
Mediaeval Universities
Universities founded in the 12th century thus : Paris
13th : Padua
14th : Erfurt
15th : Turin
The political divisions on the map are those of about 1500; although some of the universities had become extinct by that time. The figures indicate the date of founding. In Portugal the site of the university alternated between Lisbon and Coimbra till 1537, when it was fixed at the latter place.
Ar.-Arezzo, 1215? Fl.-Florence, 1349 Pa.-Padua, 1222
Pal.-Palatinate, Vic.-Vicenza. 1204
Scale 1: 22 500 000
Miles
NORTH SEA
ATLANTIC OCEAN
MEDITERRANEAN SEA
ADRIATIC SEA
BALTIC SEA
NORWAY
SWEDEN
DENMARK
SCOTLAND
IRELAND
ENGLAND
FRANCE
SPAIN
PORTUGAL
POLAND
HUNGARY
THE EMPIRE
Upsala 1477
Aberdeen 1494
St. Andrews 1413
Glasgow 1450
Copenhagen 1478
Greifswald 1456
Rostock 1419
Cambridge
Oxford
Louvain 1425
Cologne 1388
Erfurt 1379
Prague 1347
Cracow 1364
Wittenberg 1502
Leipsic 1409
Frankfort on the Oder 1506
Treves 1454
Mayence 1476
Würzburg 1402
Heidelberg 1385
Ingolstadt 1459
Presburg 1465
Vienna 1365
Buda 1389
Fünfkirchen 1367
Caen 1437
Paris
Orléans
Nantes 1460
Angers
Poitiers 1431
Bourges 1464
Freiburg im Breisgau 1455
Tübingen 1477
Besançon 1485
Basel 1459
Dôle 1422
Grenoble 1339
Valence 1459
Orange 1365
Avignon 1303
Aix 1409
Montpellier
Perpignan 1349
Bordeaux 1441
Cahors 1332
Toulouse 1230
Vercelli 1228
Pavia 1361
Turin 1405
Treviso 1318
Ferrara 1391
Bologna
Reggio
Piacenza 1248
Pisa 1343
Siena 1246
Perugia 1308
Rome 1244, 1303
Naples 1224
Salerno
Catania 1444
Palencia 1212
Huesca 1359
Lerida 1300
Valladolid
Saragossa 1474
Salamanca c. 1230
Siguenza 1489
Alcalá de Henares 1499
Barcelona 1450
Valencia 1500
Palma 1483
Coimbra 1308
Lisbon 1290
Seville
Majorca
Castile
Andalusia
Navarre
Aragon
Catalonia
Valencia
Guienne
Languedoc
Provence
Dauphiny
Savoy
Normandy
Brabant
Saxony
Bohemia
Bavaria
Austria
Switzerland
Pomerania
Brandenburg
Mecklenburg
VENETIAN REPUBLIC
PAPAL STATES
THE TWO SICILIES
Long. West 5 of Greenwich
Long. East 5 of Greenwich

Exotic baryons on the lattice

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Lattice can identify low-lying hadronic states, starting from QCD with systematically controllable approximations.

- Does QCD predict exotic states?
- If yes, what are the experimentally unkwnon quantum numbers?
- First opportunity for the lattice to predict new particles.

This talk

- Why is it not so easy?
- How to interpret the results?
- Available (still inconclusive) results.

Review: F. Csikor, Z. Fodor, S.D. Katz and TGK, hep-lat/0407033.

Hadron spectroscopy in lattice QCD

Basic object computed numerically:
Euclidean correlation function of composite op. $\mathcal{O}$:

$$C_{\mathcal{O}}(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \;=\; \sum_i \;|\; \langle 0|\mathrm{e}^{Ht}\mathcal{O}(0)\mathrm{e}^{-Ht}|i\rangle\;\langle i|\mathcal{O}^\dagger(0)|0\rangle =$$

$$\sum_i \;|\; \langle i|\mathcal{O}^\dagger(0)|0\rangle\;|^2\;\mathrm{e}^{-(E_i-E_0)t},$$

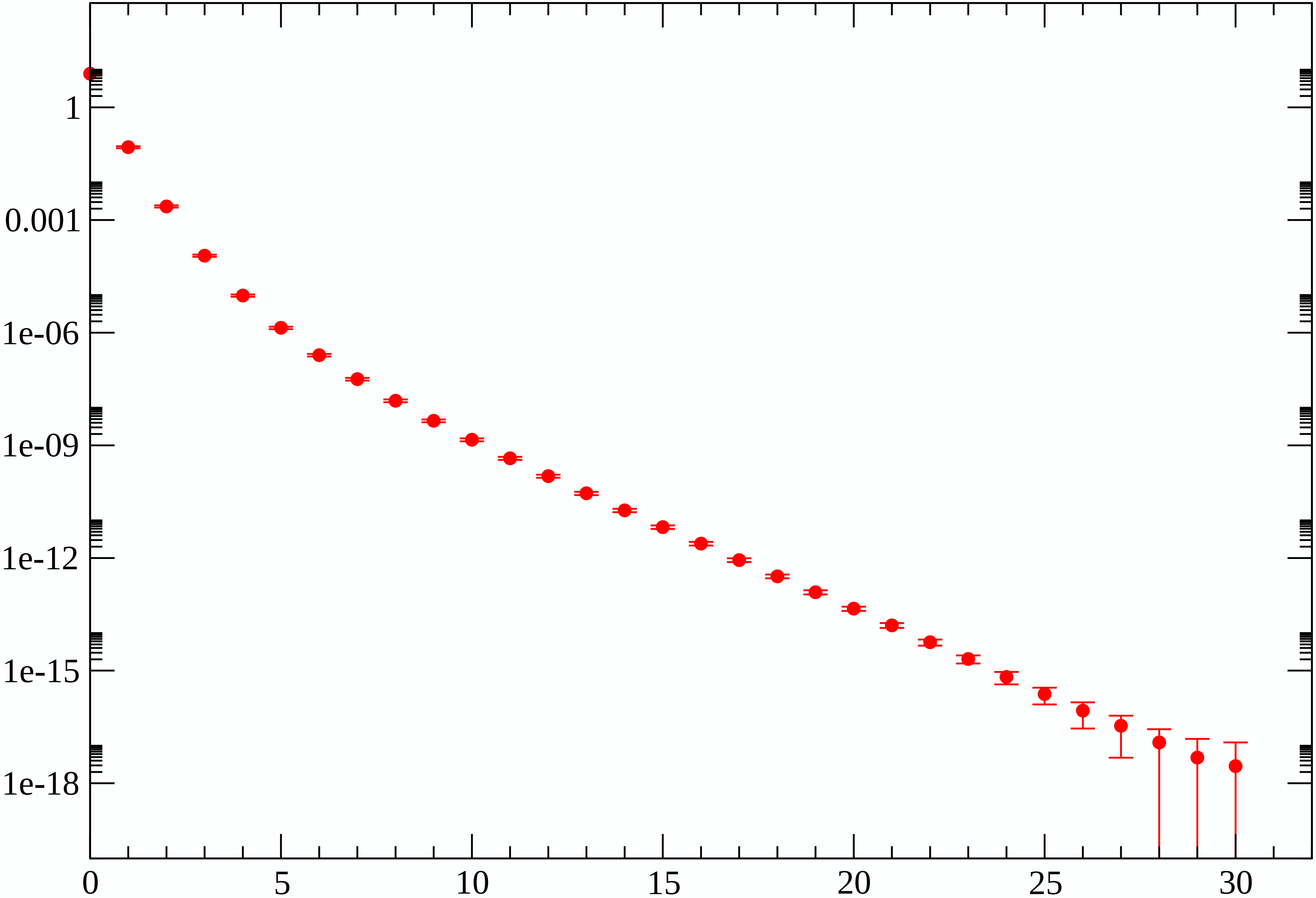
where

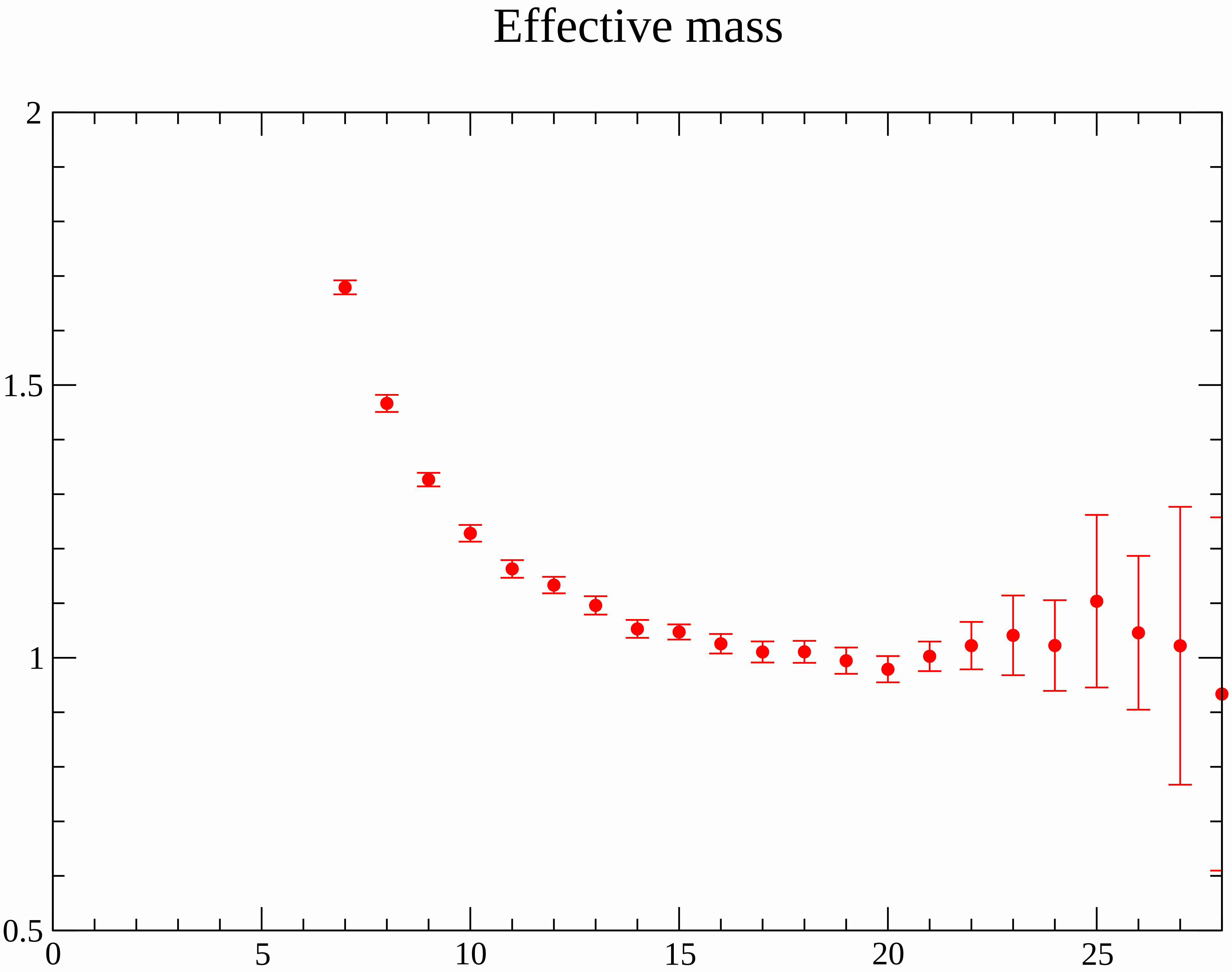
$$\mathcal{O}(t) = \mathrm{e}^{-Ht}\;\mathcal{O}\;\mathrm{e}^{Ht}.$$

and $|i\rangle$: eigenvectors of the Hamiltonian with eigenvalue E_i.

t large $\;\Rightarrow\;$ Lightest states (created by $\mathcal{O}$) dominate.
$\Rightarrow\;$ Exponential fit gives $E_i - E_0$'s

Correlator




Effective mass
2
1.5
1
0.5
0
5
10
15
20
25

How to choose $\mathcal{O}$?

Has to have quantum numbers of the desired state for $\langle i|\mathcal{O}(0)|0\rangle \neq 0$.

- Internal quantum numbers, spin (later)
- Total momentum
  Projection to zero total momentum: $\sum_{\vec{x}} \mathcal{O}(0,\vec{x})$
  - Single particle state $\Rightarrow E_1 = m$
  - 2-particle "continuum" $\Rightarrow E_1 = m+M$,
    $E_2 = (m^2+p^2)^{1/2} + (M^2+p^2)^{1/2}$, relative momentum can be $p_i = 2\pi k_i/L$.
- Large overlap with desired state $\Rightarrow$ smaller statistical errors.
- Small overlap with competing states. (Nucleon + Kaon)
- Essential to build in knowledge about the wave function!

If you see nothing, either there is nothing or you chose the wrong operator.
Lattice calculations cannot disprove the existence of a state!

Spatial wave function (why can't we do arbitrary w.f.'s?)

How is the correlator $C(t)$ computed?

Simple example: pion $\mathcal{O} = \bar{u}\gamma_5 d$

$$\langle 0|\overline{u}(x)\gamma_5 d(x) \;\; \overline{d}(0)\gamma_5 u(0)|0\rangle$$

Use Wick's theorem to decompose into sums of products of the form

$$\langle 0|d_\alpha(x)\overline{d}_\beta(0)|0\rangle = (D+m)^{-1}_{x\alpha,0\beta}$$

General wave function:

$$\mathcal{O}(0) = \int d^3x \int d^3y \; f(\vec{x},\vec{y}) \; \overline{u}(\vec{x},0)\gamma_5 d(\vec{y},0)$$

For arbitrary wave fn., quark propagators $D^{-1}(x\alpha, y\beta)$ (from any point to any other point) are needed.

$\Rightarrow$ Order 10^{13} matrix elements i.e. $\approx$ 100 Tbytes to be stored

Solution: We store only $\sum_{\vec{y}\beta}(D+m)^{-1}_{\vec{x},0,\alpha;\vec{y},0,\beta}\psi_\beta(\vec{y})$ for some ψ's

Consequences: $\mathscr{O}$ can be built only as

- a product of 1-quark wave functions
- each 1-quark wave function requires "only" 12 Dirac op. inversions
- Iterative sparse matrix techniques can be used
- Typical lattice spectroscopy uses only one source $\psi(x)$,
  all wave functions are built out of this.
- Extended "source" wave function $\Longrightarrow$ smaller overlap with higher states.
- Same propagator can be used for u and d quark.

Special problems with exotics

Spin, flavour, colour structure

Index summations exponentially more expensive with the number of quarks

- 3q baryons: summation negligible compared to inversion

$\Rightarrow$ • Simplest 5q operators: summation is order 50%

Pentaquark operators used so far on the lattice:

- All with one type of quark wave function
- $\mathscr{O}$ is a product of 1-particle states
- A few different spin, flavour, colour structures

Spin, flavour, colour (continued)

- $\mathscr{O}_{I=0/1} = \epsilon_{abc}\,[u_a^T C\gamma_5 d_b]\,\{u_c\ \bar{s}_e\gamma_5 d_e \mp (u \leftrightarrow d)\}$
  Nucleon$\times$Kaon colour structure (Csikor et al., Liu et al.)

- Diquark-diquark antiquark

$$\mathscr{O}_{I=0} = \epsilon_{adg}\,[\epsilon_{abc}\,u_b^T C\gamma_5 d_c]\ [\epsilon_{def}\,u_e^T C\gamma_5 d_f]\ C\bar{s}_g^T$$

  If the two diquarks are in the same spatial wave function
  symmetric with respect to interchanging diquarks (ad)
  $\Rightarrow$ non-trivial spatial wave-function needed
  $\Rightarrow$ Projection to spin eigenstates $\Rightarrow$ 6 propagators needed, not 1.
  (No published lattice results so far.)

- To avoid spatially antisymmetric wave-function.:

$$\mathscr{O}_{I=0} = \epsilon_{adg}\,[\epsilon_{abc}\,u_b^T C d_c]\ [\epsilon_{def}\,u_e^T C\gamma_5 d_f]\ C\bar{s}_g^T$$

  (Sasaki, Chiu & Hsieh, Ishii et al.)

Parity assignment

- $\mathscr{O}$ does not create parity eigenstates, but

$$P\mathscr{O}P^{-1} = \pm\gamma_0\mathscr{O},$$

  - $\pm$ is the internal parity of $\mathscr{O}$
  - For non-pointlike operators, can be more complicated.

- Parity projection: $\frac{1}{2}(\mathscr{O} \pm P\mathscr{O}P^{-1})$

- Compute correlators separately in both channels.

- Identify the state we are looking for by its energy.

Separating two-particle states and 5q state

The Θ^+ is above threshold $\Rightarrow$ embedded in 2-particle NK continuum.
finite V $\Rightarrow$ all states are discrete.

In realistic lattice volumes:

- Level structure:
  - $N \times K, \;\; p = \sqrt{2} * 2\pi/L$ $\quad + \; -$
  - $N \times K, \;\; p = 1 * 2\pi/L$ $\quad + \; -$
  - m_{Θ^+} $\quad$???
  - $N \times K, \;\; p = 0$ $\quad -$

- Lowest state in **–** parity channel: $m_N + m_K$ (S-wave)
- Lowest state in **+** parity channel: Θ^+ or $(m_N^2 + p^2)^{1/2} + (m_K^2 + p^2)^{1/2}$
  ($p = 2\pi/L \approx 400$ MeV, P-wave)

Pentaquark signal can be safely confirmed only after all nearby 2-particle states have been clearly identified

How to tell 2-particle state from 1-particle state?

Changing the volume

- Is there a volume dependence of the mass consistent with

$$(m_N^2 + p^2)^{1/2} + (m_K^2 + p^2)^{1/2}$$

$$p = \frac{2\pi}{L}$$

- Does the amplitude $|\langle 1|\mathscr{O}^\dagger(0)|0\rangle|^2$ go down with the volume?

How to identify several low-lying states?

- Fit with sum of exponentials: (Sasaki)
  $C_1 \mathrm{e}^{-m_1 t} + C_2 \mathrm{e}^{-m_2 t} + \ldots$
  Does not really work...

- Use cross correlators (Csikor et al.)

$$\begin{aligned} \mathscr{O}|0\rangle &\approx A_1|1\rangle + A_2|2\rangle + \ldots, \\ \mathscr{O}'|0\rangle &\approx A'_1|1\rangle + A'_2|2\rangle + \ldots \end{aligned}$$

  - Mix two operators $\mathscr{O}\cos\phi + \mathscr{O}'\sin\phi$
  - Choose ϕ to cancel lowest or 2nd lowest state
    (Eg. minimise (maximise) effective mass w.r.t. α)
  - Can be generalised to more operators
  - Expensive! Have to compute $\langle\mathscr{O}\mathscr{O}\rangle$, $\langle\mathscr{O}\mathscr{O}'\rangle$, $\langle\mathscr{O}'\mathscr{O}'\rangle$.

Extrapolations

Any lattice calculation involves two extrapolations

- Chiral extrapolation:
  Simulation at physical ud quark masses too expensive
  $\Rightarrow$ simulate heavier quarks and extrapolate to $m_\pi = 135$ MeV.

- Continuum extrapolation:
  lattice spacing $\longrightarrow$ 0.
  different lattice actions have different cut-off effects

	action	a (fm)	smallest m_π (MeV)
Csikor et al.	Wilson	0.17-0.09	420
Sasaki	Wilson	0.07	650
Mathur et al.	chiral	0.20	180
Chiu & Hsieh	chiral	0.09	400
Ishii et al.	improved Wilson	0.15	850

Results and interpretation

Lowest state seen is in $I^P = 0^-$ channel,
mass in other parity channel much higher.

- Csikor et al. & Sasaki
  Consistent: $I^P = 0^-$, however NK scattering state still not ruled out.

- Mathur et al.
  identify only NK scattering states by volume dependence of amplitudes
  $\Rightarrow$ can explain why they see only 2-particle states
  Use Nucleon$\times$Kaon operator

- Ishii et al.
  lowest state interpreted as NK scattering by using twisted boundary conditions

Lowest state seen is in $I^P = 0^+$ channel,
mass in other parity channel much higher.

- Chiu & Hsieh
  $I^P = 0^+$??? they claim, due to better chiral action
  Not very likely:
  - In all studies huge difference between two parity channels
  - No precedent for such a discrepancy
    between chiral and Wilson action

  Might be misidentified parity $\Rightarrow$ Independent cross-check needed

Warning!
Nobody sees the lowest expected scattering state in both parity channels.

Perspectives

Five independent lattice studies so far.

F. Csikor, Z. Fodor, S.D. Katz and T.G. Kovács, JHEP 0311 (2003) 070.

S. Sasaki, PRL **93** (2004) 152001.

N. Mathur et al. PRD **70** (2004) 074508.

Ting-Wai Chiu and Tung-Han Hsieh, hep-ph/0403020, hep-ph/0404007

N. Ishii, T. Doi, H. Iida, M. Oka, F. Okiharua and H. Suganuma, hep-lat/0408030.

- To be done
  - Resolve discrepancy of Chiu & Hsieh
  - Systematically map out low-lying level structure (including the expected 2-particle states).
  - Technically:
    other operators with non-trivial spatial wave function and finite volume analysis.