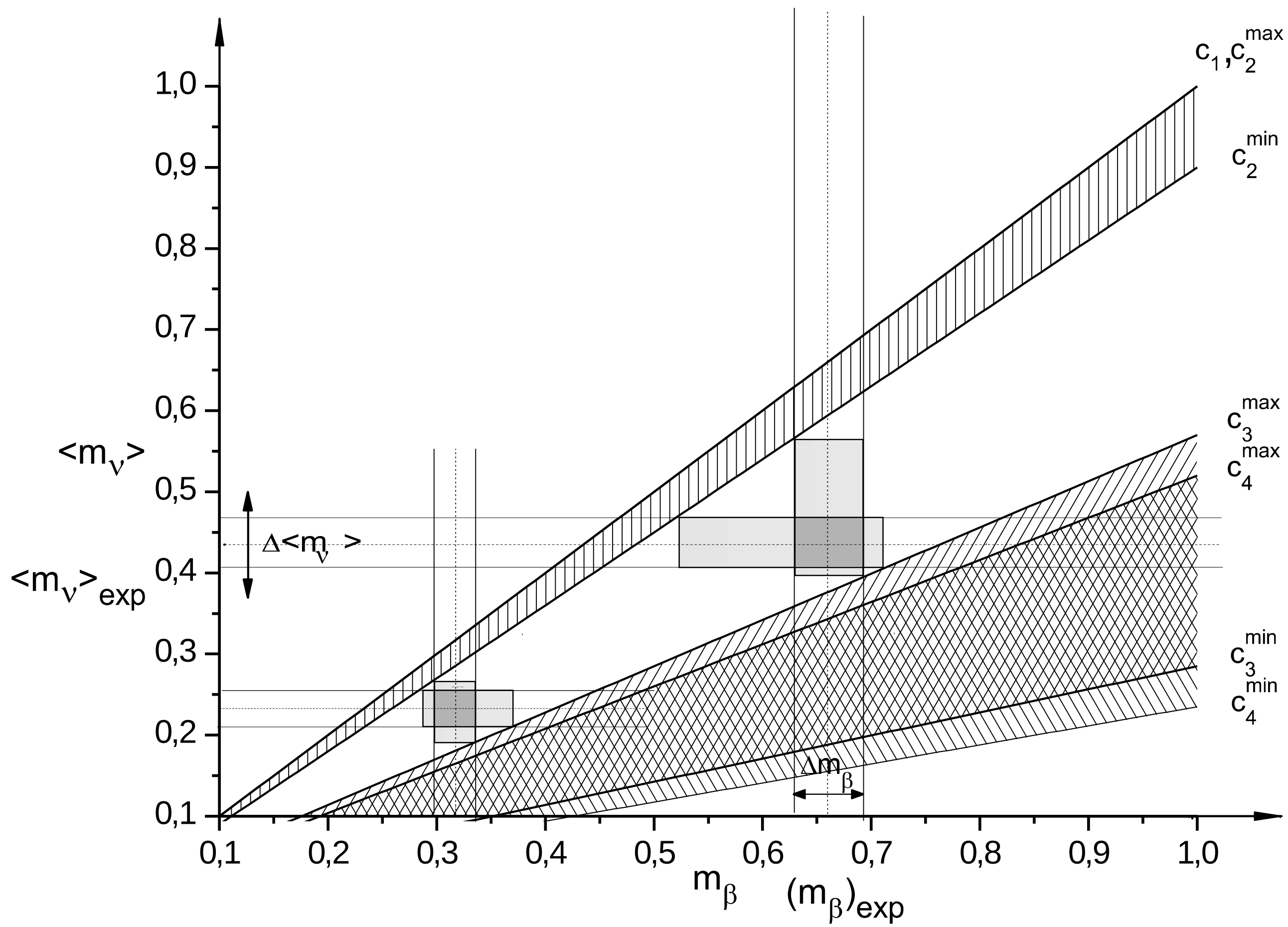
Conditions for CP symmetry conservation

- In all cases, the relation between $\langle m_\nu \rangle$ and m_β is linear: $\langle m_\nu \rangle_{(i)} = c_i m_\beta$.
- Let us assume that θ_{ij} mixing angles are known with definite precision:

$$\sin^2\theta_{ij} \in ((\sin^2\theta_{ij})_{min}, (\sin^2\theta_{ij})_{max})$$

  with central value $(\sin^2\theta_{ij})_{bestfit}$.
- For each c_i $(i = 2, 3, 4)$ we can calculate the maximal and minimal values c_i^{max}, c_i^{min}.

Visual method



A localization of the (c_i^{min} c_i^{max}) regions for the present θ_{13} and θ_{12} angles precision.

Visual method

- Let assume that in the future experiments m_β and $\langle m_\nu \rangle$ masses are determined with precision Δm_β and $\Delta\langle m_\nu \rangle$.
- Localization of the rectangle $R = (\Delta m_\beta, \Delta\langle m_\nu \rangle)$ between the lines $c_1 = 1$ and c_4^{min} decides about CP symmetry breaking.
- If the rectangle R is fully located between two lines with the c_3^{max} and c_2^{min} slopes then CP symmetry is broken.

First conditions

So, the first conditions for detecting CP violation are:

$$\Delta m_\beta < \langle m_\nu \rangle C - \Delta \langle m_\nu \rangle D,$$

$$\Delta \langle m_\nu \rangle < (m_\beta) A - (\Delta m_\beta) B.$$

where

$$A = c_2^{min} - c_3^{max},$$

$$B = \frac{c_2^{min} + c_3^{max}}{2},$$

$$C = \frac{A}{c_2^{min} c_3^{max}},$$

$$D = \frac{B}{c_2^{min} c_3^{max}}.$$

Second conditions

If this conditions are satisfied for some central values $(m_\beta)_{exp}$ and $\langle m_\nu \rangle_{exp}$ then there are further two possibilities. The rectangle R located in the point $\left((m_\beta)_{exp}, \langle m_\nu \rangle_{exp}\right)$ can be:

- fully inside two bounding lines c_2^{min} and c_3^{max},
- located partly on the first or the second line.

In the first case we can conclude that CP symmetry is broken, in the second the problem is unresolved.

Second conditions

If we translate the first point into equations we have:

$$c_3^{max}\left((m_\beta)_{exp}+\frac{\Delta m_\beta}{2}\right)<\left(\langle m_\nu\rangle_{exp}-\frac{\Delta\langle m_\nu\rangle}{2}\right)$$

and

$$\left(\langle m_\nu\rangle_{exp}+\frac{\Delta\langle m_\nu\rangle}{2}\right)<\left((m_\beta)_{exp}-\frac{\Delta m_\beta}{2}\right)c_2^{min}.$$

These all inequalities form the set of necessary conditions for CP symmetry breaking.

When can we find CP-violation?

If we now parameterize

$$\Delta\langle m_\nu\rangle = 2x\langle m_\nu\rangle, \quad \Delta m_\beta = 2y\, m_\beta,$$

- $2x$ - the relative error which measures the uncertainty coming from theoretical calculations of nuclear matrix elements and experimental measurements of $(\beta\beta)_{0\nu}$ - decay lifetime,
- $2y$ measures the relative error of the effective mass e.g. from tritium beta decay.

When can we find CP-violation?

Taking account all conditions we find that both x and y must satisfy the same inequality:

$$x,\, y \leq \frac{1-\cos 2\theta_{12\ \min} - 3\sin^2\theta_{13\ \ \max} + \sin^2\theta_{13\ \min}\cos 2\theta_{12\ \min}}{1+\cos 2\theta_{12\ \min} - \sin^2\theta_{13\ \max} - \sin^2\theta_{13\ \min}\cos 2\theta_{12\ \min}}.$$

The best circumstances to find CP violation arise for

- $\sin^2\theta_{13} \to 0$ ($x \to \tan^2\theta_{12}$)
- $\sin^2\theta_{12} \to \frac{1}{2}$.

That is opposite situation than in case of finding Dirac phase, when the $\sin^2\theta_{13}$ ruins possibility to measure it.

When can we find CP-violation?

From the same inequalities, for given relative errors x and Δm_β, we can also find the lower limit for the m_β and $\langle m_\nu \rangle$ effective masses for which measurements are still possible

$$\langle m_\nu \rangle > \frac{\Delta m_\beta}{C - 2xD}$$

and

$$m_\beta > \frac{\Delta m_\beta}{A} \left(B + \frac{2x}{C - 2xD} \right).$$

Numerical results

Using θ_{12} and θ_{13} mixing angles recently determined

(see e.g.: John N. Bahcall et al.*hep-ph/0406294*)

with 3σ precision:

$$0.22 \leqslant \sin^2\theta_{12} \leqslant 0.37, \qquad 0 \leqslant \sin^2 2\theta_{13} \leqslant 0.048$$

we obtain:

$$x < 0.2.$$

It will be a serious challenge to get such a precision.

Present situation

- Now we can check it for the isotope of Germanium ^{76}Ge where evidence for the $(\beta\beta)_{0\nu}$ decay is claimed to have been obtained

  (H. V. Klapdor-Kleingrothaus et al.,**Phys. Lett. B586**,198-212, (2004)).

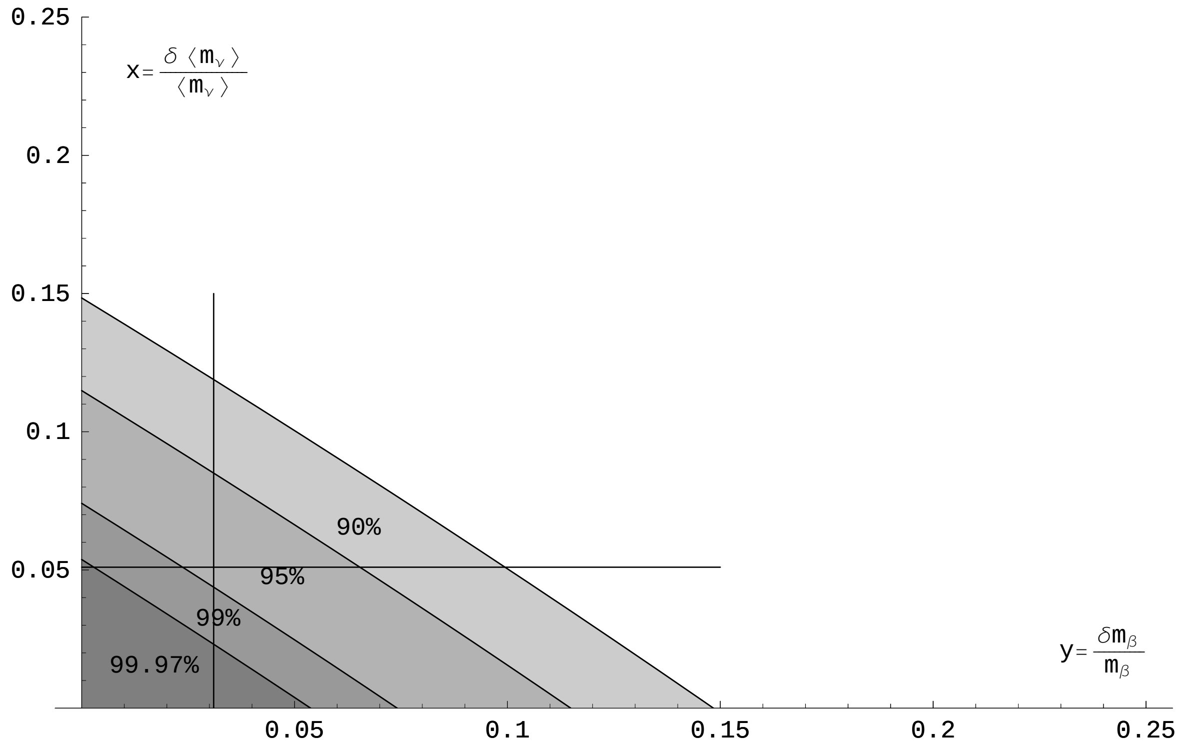
- Even assuming that precision of this measurement is much better than it is:

$$x_T = \frac{\Delta T(^{76}Ge)}{2\langle T(^{76}Ge)\rangle} \leq 0.3,$$

  and taking the new method of calculation of the NME into account, we get $x \sim 0.24$ which is still above the present necessary precision ($x < 0.2$).

Present situation

More careful analysis, taking into account the present precision of the mixing angle determination can give regions of relative errors $\frac{\Delta m_\nu}{m_\nu}$ and $\frac{\Delta m_\beta}{m_\beta}$ for which CP violation could be seen with various CL.



Future

Let us assume that during next years:

- the precision of experiments will be strongly improved,
- the best values of mixing angles will not change

$$\sin^2\theta_{12} \approx 0.28 \pm 0.01 \qquad \sin^2\theta_{13} = 0.005 \pm 0.0001,$$

- weak lensing of galaxies by large scale structure together with CMB data measure the sum of neutrino masses $\sum = m_1 + m_2 + m_3$ to an uncertainty of $0.04\, eV$. So we can expect that each individual mass is known with the precision $\Delta m_\beta = 0.015\, eV$.

Future

Now the required precision of Δm_β and $\Delta\langle m_\nu\rangle$ is

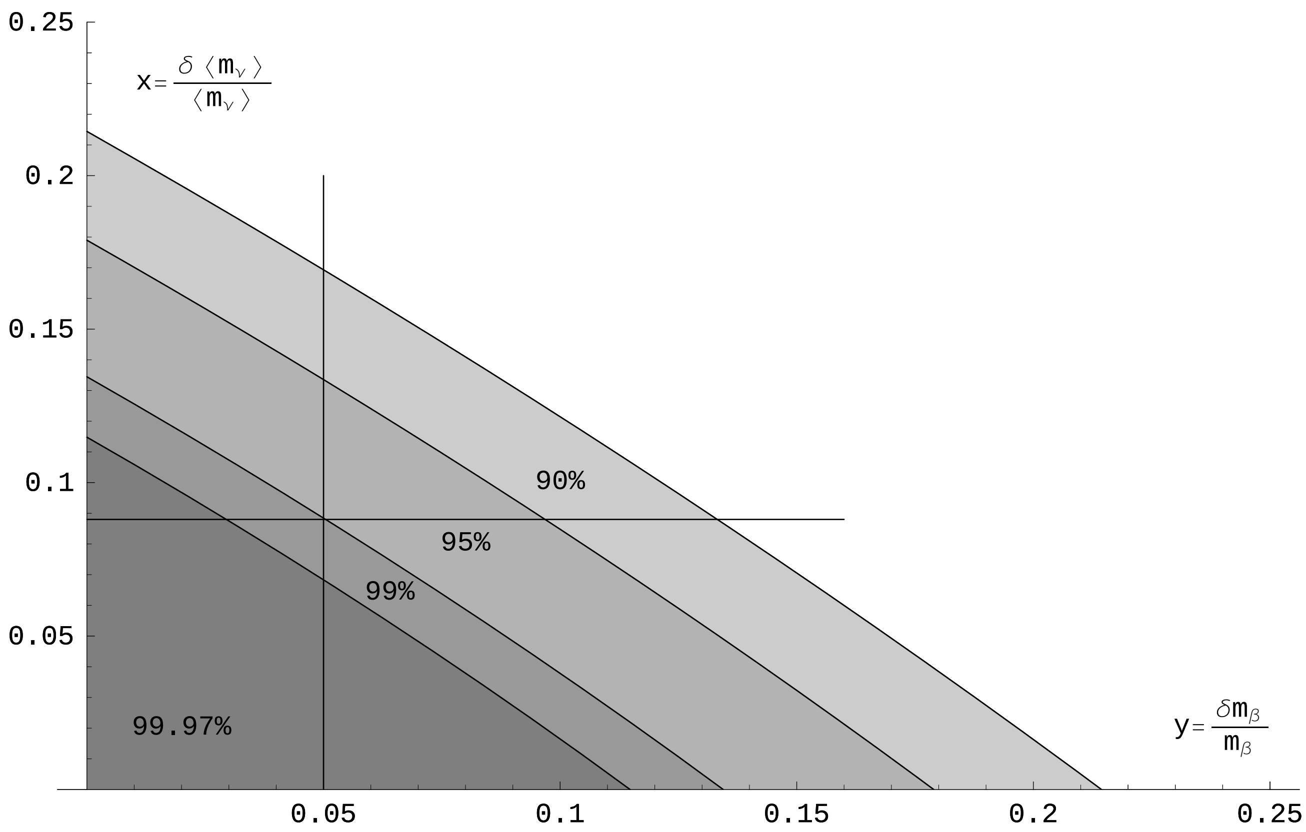
$$x,\ y < 0.36.$$

This precision will be obtained if relative experimental error for $T(^{76}Ge)$ is

$$x_T = \frac{\Delta T(^{76}Ge)}{2\langle T(^{76}Ge)\rangle} \leq 0.5,$$

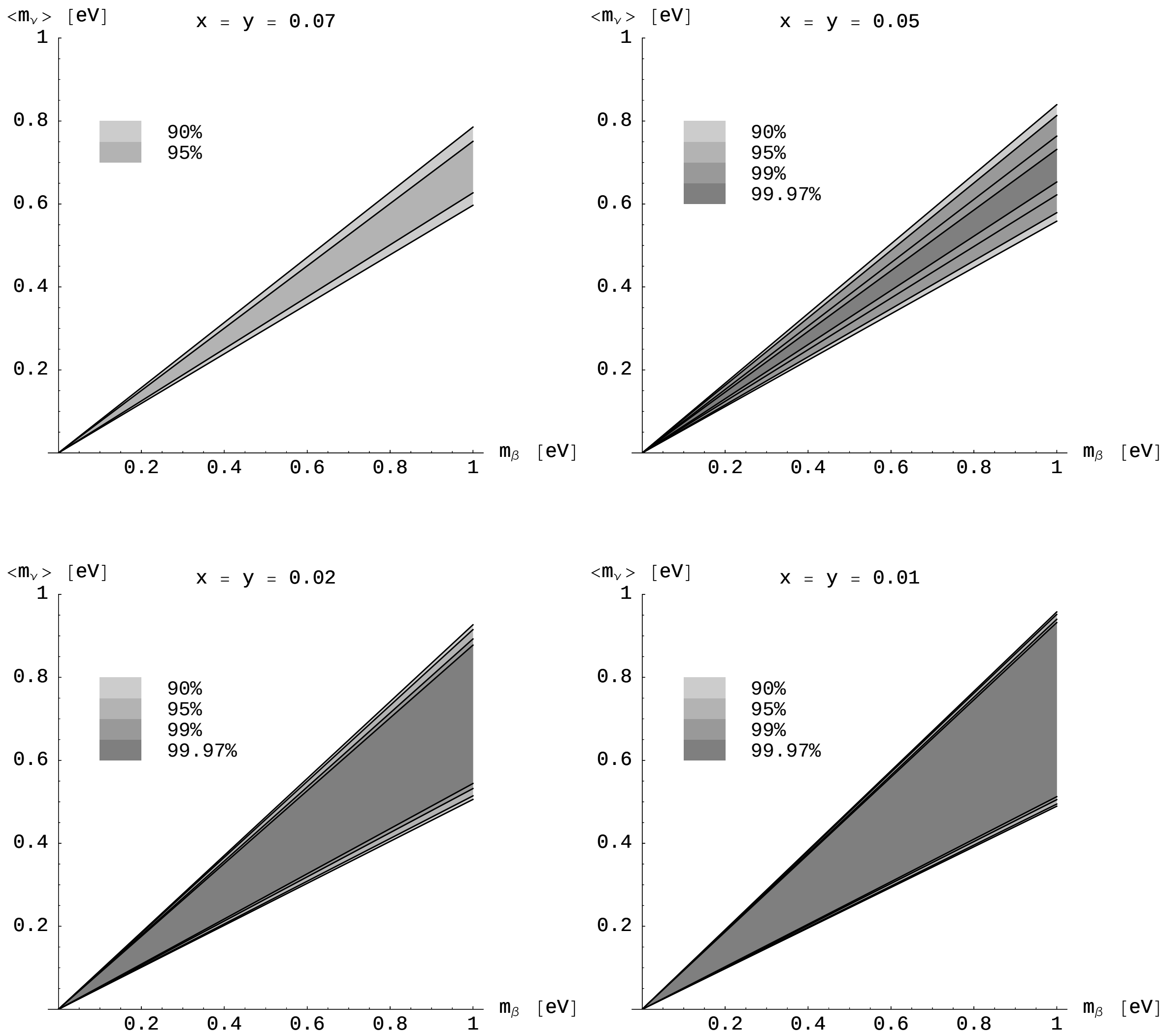
which is not a pure fantasy.

Future



Regions of relative errors $\frac{\Delta m_\nu}{m_\nu}$ and $\frac{\Delta m_\beta}{m_\beta}$ for which CP violation could be seen in future.

Future - regions of $\langle m_\nu \rangle$ and m_β



Conclusions

From presented estimations it follows that measurement of CP violation for Majorana neutrinos in neutrinoless double beta decay could be possible for almost degenerate spectrum of their masses ($m_\beta > 0.1$ eV). However, several conditions should be satisfied:

- Oscillation mixing angles should be measured with better precision e.g. $\Delta(\sin\theta_{13} \approx 0.01)$ and $\Delta(\sin\theta_{12} \approx 0.1)$ which are within the future experimental range,
- Absolute neutrino masses m_β should be measured with precision $\Delta m_\beta \approx 0.02$ eV with the central value in the range $m_\beta > 0.15$ eV, which is also not a fully fantastic dream,

Conclusions

- Neutrinoless double beta decay is discovered and the decay lifetime T is measured with precision better than 10%,
- Nuclear matrix elements of decaying isotopes are calculated with much better precision,
- There should be independent information about a full mechanism of the $(\beta\beta)_{0\nu}$ decay. Any other mechanism should give negligible contribution to the neutrinoless electrons production.