Calculation of QCD NLO Splitting Functions in the light-cone gauge: a new regularization prescription

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Collinear Factorization Theorem in light-cone gauge


According to collinear factorization theorem **Parton Shower** is a **product of Splitting Functions**.
NLO Parton Shower MC for QCD does not exist!

In this talk I discuss calculation of virtual contributions to the exclusive NLO Splitting Functions suitable for NLO Parton Shower MC in QCD [KrKMC].

Brief history of Monte-Carlo for QCD:

- LO Hard Process + LO Parton Shower
  Pythia, Herwig (1980s)

- NLO Hard Process + LO Parton Shower
  MC@NLO, PowHEG (2000s)

- NLO Hard Process + NLO Parton Shower
Splitting Functions are defined in $m = 4 + 2\epsilon$ dimensions in terms of Feynman rules (light-cone gauge) and projection operators:

$$W^{(\text{NLO})}(x, \epsilon) = \alpha_s^2 \text{Tr} \left[ \frac{\hat{q}}{4 p \cdot n} K \hat{p} \right]$$

For example, LO Splitting Function reads:

$$d_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu n_\nu + n_\mu q_\nu}{q \cdot n}$$
**Loop-Momentum Integration**

Calculation of the virtual corrections to NLO SFs requires evaluation of the following loop integrals:

\[
\int \frac{d^m l}{(2\pi)^m} \frac{\{1, l^\mu, l^\mu l^\nu\}}{l^2 (l + p_1)^2 (l + p_2)^2} \frac{1}{l \cdot n}, \quad l_+ \equiv l \cdot n
\]

where

- \( m = 4 + 2\epsilon \)
- \( n \) is a gauge-fixing constant 4-vector (in our case \( n^2 = 0 \))
- \( p_1, p_2 \) are external momenta 4-vectors

**Feynman denominators** lead to IR and UV singularities which appear in the final result as \( 1/\epsilon \) terms.

**Axial denominator** is a source of spurious (unphysical) singularities and thus needs a special regularization prescription. A common choice is Principal Value prescription.
Principal Value vs Dimensional Regularization

Dimensional Regularization

(Transition to $m$-dimensional space-time is required.)

\[
\int_0^1 dl_+ \frac{1}{l_+} \rightarrow \int_0^1 d^m l_+ \frac{1}{l_+} \stackrel{m\to 1+\epsilon}{\equiv} \int_0^1 dl_+ \frac{1}{l_+^{1-\epsilon}} = \frac{1}{\epsilon}
\]

Principal Value Prescription

\[
\int_0^1 dl_+ \frac{1}{l_+} \rightarrow \int_0^1 dl_+ \left( \frac{1}{l_+} \right)_{PV} \equiv \int_0^1 dl_+ \frac{l_+}{l_+^2 + \delta^2} = -\ln \delta
\]

In contrast to Dim Reg, Pr Val works in 4 dimensions.

That makes it possible to use expressions calculated in PV for Monte-Carlo simulations.

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Standard Principal Value Approach

Use Principal Value prescription for axial denominators ONLY...

\[ d^{\mu\nu} = g^{\mu\nu} - (l^\mu n^\nu + n^\mu l^\nu) \frac{1}{l^+} \rightarrow d_{PV}^{\mu\nu} = g^{\mu\nu} - (l^\mu n^\nu + n^\mu l^\nu) \frac{l^+}{l^+_2 + \delta^2} \]

... and Dimensional Regularization for all others!

\[ \frac{1}{l^+} \rightarrow \frac{1}{l^{1-\epsilon}^+} \]

\[ \int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2 (l+p)^2 (l+k)^2} = \int_0^1 dz_1 \int_0^1 dz_2 \int \frac{d^m l}{(2\pi)^m} \frac{1}{(l^2 + l \cdot A + B^2)^3} \approx \frac{1}{\epsilon^2} \]

\[ k^2 \neq 0 \quad p^2 = (p - k)^2 = 0 \]
New Principal Value Approach

Solution of the $1/\epsilon^2$ problem!

Use Principal Value prescription for all singularities in $l_+$!

(not only for axial denominators)

$$\frac{1}{l_+} \rightarrow \frac{l_+}{l_+^2 + \delta^2} \quad \text{and} \quad \frac{1}{l_+^{1-\epsilon}} \rightarrow \frac{l_+}{l_+^2 + \delta^2} \left(1 + \epsilon \ln l_+ + O(\epsilon^2)\right)$$

$$\int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2(l + p)^2(l + k)^2} = \int_0^1 dz_1 dz_2 \int \frac{d l_+^{PV} dl_- d^{m-2} l_\perp}{(2\pi)^m} \frac{1}{(l^2 + l \cdot A + B^2)^3} \approx \frac{\ln \delta}{\epsilon}$$

$k^2 \neq 0 \quad p^2 = (p - k)^2 = 0$
Example: three-point scalar Feynman integral

\[ I_3 = \int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2(l + k)^2(l + p)^2} \]

Principal Value approach

\[ I_3^{\text{PV}} = \frac{i}{(4\pi)^2 |k^2|} \left( \frac{4\pi}{|k^2|} \right)^\epsilon \Gamma(1 + \epsilon) \left( -\frac{1}{\epsilon^2} + \frac{\pi^2}{6} \right) \]

New Principal Value approach

\[ I_3^{\text{NPV}} = \frac{i}{(4\pi)^2 |k^2|} \left( \frac{4\pi}{|k^2|} \right)^\epsilon \Gamma(1 + \epsilon) \left( -\frac{2 \ln \delta + \ln(1 - x)}{\epsilon} \right) \]
\[ + 2 \ln^2 \delta - 2 \ln \delta \ln(1 - x) + \frac{\ln^2(1 - x)}{2} + \frac{\pi^2}{6} \], \quad x \equiv k \cdot n
Properties of the Solution

- NLO SFs suitable for Parton Shower MC
- No higher-order $\epsilon$ poles in real and virt diagrams
- No dependence on the hard scale $Q$
- Full agreement with inclusive calculations

[G.Curci et al. 1980; G.Heinrich 1998]
Axiloop: Features

Fully automated tool for symbolic calculation of Splitting Functions in light-cone gauge written in Mathematica.

- Index contraction and trace calculation
- One-loop integration with various prescriptions in light-cone and covariant gauges
- Passarino-Veltman reduction of tensor integrals with separated IR and UV poles
- One-particle final state integration
Non-Singlet Splitting Functions

For the all depicted non-singlet graphs we calculated:

- **UV counter-term**
- **Inclusive SF**
  (crosscheck with [G.Curci et al. 1980, G.Heinrich 1998])
- **Exclusive SF** – for Monte Carlo!
Cross-check with CFP: Inclusive SF

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<th>Single poles</th>
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</tr>
<tr>
<td>$1 - x$</td>
<td>-5/2</td>
</tr>
<tr>
<td>$1 + x$</td>
<td>-1/2</td>
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<tr>
<td>$p_{qq} \ln x$</td>
<td>0</td>
</tr>
<tr>
<td>$(1 - x) \ln x$</td>
<td>2</td>
</tr>
<tr>
<td>$(1 + x) \ln x$</td>
<td>0</td>
</tr>
<tr>
<td>$p_{qq} \ln(1 - x)$</td>
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</tr>
<tr>
<td>$(1 - x) \ln(1 - x)$</td>
<td>4</td>
</tr>
<tr>
<td>$p_{qq} \ln^2 x$</td>
<td>2</td>
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<tr>
<td>$p_{qq} \ln x \ln(1 - x)$</td>
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<tr>
<td>$p_{qq} \ln^2(1 - x)$</td>
<td>4</td>
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<tr>
<td>$p_{qq} \text{Li}_2(1 - x)$</td>
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<tr>
<td>$p_{qq} \text{Li}_2(1)$</td>
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<tr>
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<tr>
<td>$(1 - x) l_0$</td>
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<td>4</td>
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<tr>
<td>$p_{qq} l_0 \ln(1 - x)$</td>
<td>12</td>
</tr>
</tbody>
</table>

Sum of real and virt corrections to NLO SF calculated in NPV agrees with inclusive results from [G.Curci et al. 1980].
Summary

Done:

- Complete calculation of non-singlet case (virtual + real)
- New IR regularization scheme defined in 4 dimensions
- Resulting exclusive SFs are better suitable for MC!
- Axiloop package written in Mathematica for fully automatic analytical calculations

In progress:

- Singlet splitting functions

Future:

- Coefficient functions (hard process)
- Two-loop virtual corrections